Review on black hole evaporation and related topics

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Contents

- Black hole thermodynamics and evaporation:
  Easy applications
- Origin of Hawking radiation
- Applications of Entropy
- Information loss problem

Black hole thermodynamics and evaporation

Easy applications

Bardeen-Carter-Hawking, 1973

Stationary black holes are parametrized by mass M, charge Q, and angular momentum J.

Bardeen-carter-Hawking, 1973

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For slow-varying processes, the following law is satisfied:

(Φ: electrostatic potential, Ω: angular frequency)

$$\delta M = \frac{\kappa}{8\pi}\delta A + \Phi\delta Q + \Omega\delta J$$

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working term

Analogous with the first law of thermodynamics:

$$dE = TdS - pdV$$

Bekenstein, 1974

Is the area of the event horizon thermodynamic entropy?

$\delta A \geq 0$ (Hawking, 1971)

$\delta A \propto \delta S$ (Bekenstein, 1974)

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Analogous with the second law of thermodynamics:

$$dS \geq 0$$

Hawking, 1975

A black hole emits thermal radiation.

$$\langle n_\omega \rangle \propto \frac{1}{e^{2\pi\omega/\kappa} - 1}$$

Hawking, 1975

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The temperature, a.k.a. Hawking temperature, and the thermodynamic entropy, a.k.a. Bekenstein-Hawking entropy, are determined.

$$T = \frac{\kappa}{2\pi}$$

$$S = \frac{A}{4}$$

Black hole thermodynamics and evaporation

Easy applications

Just memorize

For static spherical symmetric black holes:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega^2$$

where $f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{3}\Lambda.$

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horizon

$$f(r_+) = 0$$

entropy

$$S = \pi r_+^2$$

Hawking temperature

$$T = \frac{1}{4\pi}\left|\frac{df}{dr}\right|_{r_+}$$

Exercise 1: Schwarzschild BH

Horizon: $r_+ = 2M$

Entropy: $S = 4\pi M^2$

Hawking temperature: $T = \dfrac{1}{8\pi M}$

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As a check:

$$\delta S = 8\pi M \delta M = \frac{\delta M}{T}$$

So, that's ok! ☺

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What are the consequences according to black body radiation?

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What are the consequences according to black body radiation?

According to the Stefan-Boltzmann law:

$$\frac{dM}{dt} = \alpha NAT^4 \propto \frac{1}{M^2}$$

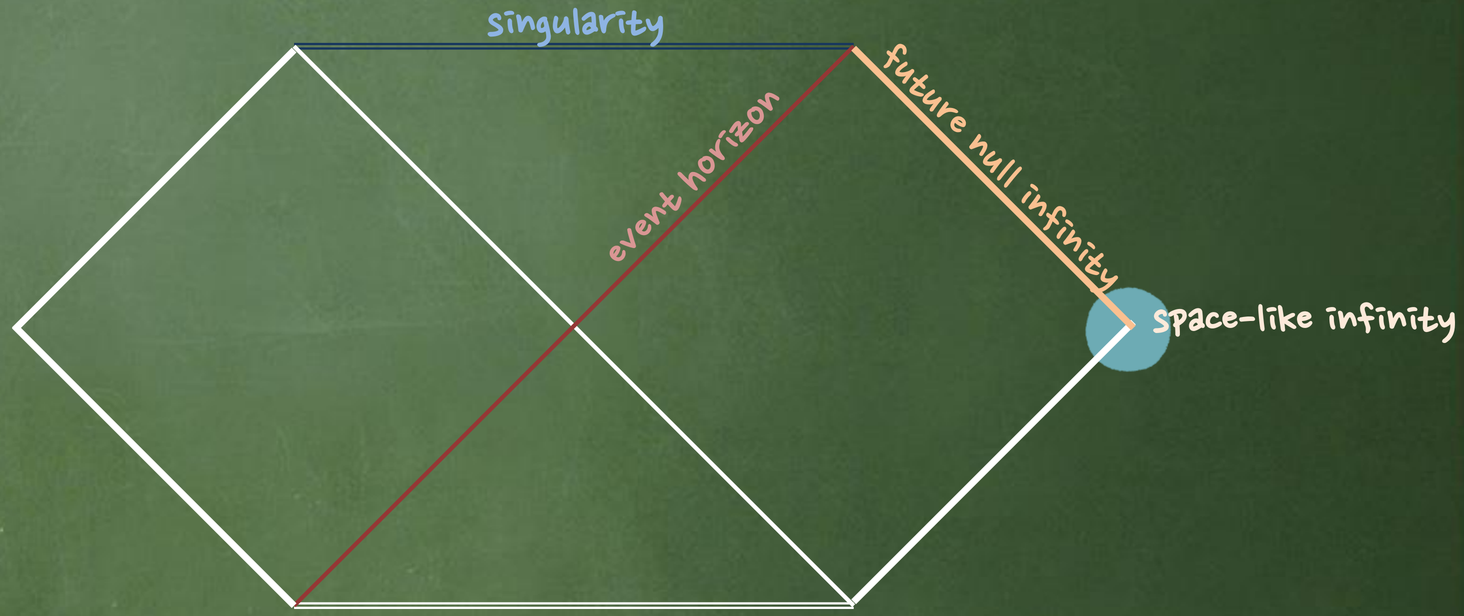
Therefore, the lifetime of a black hole is

$$t_{BH} \sim M^3.$$

Hence, black hole will disappear in finite time.

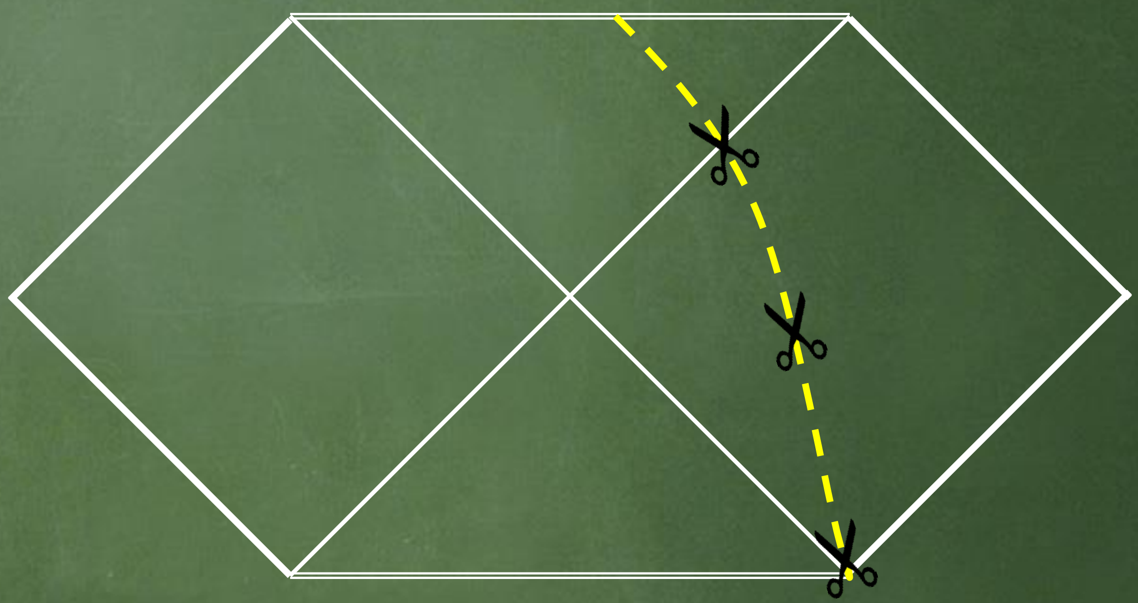
Exercise 1: Schwarzschild BH

Penrose diagram of the Schwarzschild black hole



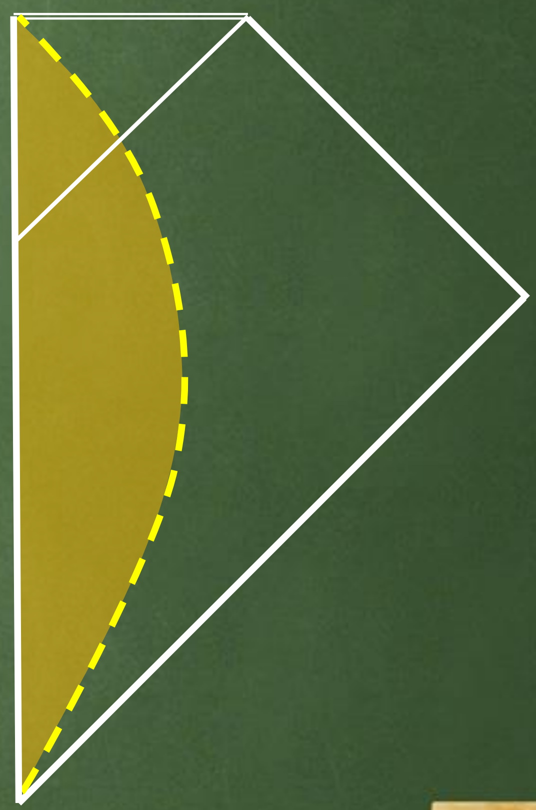
Exercise 1: Schwarzschild BH

Matching with the star-interior (time-like surface)



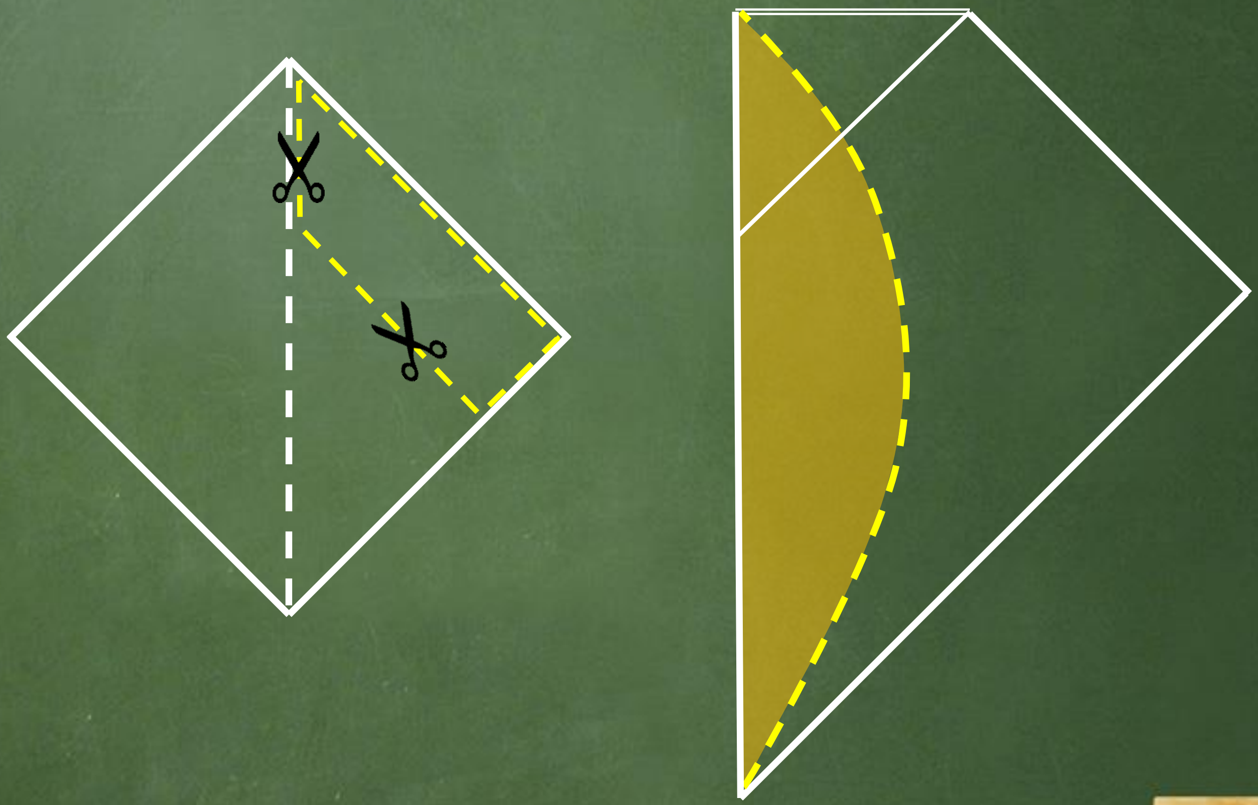
Exercise 1: Schwarzschild BH

Dynamical formation of a black hole



Exercise 1: Schwarzschild BH

After the evaporation, paste Minkowski



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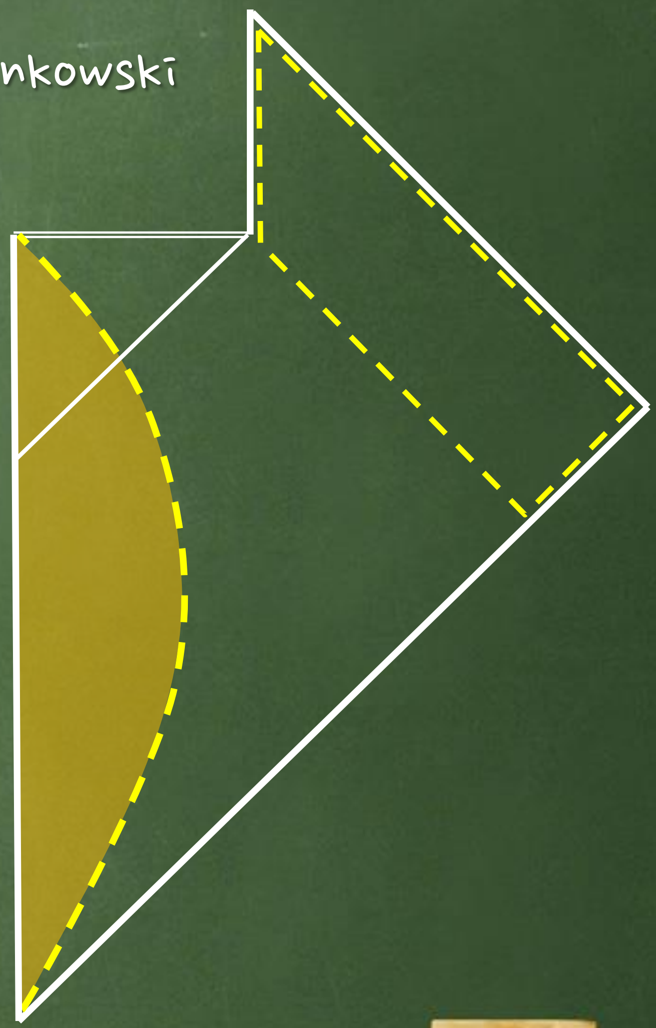


Exercise 1: Schwarzschild BH

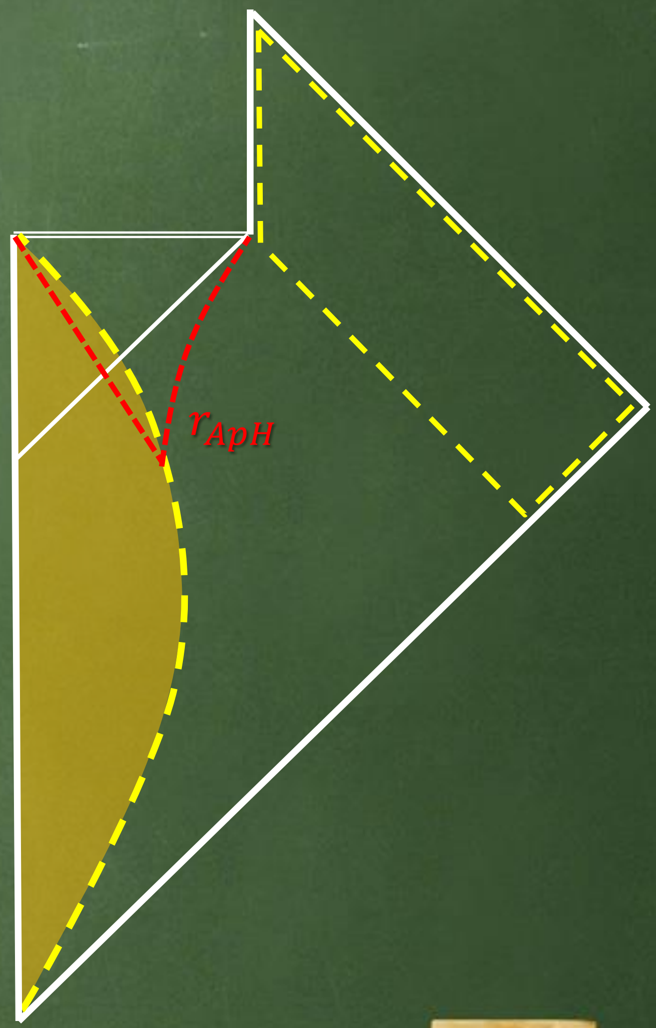
For dynamical black holes,
a locally defined horizon is
more meaningful.

For example, in the vaidya metric,

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2$$

the apparent horizon is

$r_{ApH} = 2M(v)$.



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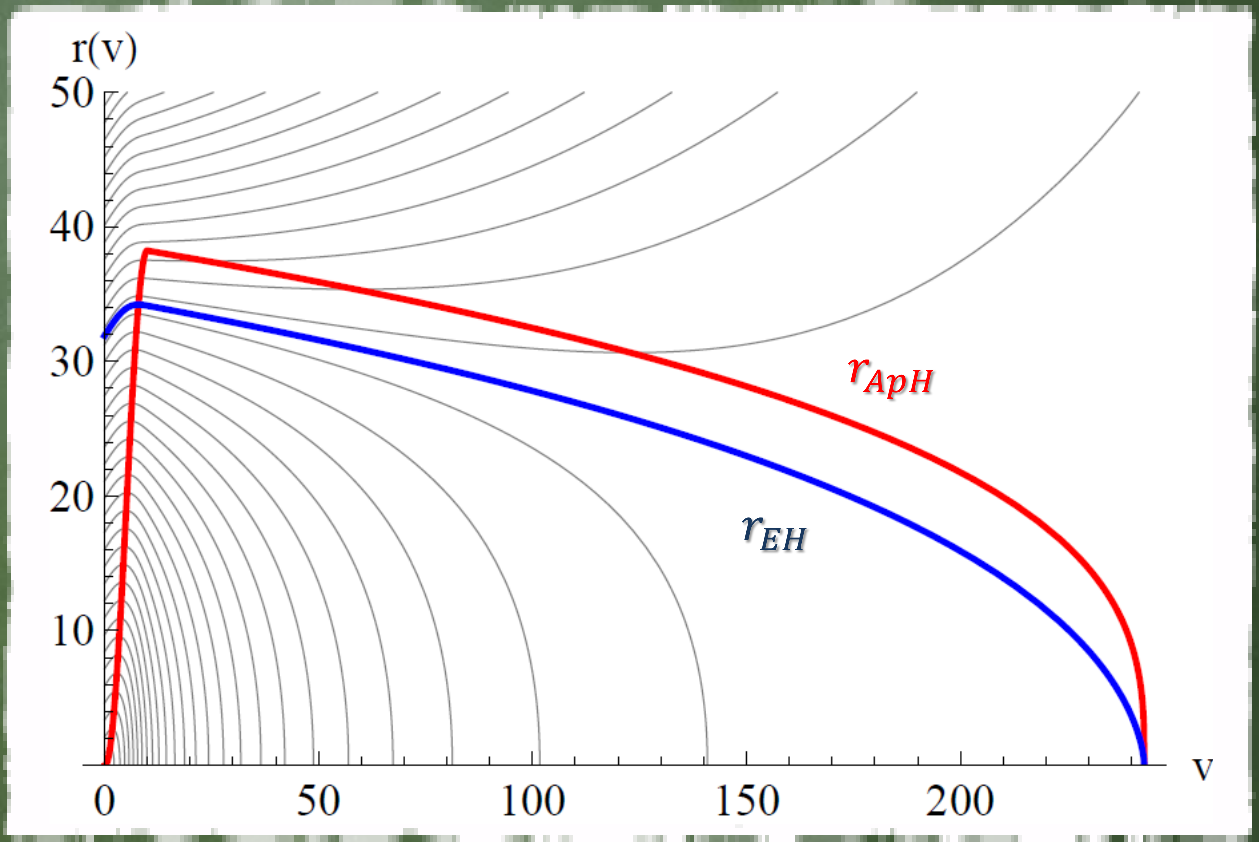

Exercise 2: Charged BH

Horizon: $r_\pm = M \pm \sqrt{M^2 - Q^2}$

Entropy: $S = \pi(M + \sqrt{M^2 - Q^2})^2$

Hawking temperature: $T = \frac{r_+ - M}{2\pi r_+^2}$

As a check:

$$\delta S = \frac{1}{T}\delta M - \frac{\Phi}{T}\delta Q$$

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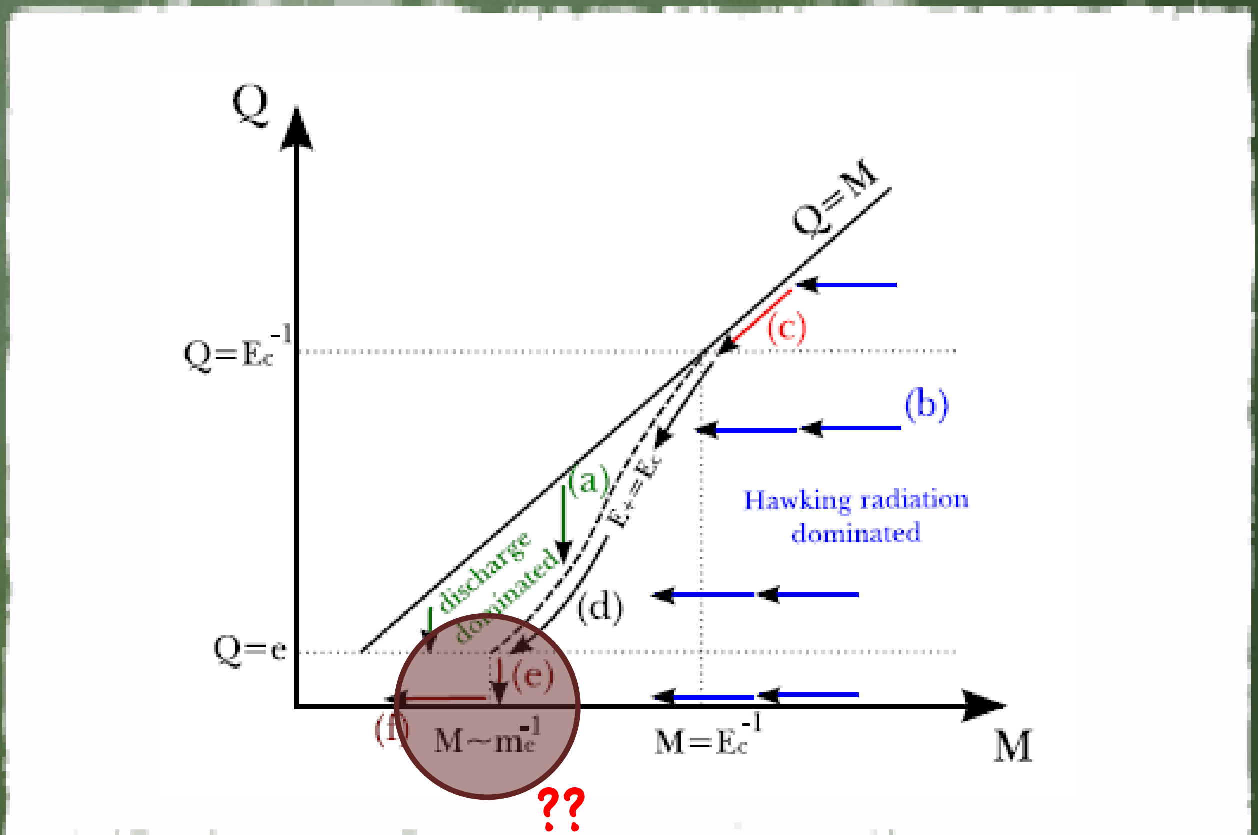
As time goes on, M approaches to Q by Hawking radiation.

As time goes on, Q decreases due to the Schwinger effect,

where $\frac{dQ}{dt} \propto e^{-E_c/E_+}$ $(E_c = \frac{\pi m_e^2}{e},\ E_+ = \frac{Q}{r_+^2})$.

Exercise 2: Charged BH



Hong, Hwang, Stewart and DY, 2010

Exercise 2: Charged BH

What is the correct causal structure for the formation of a charged black hole?

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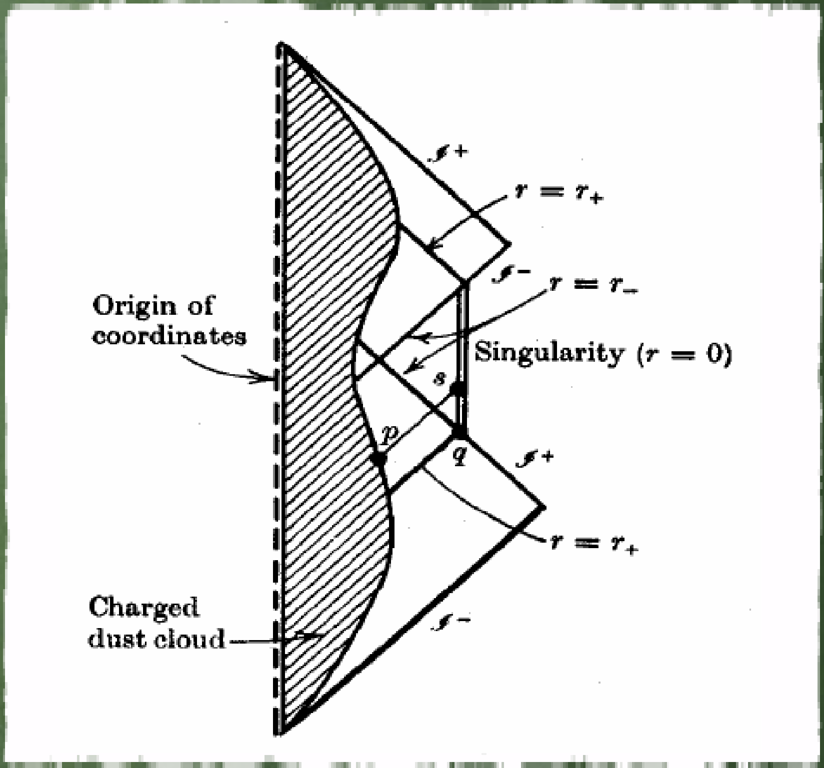
Hawking-Ellis, 1973



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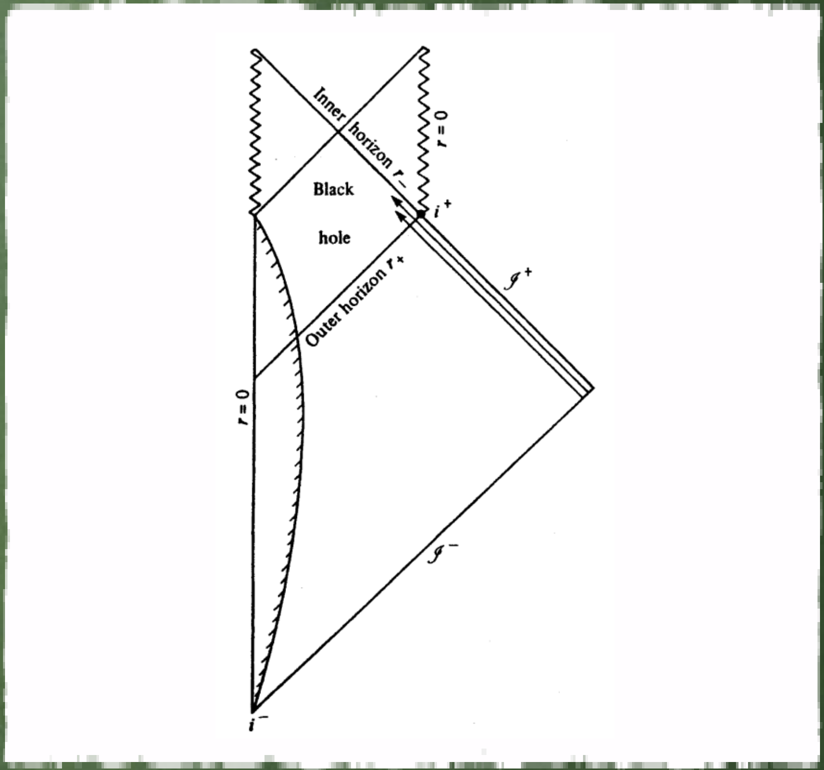
Birrell-Davies, 1982



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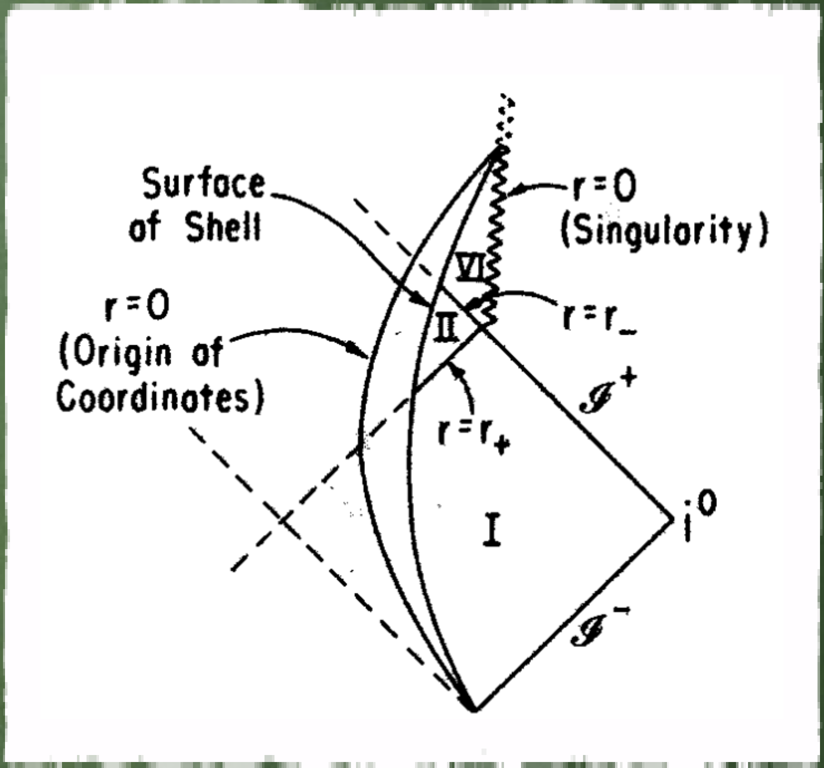
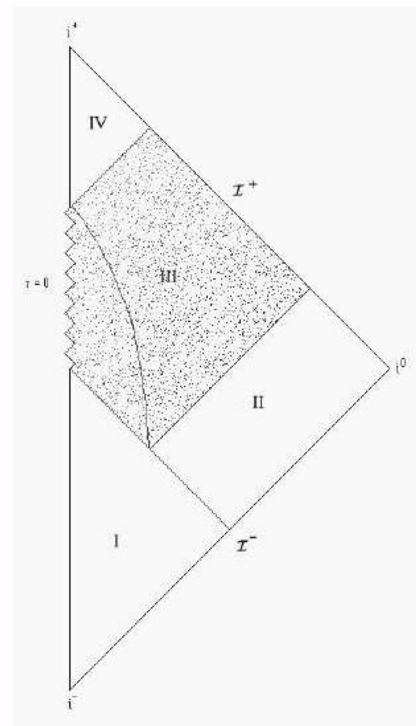
Wald, 1984



Exercise 2: Charged BH

What is the correct causal structure for the formation of a charged black hole?

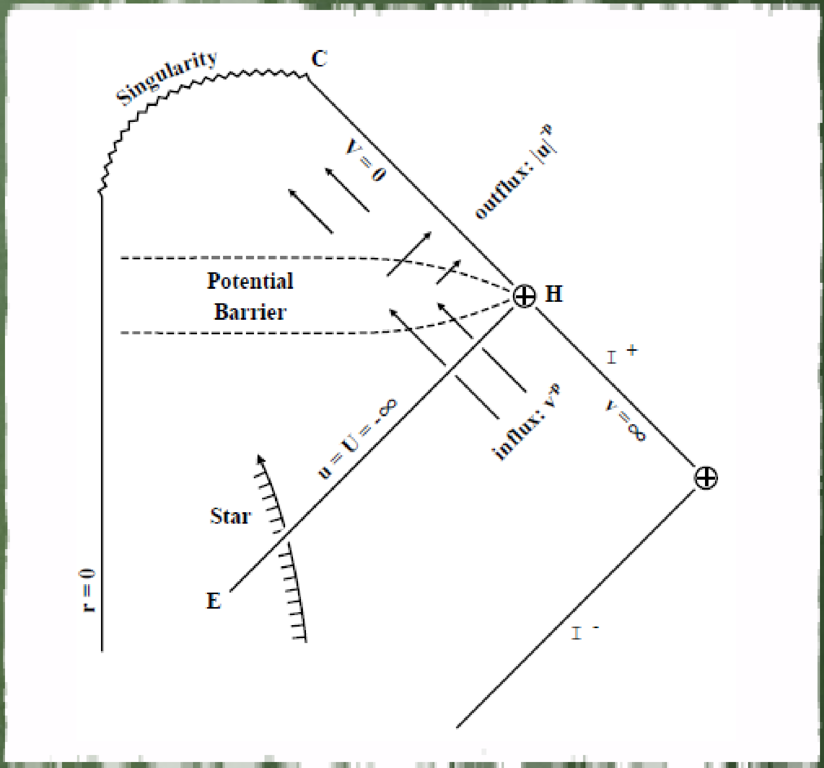
Parikh-Wilczek, 1998



Exercise 2: Charged BH

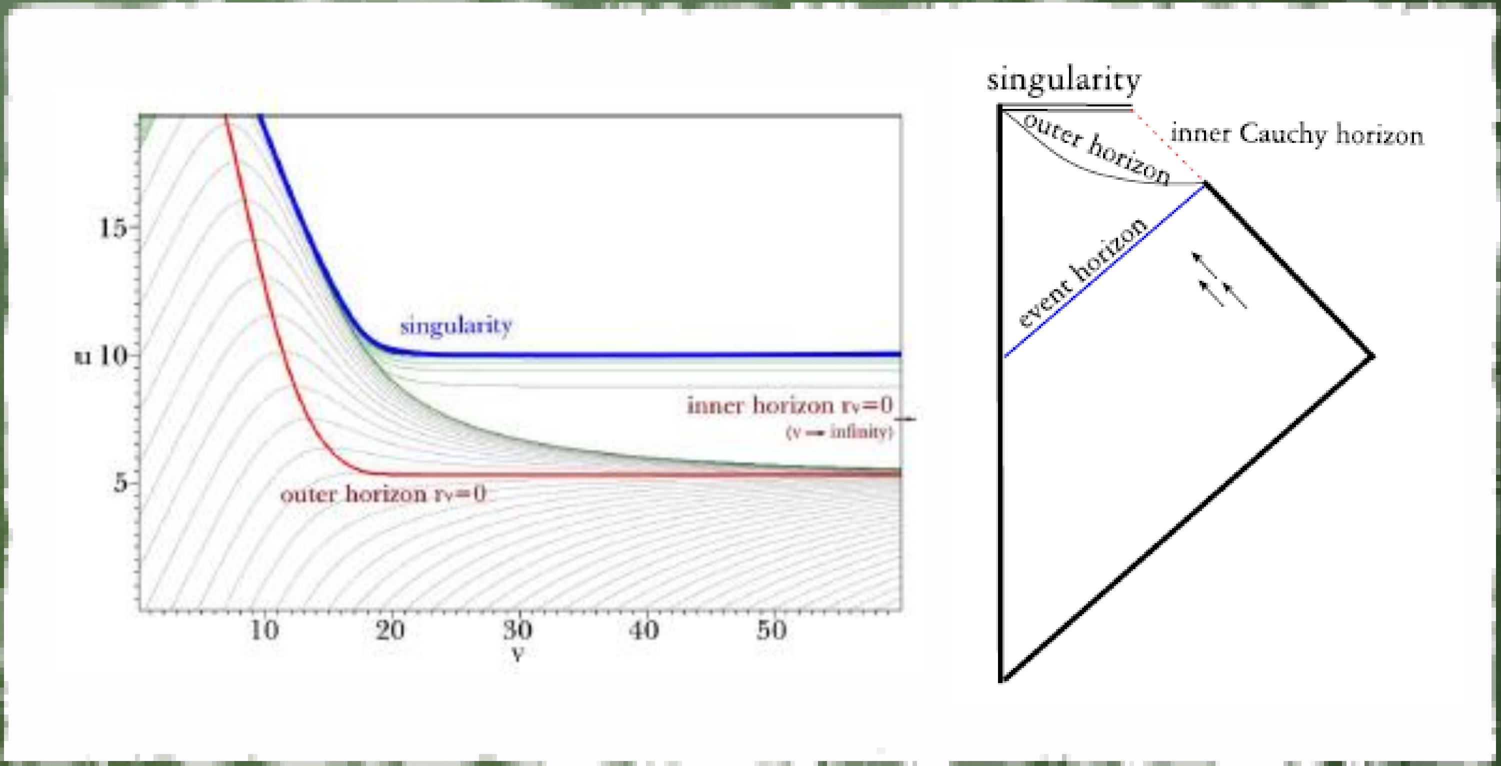
Due to mass inflation, there appears a space-like singularity.

Bonanno-Droz-Israel-Morsink, 1994



Exercise 2: Charged BH

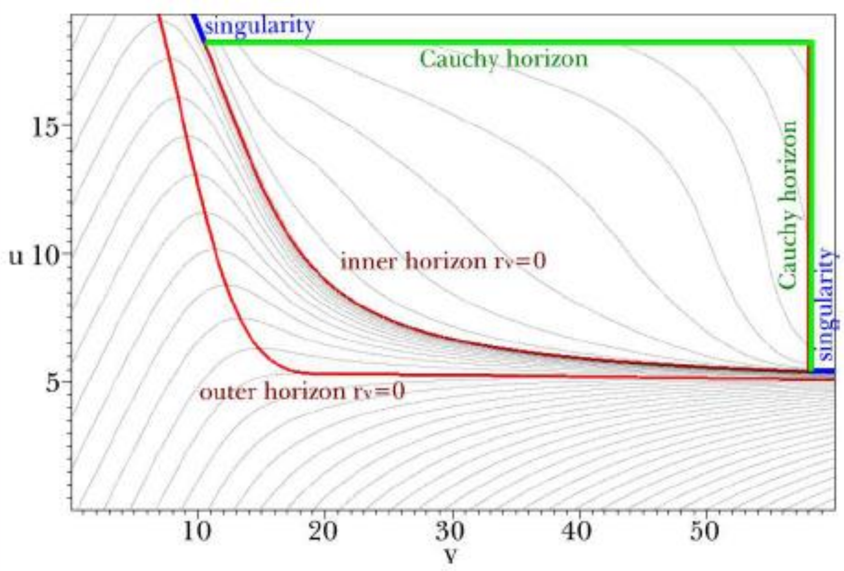
confirmed by numerical calculations ☺

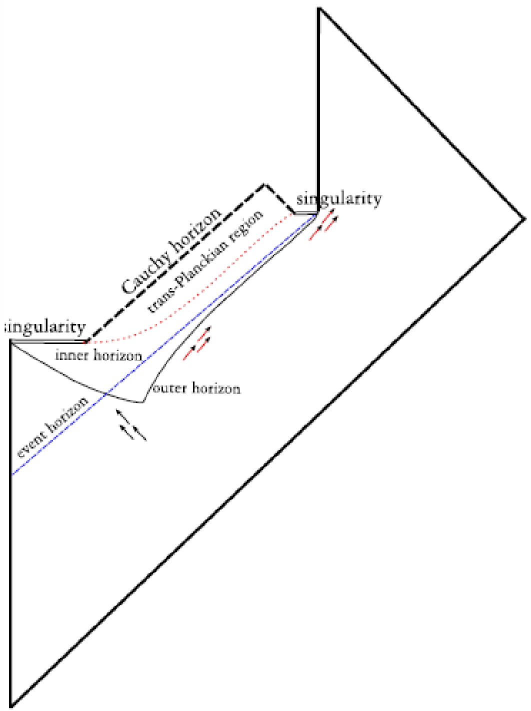


Hod and Piran, 1998; Hong, Hwang, Stewart and DY, 2010

Exercise 2: Charged BH

Including Hawking radiation and discharging effect





Sorkin and Piran, 2001; Hong, Hwang, Stewart and DY, 2010

Exercise 2: Charged BH

Further on charged black holes:

if a dilaton field is coupled to charge, then there may or may not be a cauchy horizon depending on details of coupling as well as the potential of the dilaton field.

Borokowska (Nakonieczna), Rogatko and Moderski, 2011

Hansen and DY, 2014; 2015

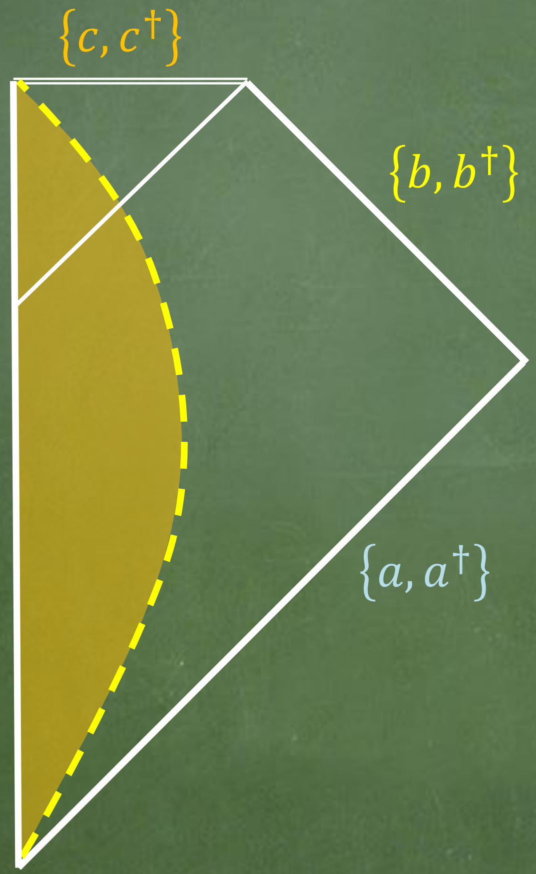
Nakonieczna and DY, 2016

Origin of Hawking radiation

Three ways to Hawking radiation

First way: Bogoliubov transformation

Hawking, 1975



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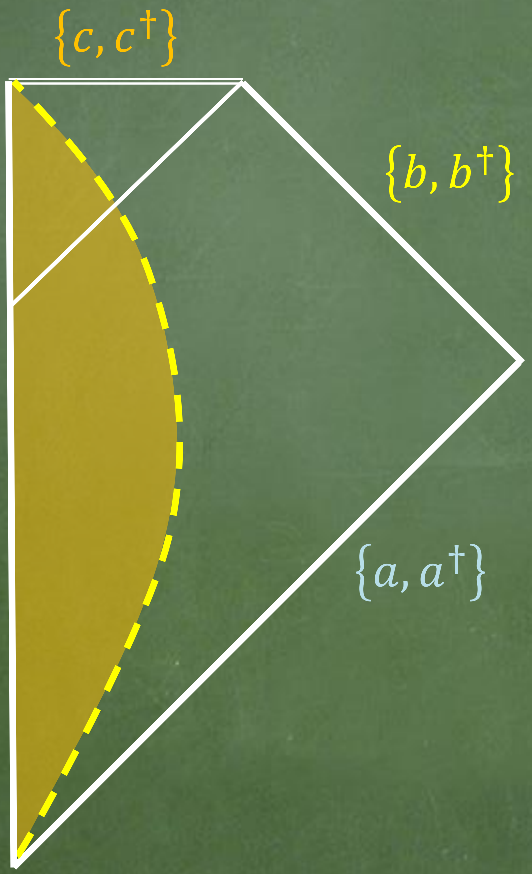
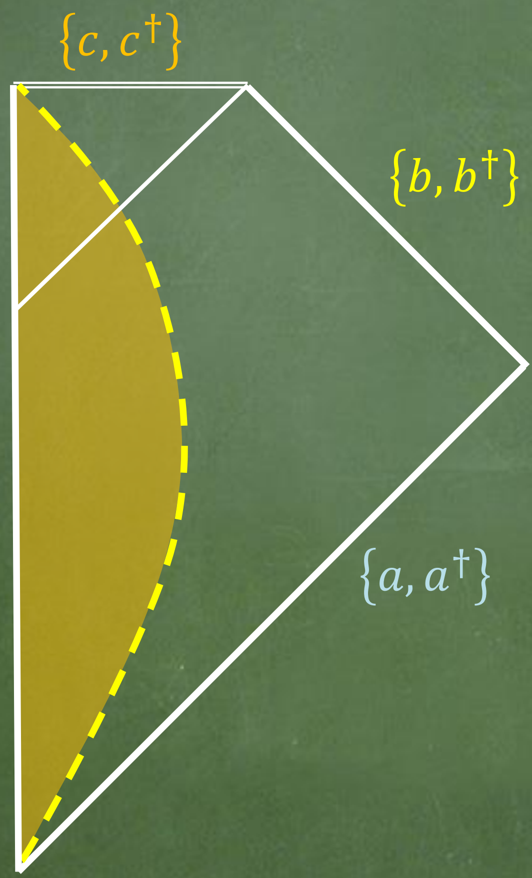


unitary transformation between operators:

First way: Bogoliubov transformation

Hawking, 1975



unitary transformation between operators:

from

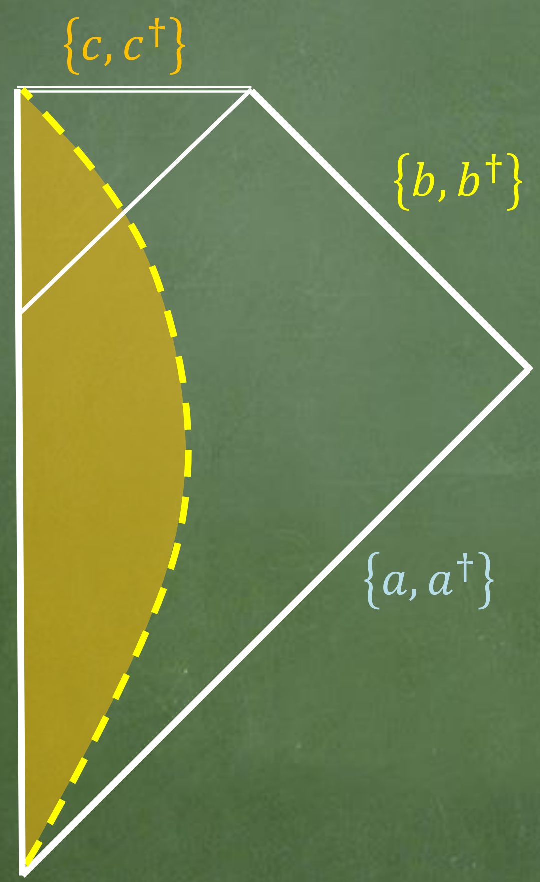
$$\phi(x) = \sum_i [a_i f_i(x) + a_i^\dagger f_i^*(x)]$$

to

$$\phi(x) = \sum_i [b_i p_i(x) + b_i^\dagger p_i^*(x) + c_i q_i(x) + c_i^\dagger q_i^*(x)]$$

First way: Bogoliubov transformation

Hawking, 1975



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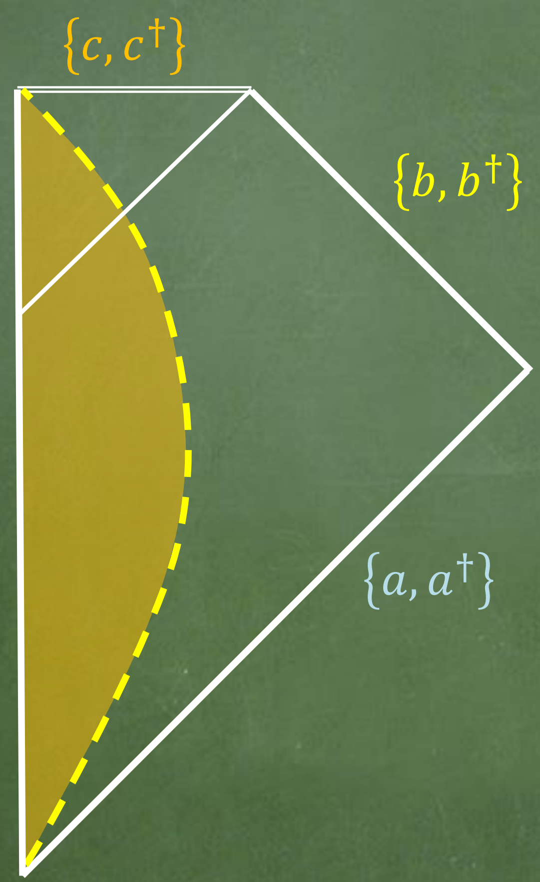
so to speak,

$$p_i = \sum_j [\alpha_{ij} f_j + \beta_{ij} f_j^*]$$

$$q_i = \sum_j [\gamma_{ij} f_j + \eta_{ij} f_j^*]$$

First way: Bogoliubov transformation

Hawking, 1975



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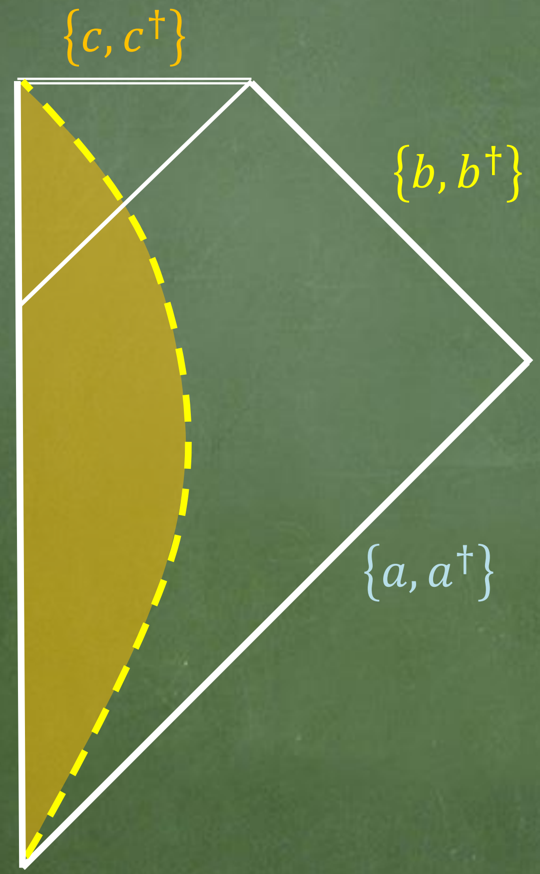
Equivalently,

$$b_i = \sum_j [\alpha_{ij}^* a_j - \beta_{ij}^* a_j^\dagger]$$

$$c_i = \sum_j [\gamma_{ij}^* a_j - \eta_{ij}^* a_j^\dagger]$$

First way: Bogoliubov transformation

Hawking, 1975



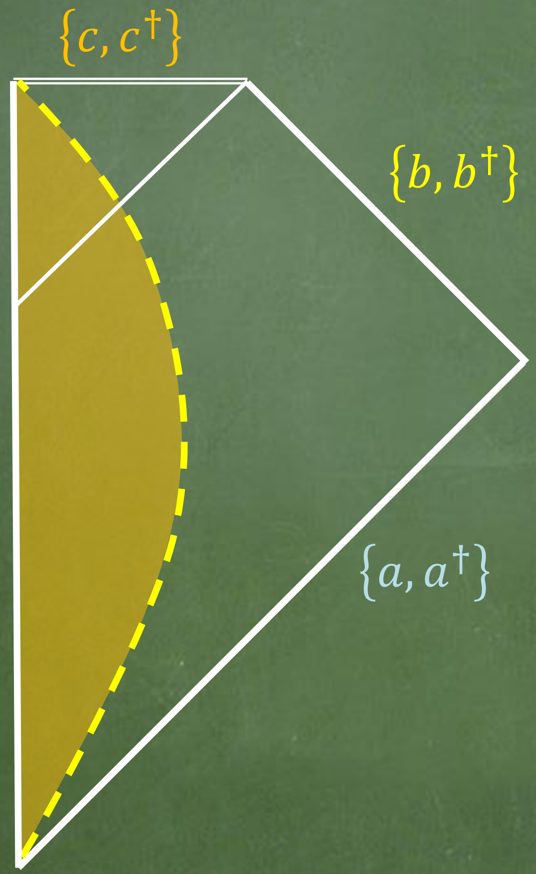
Therefore, number of particles are non-trivial!

Even though we start from the vacuum

$$a_\omega|0\rangle = 0$$

First way: Bogoliubov transformation

Hawking, 1975



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by using

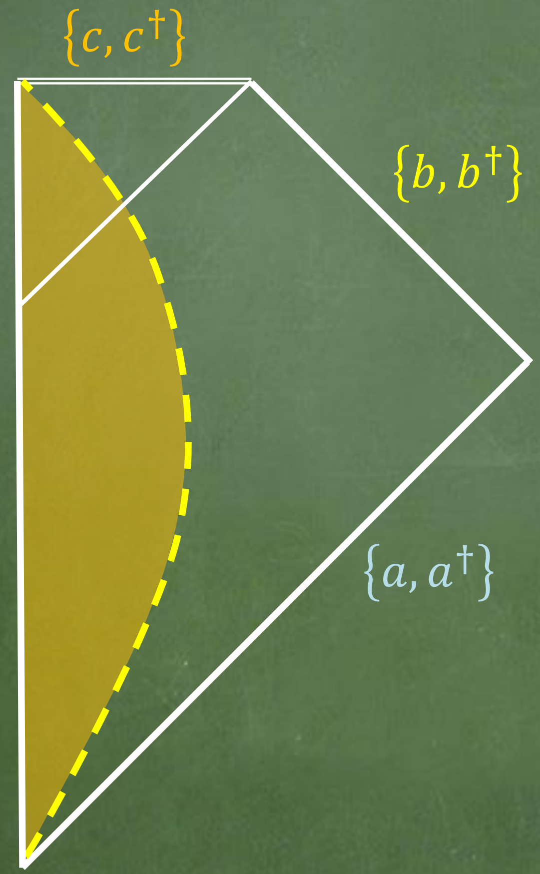
$$b_\omega = \sum_{\omega'} [\alpha^*_{\omega\omega'} a_{\omega'} - \beta^*_{\omega\omega'} a^\dagger_{\omega'}]$$

we obtain

$$\langle n_\omega \rangle = \langle b^\dagger_\omega b_\omega \rangle = \sum_{\omega'} |\beta_{\omega\omega'}|^2$$

First way: Bogoliubov transformation

Hawking, 1975



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This may be non-zero.

First way: Bogoliubov transformation

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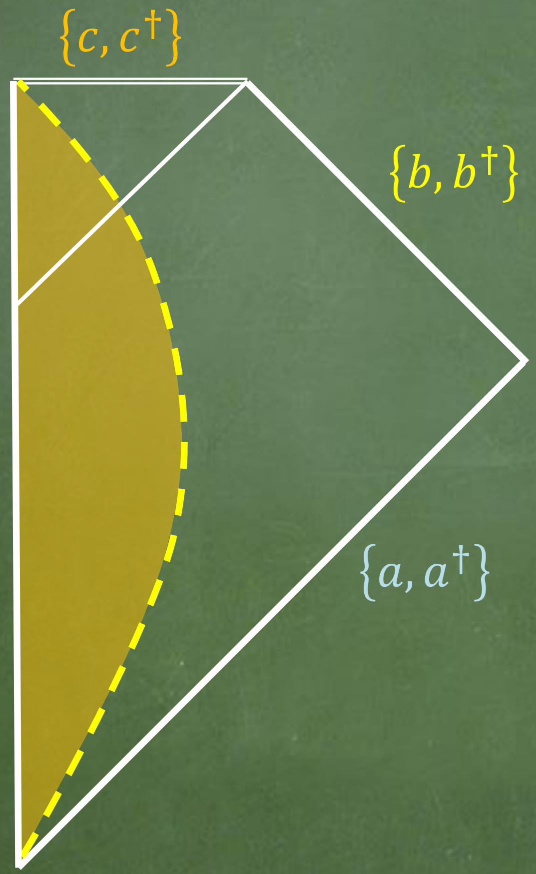

Then how to calculate β?

First way: Bogoliubov transformation

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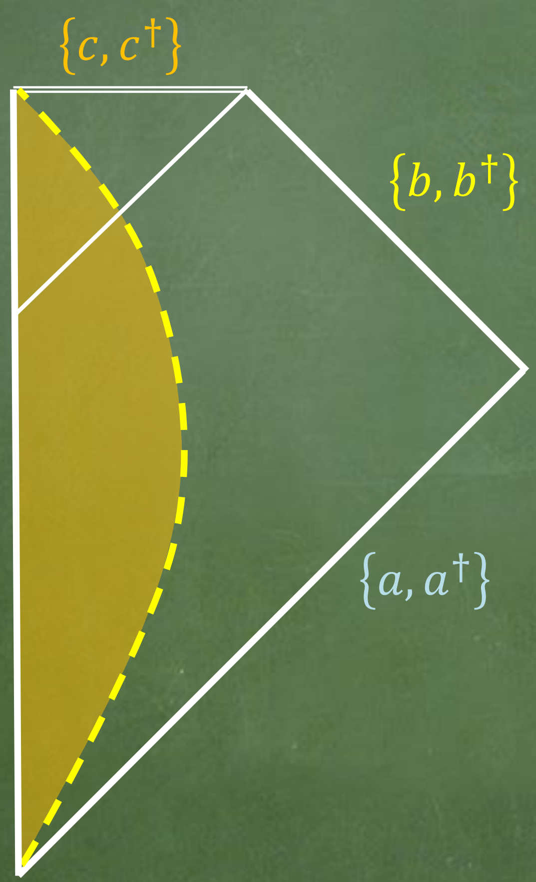


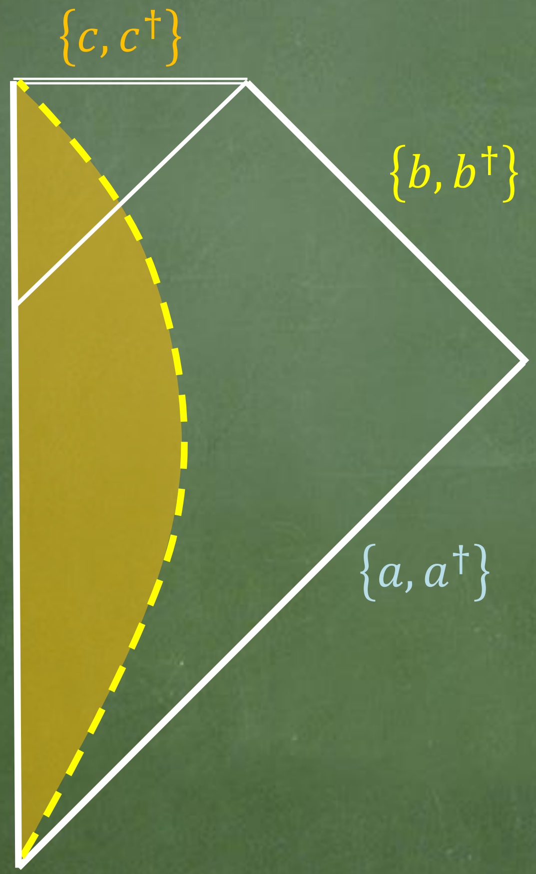
Then how to calculate β?

The essence of nontrivial β is the change between in-going mode and out-going mode.

$$p_\omega = \sum_{\omega'} [\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'}]$$

First way: Bogoliubov transformation

Hawking, 1975



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Here, $f_{\omega'}$ is the mode function of the past infinity (mainly in-going) and p_ω is the mode function of the future infinity (mainly out-going).

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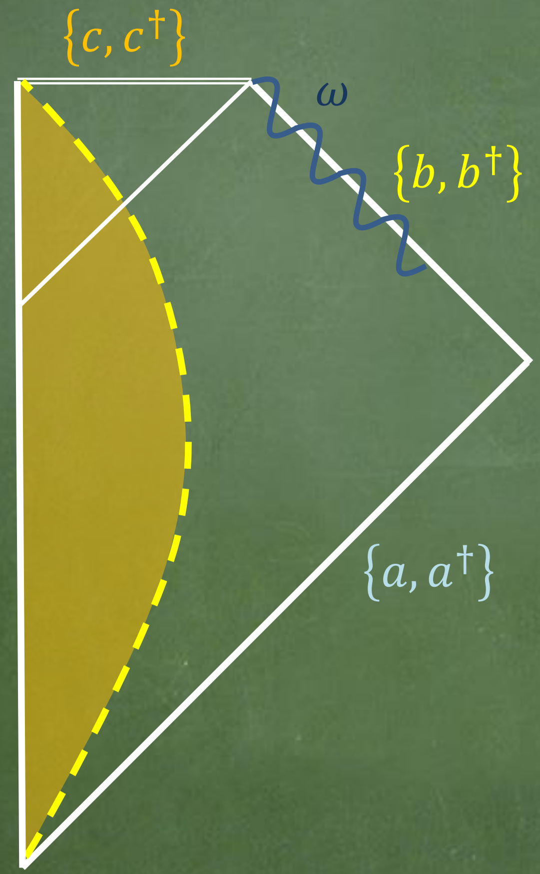

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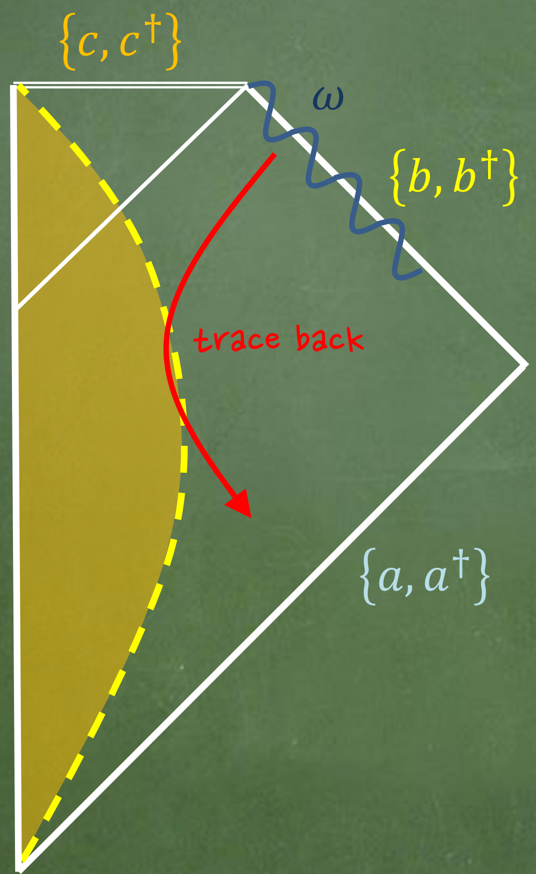

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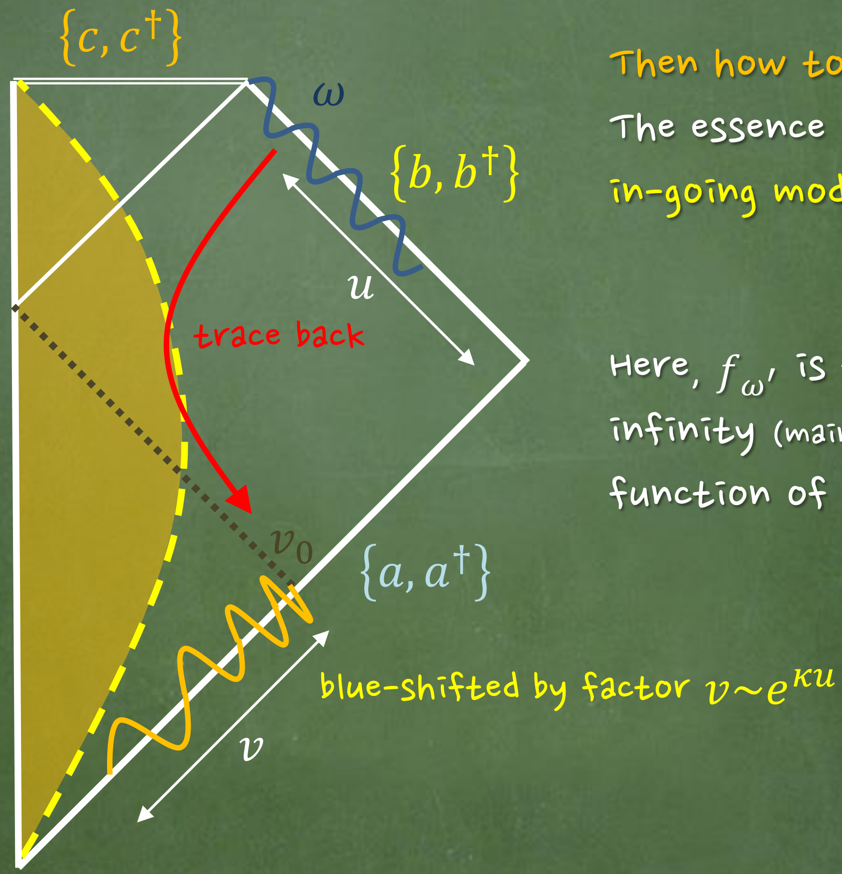

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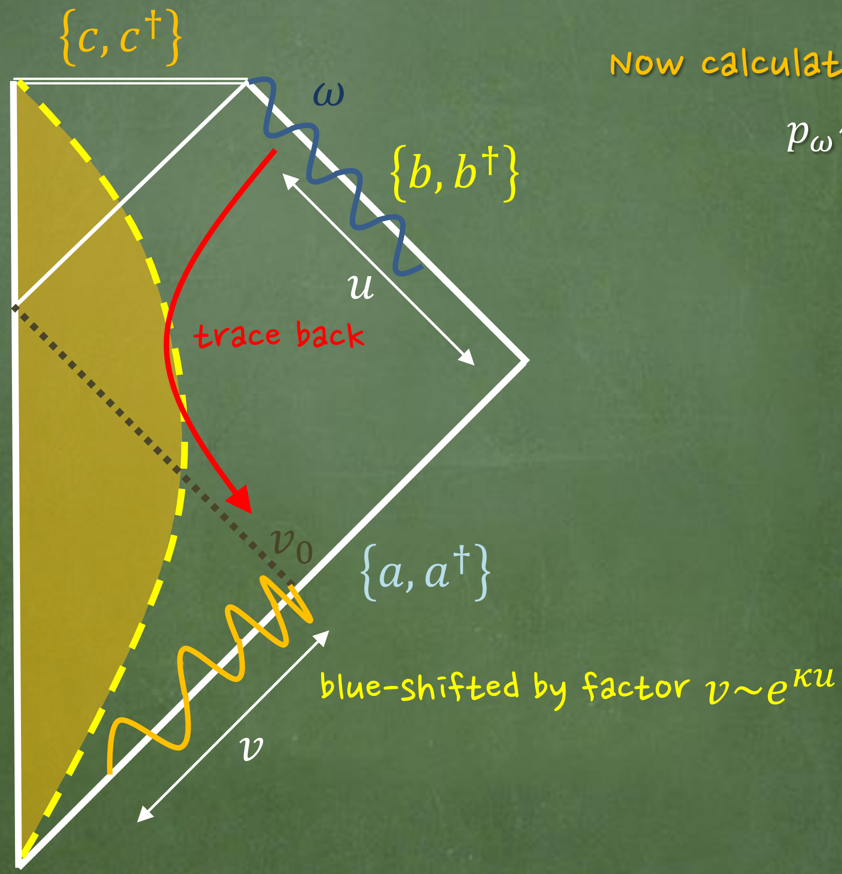
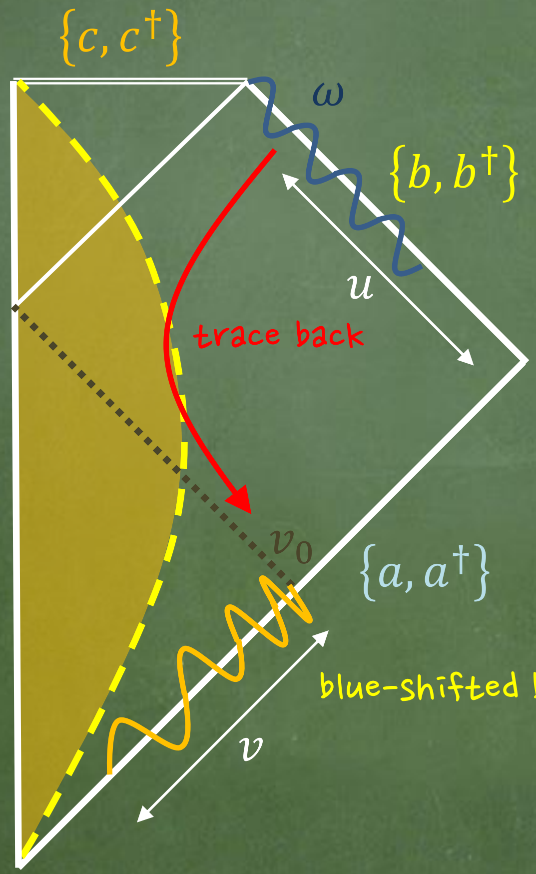


Now calculate!

$p_\omega \sim e^{i\omega\kappa^{-1}\ln[(v_0 - v)/c]}$ (for $v < v_0$)

First way: Bogoliubov transformation

Hawking, 1975



Now calculate!

$$p_\omega \sim e^{i\omega\kappa^{-1}\ln[(v_0 - v)/c]} \text{ (for } v > v_0\text{)}$$

This is a function of v. Hence, by using the Fourier transformation, we can match the coefficients α and β

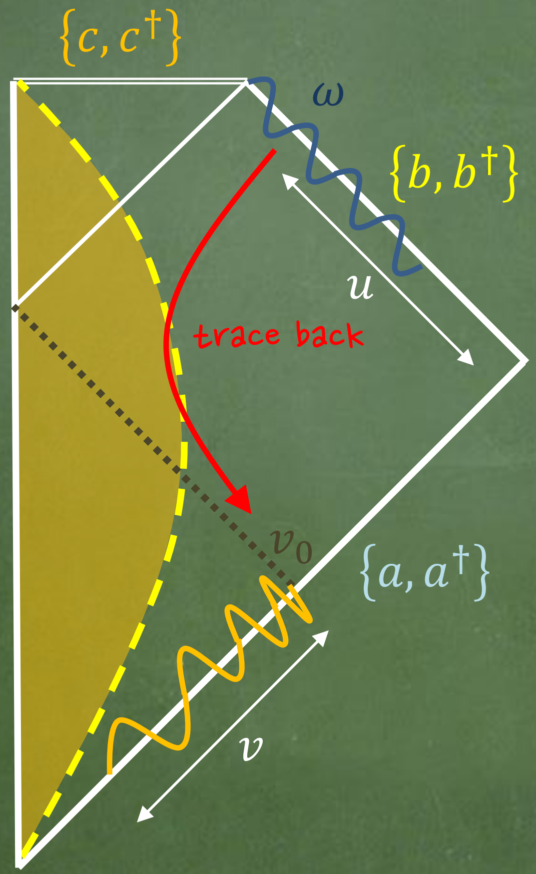
$$p_\omega = \sum_{\omega'} [\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} f^*_{\omega'}]$$

where $f_\omega \sim e^{i\omega v}$.

blue-shifted by factor $v \sim e^{\kappa u}$

First way: Bogoliubov transformation

Hawking, 1975



We obtain the relation

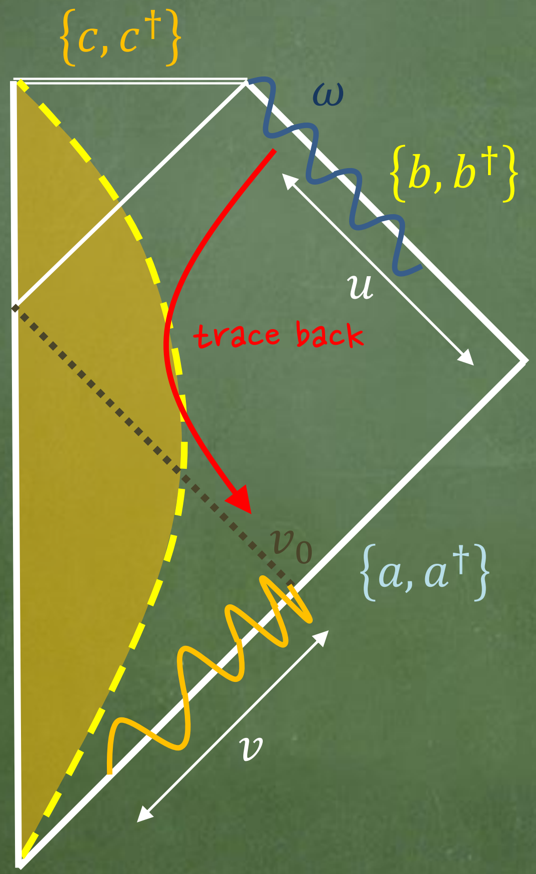
$$|\alpha_{\omega\omega'}|^2 = e^{2\pi\omega/\kappa}|\beta_{\omega\omega'}|^2$$

In addition, there is a normalization condition

$$\sum_{\omega'}(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) = 1$$

First way: Bogoliubov transformation

Hawking, 1975



We obtain the relation

$$|\alpha_{\omega\omega'}|^2 = e^{2\pi\omega/\kappa}|\beta_{\omega\omega'}|^2$$

In addition, there is a normalization condition

$$\sum_{\omega'}(|\alpha_{\omega\omega'}|^2 - |\beta_{\omega\omega'}|^2) = 1$$

In conclusion,

$$\langle n_\omega \rangle \propto \frac{1}{e^{2\pi\omega/\kappa} - 1}$$

Second way: Renormalized EM tensor

We want to solve the equation

$$G_{\mu\nu} = 8\pi \langle T_{\mu\nu} \rangle$$

where the energy-momentum tensor has ambiguities.

Moreover, the expectation values are divergent in general:

$$\langle \phi(x)\phi(x) \rangle$$

Second way: Renormalized EM tensor

Birrell and Davies, "Quantum fields in curved space", 1982

In order to resolve these problems, renormalization techniques were developed.

Step 1. obtain finite two-point correlation function, e.g., by the point splitting method:

$$\lim_{x \to x'} \langle \phi(x)\phi(x') \rangle = \frac{c}{x - x'} + (\text{finite terms})$$

Second way: Renormalized EM tensor

Birrell and Davies, "Quantum fields in curved space", 1982

In order to resolve these problems, renormalization techniques were developed.

Step 2. Using the two-point correlation function, we obtain the energy-momentum tensor.

$$\langle T_\mu^\nu \rangle = \langle (\tfrac{2}{3}\phi_{;\mu}\phi_;{}^\nu - \tfrac{1}{6} g_\mu^\nu \phi_{;\alpha}\phi_;{}^\alpha - \tfrac{1}{3}\phi\phi_{;\mu}{}^\nu) \rangle$$

$$G(x,x') = i\langle \phi(x)\phi(x') \rangle$$

$$\langle T_\mu^\nu \rangle_{\mathrm{REN}} = \lim_{x' \to x} \{ -i[\tfrac{1}{3}(G_{;\mu\alpha'} g^{\alpha'\nu} + G_;{}^\nu{}_{\alpha'} g^{\alpha'}{}_\mu) - \tfrac{1}{6} G_{;\alpha\beta'} g^{\alpha\beta'} g_\mu^\nu - \tfrac{1}{6}(G_{;\mu}{}^\nu + G_{;\alpha'\beta'} g^{\alpha'}{}_\mu g^{\beta'\nu})] - \langle T_\mu^\nu \rangle_{\mathrm{subtract}} \}$$

Howard and Candelas, 1984

Second way: Renormalized EM tensor

Davies, Fulling and Unruh, 1976

For two-dimensional cases, we can obtain the simpler form:

$$\langle T_{\mu\nu}\rangle = \frac{P}{\alpha^2}\begin{pmatrix} (\alpha\alpha_{,uu} - 2\alpha_{,u}^2) & -(\alpha\alpha_{,uv} - \alpha_{,u}\alpha_{,v}) \\ -(\alpha\alpha_{,uv} - \alpha_{,u}\alpha_{,v}) & (\alpha\alpha_{,vv} - 2\alpha_{,v}^2) \end{pmatrix}$$

where $ds^2 = -\alpha^2 du dv$.

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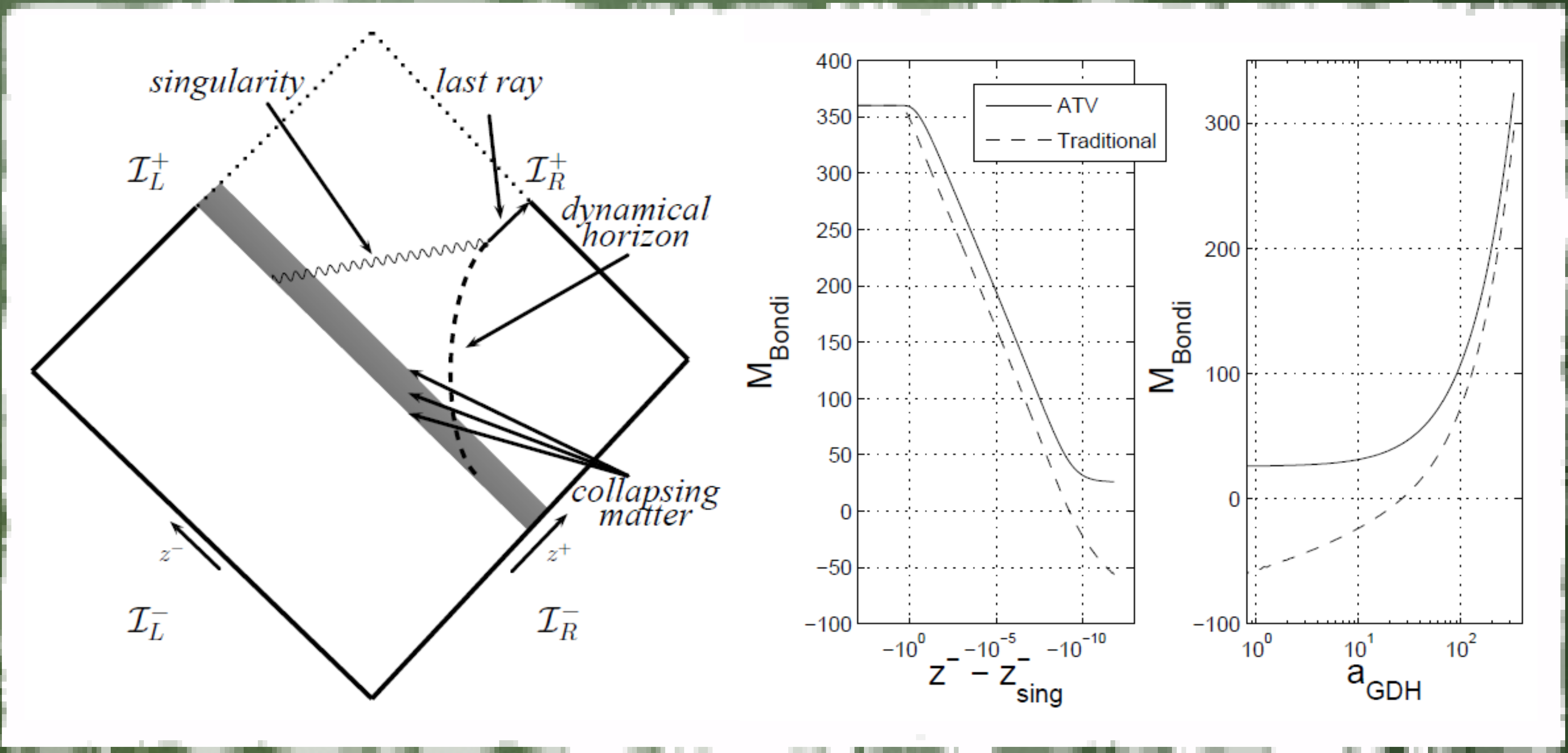
where $ds^2 = -\alpha^2 dudv$.

For dilaton black holes, there can be a back-reaction even for 2D: CGHS model.

Callan, Giddings, Harvey and Strominger, 1991

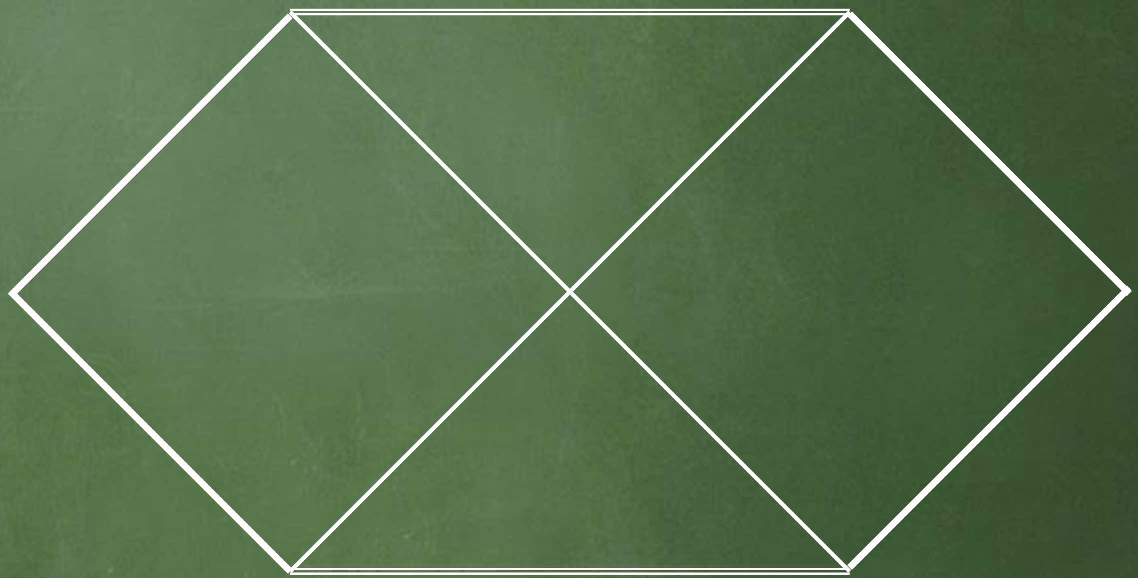
Second way: Renormalized EM tensor

Ashtekar, Pretorius and Ramazanoglu, 2011



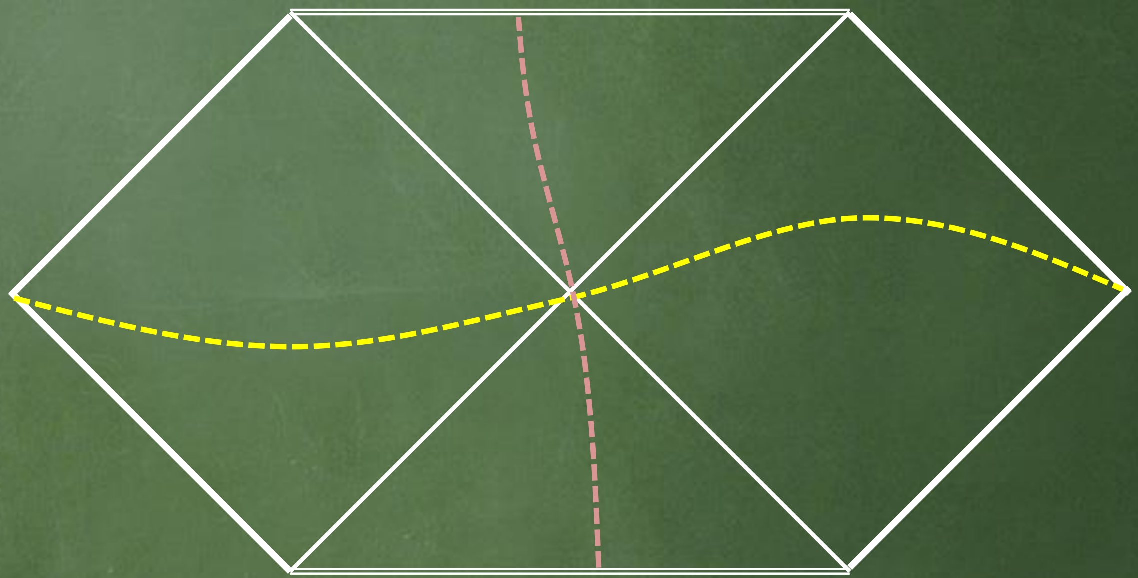
Third way: Particle tunneling

causal structure of a Schwarzschild black hole.



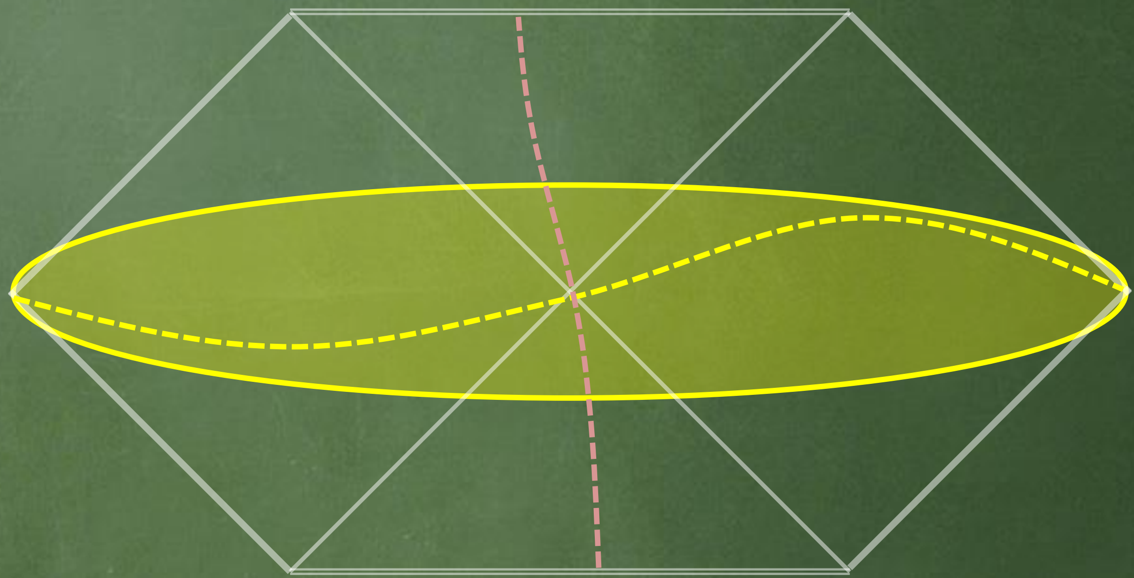
Third way: Particle tunneling

constant time hypersurfaces



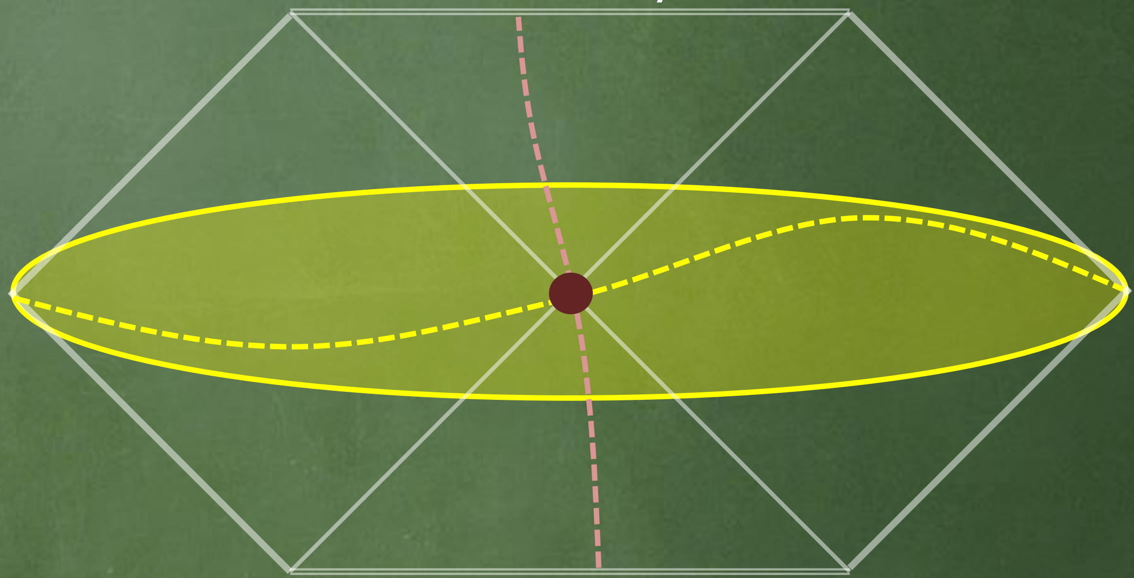
Third way: Particle tunneling

Euclidean analytic continuation



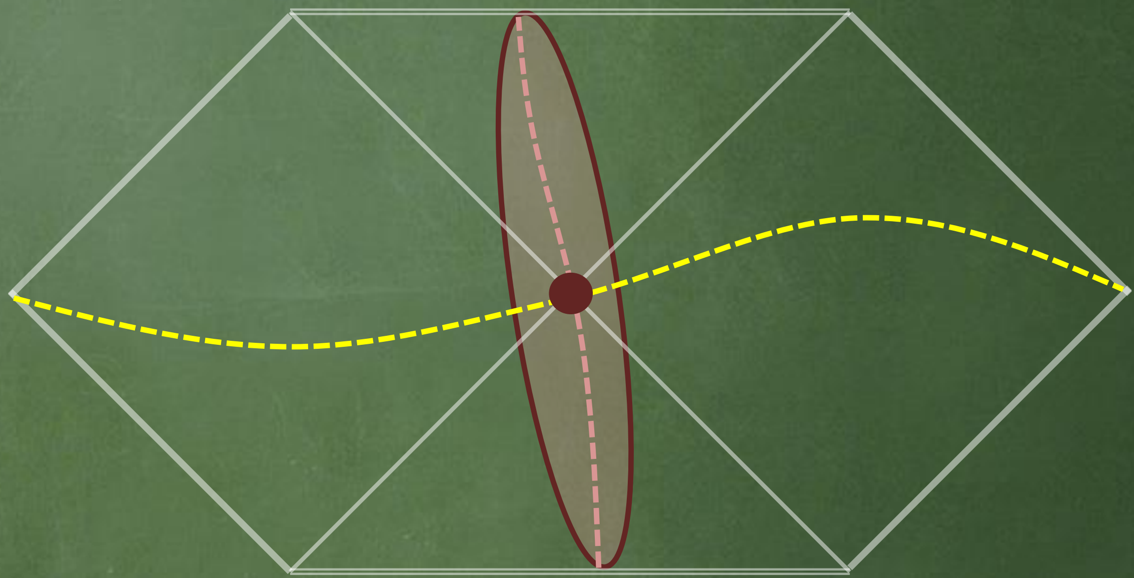
Third way: Particle tunneling

In order to remove a cusp singularity, we need to choose the Euclidean time period $\tau = 8\pi M = 1/T$.



Third way: Particle tunneling

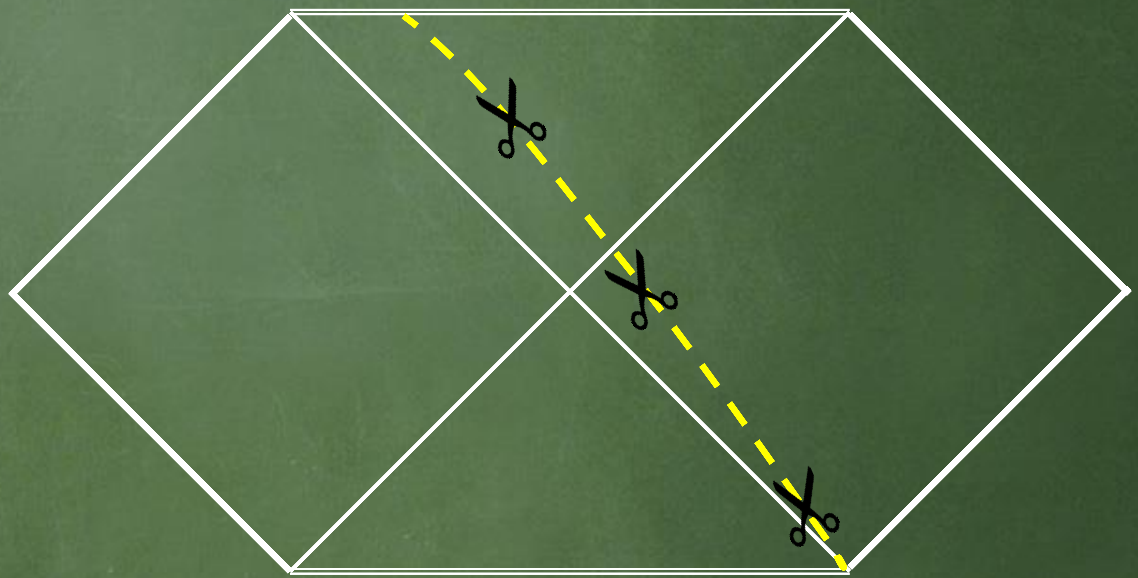
One can do a similar analytic continuation inside the horizon, although the signature becomes (++--).



Third way: Particle tunneling

Hartle and Hawking, 1976

Hartle and Hawking considered a particle tunneling from inside to outside the horizon.



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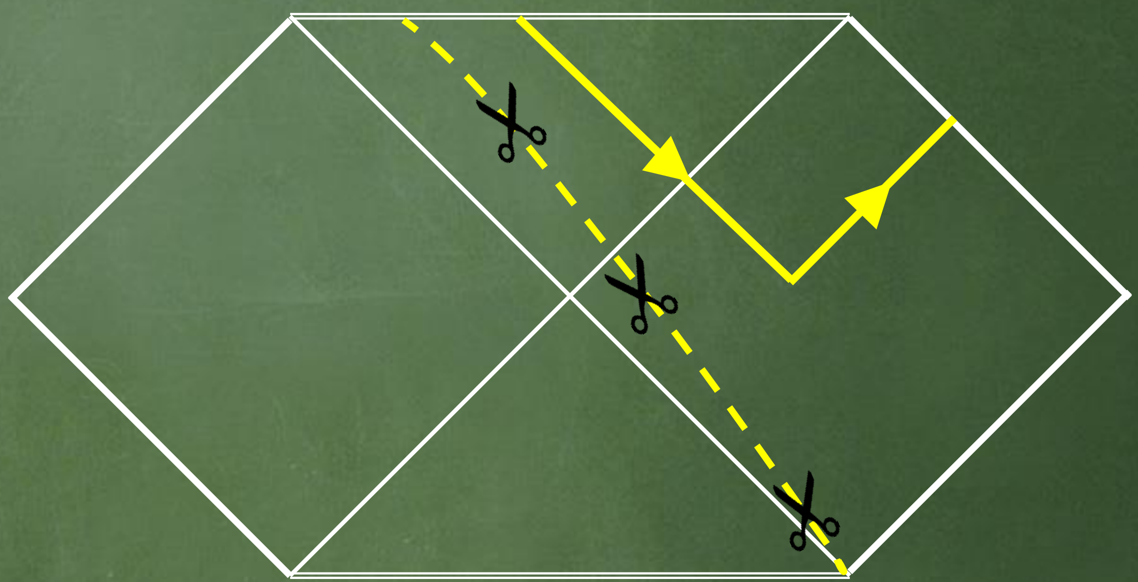
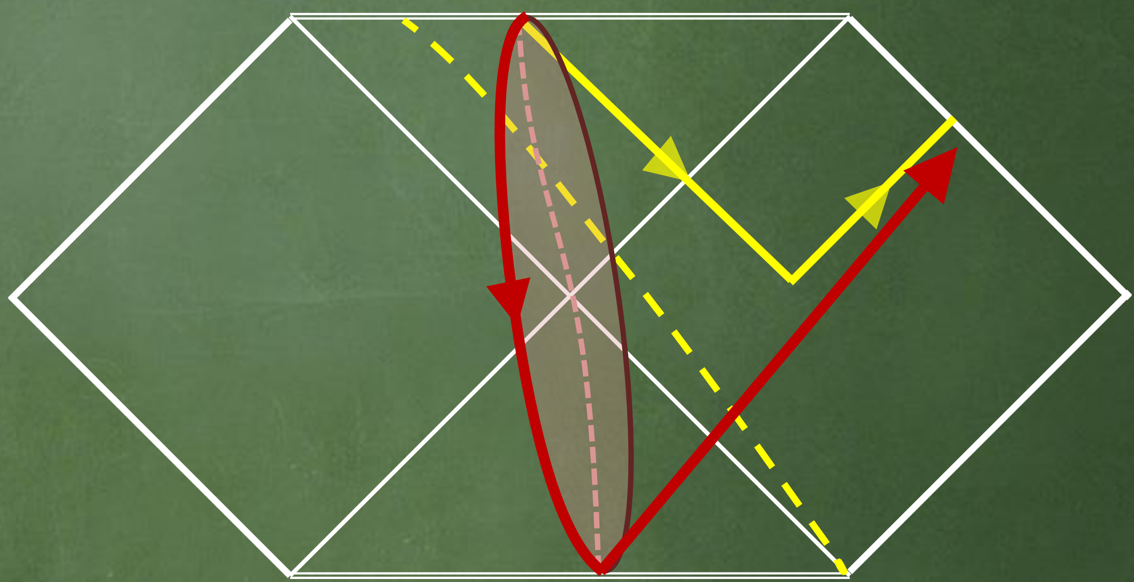


Third way: Particle tunneling

Hartle and Hawking, 1976

Using the analytic continuation, one can calculate the emission rate.

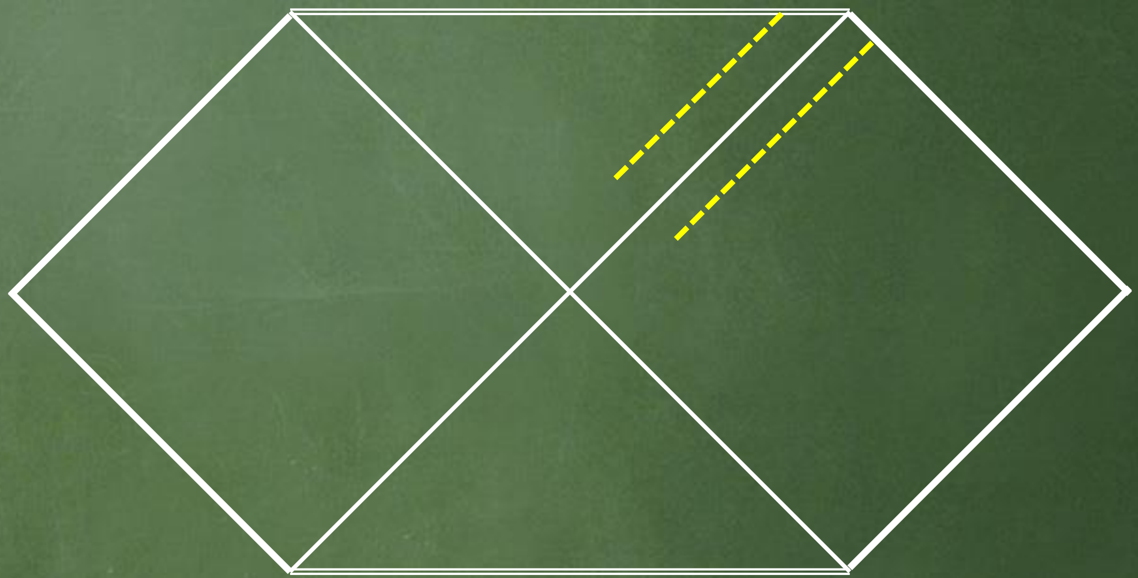


(probability to emit a particle with energy E) = $e^{-\frac{2\pi E}{\kappa}}$ × (probability to absorb a particle with Energy E)

Third way: Particle tunneling

Parikh and Wilczek, 2000

One can also calculate a tunneling between two **null geodesics**.



Third way: Particle tunneling

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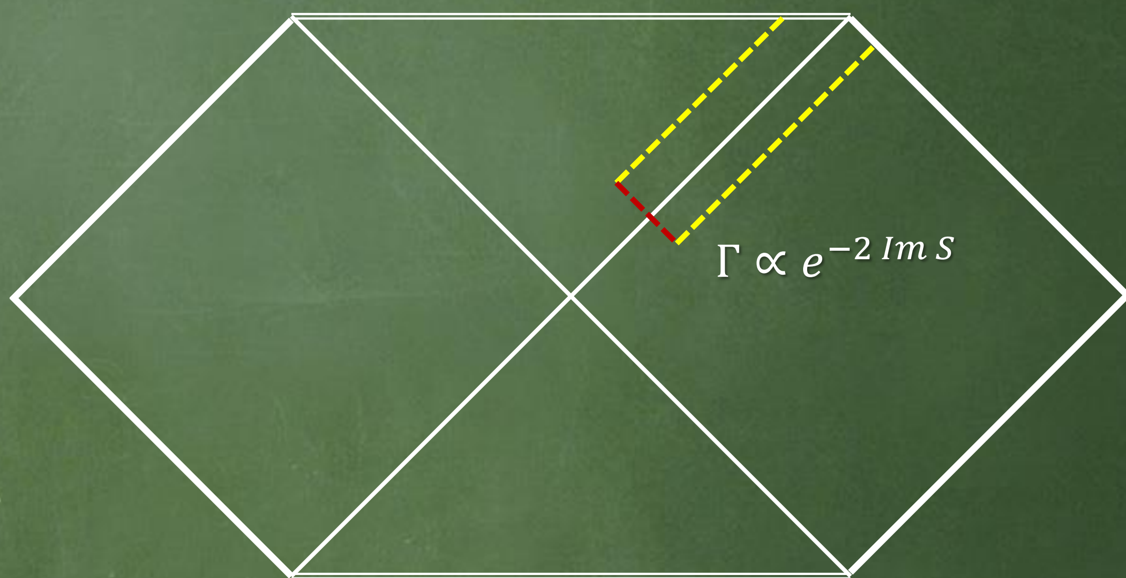


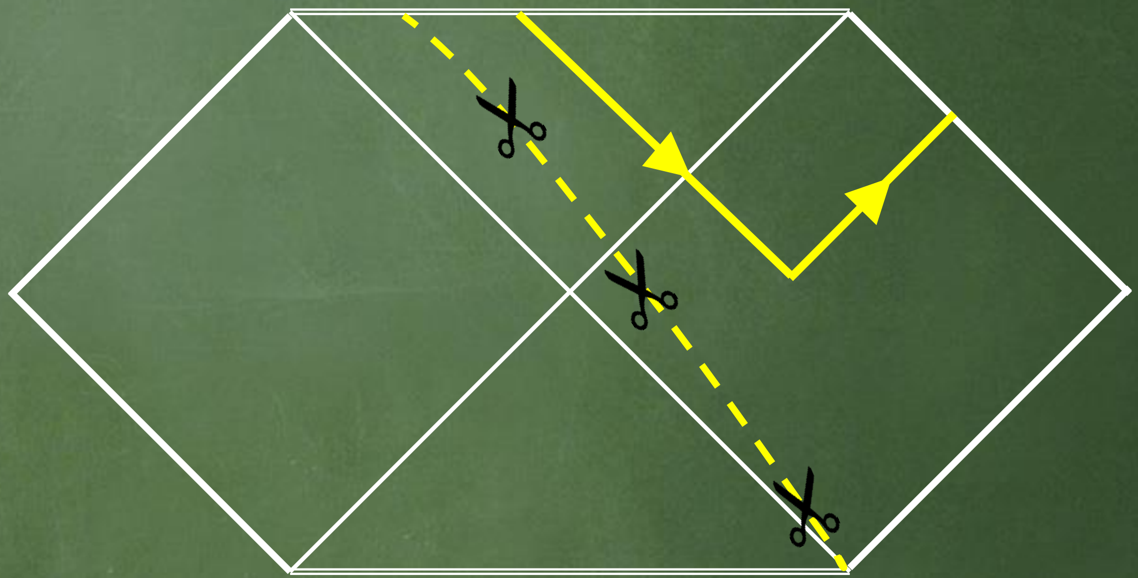
$$Im\,S = Im\int_{r_{in}}^{r_{out}} p_r dr = Im\int_{M}^{M-\omega}\int_{r_{in}}^{r_{out}} \frac{dr}{\dot{r}}dH = 4\pi\omega\left(M - \frac{\omega}{2}\right)$$

$$(H = M - \omega')$$

Fourth way?: Using instantons?

Chen, Domenech, Sasaki and DY, in preparation

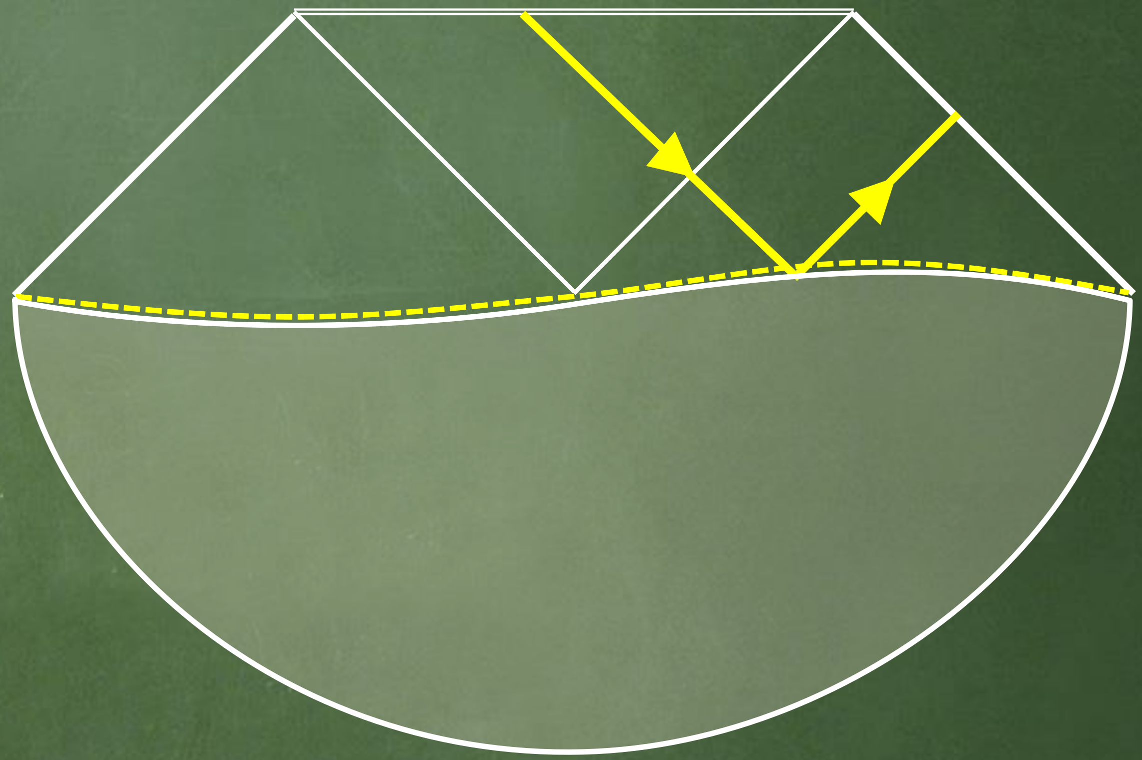
Can this particle tunneling process generalize to instanton picture of a field?



Fourth way?: Using instantons?

Chen, Domenech, Sasaki and DY, in preparation

Find a scalar field solution of the Euclidean-Lorentzian manifold. How to calculate a consistent probability?



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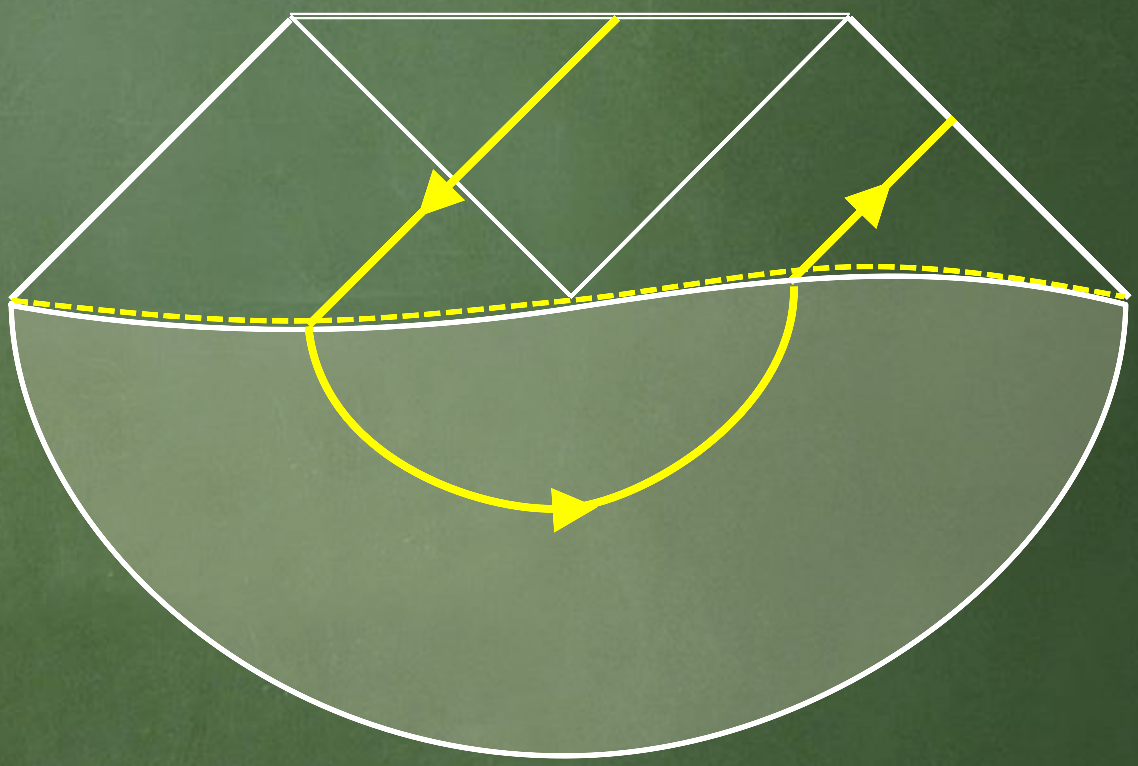

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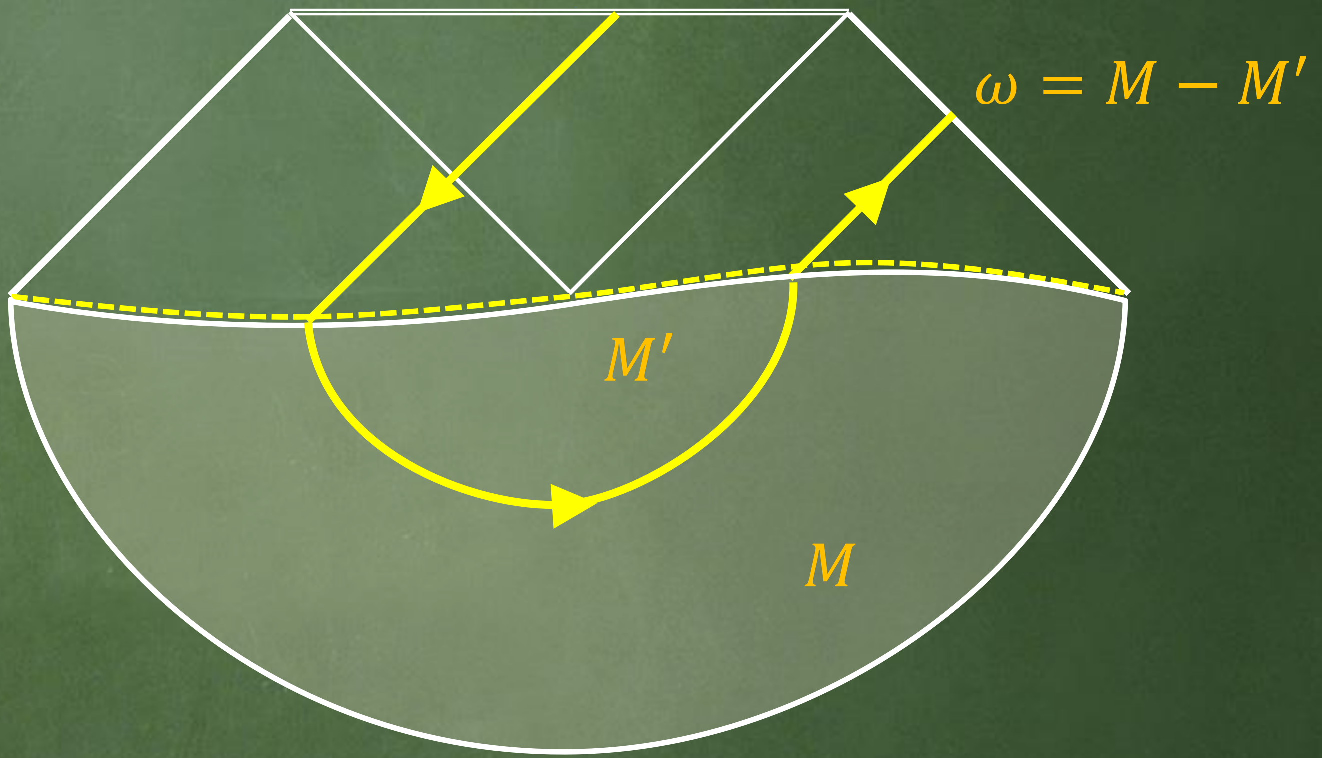

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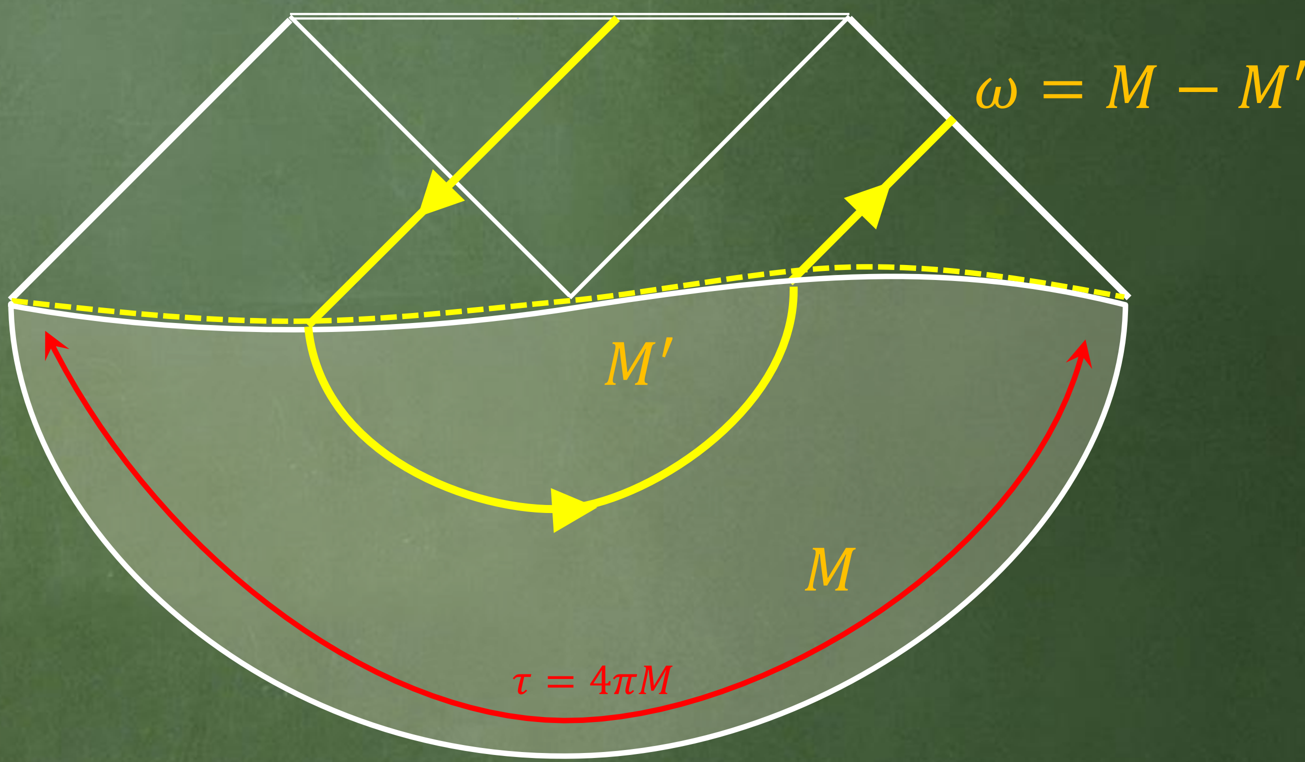
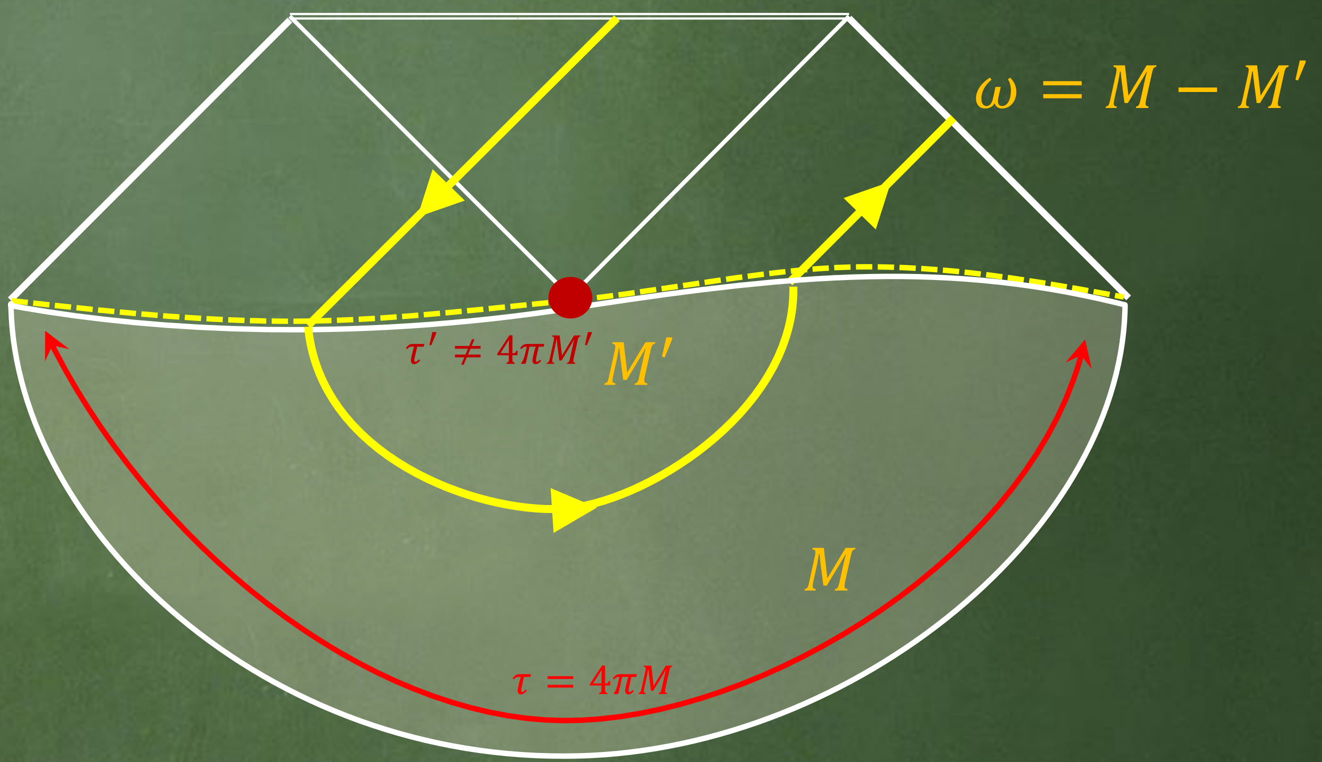


Fourth way?: Using instantons?

Chen, Domenech, Sasaki and DY, in preparation

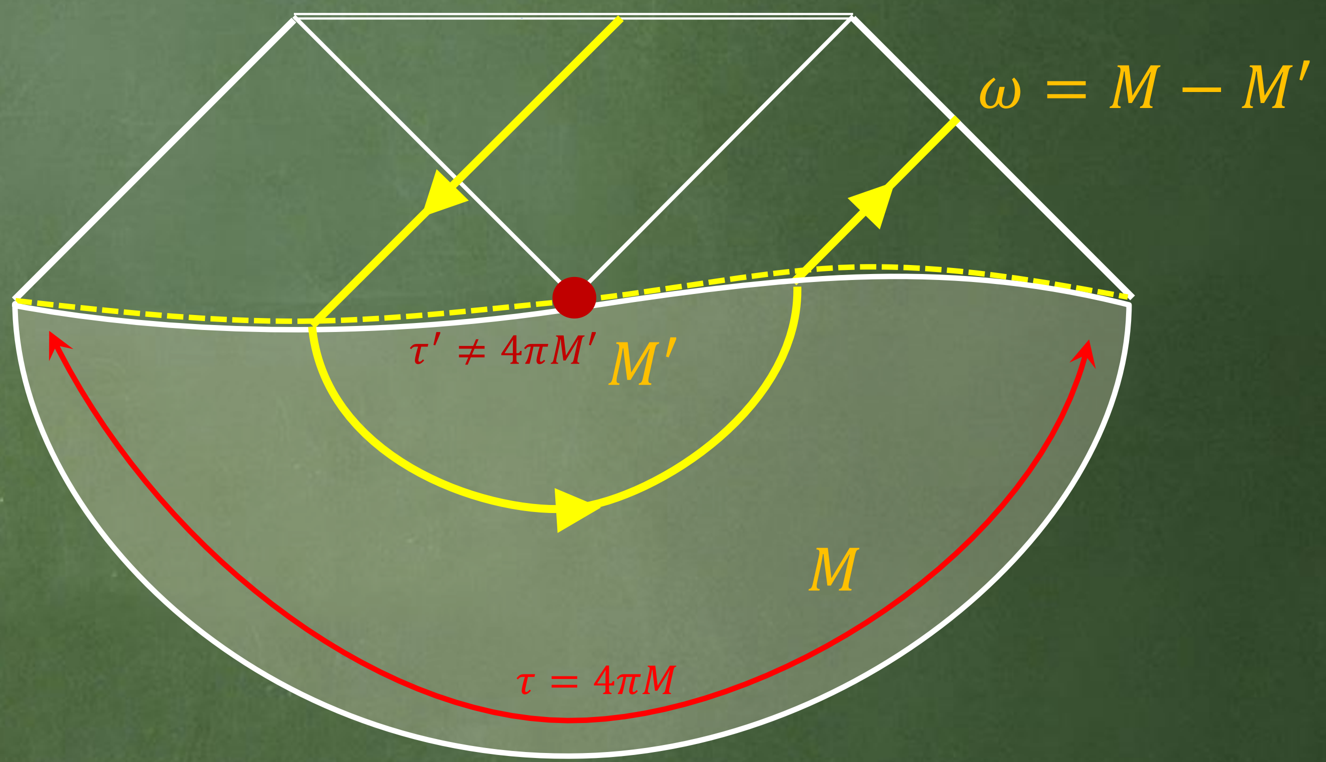
Obstacles: there should be a cusp singularity!



Fourth way?: Using instantons?

Chen, Domenech, Sasaki and DY, in preparation

We may overcome this problem: Chen, Hu and DY, 2015.



Applications of Entropy

various notions of entropy

Information and entropy

A less probable event/state has more information.

$$I_i = -\log p_i$$

Information and entropy

A less probable event/state has more information.

$$I_i = -\log p_i$$

Entropy = Expectation value of information

= capacity of information

$$S \equiv \langle I \rangle = -\sum_i p_i \log p_i$$

From statistical mechanics

Density matrix: $\rho \equiv |\psi\rangle\langle\psi| = \sum_i p_i \, |i\rangle\langle i|$.

Von Neumann entropy

$$S \equiv -Tr\rho \log \rho = -\sum_i p_i \log p_i$$

From statistical mechanics

Density matrix: $\rho \equiv |\psi\rangle\langle\psi| = \sum_i p_i \, |i\rangle\langle i|$.

Von Neumann entropy

$$S \equiv -Tr\rho \log \rho = -\sum_i p_i \log p_i$$

Boltzmann entropy: in thermal equilibrium $\rho_{eq} = \sum_i \frac{1}{N} |i\rangle\langle i|$

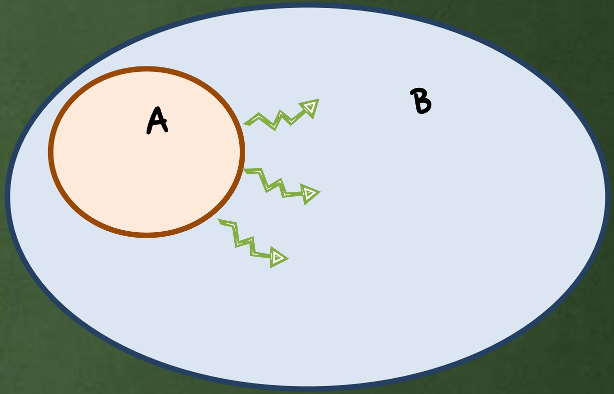
$$S_{eq} = N \times \left(-\frac{1}{N}\log\frac{1}{N}\right) = \log N$$

The possible largest entropy.

Entropy of subsystem

Let us assume that we consider:

1. A closed system with the total number of states N
2. Initially total information of the system is log N
3. It was concentrated in A in the beginning.



Entropy of subsystem

Let us assume that we consider:

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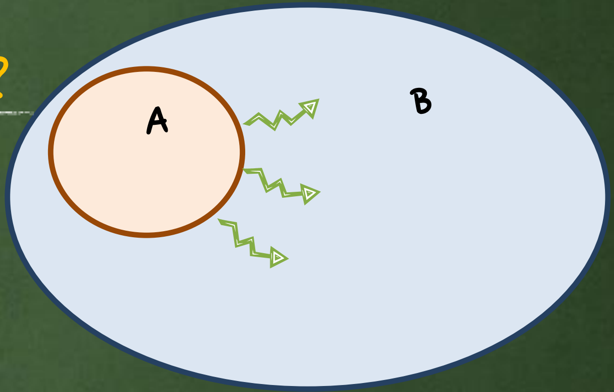
2. Initially total information of the system is log N
3. It was concentrated in A in the beginning.

Now I want to move particles from A to B.

Will the total information be conserved?



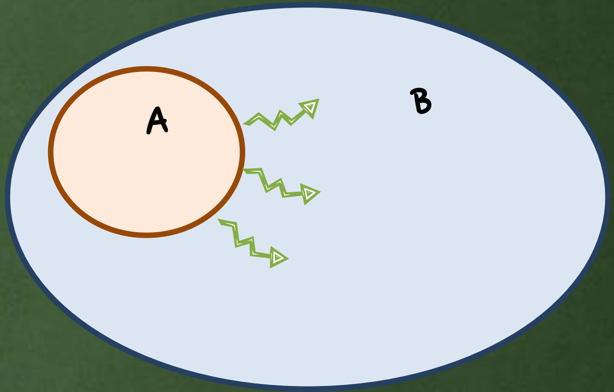
Entropy of subsystem

Page, 1993; Page, 2013

Let us define a measure of information:

$$I = S(B) - S(B|A)$$

where $S(B|A) = -Tr_B \rho_A \log \rho_A$ is the entanglement entropy and $\rho_A = Tr_A \rho$.

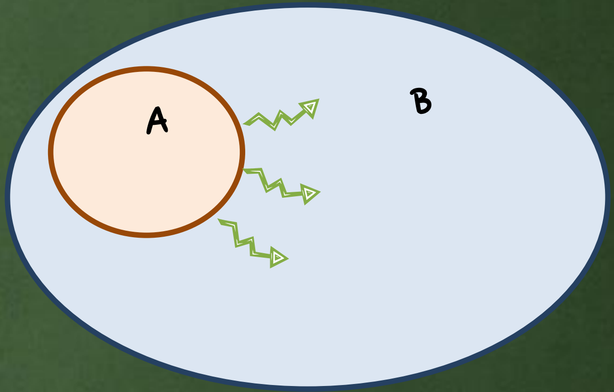


Estimation of entanglement entropy

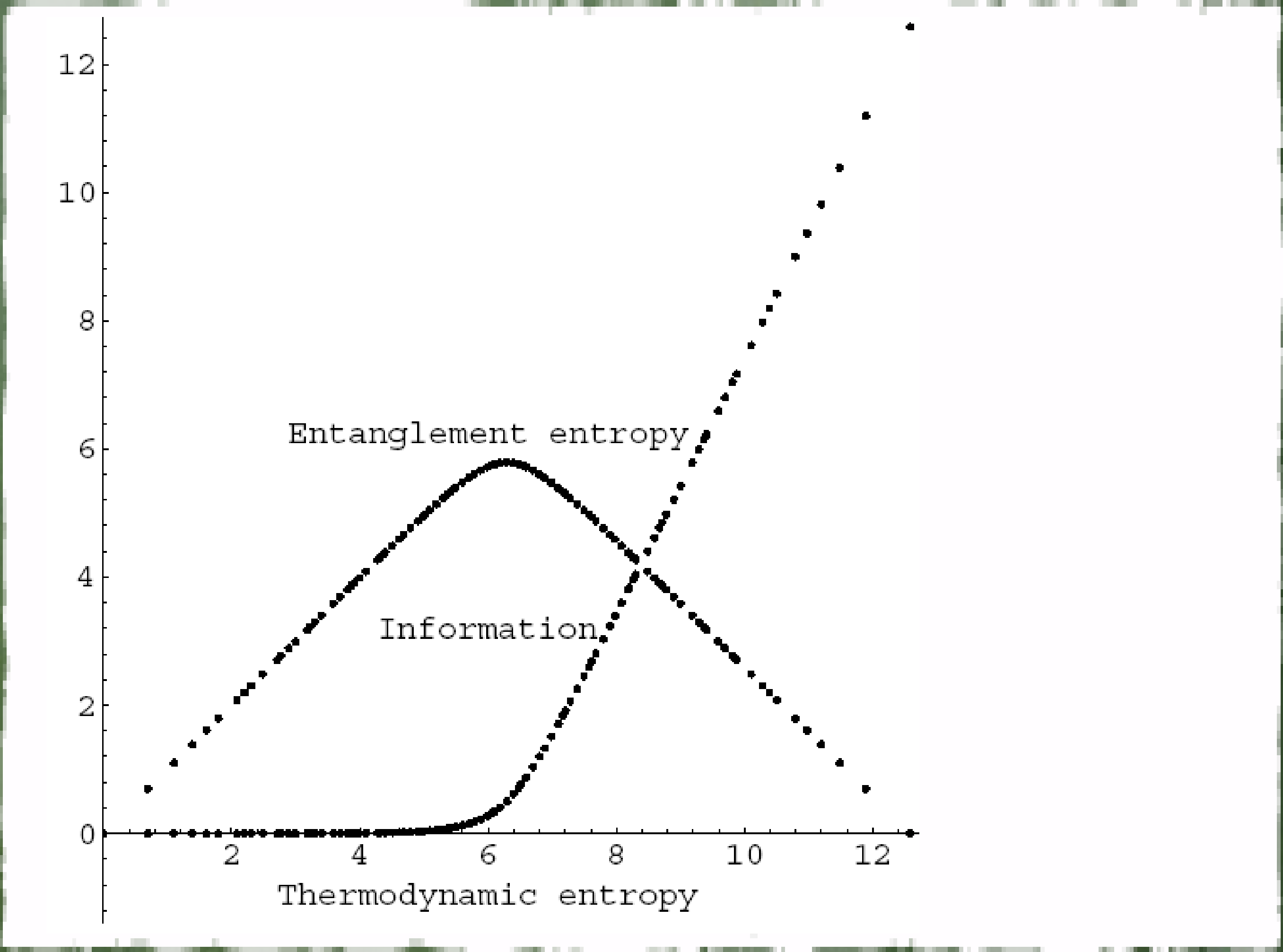
Page, 1993; Page, 2013

For a pure and random system, we can estimate the entanglement entropy (m<n):

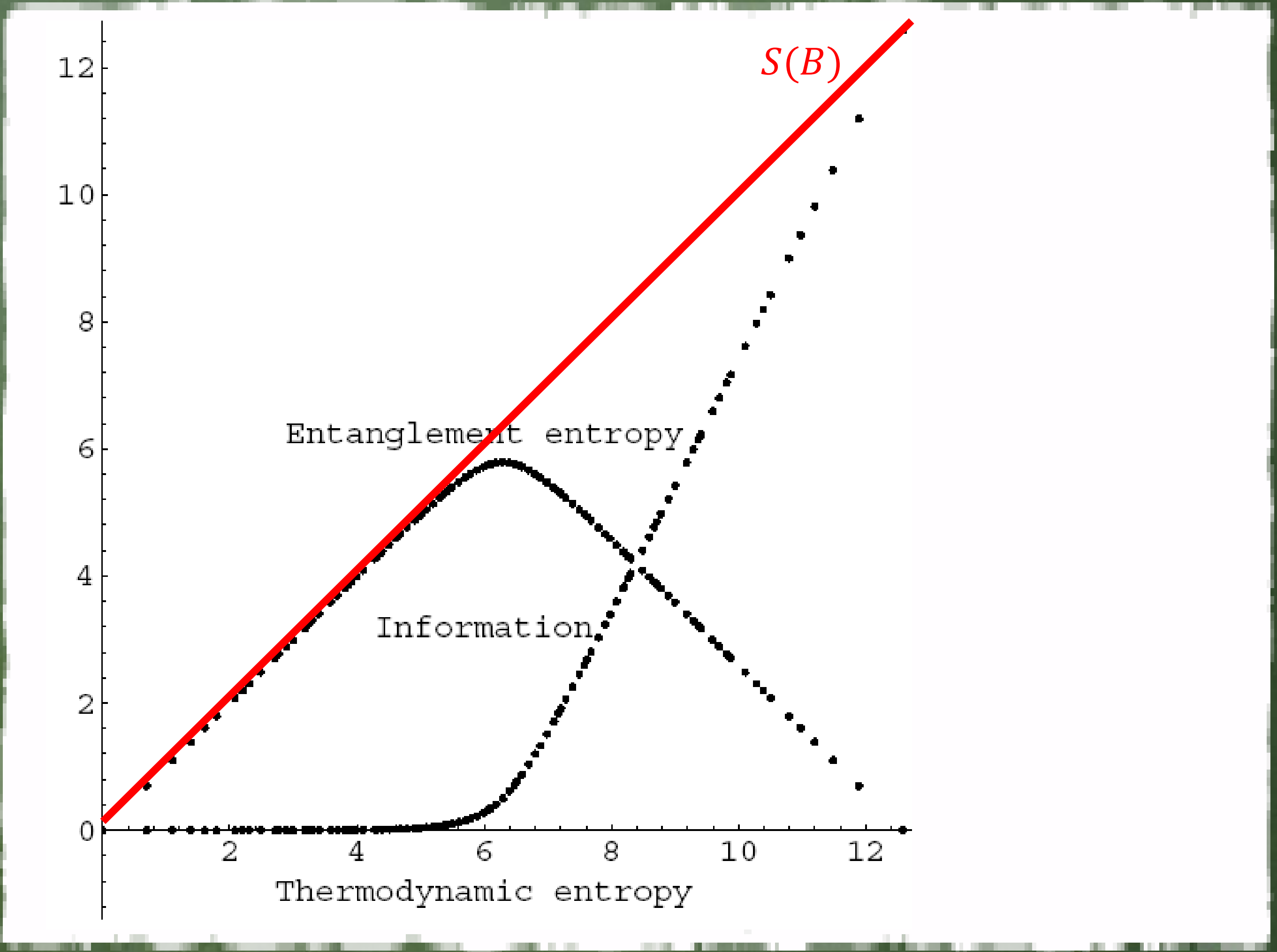
$$S(B|A) = \sum_{k=n+1}^{mn} \frac{1}{k} - \frac{m-1}{2n}$$



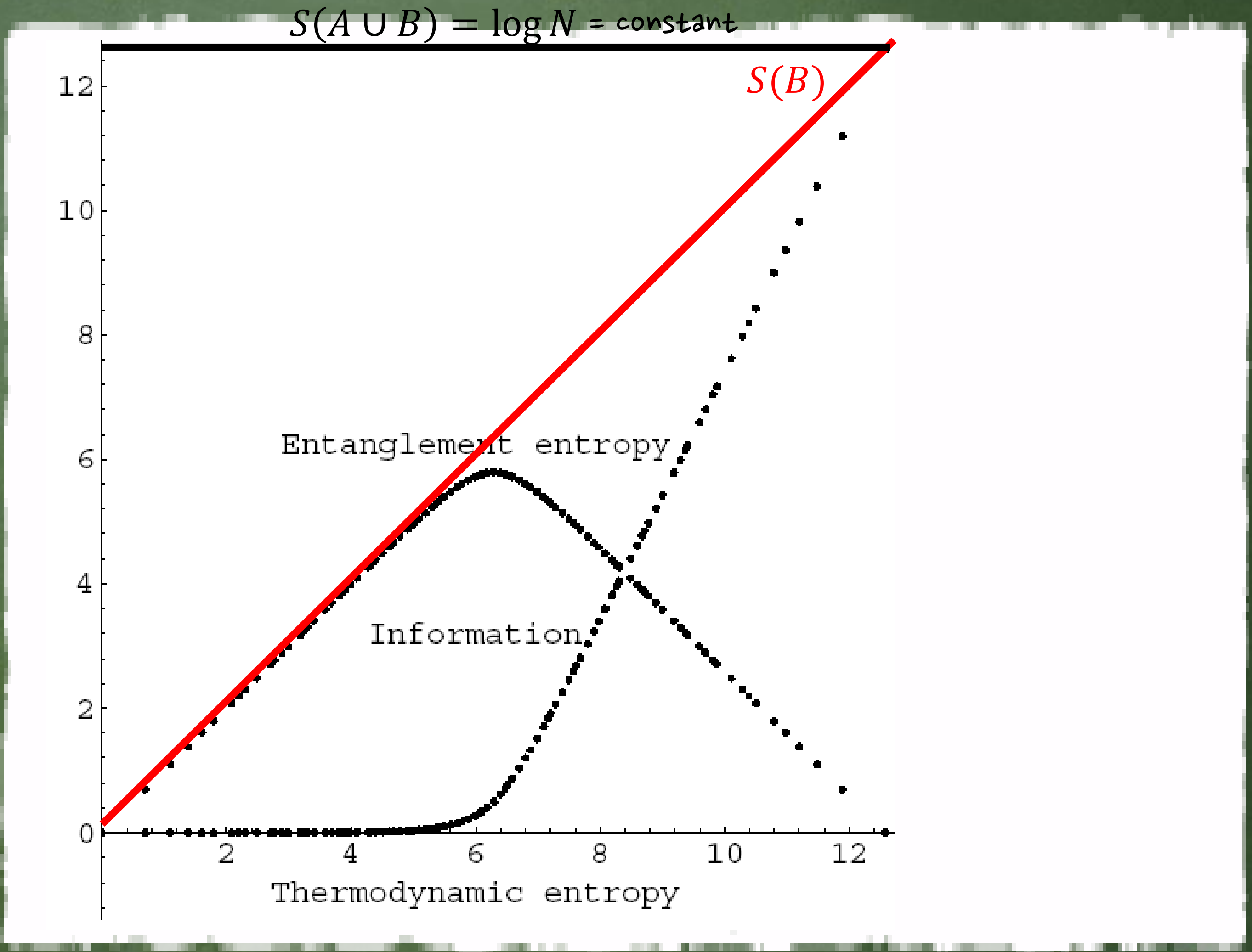
Estimation of entanglement entropy



Estimation of entanglement entropy



Estimation of entanglement entropy



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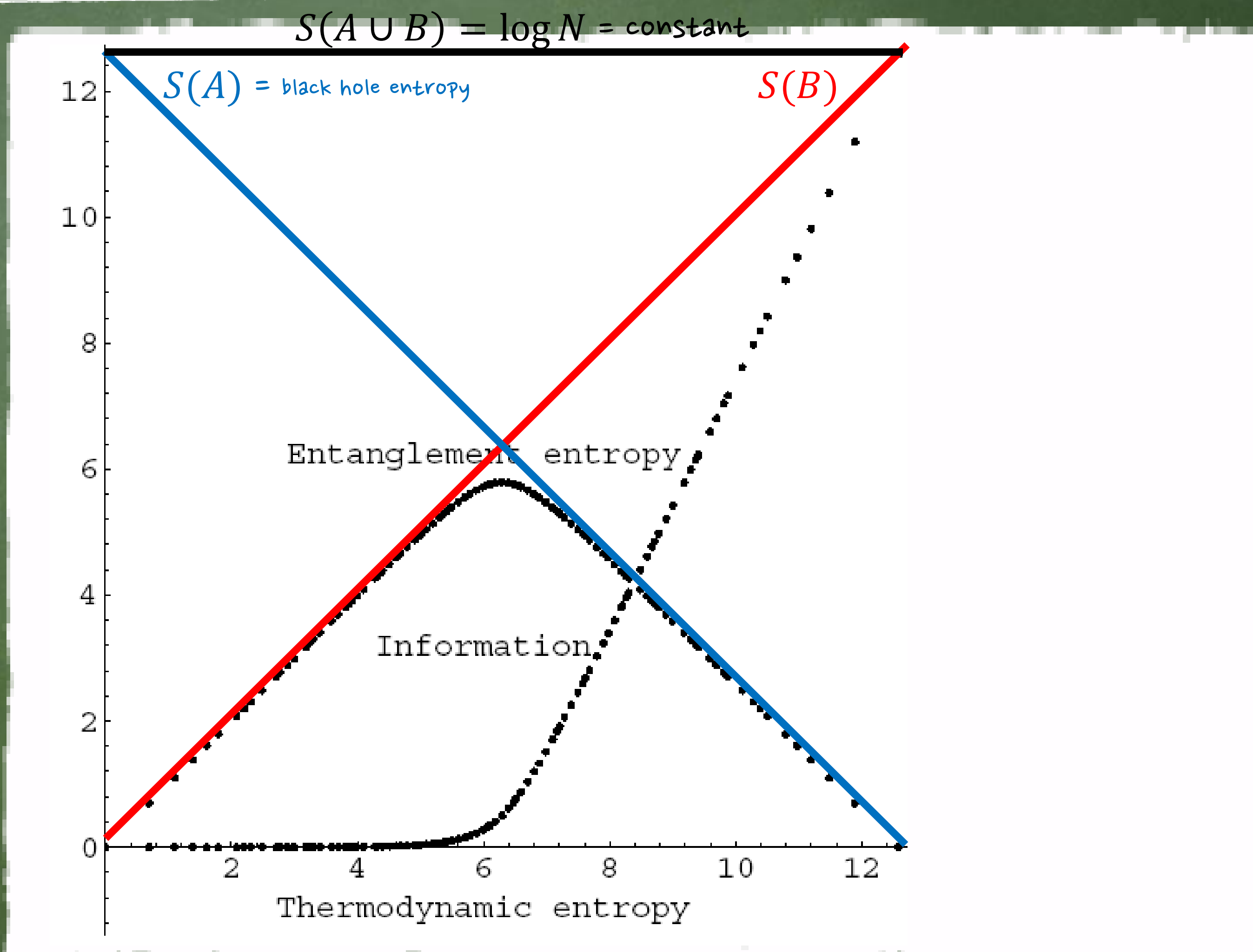

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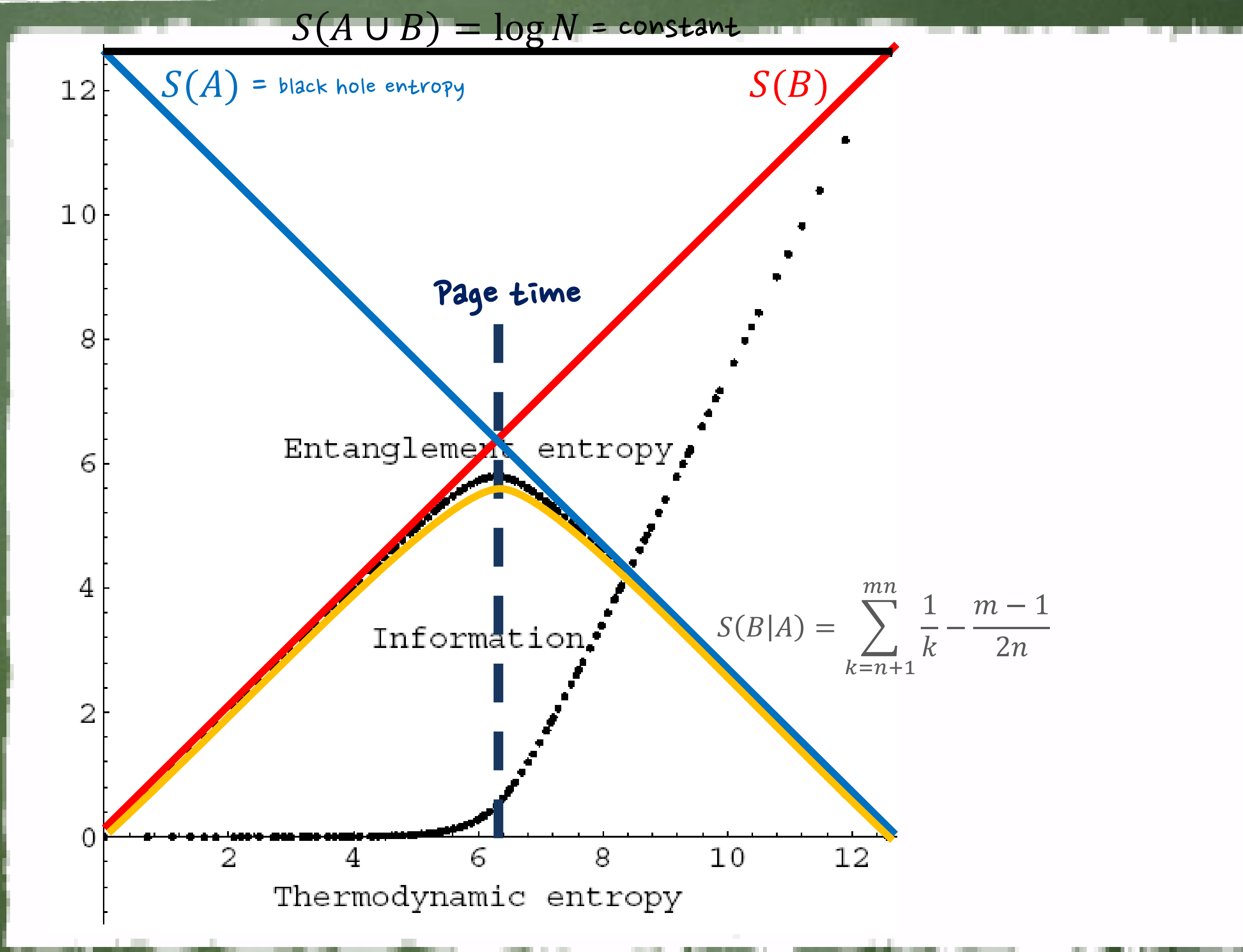
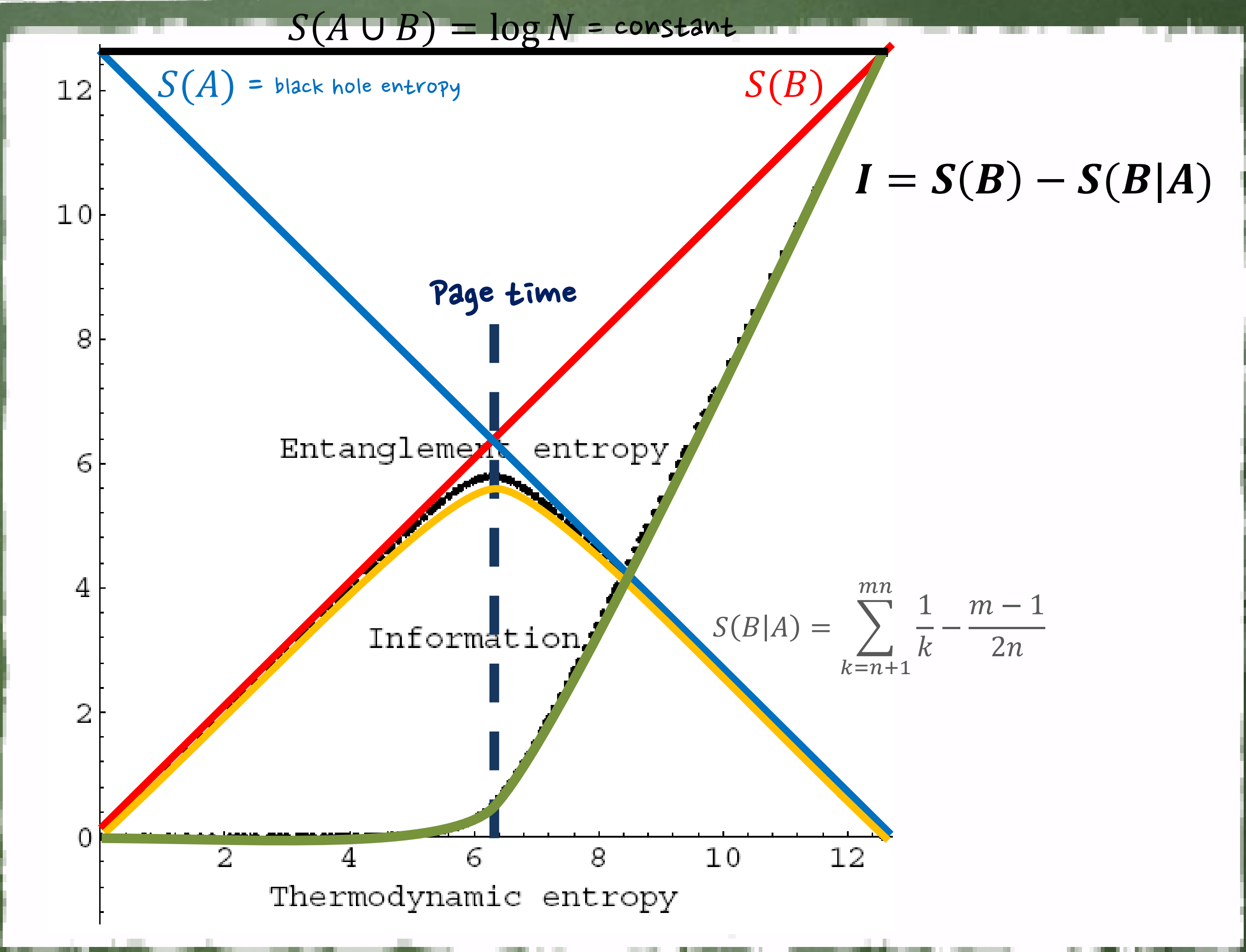
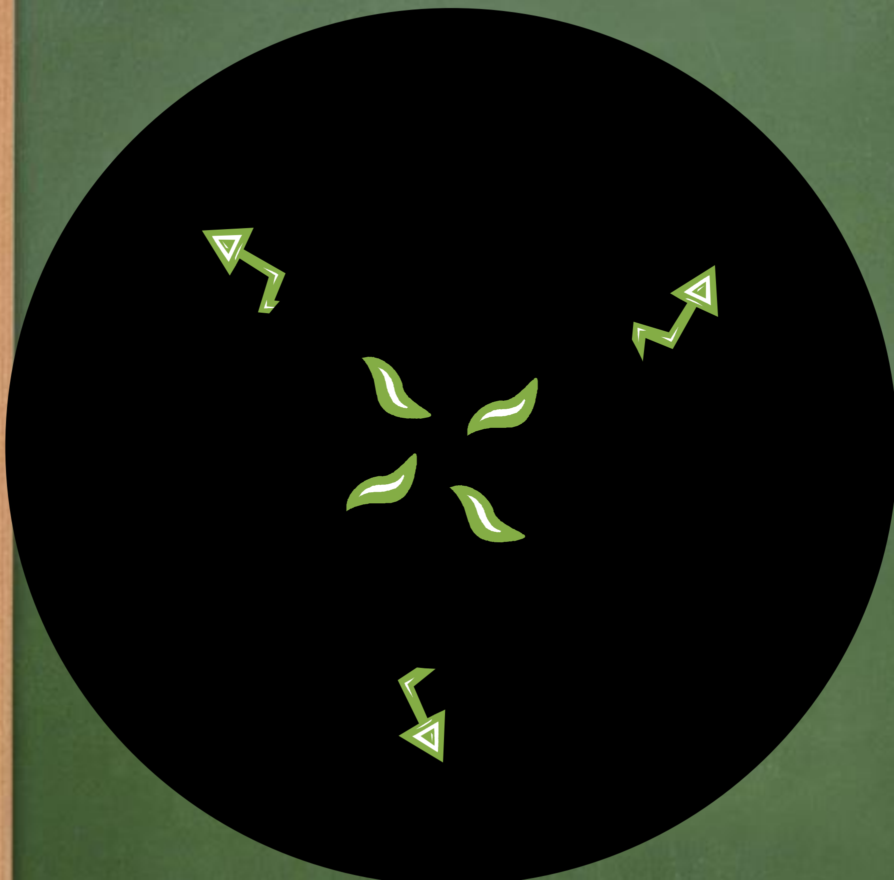
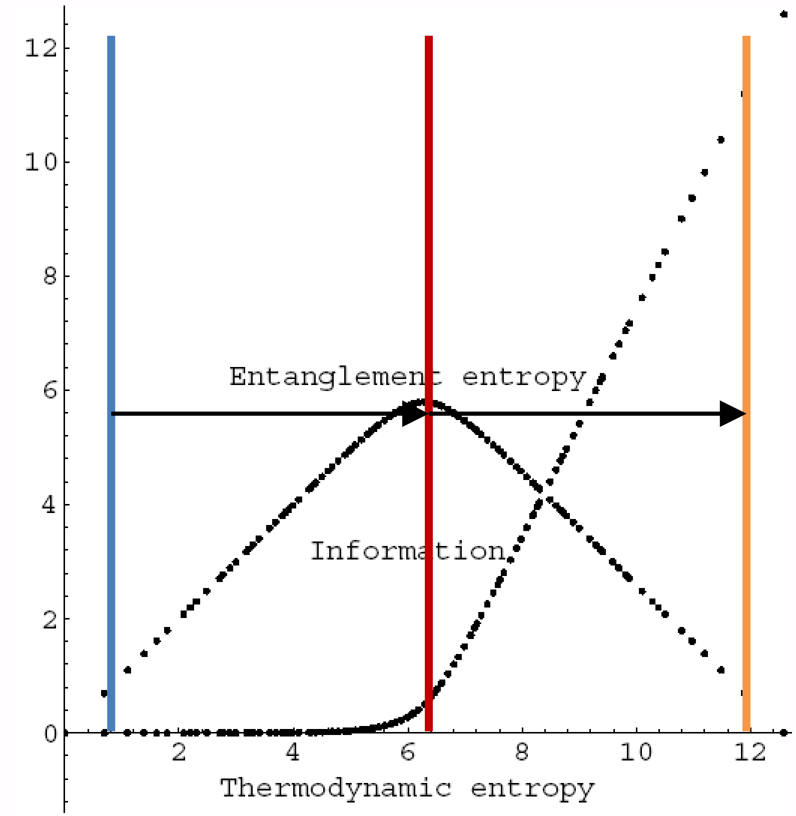


Estimation of entanglement entropy



Emission of information





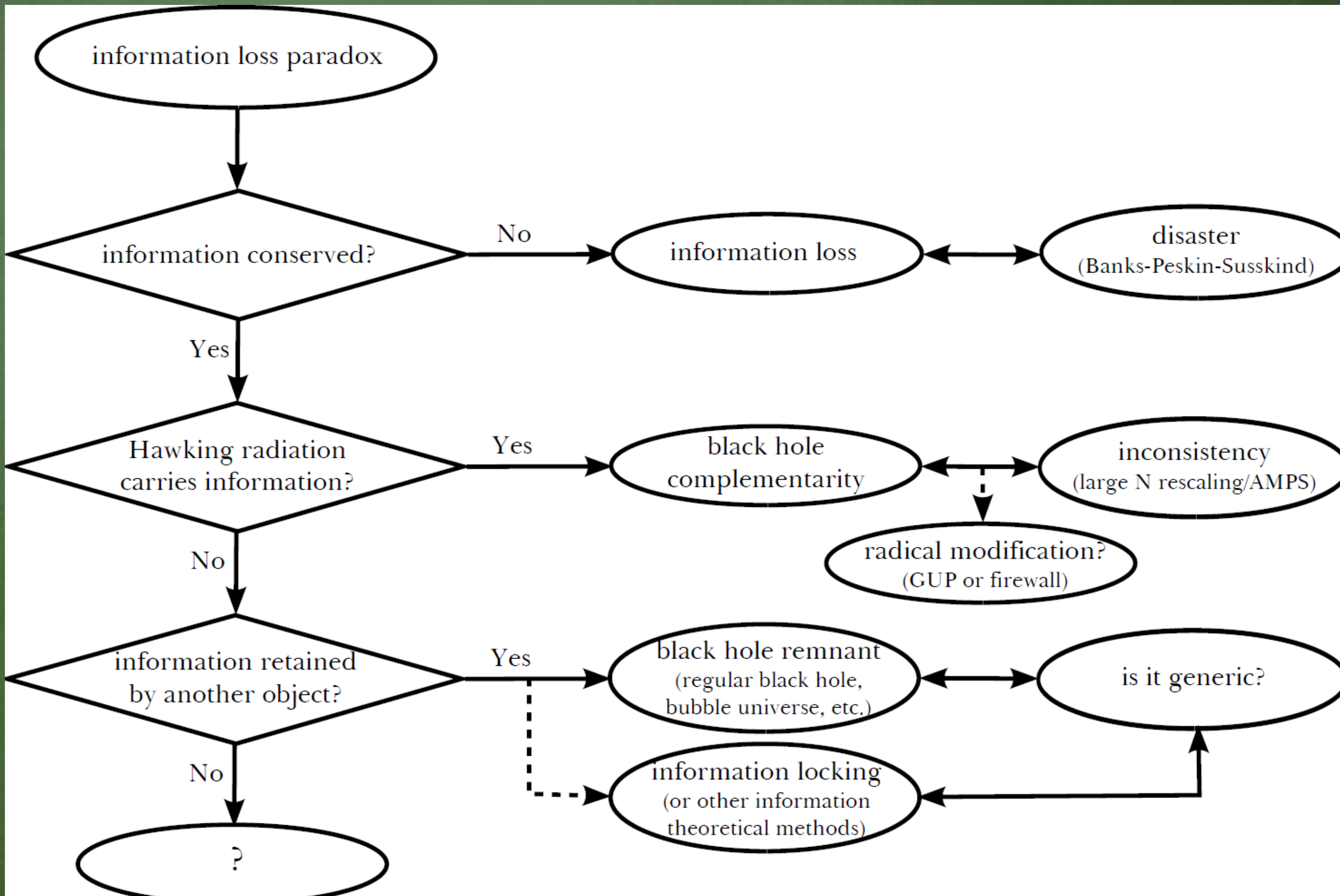
If $S(A) \propto$ Area, then Hawking radiation should contain information.

Information loss problem

A bird's-eye view

A bird's-eye view

chen, ong and DY, 2014



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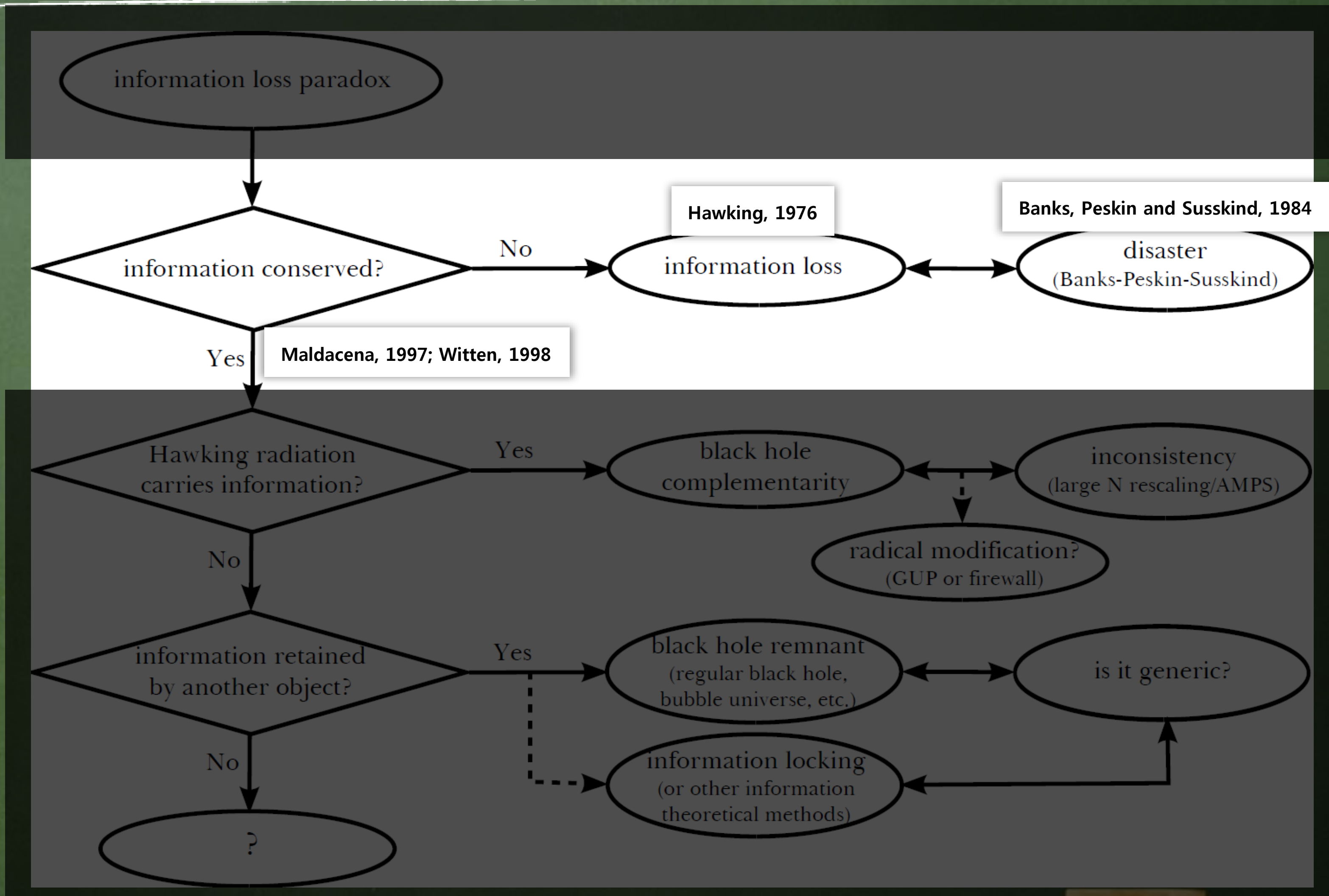
chen, Ong and DY, 2014



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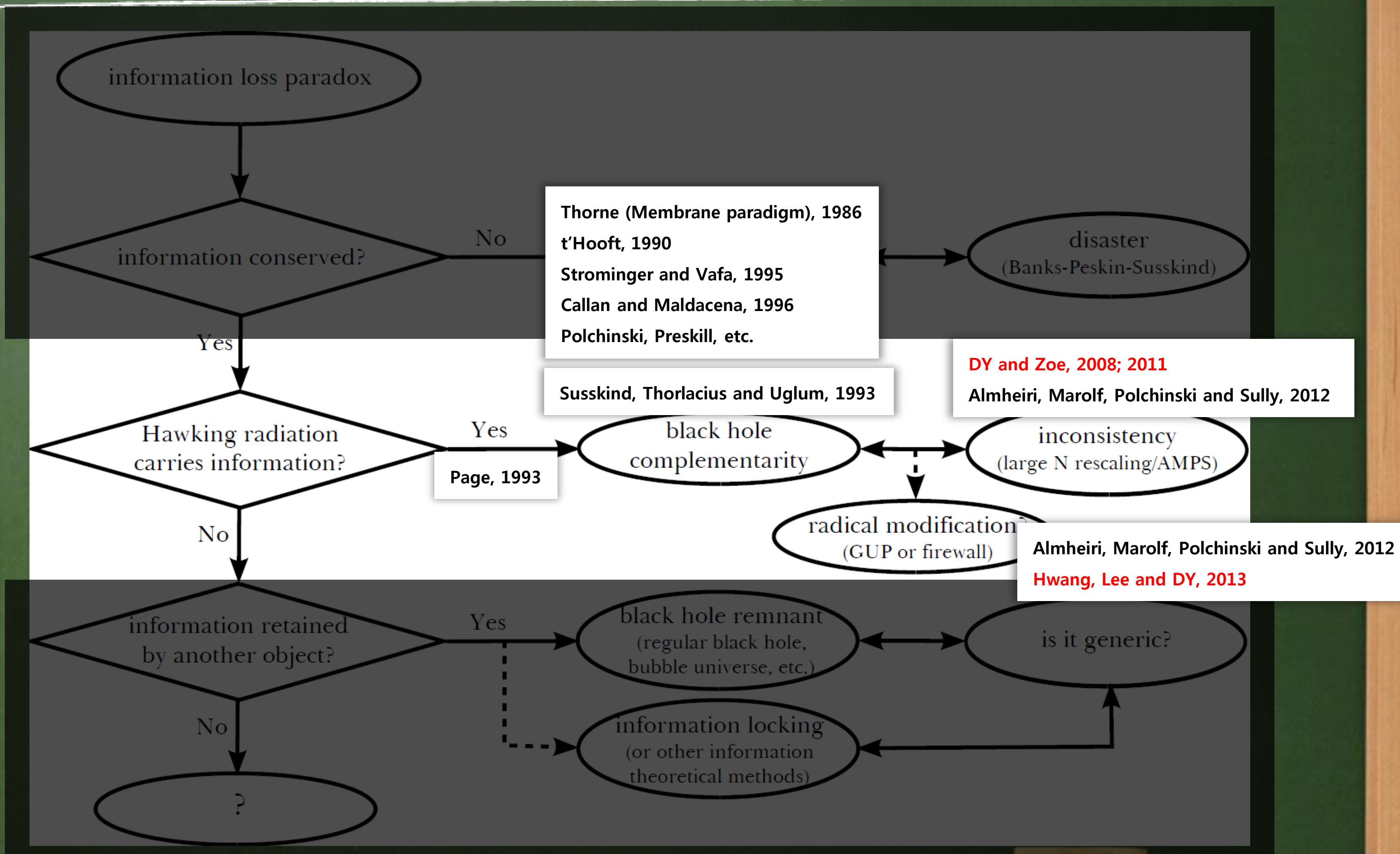

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