A Review of Higher Spin Field Theory

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Higher Spins

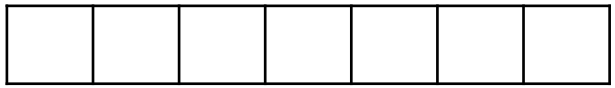
- Hadronic excitations
- Higher excitations in String Theory

Can a better field-theoretical understanding of higher spins give useful lessons to String (Field) Theory?

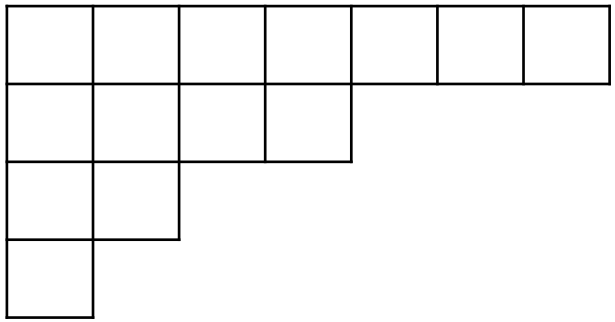
Massive Higher Spins

Around **flat d** dimensions, massive little group: SO(d-1)

In d=4, only **symmetric** YD rep

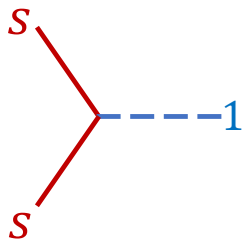


In d>4, various **mixed-symmetry** YD rep

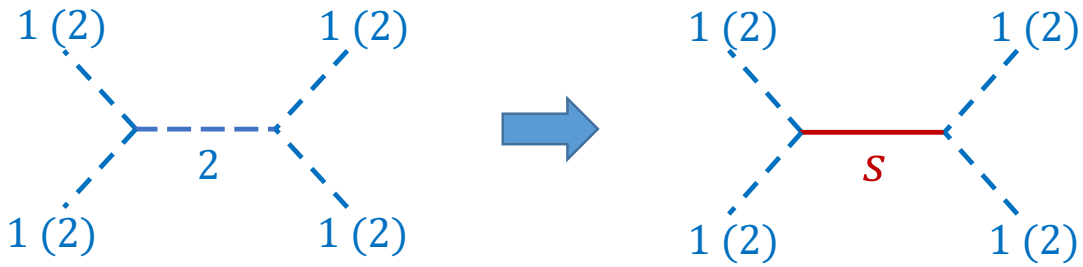


Let us focus first on **symmetric massive Higher Spin** fields

- ❖ Free Lagrangian by Singh Hagen in ’74
- ❖ Consistent Interactions?
  - Minimal EM interaction to charged massive HS → Causality Problem
    - ✓ Requires non-minimal $F_{\mu\nu}$ interaction
    - ✓ String theory reproduces also higher F^n interactions



  - Causality of $F_{\mu\nu}$ or $R_{\mu\nu\varrho\sigma}$ interactions requires infinitely many **massive HS**



★ **Lesson**: HS interactions ↔ Higher derivatives (dim.ful parameter)

Other consistency of **massive** interactions?

Even, consistency of DoF is not obvious

- Simplest example of "higher spin" → spin two
  - Massive spin-two interaction problem → Massive Gravity
  - Consistent massive gravity potential term is very restrictive,

    but it turned out to be natural ones from massless gravity viewpoint

- Massless theory may give a hint

Basic features and Novelties of
massless higher spin dynamics

Massless Higher Spins
and
Their Interactions

Massless (symmetric) Higher Spins

❖ Free Lagrangian by Fronsdal '78 (via massless limit of massive HS)

$$S_{\text{Fronsdal}} = \int d^d x \, \varphi^{\mu_1 \cdots \mu_s} \left(\Box + \cdots \right) \varphi_{\mu_1 \cdots \mu_s}$$

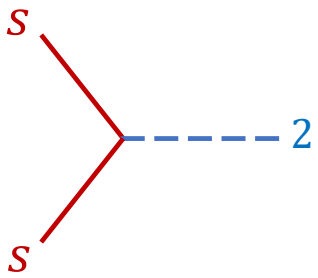
- Gauge Symmetry: $\delta \varphi_{\mu_1 \cdots \mu_s} = \partial_{(\mu_1} \varepsilon_{\mu_2 \cdots \mu_s)}$
- Subtlety of Trace Constraints
  - Equivalent formulation w/o trace constraints (inspired by SFT)

❖ Consistent **Interactions**?

- Various **problems** such as Weinberg '64 (No long range interaction of HS)
- **Gauge Invariance** (with a nonlinear deformation)

Gravitational minimal interaction of massless spin s

→ Fronsdal Lagrangian w/ covariant derivatives: $\mathcal{L}_{\text{Fronsdal}}(\varphi, \nabla\varphi)$



✓ Manifestly invariant under diffeomorphism

✓ Invariance under HS gauge transform?

- In principle, terms from $[\nabla_\mu, \nabla_\nu] \sim R_{\mu\nu\rho\sigma}$
- $s=1$: no such term → Spin 1 is 'matter' w.r.t Gravity
- $s=3/2$: term prop to $R_{\mu\nu}$ → Compensate by $\delta(\sqrt{g}\,R)$ with $\delta g_{\mu\nu}$

➡ Spin 3/2 and 2 in a SUSY multiplet

- $s>2$: term prop to $R_{\mu\nu\varrho\sigma}$ → Impossible to save HS gauge sym

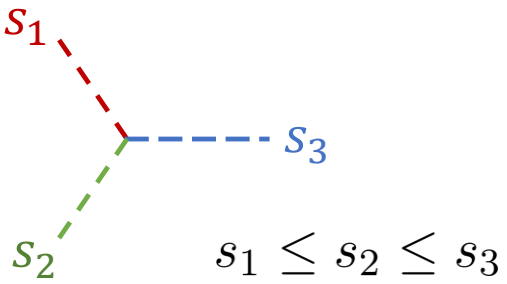
Incompatibility between **Diffeomorphism** and **HS gauge symmetry**

❖ Construction of gauge invariant interaction vertices

- Perturbative expansion

$$S = S_0 + S_1 + \cdots$$

$$\delta\varphi = \partial\varepsilon + T_1(\varphi, \varepsilon) + \cdots$$



- Gauge invariant cubic vertices

$$S_1 = \sum_{n=0}^{s_1} g_{s_1+s_2+s_3-2n}\, V_{s_1+s_2+s_3-2n}$$

of ∂

Diffeomorphism

2-2-2 $V_2 \sim R$ $\quad V_4 \sim R^{\mu\nu}{}_{\rho\sigma}\, R^{\rho\sigma}{}_{\mu\nu}$ $\quad V_6 \sim R^{\mu\nu}{}_{\rho\sigma}\, R^{\rho\sigma}{}_{\alpha\beta}\, R^{\alpha\beta}{}_{\mu\nu}$

0-0-2 $V_2 \sim (\nabla\phi)^2$

1-1-2 $V_2 \sim g^{\mu\rho}\, g^{\nu\sigma}\, F_{\mu\nu}\, F_{\rho\sigma}$ $\quad V_4 \sim R^{\mu\rho\nu\sigma}\, F_{\mu\nu}\, F_{\rho\sigma}$

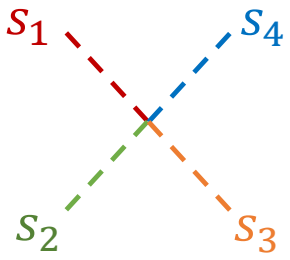
General covariance of spin 0 and 1

s-s-2 V_{2s-2} $\quad V_{2s}$ $\quad V_{2s+2}$

Loose general covariance

➡ Consistent cubic interactions are higher derivative types

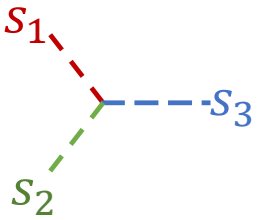
❖ Consistent quartic vertices: seemingly, nothing local



- One way to see this → Global Sym
  - ✓ Generator fixed by Killing eq, $\partial\varepsilon = 0$

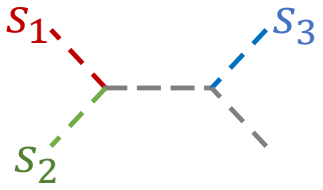
$$K_s \sim \underbrace{\square\!\square\!\square}_{s-1} \quad \underbrace{\begin{array}{l}\square\!\square\!\square\\ \square\end{array}}_{s-1} \quad \underbrace{\begin{array}{ll}\square\!\square\!\square & \\ \square & \end{array}}_{s-1} \quad \cdots \quad \underbrace{\begin{array}{ll}\square\!\square\!\square & \\ \square & \square\end{array}}_{s-1}$$

  - ✓ Bracket fixed by cubic vertices



$$[K_{s_1}, K_{s_2}] = (\cdots)\, K_{s_3}$$

  - ✓ Quartic consistency → Jacobi idenitity



$$[[K_{[s_1}, K_{s_2}], K_{s_3}]] = 0$$

Impossible → No massless HS interactions in **FLAT SPACE**

Remind

★ **Lesson**: HS interactions ↔ Higher derivatives (dim.ful parameter)

➡ Massive HS and/or Massless HS in **(A)dS**

Mass and Cosmological Const. play somewhat similar role

❖ Gauge invariant cubic vertices in (A)dS

$$S_1 = \sum_{n=0}^{s_1} g_{s_1+s_2+s_3-2n}\, V_{s_1+s_2+s_3-2n}$$

Max # of ∇

- Very analogous to flat space case
- Important **difference**:

General covariance of spin s !!

s -s -2 $\quad V_{2s-2} \quad V_{2s} \quad V_{2s+2}$

$$\sim (\nabla\varphi_s)^2 + \frac{W}{\Lambda}\left(\varphi_s\,\nabla^2\,\varphi_s\right) + \cdots + \frac{W}{\Lambda^{s-2}}\left(\varphi_s\nabla^{2s-4}\varphi_s\right)$$

➡ Fields of diff spins in a multiplet of HS sym

- Higher spins can be charged under lower spins (e.g. colored gravity)

Towards a Full Nonlinear Theory
of Massless Higher Spins

Higher Spin Gravity

Higher Spin Algebra

❖ What is it? : Lie algebra generated by

$$\bigcup_s \left\{ \begin{array}{|c|c|c|}\hline & s-1 & \\ \hline \end{array} \quad \begin{array}{|c|c|c|}\hline & s-1 & \\ \hline \multicolumn{1}{|c}{} & \multicolumn{1}{|c|}{} & \multicolumn{1}{c}{} \\ \cline{2-2} \end{array} \quad \begin{array}{|c|c|c|}\hline & s-1 & \\ \hline \multicolumn{1}{|c}{} & \multicolumn{1}{|c|}{} & \multicolumn{1}{c}{} \\ \cline{2-3} \end{array} \quad \cdots \quad \begin{array}{|c|c|c|}\hline & s-1 & \\ \hline \multicolumn{1}{|c}{} & \multicolumn{2}{|c|}{} \\ \cline{2-3} \end{array} \right\}$$

❖ Does this sym exists? If not, the quartic consistency would fail even in (A)dS

❖ **Vasiliev's HS Algebra** '87

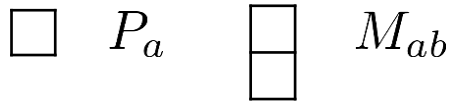
- Various Equivalent Definitions
  - ➢ Star product algebra in a certain oscillator space
  - ➢ Maximal quotient of UEA of so(2,d) (relation to min orbit)
  - ➢ Maximal symmetry of free **conformal scalar** in d-1 dim!
- Contains all even (and odd) spin generators
- Flato-Fronsdal

$$\mathrm{Rac} \otimes_{(\mathrm{sym})} \mathrm{Rac} = \bigoplus_{\text{even } s,(\text{odd } s)} D(s+d-2,s)$$

massless spin s rep

1st order formulation of Higher Spins

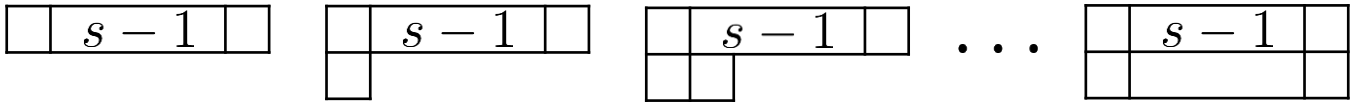
❖ 1st order formulation of **Gravity**



1-form connection e^a ω^{ab}

2-form curvature T^a =0 → fix R^{ab}

❖ 1st order formulation of **massless spin s**



1-form $A^{a_1\cdots a_{s-1}}$ $A^{a_1\cdots a_{s-2}b}$ $A^{a_1\cdots a_{s-3}b_1b_2}$ $A^{a_1\cdots a_{s-k}b_1\cdots b_{k-1}}$

=0 → fix =0 → fix =0 → ⋯ → fix

2-form $F^{a_1\cdots a_{s-1},b_1\cdots b_{s-1}}$

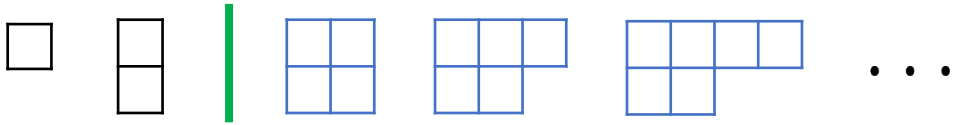
- This step also gives **EoM** → difficult to disentangle **EoM** and **Constraints**
- Fradkin Vasiliev construction $S = \int \sum_{r=0}^{s-1} \frac{a_r}{\Lambda^r} (F^{(s-1,r)})^2$

✓ Use of **nonlinear** curvature → Gauge invariant cubic interactions

Unfolded Equations

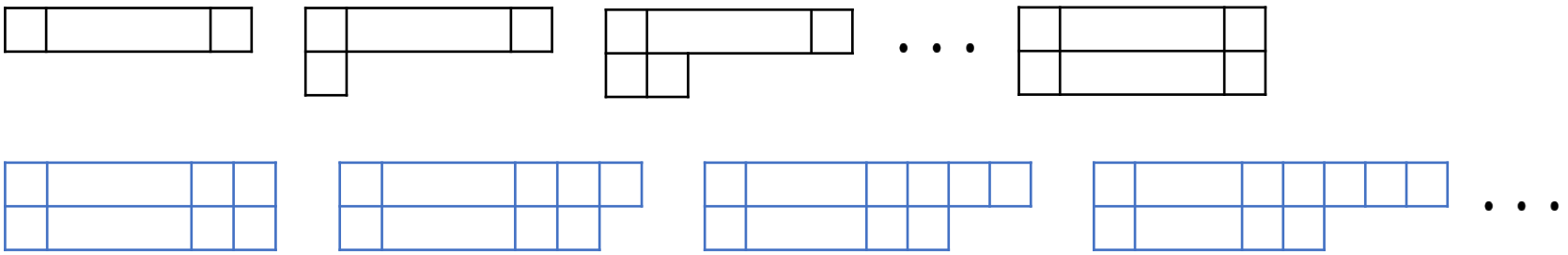
- Motivation: no privilege to metric
- Universal treatment of **EoM** & **Constraints**
- Gravity ex.

Fields: e^a	ω^{ab}	$C^{ab,cd}$	$\cdots$
Eqns: $De^a = 0$	$D\omega^{ab} = e_c\, e_d\, C^{ac,bd}$	$D\, C^{ab,cd} = e_e\, C^{abe,cd} + \cdots$	

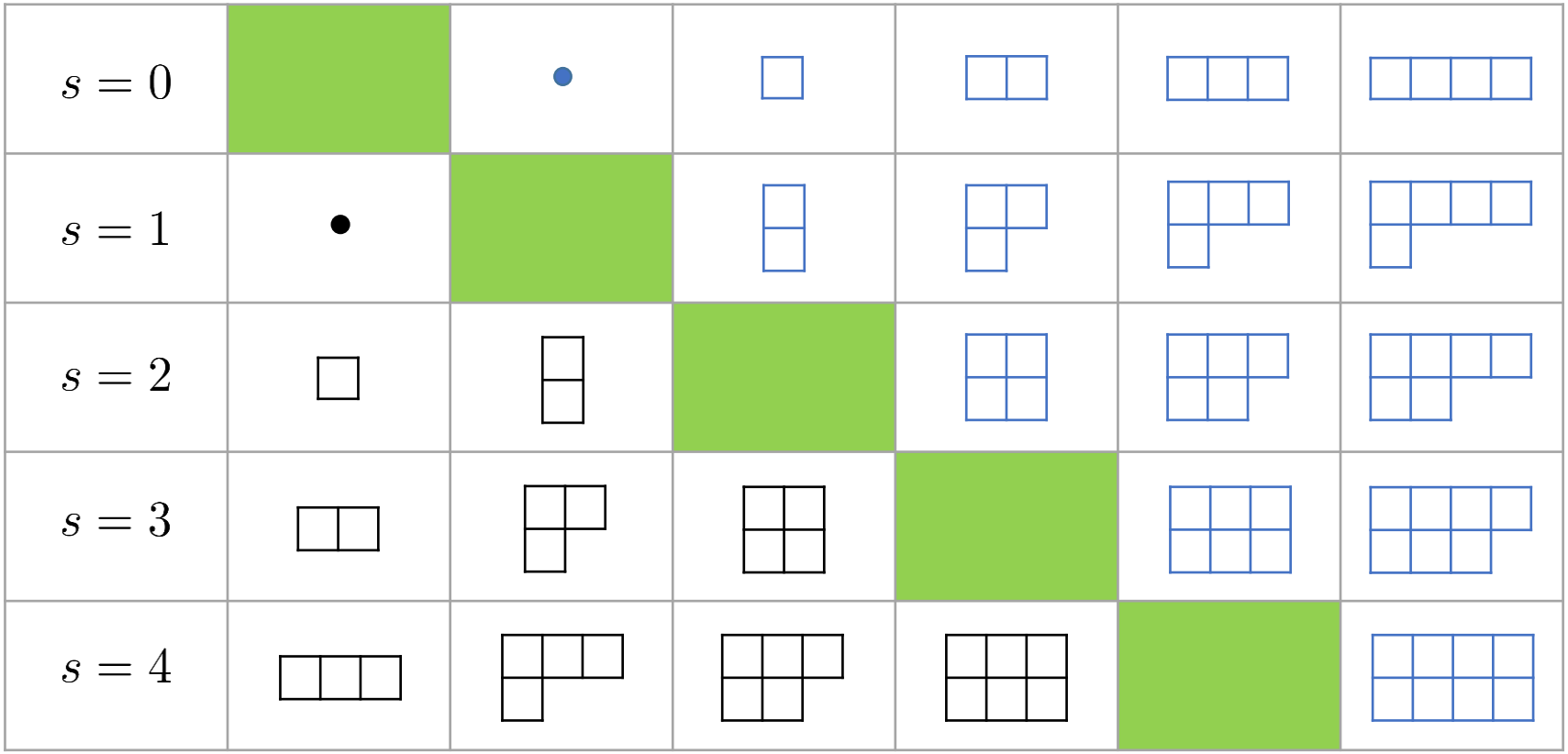


1-form **0-form**

- Spin s



0-form field C_I **Twisted Adj** rep generated by K^I



1-form field A^I **Adjoint** rep of HS algebra generated by K_I

❖ Ansatz for nonlinear HS equations

$$d\,A^I + f^I_{JK}(C)\,A^J\,A^K = 0$$
$$d\,C_I + g^J_{IK}(C)\,C_J\,A^K = 0$$

- Frobenius condition → impose conditions on $f^I_{JK}(C)$ and $g^J_{IK}(C)$
  - ➢ Free Differential Algebra (FDA)
  - ➢ Infinite dimensional Lie Algebroid
- Vasiliev identified on $f^I_{JK}(C)$ and $g^J_{IK}(C)$ up to $O(C^3)$ [‘88, ‘89]

❖ “**Vasiliev’s Equations**” [‘90]

- Similar eqns which **GENERATE** $f^I_{JK}(C)$ and $g^J_{IK}(C)$
- Key ideas: extend (“double”) the fiber, s.t “f^I_{JK} and g^J_{IK}” become cnst
  but, fields are subjects to algebraic constraints

Vasiliev's Equation in 4d

- HS algebra realized by oscillators $Y_A Y_B$, $Y_A Y_B Y_C Y_D$, …

$$A^I \quad \to \quad A(Y) \qquad\qquad C_I \quad \to \quad C(Y)$$

- Doubling of oscillator space: $A(Y,Z)\,, \quad C(Y,Z)\,, \quad S_A(Y,Z)$

new fields

- The Equations

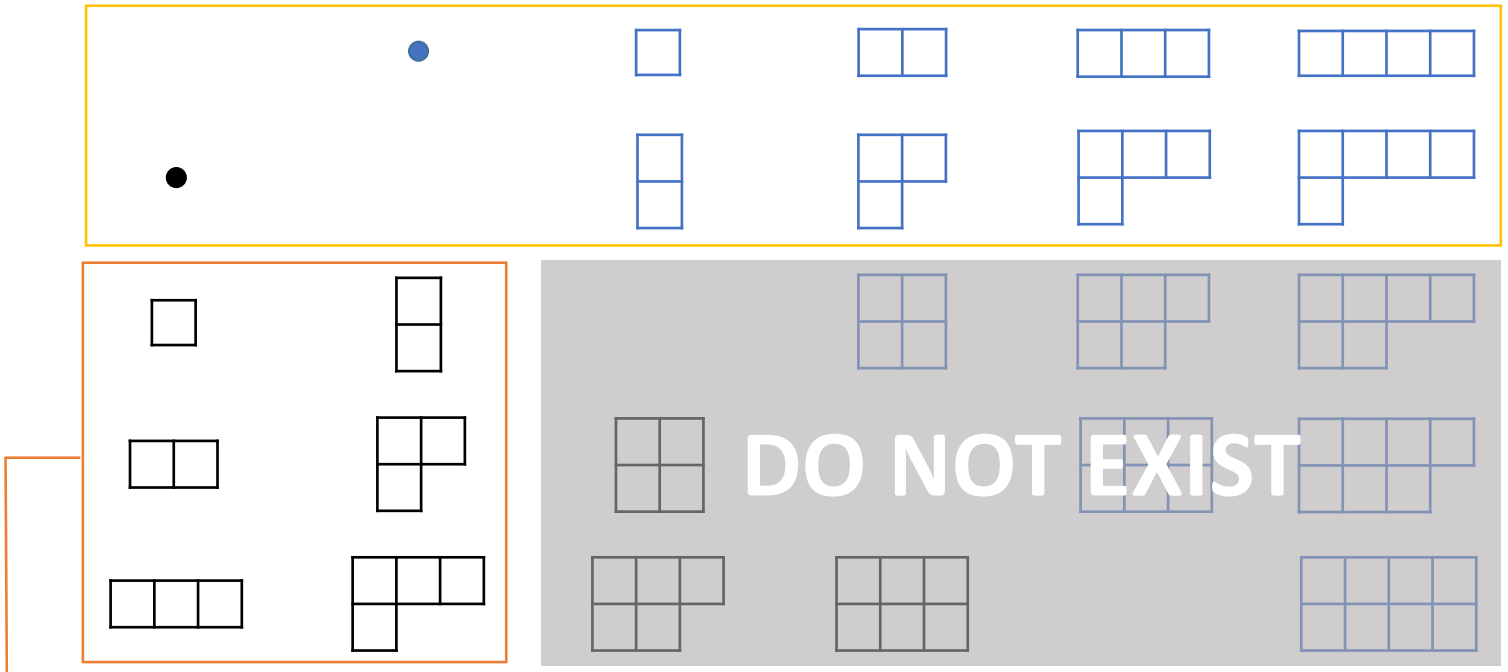
$$d\,A + A \star A = 0 \qquad d\,C + [A\,\stackrel{\star}{;}\,C] = 0 \qquad d\,S_\alpha + [A\,\stackrel{\star}{;}\,S_\alpha] = 0$$

Algebraic constraints

$$[C\,\stackrel{\star}{;}\,S_\alpha] = 0 \qquad [S_\alpha\,\stackrel{\star}{;}\,S_\beta] = \epsilon_{\alpha\beta}(1 + e_\star^{i\,\Theta_\star(C)} \star C)$$

- Interaction Ambiguity $\Theta_\star(C) = \theta_0 + \theta_2\, C^{\star 2} + \cdots$
  - Parity invariance → $\theta_0 = 0$ or π , $\theta_{n>0} = 0$
- A few exact solutions
- No action principle yet

Higher Spin Theories in 3d



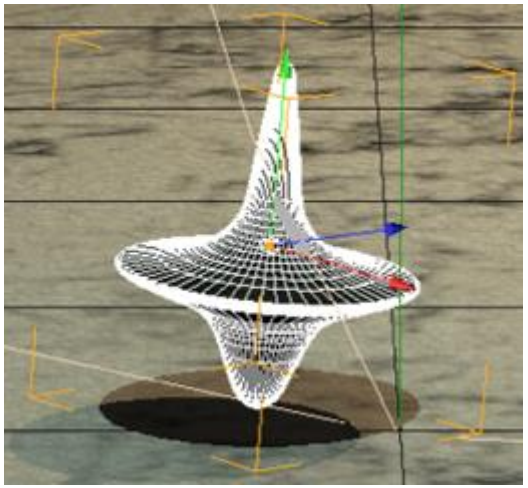
two scalars

- $sl(n,R) \oplus sl(n,R)$ CS → Theory of massless spin 2,3,...,n
- 3d Vasiliev Eq → Theory of massless spin 2,3,...,∞ and **two scalars**

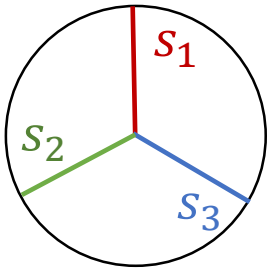
Can be viewed as CS gauge sector coupled to matter sector

AdS/CFT conjectures
Involving higher spin gravity

Higher Spin Holography



AdS_{d+1}	CFT_d
Field contents	Single trace operators
• Fields of mass M & spin s	• Operators of dimension Δ & spin s
• Massless spin s fields	• Spin s conserved current operators
Cubic Interactions	3pt functions



Classifications match

$$\langle \mathcal{O}_{s_1}(x_1)\,\mathcal{O}_{s_2}(x_2)\,\mathcal{O}_{s_3}(x_3)\rangle$$

❖ Holography for HS gravity in AdS_{d+1} with $d \geq 3$

- HS sym: max sym of conf scalar (CFT_3 with HS sym → only free scalar/spinor)
- Flato-Fronsdal: operators bilinear in ϕ → Conserved currents of any spins

Vector Model

❖ $AdS_{d+1}/CFT_d \quad (d \geq 3)$

- U(N)/O(N) Scalar Vector Model ⇔ (Non)-minimal Vasiliev Theory

❖ AdS_4/CFT_3

- Spinor Vector Model ⇔ Vasiliev Theory with $\theta_0 = \pi$
  - ✓ Test for a large class of 3pt functions
- Critical Models ⇔ AdS scalar with different BC
- Parity violating Vasiliev Theory with θ_0 ⇔ CS coupling to 3d CFT
  - ✓ Test for a few 3pt fns, but not conclusive
- Open question: Vasiliev theory with other θ_n ?

❖ AdS_3/CFT_2

- Background of 3d Vasiliev theory, parametrized by λ
- HS sym: $hs(\lambda) = UEA(sl_2)/C_2(\lambda)$
- Asymptotic symmetry: $W_\infty(\lambda)$
  - Nonlinear sym (not a Lie algebra)
  - Does not contain $hs(\lambda)$
- Duality: Vasiliev theory ⇔ W_N minimal model CFT

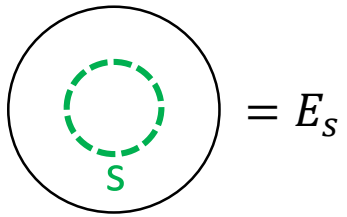
  ✓ Test for spectrum and a few 3pt functions
- BH-like solutions in HS CS theory

1 Loop Test for HS AdS/free CFT

❖ Dictionary: 1 Loop in AdS ⇔ 1/N in CFT

Free CFT	HS Gravity in AdS
No 1/N Correction	No Loop Correction?

❖ Test for **Vacuum Energy**



- Total vacuum energy: sum of ∞ contributions
  - For non-minimal model: $\sum_{all\ s} E_s = 0$
  - For minimal model: $\sum_{even\ s} E_s = E_{bd\ scalar}$

    ➡ Shift of HS Coupling Const. N→N-1
- 5d HS Gravity dual to free spin 1 in 4d : N→N-2

Comments on
Other Topics / Recent Progress

Other Topics

- Other Formulation
  - BRST related
  - Tensorial space
  - Other metric-like form.
  - World-Line formalism
- Other Spectra
  - Mixed Sym HS
  - Partially Massless and Massive HS
  - Conformal HS
  - Non-Relativistic HS
- Extensions of Vasiliev's Eq
  - SUSY
  - Color Decoration
  - Higher Form
- Relation to String Theory
  - Tensionless Limit
  - WS proposal

Recent Progress

- Better understanding of Vasiliev's Equation
  - Explicit derivation of cubic vertices
  - Holographic identification of one nonlocal quartic vertex

- Generalizations
  - Extension of HS Algebras to Multi-Ptcl. & Partially Massless ones
  - Various Properties of Conformal HS
  - Holography for Stringy Extensions
  - Rainbow Vacua of Colored (HS) Gravity

Thank you for your attention