Duality and string/M-theory

Work in progress with Jeong-Hyuck Park and Yoonji Suh

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Introduction

Duality manifest formulations of M-theory

U-gravity

Outlook

String/M-theory and duality

• There are five kinds of string theories and M-theory.

Type I string theory $\rightarrow$ Type I supergravity
Type IIA string theory $\rightarrow$ Type IIA supergravity
Type IIB string theory $\rightarrow$ Type IIB supergravity
SO(32) heterotic string theory $\rightarrow$ SO(32) heterotic supergravity
$E_8 \times E_8$ heterotic string theory $\rightarrow$ $E_8 \times E_8$ heterotic supergravity

M-theory $\rightarrow$ Eleven-dimensional supergravity

• String theories and M-theory are related by dualites.

S-duality: $g_s \leftrightarrow 1/g_s$, T-duality: $R \leftrightarrow 1/R$, U-duality

• In string theory, because the fundamental objects are strings, not point particles, in addition to the conventional momentum modes, there are also winding modes. T-duality works by exchanging momentum modes with winding modes.

• While dualities are genuine stringy effect in this sense, they cannot be seen as a conventional Noether symmetry of action or equations of motion.

• For string theory, by doubling the spacetime dimensions, double field theory could realize T-duality as O(10,10) global symmetry.

Question

Can we have duality manifest formulation of M-theory and capture genuine stringy physics?

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U-duality

[Cremmer, Julia, 1979]

• Eleven-dimensional supergravity on T^d reduces to (11-d)-dimensional supergravity with global symmetry. The global symmetry is called *U-duality*.

T^d	(11-d)-dim supergravity	Global symmetry
T^8	$3-dim$	$E_{8(8)}$
T^7	$4-dim$	$E_{7(7)}$
T^6	$5-dim$	$E_{6(6)}$
T^5	$6-dim$	$SO(5,5)$
T^4	$7-dim$	$SL(5)$

Question

• Can we formulate M-theory to manifest U-duality?

U-duality manifest formulations of M-theory

• Tensor hierarchy and generalized vielbein postulate
[de Wit, Nicolai, 1986, 2013], [Godazgar, Godazgar, Nicolai, 2013, 2014]

• Duality invariant M-theory and SL(N) U-gravity
[Berman, Perry, 2010], [Berman, Godazgar, Perry, 2011], [Berman, Godazgar, Godazgar, Perry, 2011], [Berman, Godazgar, Perry, West, 2011], [Jeong-Hyuck Park, Yoonji Suh, 2013, 2014]

• Generalized geometry via Hitchin and Gualtieri
[Coimbra, Stickland-Constable, Waldram, 2011, 2012]

• Exceptional field theory
[Hohm, Samtleben, 2013, 2014, 2015], [Godazgar, Godazgar, Hohm, Nicolai, Samtleben, 2014], [Musaev, Samtleben, 2014], [Abzalov, Bakhmatov, Musaev, 2015]

SL(5) duality invariant theory

[Berman, Perry, 2010], [Berman, Godarzgar, Perry, West, 2011]

- We define coordinates,

$$x^{ab} = -x^{ba},$$

where $a, b = 1, \ldots, 5$ are in the fundamental representation of SL(5). Hence, it is formally $\frac{5\times4}{2} = 10$-dimensional theory. We also have

$$\partial_{ab} = -\partial_{ba}.$$

- The section condition is

$$\partial_{[ab}\partial_{cd]} = 0.$$

A solution of this section condition is

$$\partial_{\mu\nu} = 0, \qquad \partial_{\mu 5} \neq 0,$$

where $\mu, \nu, = 1, \ldots 4$. Hence, it reduces to four-dimensional theory.

• They construct action which is invariant under the SL(5) global symmetry manifestly and the generalized diffeomorphism implicitly,

$$\mathcal{L} = \frac{1}{12} M^{MN} \partial_M M^{KL} \partial_N M_{KL} - \frac{1}{2} M^{MN} \partial_M M^{KL} \partial_K M_{NL} - \frac{1}{4} \partial_M M^{MQ} (M^{RS} \partial_P M_{RS}) + \frac{1}{12} M^{MN} (M^{RS} \partial_M M_{RS})(M^{KL} \partial_N M_{KL}) .$$

They parametrize the generalized metric which contains the metric and three-form gauge field of eleven-dimensional supergravity,

$$M_{MN} = M_{ab,cd} = \begin{pmatrix} M_{\mu 5, \nu 5} & M_{\mu 5, \alpha\beta} \\ M_{\gamma\delta, \nu 5} & M_{\gamma\delta, \alpha\beta} \end{pmatrix} = \begin{pmatrix} g_{\mu\nu} + \frac{1}{2} C_\mu{}^{\rho\sigma} C_{\nu\rho\sigma} & -\frac{1}{2\sqrt{2}} C_\mu{}^{\rho\sigma} \varepsilon_{\rho\sigma\alpha\beta} \\ -\frac{1}{2\sqrt{2}} C_\nu{}^{\rho\sigma} \varepsilon_{\rho\sigma\gamma\delta} & g^{-1} g_{\gamma\delta, \alpha\beta} \end{pmatrix},$$

where $g_{\alpha\beta,\mu\nu} = \frac{1}{2}(g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu})$.

• After applying the section condition, the action reduces to four-dimensional reduction of M-theory, where $F = dC$,

$$S = \int d^4 x \sqrt{g} \left(R - \frac{1}{48} F^2 \right) .$$

• "We ignore the dependence of all fields on directions orthogonal to the four-manifold. This is opposite to kaluza-Klein reduction."

Unusually, U-duality is manifest in the internal space, not in the external spacetime.

$d = 4$, SL(5): [Berman, Perry, 2010]
$d = 5$, SO(5,5): [Berman, Godazgar, Perry, 2011]
$d = 6$, $E_{6(6)}$; $d = 7$, $E_{7(7)}$: [Berman, Godarzgar, Perry, West, 2011]

T^d	(11-d)-dim supergravity	Global symmetry
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T^4	$7-dim$	$SL(5)$

Scherk-Schwarz reduction and gauged supergravity

[Berman, Musaev, Thompson, 2012], [Musaev, 2013]

- Introduce a new generalized metric as

$$M_{MN} = M_{mn,pq} = m_{mp}m_{nq} - m_{mq}m_{np}\,,$$

and, hence,

$$m_{mn} = \begin{pmatrix} g^{-1/2}g_{\mu\nu} & V_{\nu} \\ V_{\mu} & g^{1/2}(1 + V_{\mu}V_{\nu}g^{\mu\nu}) \end{pmatrix},$$

where $V^{\mu} = \frac{1}{6}\varepsilon^{\mu\nu\rho\sigma}C_{\nu\rho\sigma}$. Then, we rewrite everything in terms of m_{mn}.

• Perform the Scherk-Schwarz reduction by

$$m_{mn} = V_m^{\bar{a}} m_{\bar{a}\bar{b}} V_n^{\bar{d}} ,$$

where V are twist matrices.

Then, surprisingly, from the action, we obtain the scalar potential of gauged supergravity in seven dimensions formulated by embedding tensor, [Samtleben, Weidner, 2005]

$$V = \frac{1}{64}\left(2m^{\bar{a}\bar{b}} Y_{\bar{b}\bar{c}} m^{\bar{c}\bar{d}} Y_{\bar{d}\bar{a}} - \left(m^{\bar{a}\bar{b}} Y_{\bar{a}\bar{b}}\right)^2\right)$$
$$+ Z^{\bar{a}\bar{b},\bar{c}} Z^{\bar{d}\bar{e},\bar{f}} \left(m_{\bar{a}\bar{d}} m_{\bar{b}\bar{e}} m_{\bar{c}\bar{f}} - m_{\bar{a}\bar{d}} m_{\bar{b}\bar{c}} m_{\bar{e}\bar{f}}\right) ,$$

where Y and Z are in terms of m_{mn} and $V_m^{\bar{a}}$, and they corresponds to objects from embedding tensor formulation.

• However, we cannot obtain the kinetic terms of gauged supergravity, as we do not have the external spacetime.

• The Scherk-Schwarz reduction was done for SL(5), SO(5,5), $E_{6(6)}$ theories, and obtained the scalar potentials of gauged supergravity.

Differential geometry of U-duality invariant theories

[Jeong-Hyuck Park, Yoonji Suh, 2013, 2014]

• The U-duality invariant theories of Berman, Perry and collaborators lack the geometric structure, i.e., covariant derivative, connection, Ricci tensor, as double field theory of Hohm, Hull and Zwieback did.

For instance, action is explicitly written by the generalized metric itself, and all derivatives are simple partial derivatives.

• Analogous to the semi-covariant formulation of double field theory, differential geometry for SL(5)-invariant theory was constructed, and generalized to SL(N) theories. They are called U-gravity.
[Jeong-Hyuck Park, Yoonji Suh, 2013, 2014]

Summary so far

• Duality invariant theories have benefits.

First, they manifest the U-duality of M-theory.

Second, they reproduce the scalar potentials of gauged supergravity.

• However, duality invariant theories have two problems.

First, they lack the geometric structure.

Second, they do not have dependence on external spacetime.

• In the rest of this talk, we generalize them to SL(11) U-gravity and see what comes out.

Also SL(11) is a subgroup of E_{11}, which is believed to be the ultimate U-duality group of M-theory. SL(11) may give us some hints of it.

Introduction

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Outlook

SL(11) U-gravity

[Jeong-Hyuck Park, Yoonji Suh, 2014]

- We define coordinates,

$$x^{ab} = -x^{ba}\,,$$

where $a, b = 0, \ldots, 10$. Hence, it is $\frac{11\times 10}{2} = 55$-dimensional theory. We also have

$$\partial_{ab} = -\partial_{ba}\,.$$

- The section condition is

$$\partial_{[ab}\partial_{cd]} = 0\,.$$

A solution of this section condition is

$$\partial_{\mu\nu} = 0, \qquad \partial_{\mu 10} \neq 0,$$

where $\mu, \nu, = 0, \ldots 9$. Hence, it reduces to ten-dimensional theory.

• Under the generalized diffeomorphism, fields transform as

$$\delta_X T^{a_1 \ldots a_p}{}_{b_1 \ldots b_q} = \hat{\mathcal{L}}_X T^{a_1 \ldots a_p}{}_{b_1 \ldots b_q} \,,$$

where the generalized Lie derivative is defined by

$$\begin{aligned}\hat{\mathcal{L}}_X T^{a_1 \ldots a_p}{}_{b_1 \ldots b_q} =& \frac{1}{2} X^{cd} \partial_{cd} T^{a_1 \ldots a_p}{}_{b_1 \ldots b_q} + \left(\frac{p}{4} - \frac{q}{4} + \frac{w}{2} \right) \partial_{ab} X^{cd} T^{a_1 \ldots a_p}{}_{b_1 \ldots b_q} \\ & - T^{a_1 \ldots c \ldots a_p}{}_{b_1 \ldots b_q} \partial_{cd} X^{a_i d} + \partial_{b_j d} X^{cd} T^{a_1 \ldots a_p}{}_{b_1 \ldots c \ldots b_q} \,,\end{aligned}$$

and w is weight of the field.

• We define the semi-covariant derivative,

$$\nabla_{cd} T^{a_1\dots a_p}{}_{b_1\dots b_q} = \partial_{cd} T^{a_1\dots a_p}{}_{b_1\dots b_q} + \left(\frac{p}{4} - \frac{q}{4} + \frac{w}{2}\right) \Gamma_{cde}{}^{e} T^{a_1\dots a_p}{}_{b_1\dots b_q}$$
$$- T^{a_1\dots e\dots a_p}{}_{b_1\dots b_q} \Gamma^{a_i}_{cde} + \Gamma^{e}_{cdb_j} T^{a_1\dots a_p}{}_{b_1\dots e\dots b_q} ,$$

where w is weight of the field. Implying metric compatibility, $x^{(ab)} = 0$, compatibility with the generalized Lie derivative, and uniqueness, we determine the connection, uniquely,

$$\Gamma_{abcd} = A_{abcd} + \frac{1}{2}(A_{acbd} - A_{adbc} + A_{bdac} - A_{bcad})$$
$$+ \frac{1}{9}(M_{ac}A^{e}{}_{(bd)e} - M_{ad}A^{e}{}_{(bc)e} + M_{bd}A^{e}{}_{(ac)e} - M_{bc}A^{e}{}_{(ad)e}) ,$$

where

$$A_{abcd} = -\frac{1}{2}\partial_{ab}M_{cd} + \frac{1}{14}M_{cd}\partial_{ab}\ln|M| .$$

When the free indices are antisymmetrized, the semi-covariant objects become covariant.

• Now we define the semi-covariant Riemann tensor,

$$
\begin{aligned}
R_{abcd} = {} & 3\partial_{[ab}\Gamma_{e][cd]}{}^{e} + 3\partial_{[cd}\Gamma_{e][ab]}{}^{e} + \frac{1}{4}\Gamma_{abc}{}^{e}\Gamma_{cdf}{}^{f} + \frac{1}{2}\Gamma_{abe}{}^{f}\Gamma_{cdf}{}^{e} \\
& + \Gamma_{ab[c}{}^{e}\Gamma_{d]ef}{}^{f} + \Gamma_{cd[a}{}^{e}\Gamma_{b]ef}{}^{f} + \Gamma_{ea[c}{}^{e}\Gamma_{d]fb}{}^{e} - \Gamma_{eb[c}{}^{f}\Gamma_{d]fa}{}^{e} .
\end{aligned}
$$

The semi-covariant Ricci tensor and Ricci scalar are, respectively,

$$
R_{ab} = R_{acb}{}^{c} = R_{ba} , \qquad R = M^{ab}R_{ab} = R_{ab}{}^{ab} .
$$

Now we have the action for U-gravity,

$$
S = \int_{\Sigma} |M|^{-\frac{1}{7}} R .
$$

SL(11) Supersymmetric U-gravity

[Work in progress with Jeong-Hyuck Park and Yoonji Suh]

• As we saw, the section condition is

$$\partial_{[ab}\partial_{cd]} = 0\,,$$

and there are two solutions. One is a ten-dimensional section,

$$\partial_{\mu\nu} = 0\,, \qquad \partial_{\mu 10} \neq 0\,,$$

where $\mu, \nu = 0, \ldots, 9$. There is also a three-dimensional section, but we do not consider today.

• The field content of supersymmetric theory is $(M_{ab}, \Psi_a, fluxes)$, and we consider $fluxes = 0$ for today.

• We just present the results. The action of supersymmetric U-gravity for (M_{ab}, Ψ_a, *fluxes* $= 0$) is

$$S = \int_\Sigma |M|^{-\frac{1}{7}} \Big[R - \frac{11}{196} \overline{\Psi}_a \Gamma^{abcd} D_{bc} \Psi_d - \frac{50}{98} \overline{\Psi}^a \Gamma^{bc} D_{ab} \Psi_c + \frac{19}{98} \overline{\Psi}^a \Gamma^{bc} D_{bc} \Psi_a - \frac{137}{98} \overline{\Psi}^a D_{ab} \Psi^b \Big] .$$

The supersymmetry transformations are

$$\delta M_{ab} = 2\bar{\varepsilon} \Gamma_{(a} \Psi_{b)} ,$$

$$\delta \Psi_a = \frac{1}{4} \Gamma^{bc} \Gamma_a D_{bc} \varepsilon .$$

• Now, for the ten-dimensional section, we parametrize the generalized metric and the fermion field as

$$M_{ab} = e^{8\phi/5}\begin{pmatrix} g_{\mu\nu}/\sqrt{|g|} & v_\mu \\ v_\nu & \sqrt{|g|}(v^2 - e^{5\phi}) \end{pmatrix},$$

and

$$\Psi_{\bar{\mu}} = e^{-4\phi/5}\left(\psi_{\bar{\mu}} - \frac{1}{2}\gamma_{\bar{\mu}}\gamma_{\bar{\nu}}\psi^{\bar{\nu}} - \frac{4}{5\sqrt{2}}\lambda\right),$$
$$\Psi_{\overline{10}} = e^{-4\phi/5}\left(\frac{1}{2}\gamma_{\bar{\nu}}\psi^{\bar{\nu}} + \frac{17}{10\sqrt{2}}\lambda\right).$$

• From the equations of motion, we obtain a condition,

$$\nabla_\mu v^\mu = m\, e^{-5\phi},$$

where m is a constant.

• Then, surprisingly, we precisely reproduce *massive type IIA supergravity* [Romans, 1986] with ($g_{\mu\nu}$, ϕ, ψ_μ, λ, *fluxes* $= 0$).

$$
\begin{aligned}
e^{-1}\mathcal{L} = & -\frac{1}{4}R + \frac{1}{2}D_\mu\phi D^\mu\phi + \frac{1}{8}m^2 e^{-5\phi} \\
& + \frac{1}{2}\overline{\psi}_\mu\gamma^{\mu\nu\lambda}D_\nu\psi_\lambda + \frac{1}{2}\overline{\lambda}\gamma^\mu D_\mu\lambda - \frac{1}{\sqrt{2}}\partial_\nu\phi\overline{\lambda}\gamma^\mu\gamma^\nu\psi_\mu \\
& + m\,e^{-5\phi/2}\left(\frac{1}{8}\overline{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + \frac{5}{8\sqrt{2}}\overline{\lambda}\gamma^\mu\psi_\mu - \frac{21}{32}\overline{\lambda}\lambda\right).
\end{aligned}
$$

The supersymmetry transformations are reproduced

$$
\begin{aligned}
\delta g_{\mu\nu} &= 2\overline{\varepsilon}\gamma_{(\mu}\psi_{\nu)}\,, & \delta\phi &= \frac{1}{\sqrt{2}}\overline{\varepsilon}\lambda\,, \\
\delta\psi_\mu &= D_\mu\varepsilon - \frac{1}{32}m\,e^{-5\phi/2}\gamma_\mu\varepsilon\,, & \delta\lambda &= \frac{1}{\sqrt{2}}\partial_\mu\gamma^\mu\varepsilon - \frac{5}{8\sqrt{2}}me^{-5\phi/2}\varepsilon\,.
\end{aligned}
$$

Also, recall how we obtained the Romans mass, m, from vector, v^μ.

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Outlook

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• Flux part of SL(11) supersymmetric U-gravity is a work in progress.

• Why would we obtain massive type IIA supergravity from the SL(11) invariant theory?

• Would it give us some hint on the old puzzle of M-theory origin of massive type IIA supergravity?

Thank you.