5d descriptions of 6d SCFTs

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based on various collaborations with

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arXiv:1504.03672
arXiv:1505.04439
arXiv:1509.03300
arXiv:1512.08239

Introduction

- 6d **N=(2,0)** theory <—> 5d Max. SYM (N=2)

[Lambert, Papageorgakis, Schmidt-Sommerfeld ’10]
[Douglas ’10]

- 6d **N=(1,0)**
  E-string theory <—> 5d N=1 SU(2), Nf=8 flavors

6d theory on S^1 : 5d description (UV completion)

radius <—> gauge coupling
KK momentum <—> 5d instanton particle

Introduction

F-theory classifications

[Heckman-Morrison-Vafa '13]

[Del Zotto-Heckman-Tomasiello-Vafa '14]

6d SCFTs from F-theory on elliptic CY3

Like the E-string theory case,

Many other N=(1,0) SCFTs → 5d descriptions? (Lagrangian)

New perspectives on 6d and 5d SCFTs

5d dualities, Index functions

[Bergman-Zafrir '13-'15]

Introduction

Idea:

6d (1,0) SCFTs

⇓ Tensor branch

IIA brane configurations

[Hanany, Zaffaroni '97, Brunner, Karch '97]

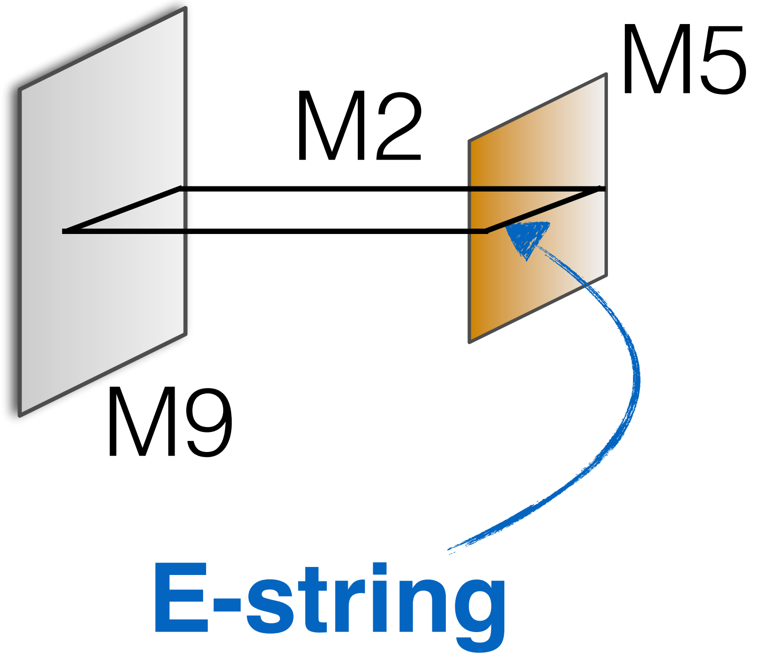
⇓ on S^1 and T-dual

IIB (p,q) brane web diagrams

: read off 5d descriptions
not just one but **many 5d theories**
(S-duality)

Contents

6d N=(1,0) E-string theory

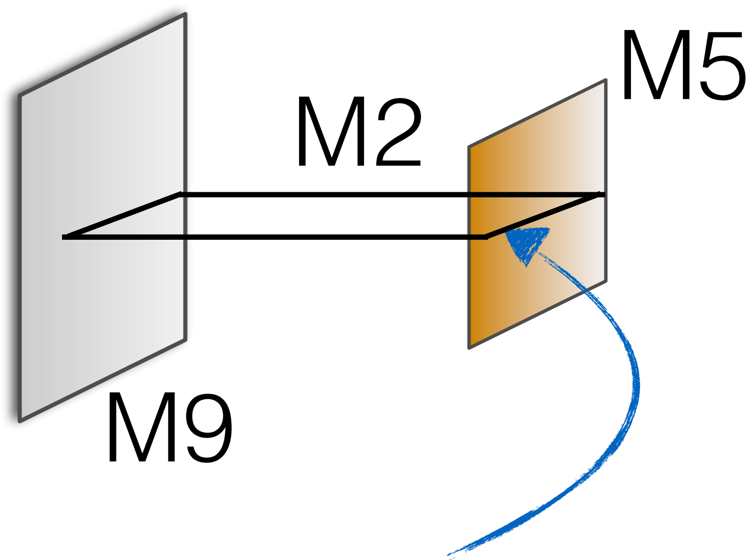


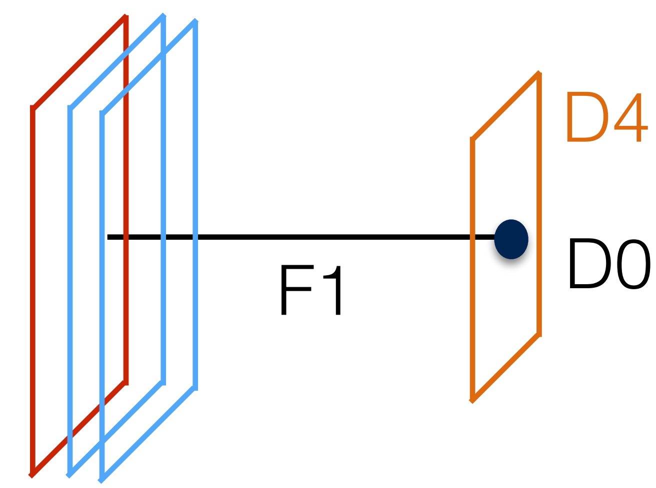
E-string partition function
(elliptic genus)

['14 Chiung Hwang, Joonho Kim, Seok Kim, Jaemo Park]

['14 Seok Kim, Joonho Kim, Kimyeong Lee, Jaemo Park, Vafa]

The UV completion of 5d SU(2) w/ Nf=8 flavors



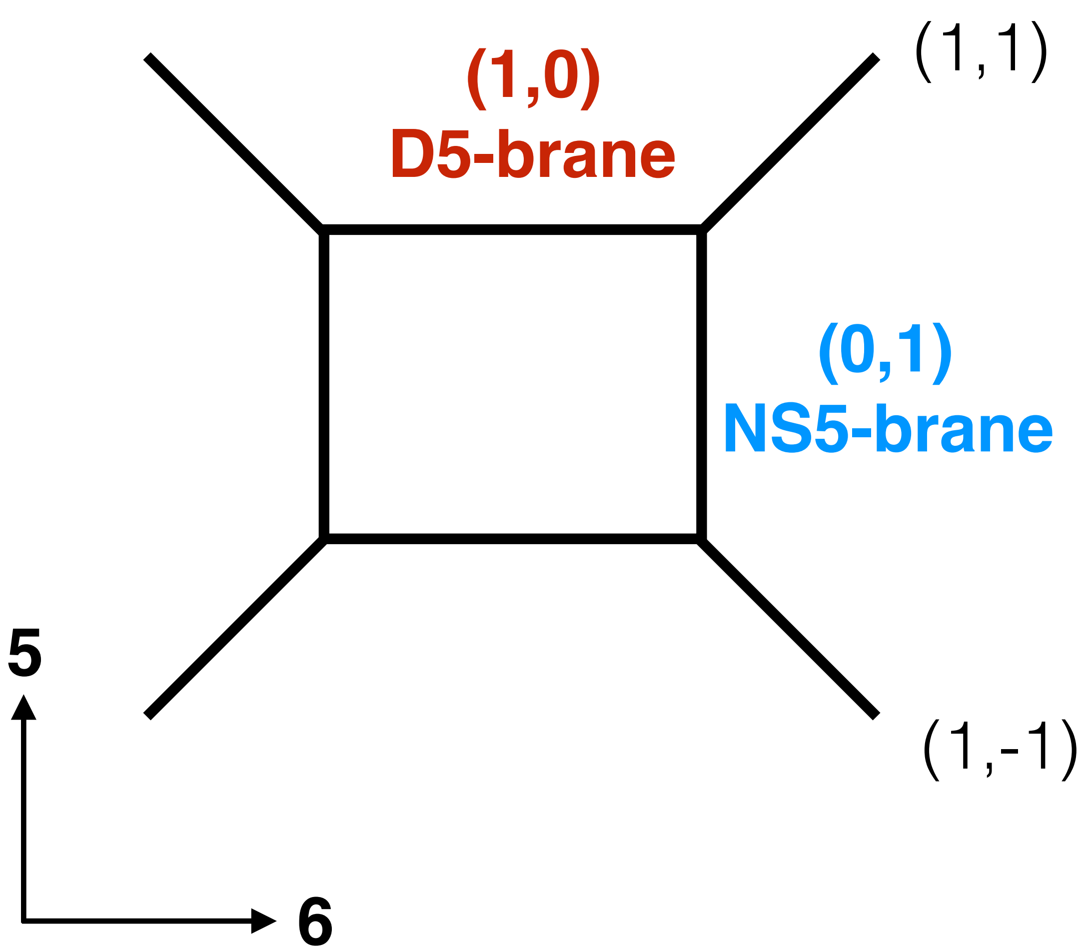


E-string theory on a circle = **5d SU(2) theory with Nf=8**

KK modes = **Instantons**

5d SU(2) theory and **IIB brane picture**

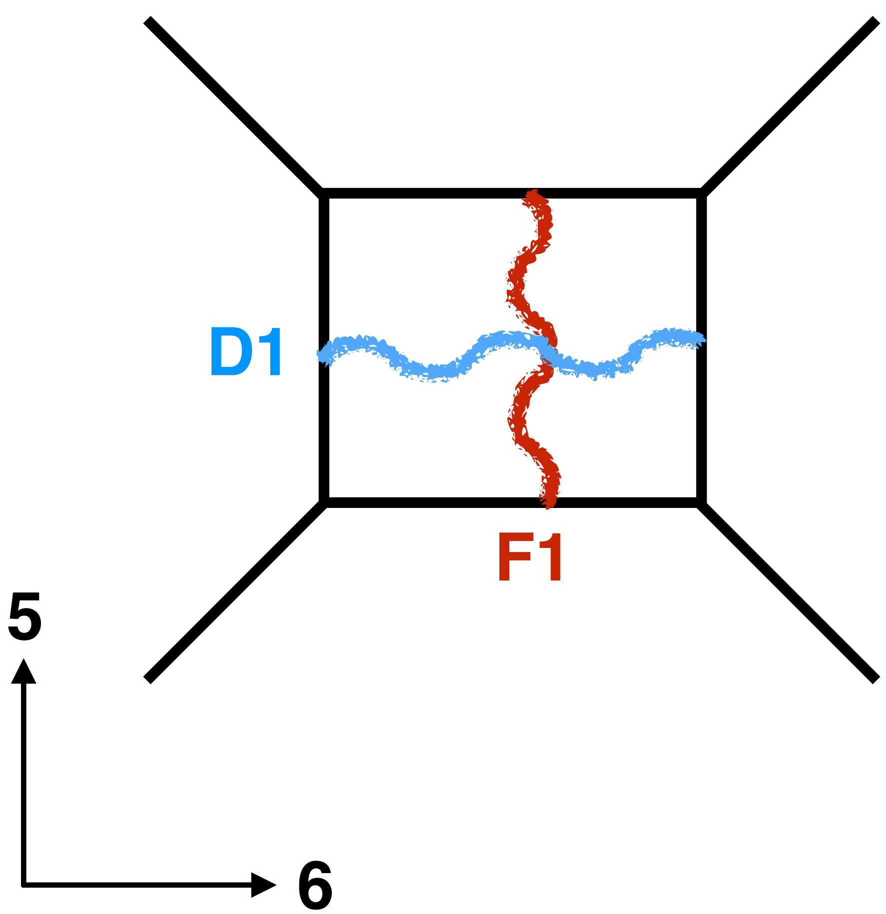
[Aharony-Hanany, '97]



	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D5	–	–	–	–	–		–			
(1,1)	–	–	–	–	–	-	-			

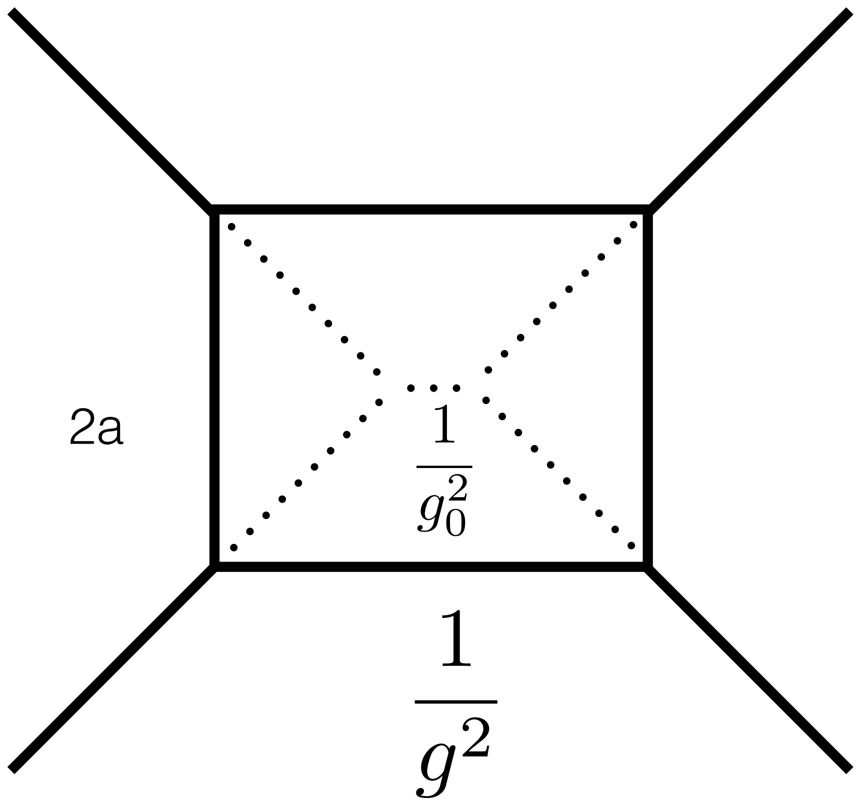
5d SU(2) theory and IIB brane picture

[Aharony-Hanany, '97]



	0	1	2	3	4	5	6	7	8	9
NS5	–	–	–	–	–	–				
D5	–	–	–	–	–		–			
(1,1)	–	–	–	–	–	-	-			

5d pure SU(2) SYM



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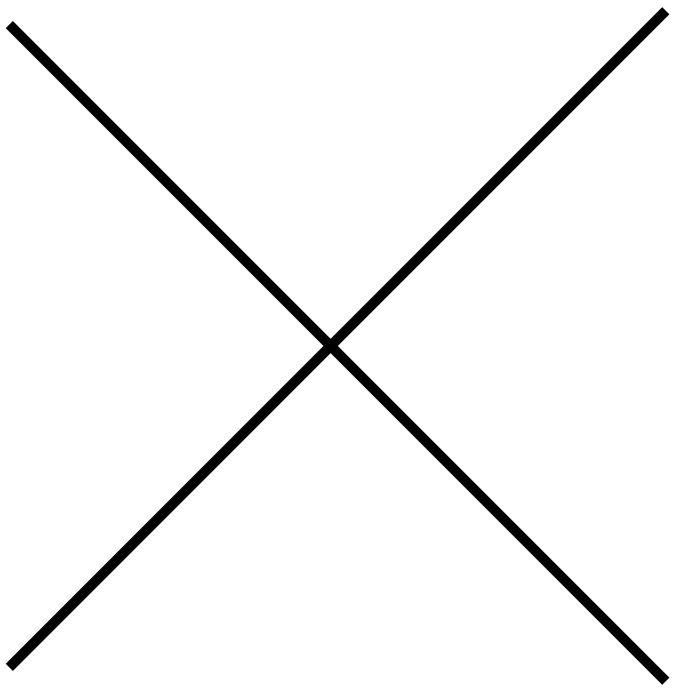


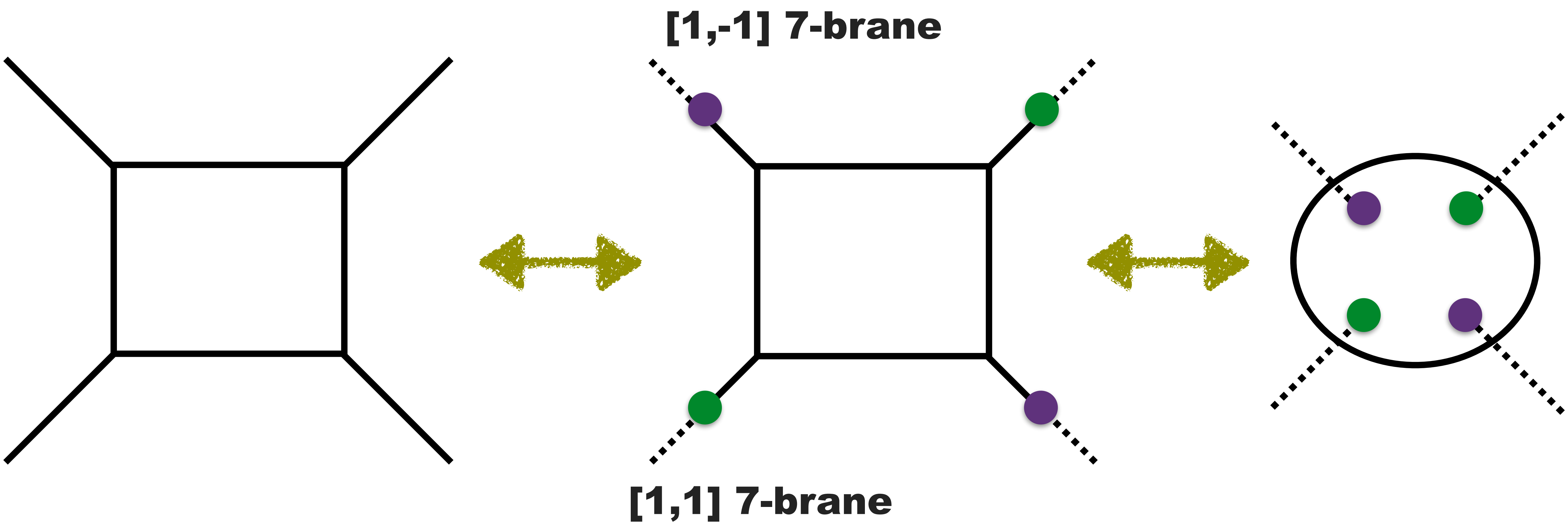
UV

Strong coupling,

Vanishing Coulomb moduli

E_1 theory

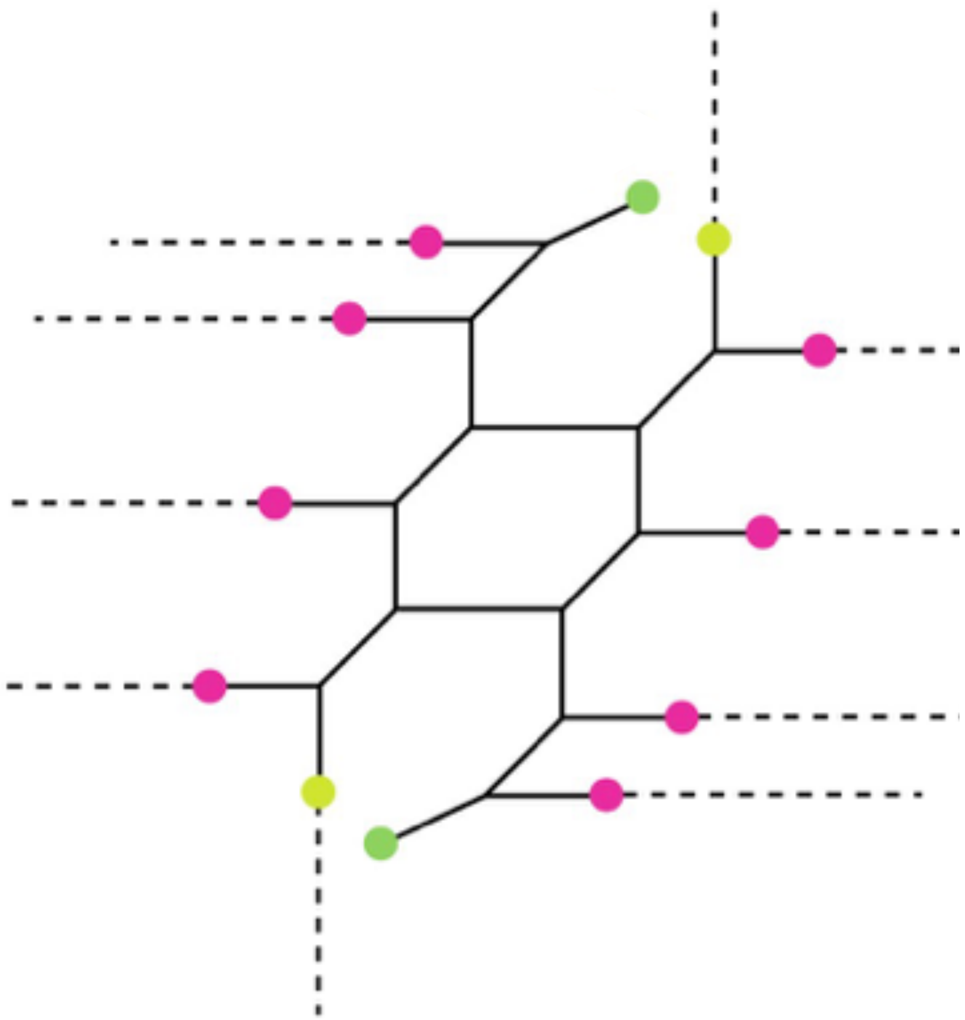
5d SU(2) theory and 7-branes



[DeWolfe et al. '99]

	0	1	2	3	4	5	6	7	8	9
NS5	—	—	—	—	—	—	.			
D5	—	—	—	—	—	.	—			
D7	—	—	—	—	—	.	.	—	—	—

Brane configuration
for 5d **SU(2)** theory with **Nf=8 flavors**

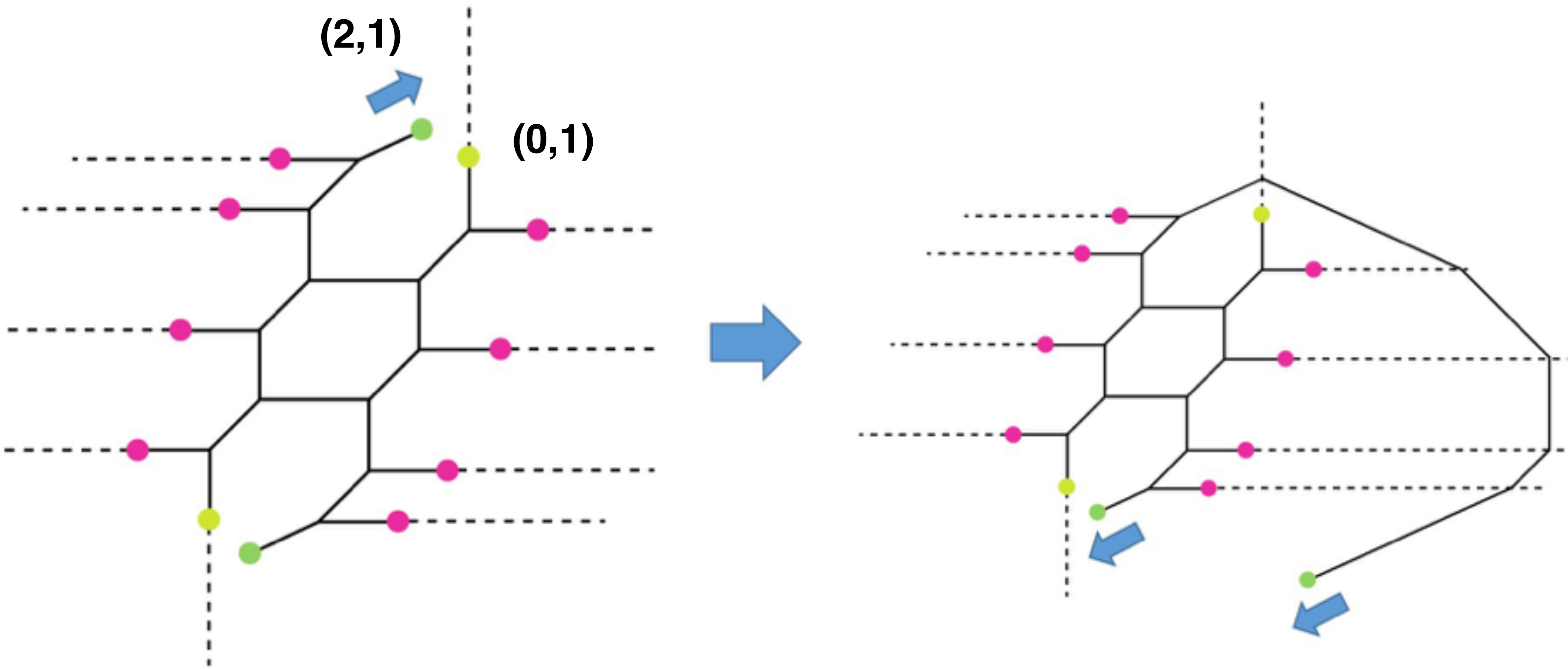


SU(2) = one Coulomb moduli

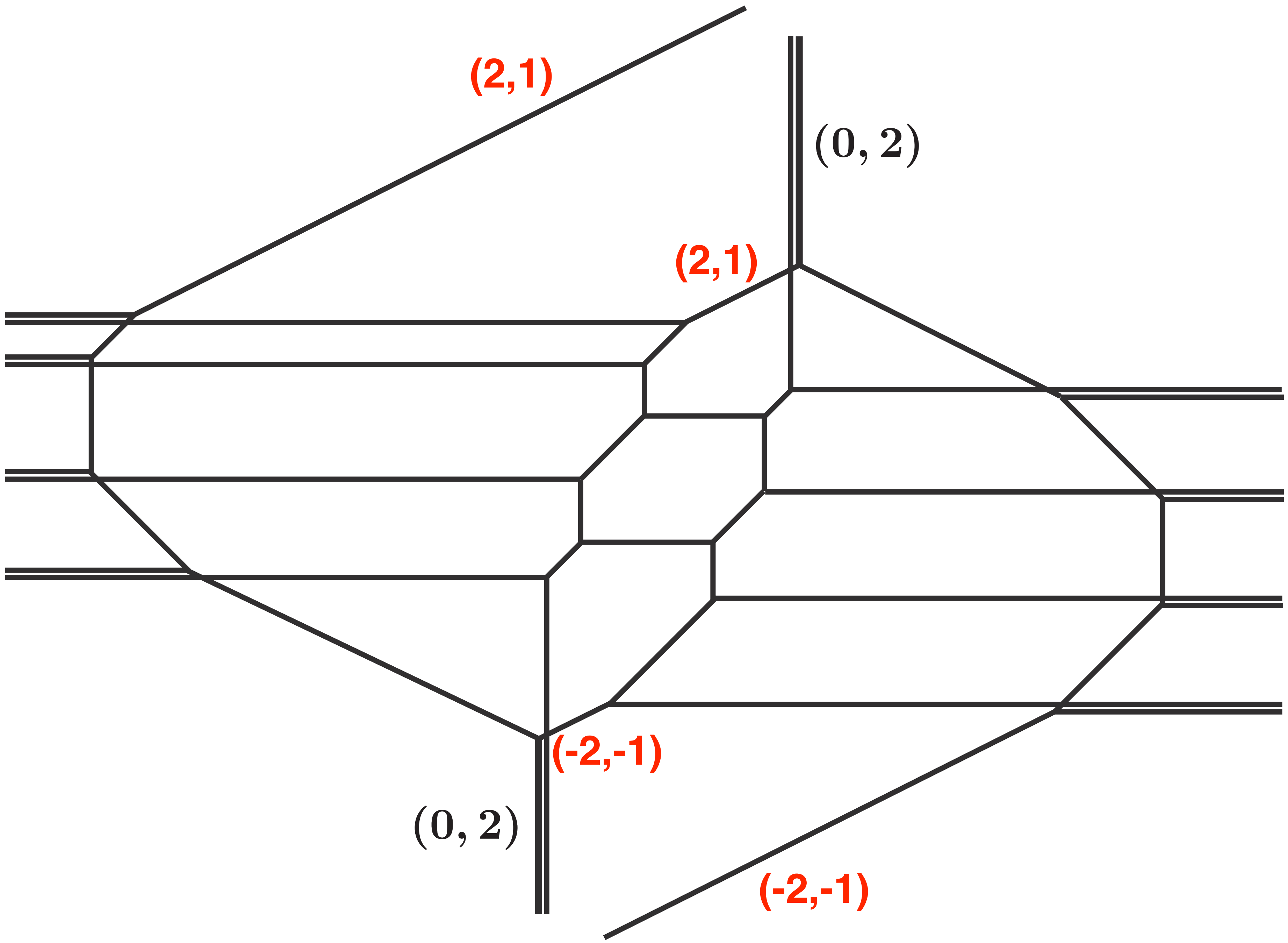
8 Flavors = 8 D7 branes

5-brane web = pulling out
all the 7-branes
through
the branch cuts

Spiral structure of the web diagram



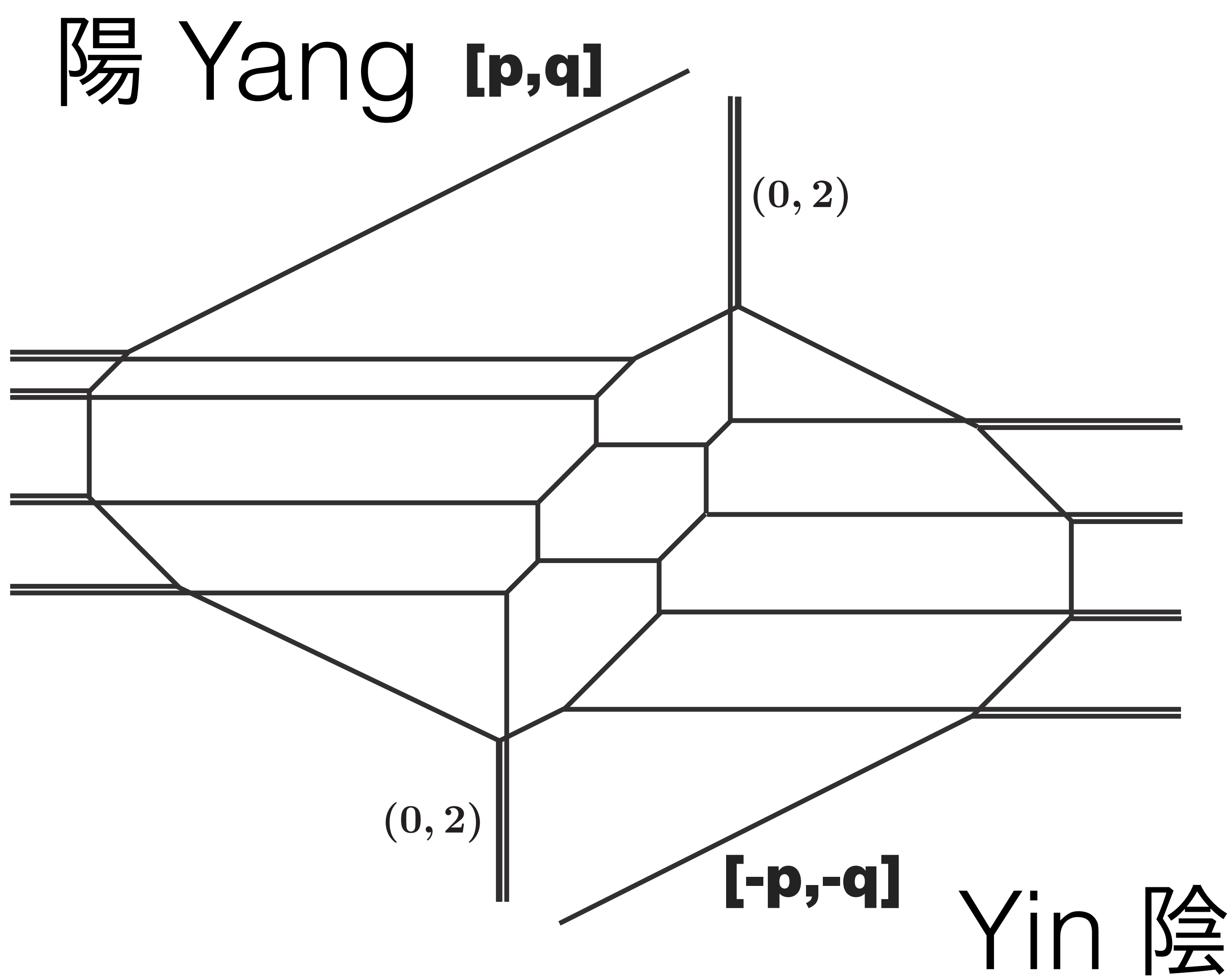
By pulling out 7-branes to infinity

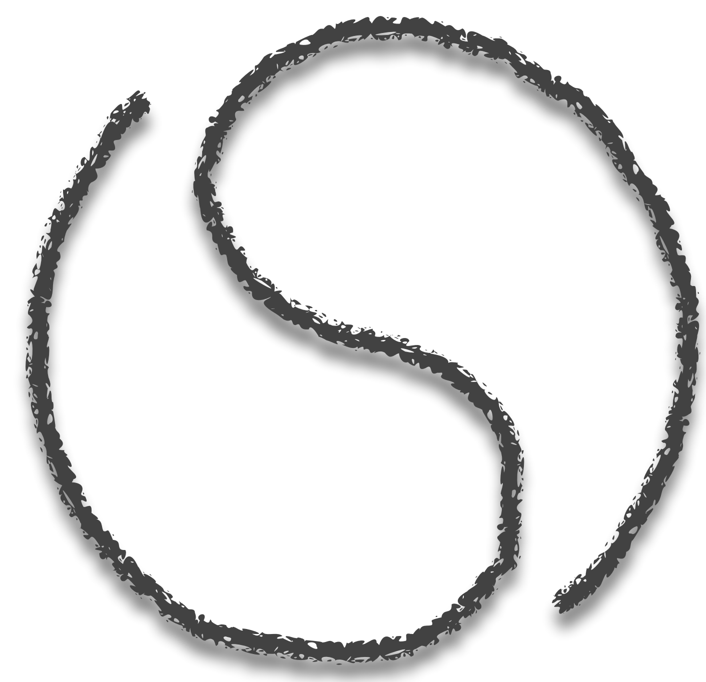


Spirally rotating!   One revolution: charges remain the same.

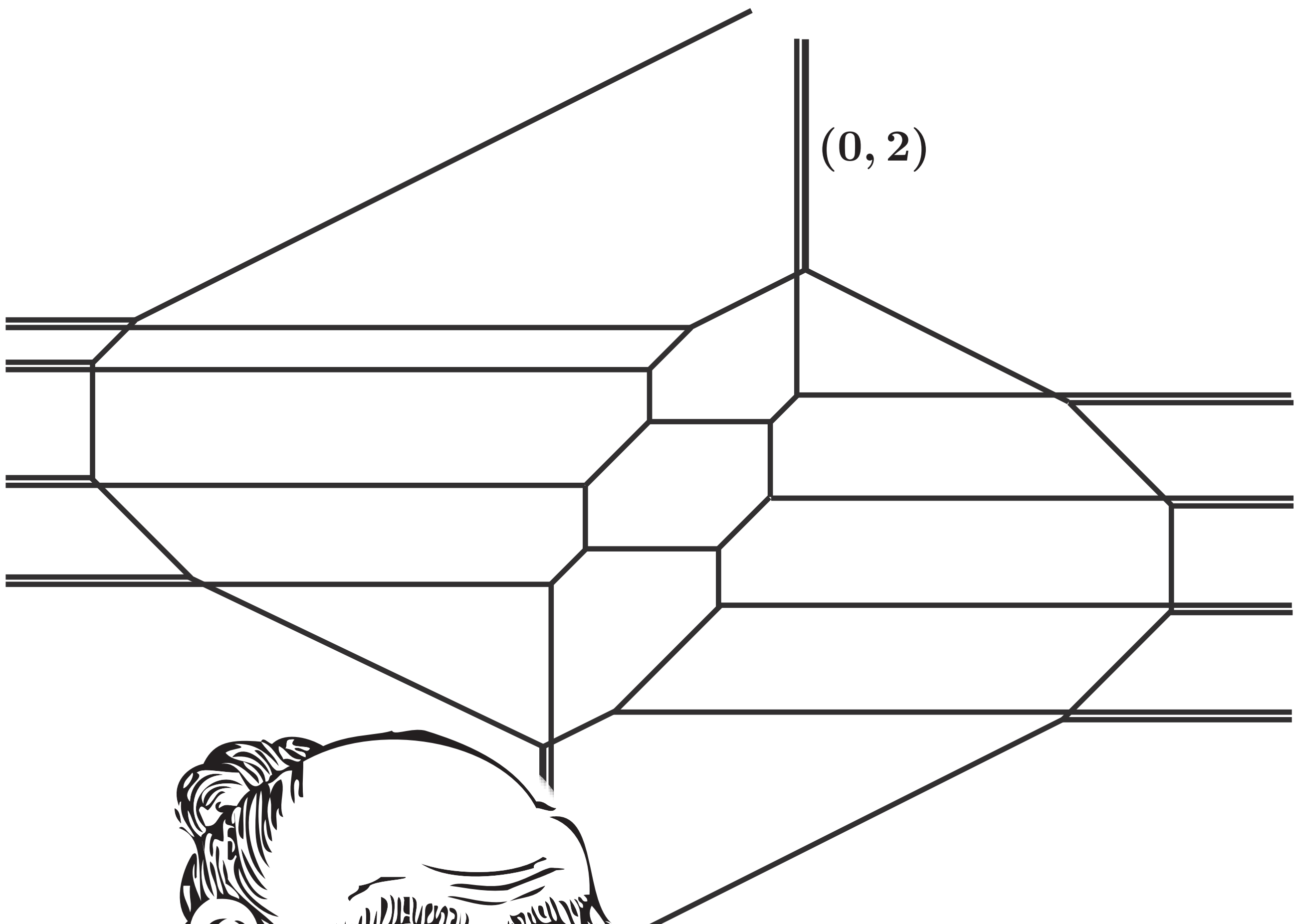
Infinitely rotating spiral diagram

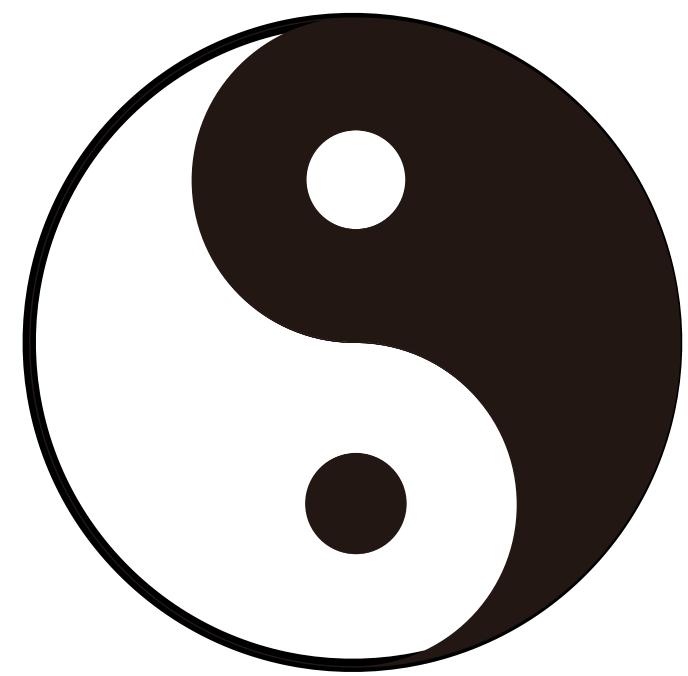
The shape looks like





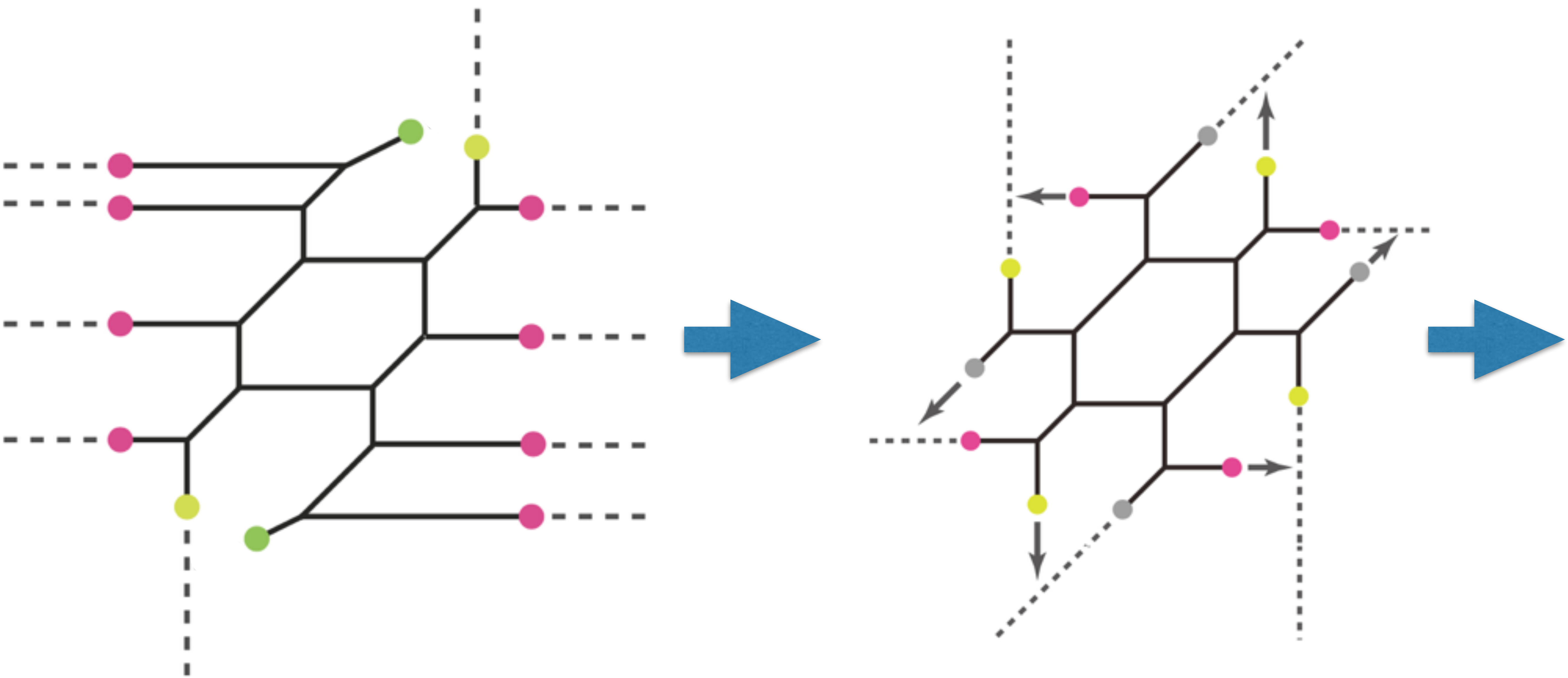
Call it **Tao diagram**…



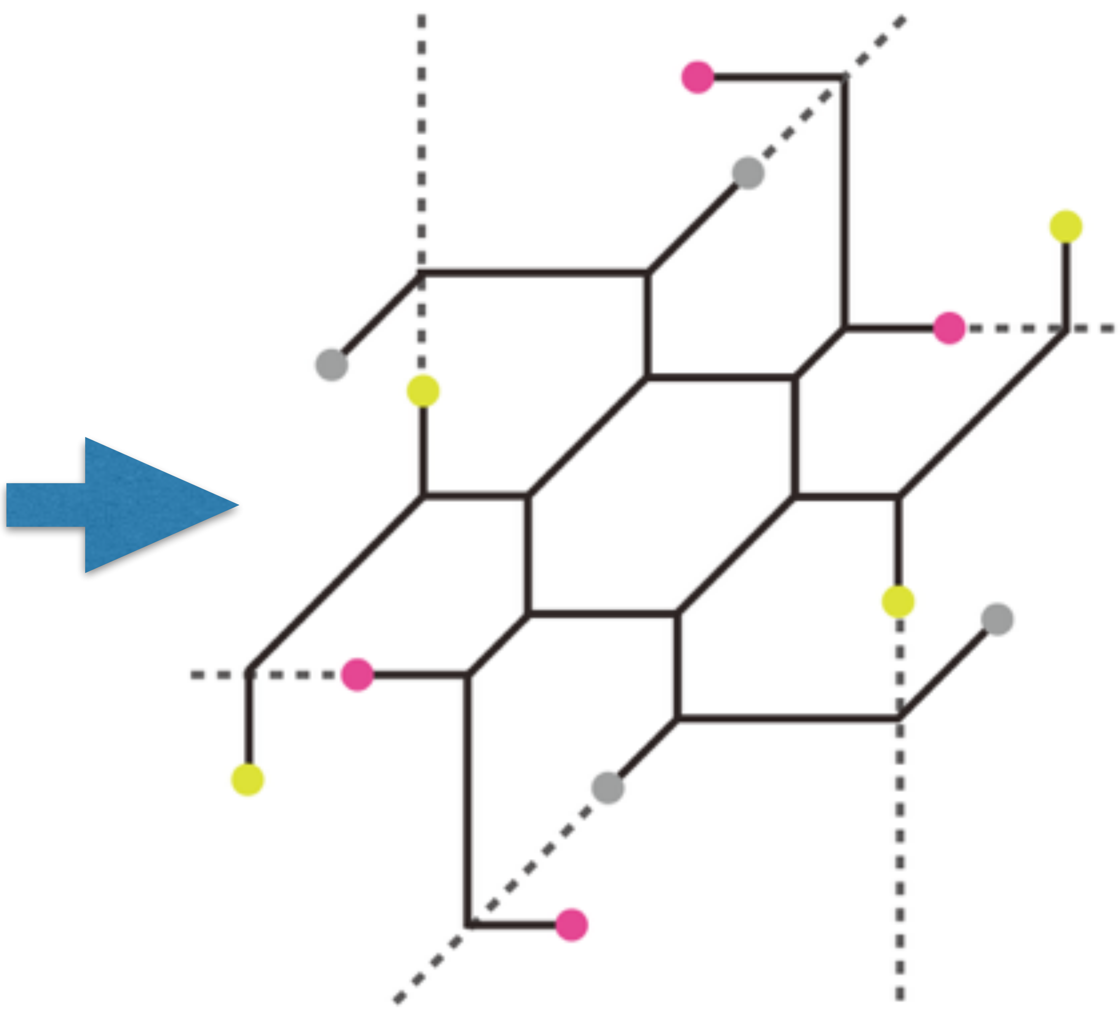


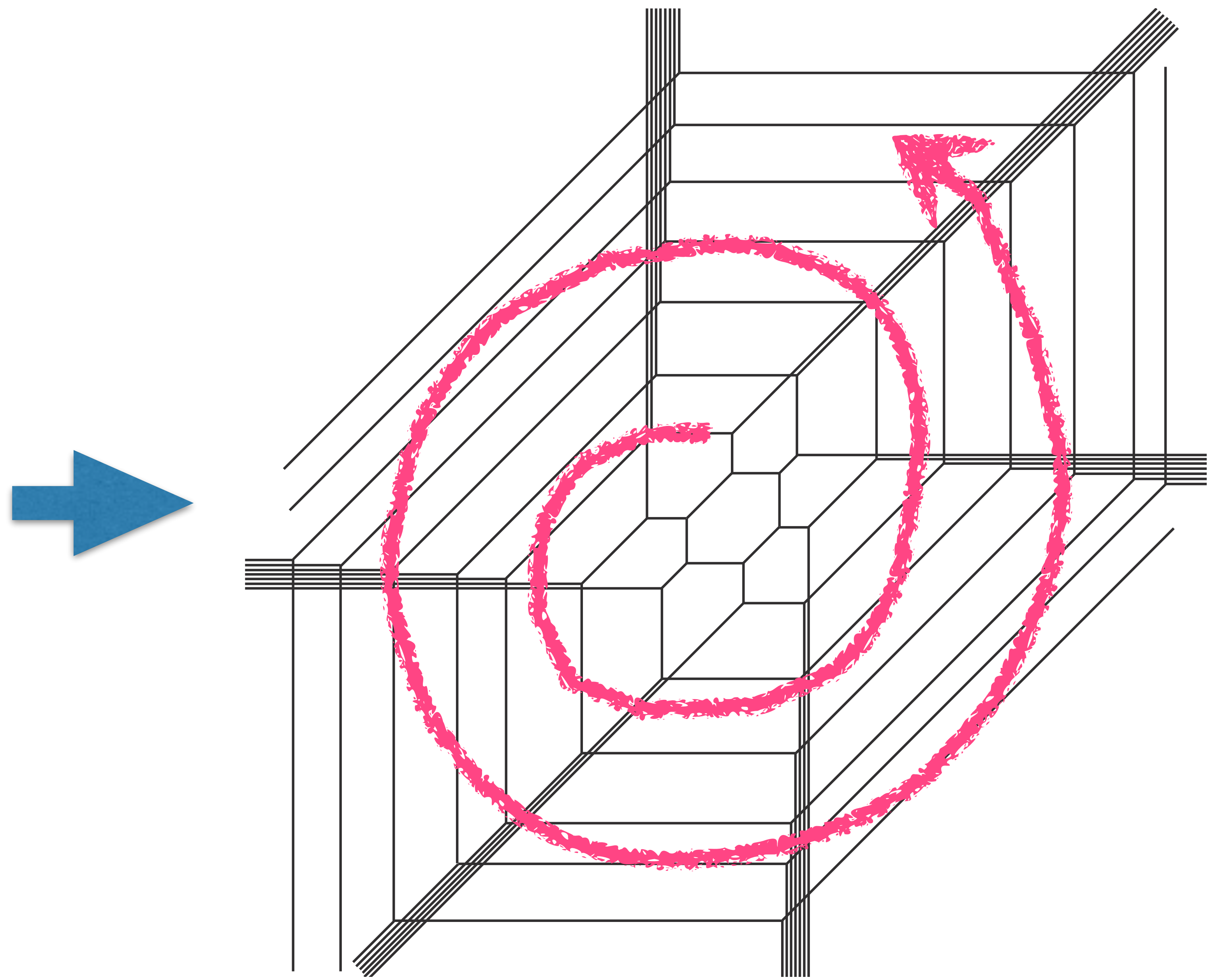
Laozi 老子

Equivalent Tao diagram



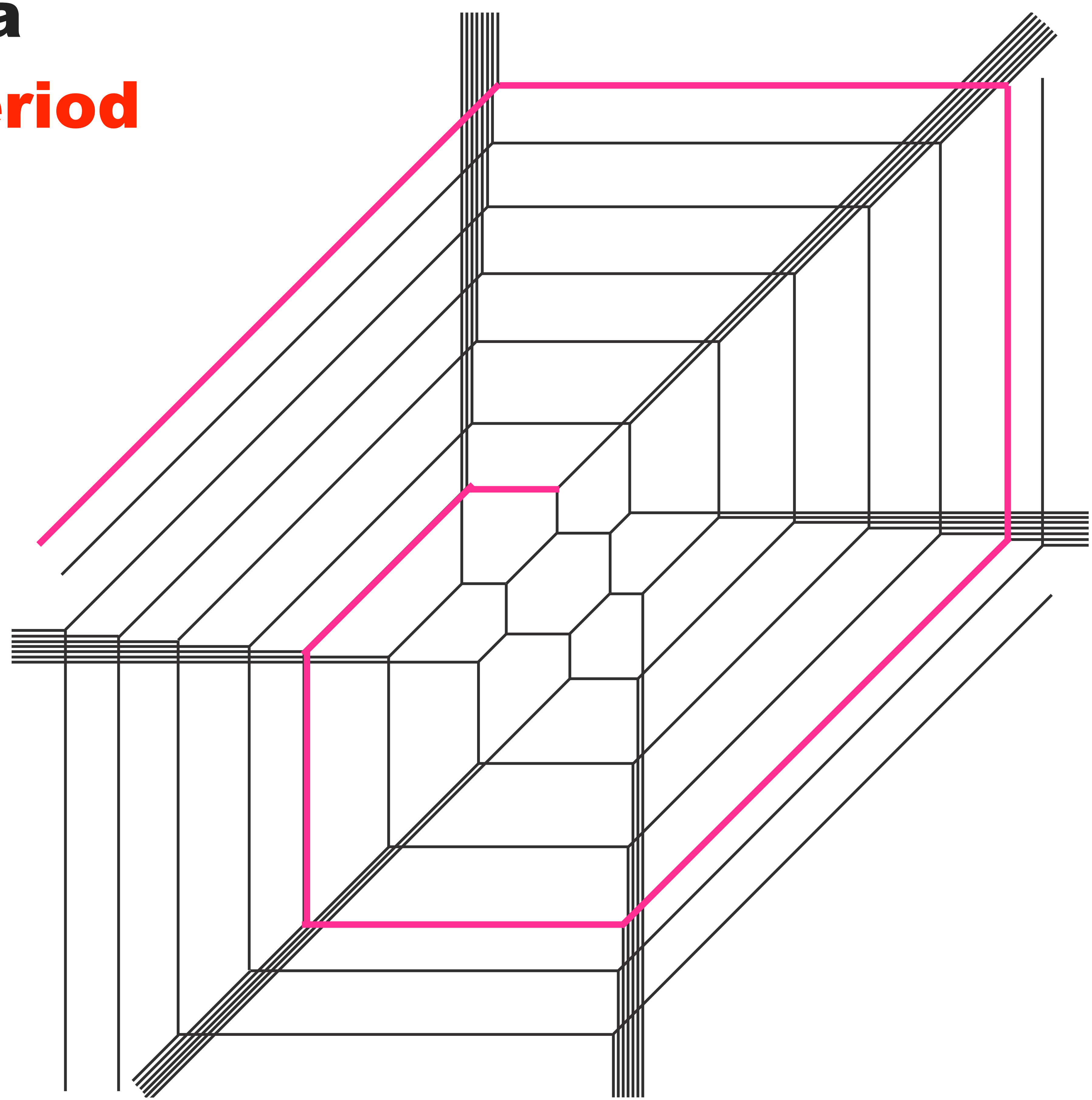
Equivalent Tao diagram





Web diagram realization of E-string theory?

spiral with a
constant period



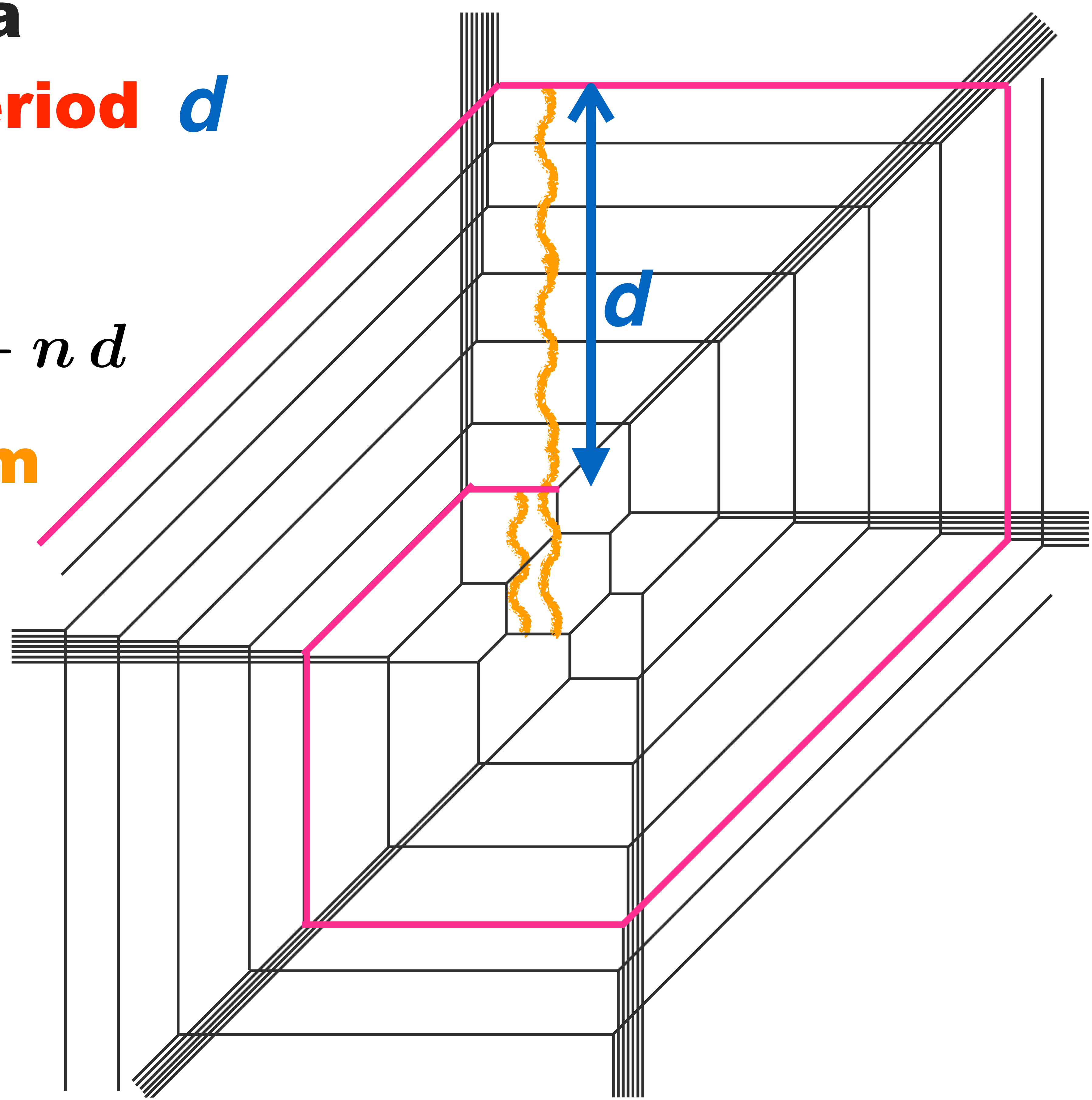
6-th direction generated?!

spiral with a

constant period d

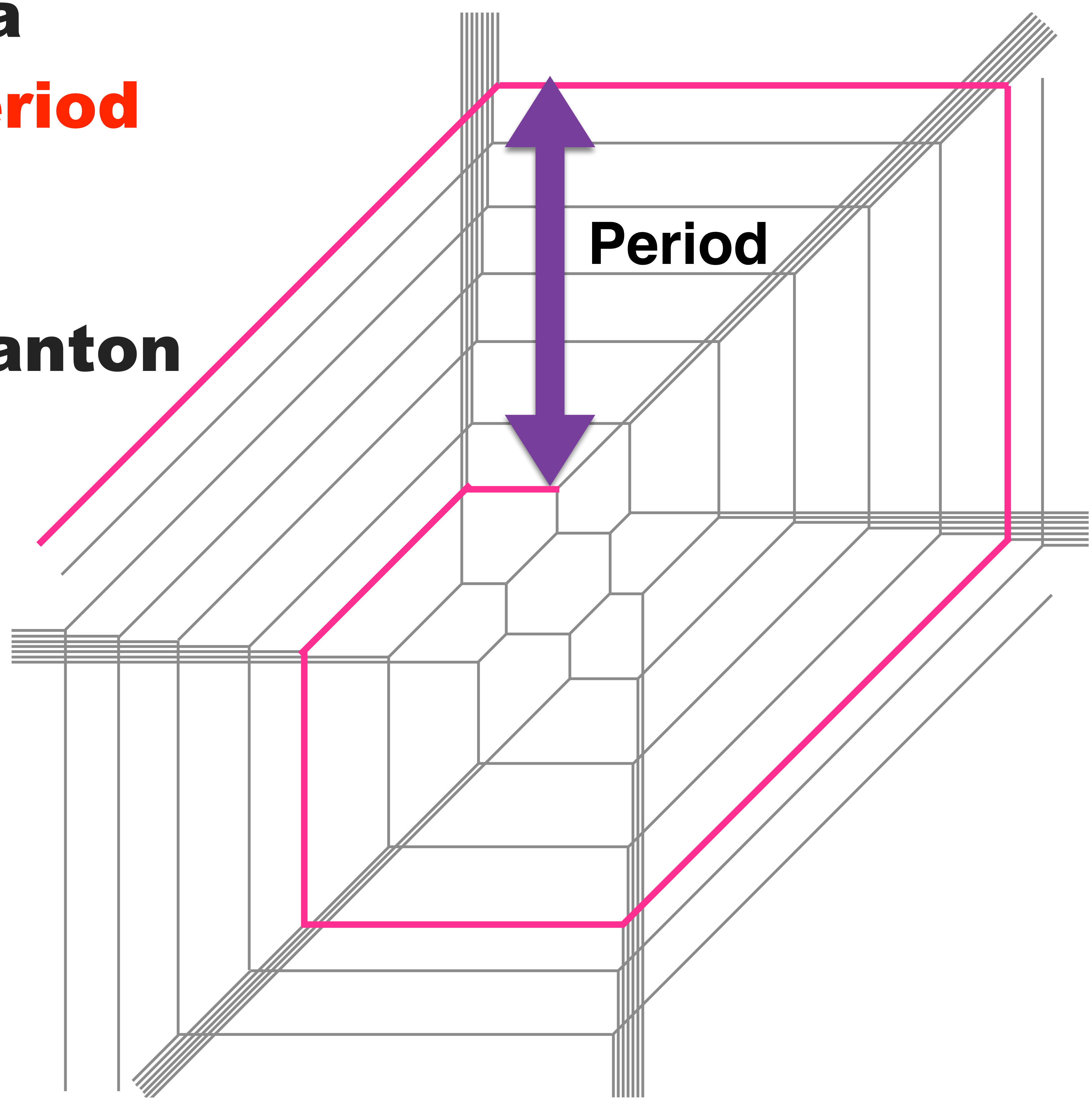
$$m_{(n)} = m_{(0)} + n\, d$$

KK spectrum



spiral with a
constant period

period = instanton
$\sim R^{-1}$



Tao diagrams

Infinite spirals (KK spectrum)
constant period (compactified radius)

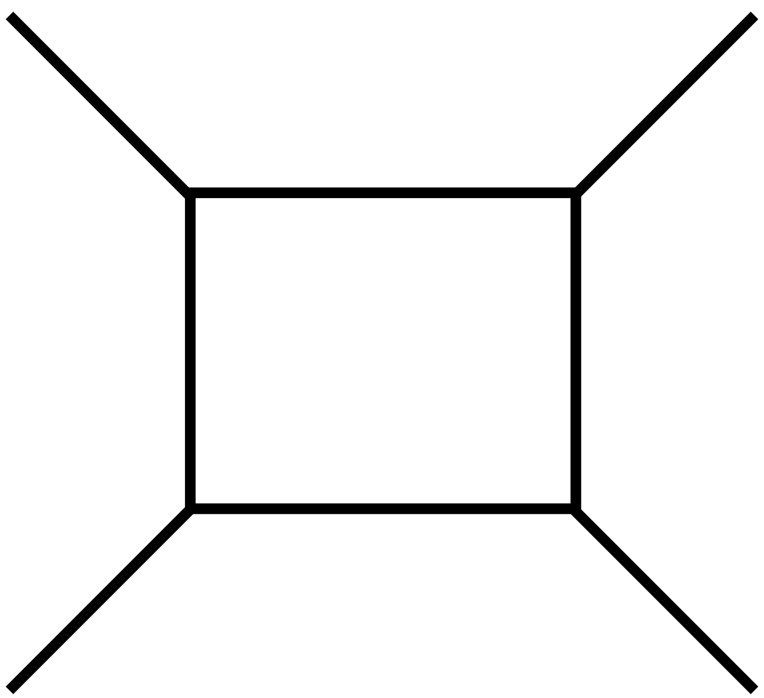
Naturally identified as **a 6d theory on a circle**
(compactification radius emerges as a spiral)

- Computational tool:
  Partition function

Topological Vertex formalism

[Vafa et al.]

web diagram

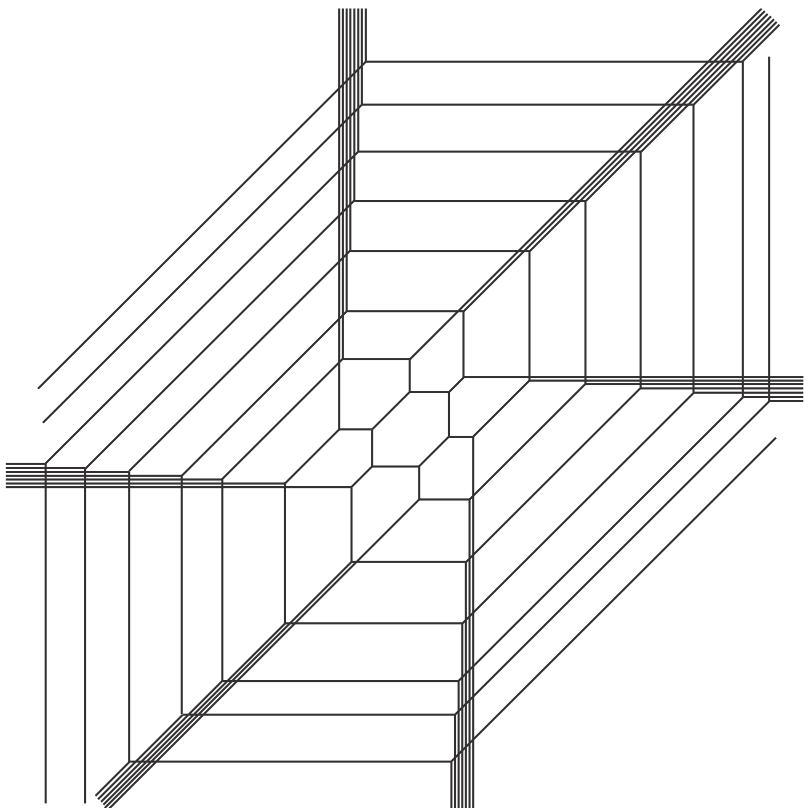


BPS partition function

$$Z = \cdots$$

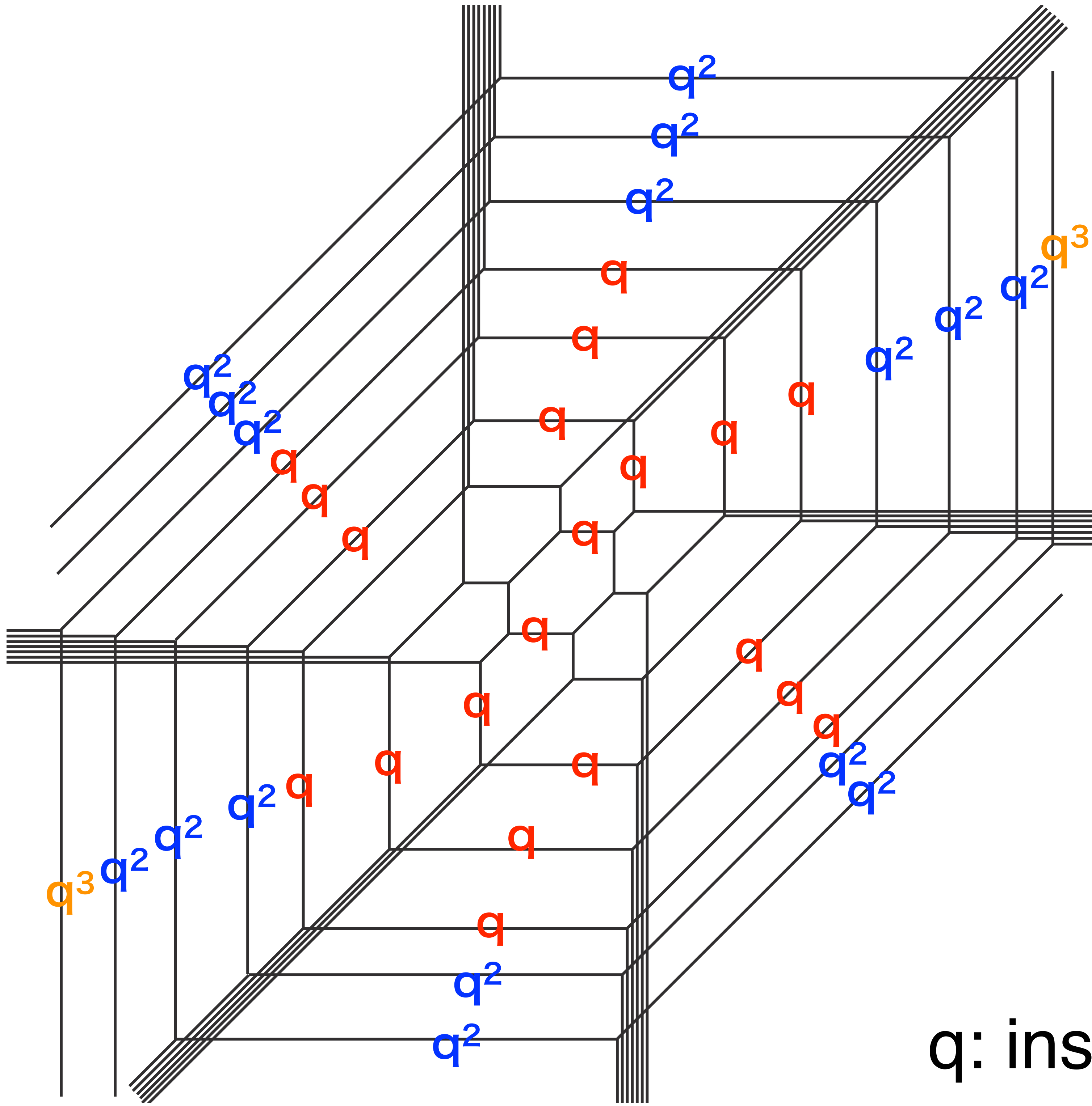
Topological Vertex formalism

Tao web diagram



BPS partition function

$$Z \overset{?}{=} \cdots$$



q: instanton factor

Compute order-by-order in **q**

1-instanton, **2-instanton**, … , up to **q^k** order

Partition function from Tao diagram

$$Z_{E\text{-}string} = \mathrm{PE}\Big[\sum_{m=0}^{\infty}\mathcal{F}_m(y,A,q)\mathfrak{q}^m\Big] = \mathrm{PE}\Big[\frac{1}{(1-q)(1-q^{-1})}\sum_{n=1}^{\infty}\tilde{f}_n A^n\Big]$$

$$\begin{aligned}
\tilde{f}_1 =\ & \chi^{(1)} + \chi_{\mathrm{c}}\,\mathfrak{q} + \Big(2\chi_{\mathbf{2}}(q)\chi^{(1)} + \chi^{(3)} + \chi^{(1)}\Big)\mathfrak{q}^2 + \Big(\chi^{(1)}\chi_{\mathrm{s}} + 2\chi_{\mathbf{2}}(q)\chi_{\mathrm{c}}\Big)\mathfrak{q}^3 \\
& + \Big(3\chi_3(q) + 4\chi_2(q) + 2)\chi^{(1)} + 2\chi_2(q)\chi^{(3)} + \chi^{(5)} + \chi^{(1)}\chi^{(2)}\Big)\mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5), \qquad (4.54)
\end{aligned}$$

$$\begin{aligned}
\tilde{f}_2 =\ & -2 - 2\chi_{\mathrm{s}}\,\mathfrak{q} - \Big(2\chi^{(4)} + (3\chi_{\mathbf{2}}(q)+2)\chi^{(2)} + 4(\chi_{\mathbf{3}}(q) + \chi_{\mathbf{2}}(q) + 1)\Big)\mathfrak{q}^2 \\
& - \Big(2\chi^{(2)}\chi_{\mathrm{s}} + 3\chi_{\mathbf{2}}(q)\chi^{(1)}\chi_{\mathrm{c}} + 4(\chi_{\mathbf{3}}(q) + \chi_{\mathbf{2}}(q) + 1)\chi_{\mathrm{s}}\Big)\mathfrak{q}^3 \\
& + \Big((5\chi_4(q) + 6\chi_3(q) + 11\chi_2(q) + 8)\chi^{(2)} + (4\chi_3(q) + 4\chi_2(q))\chi^{(4)} + (3\chi_2(q) - 2)\chi^{(6)} \\
& \quad + (4\chi_3(q) + 3\chi_2(q) + 2)(\chi^{(1)})^2 + 3\chi_2(q)\chi^{(1)}\chi^{(3)} + 2\chi^{(1)}\chi^{(5)} + 2(\chi^{(2)})^2 + 2(\chi_s)^2 \\
& \quad + (6\chi_5(q) + 8\chi_4(q) + 16\chi_3(q) + 20\chi_2(q) + 10)\Big)\mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5). \qquad (4.55)
\end{aligned}$$

arXiv:1504.03672

reproduces the E-string partition function (elliptic genus)

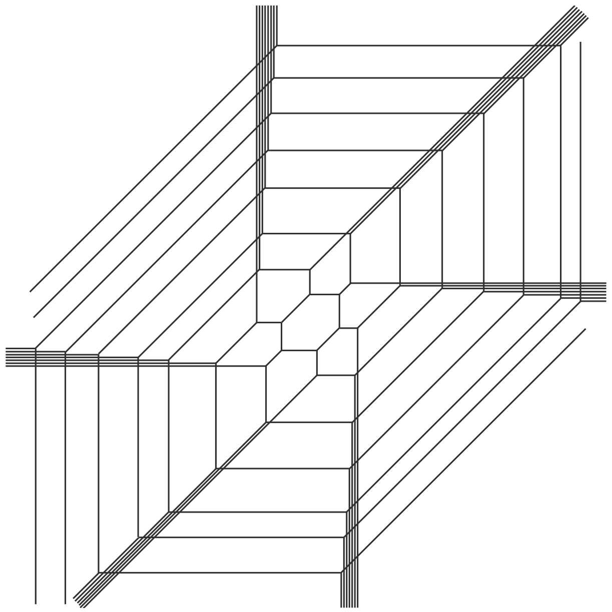
by (up to 4 instantons) ['14 Kim, Kim, Lee, Park, Vafa]

Tao diagram indeed sees the E-string theory on a circle

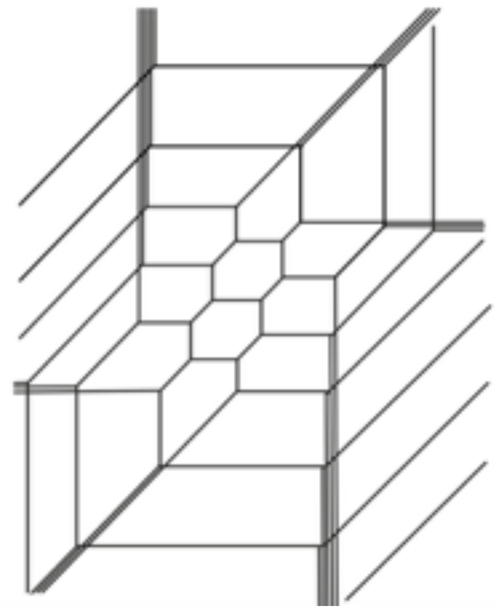
Claim:

A **Tao web diagram** implies that
a 5d theory has UV completion as **6d SCFT**

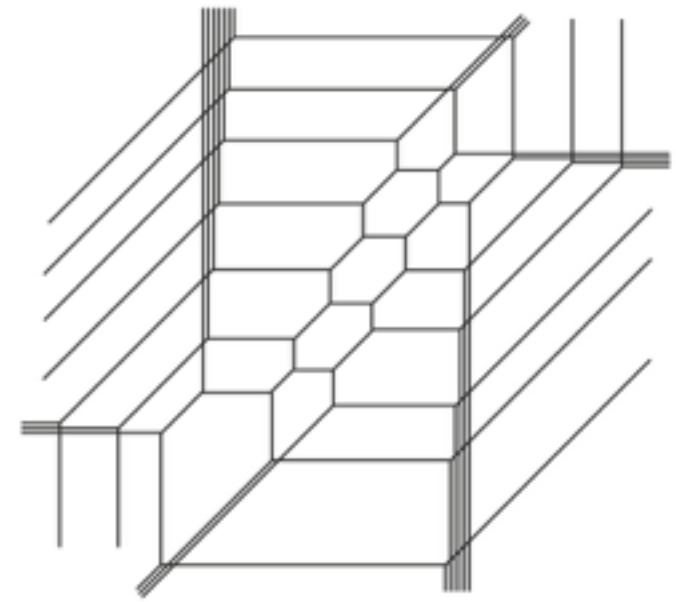
Many more Tao web diagrams



$N = 2$

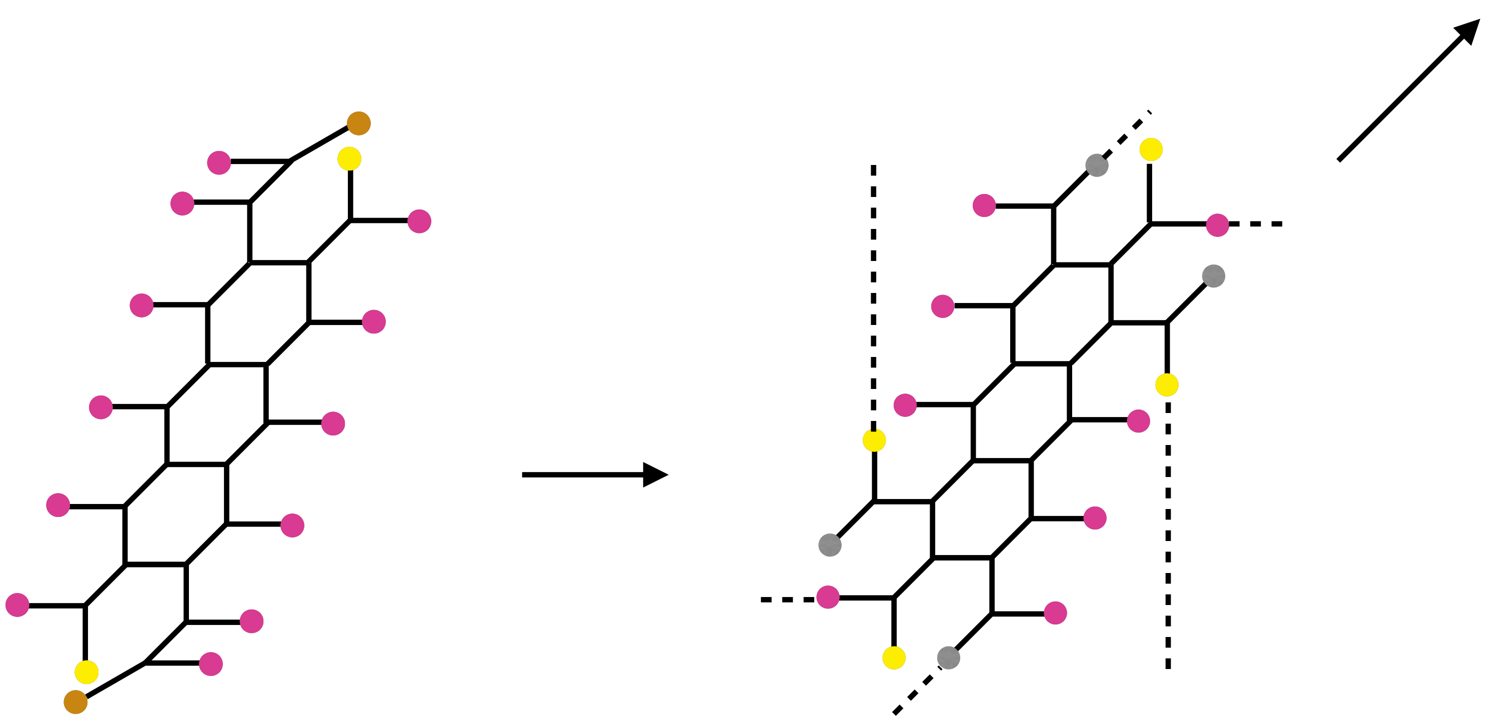


$N = 3$



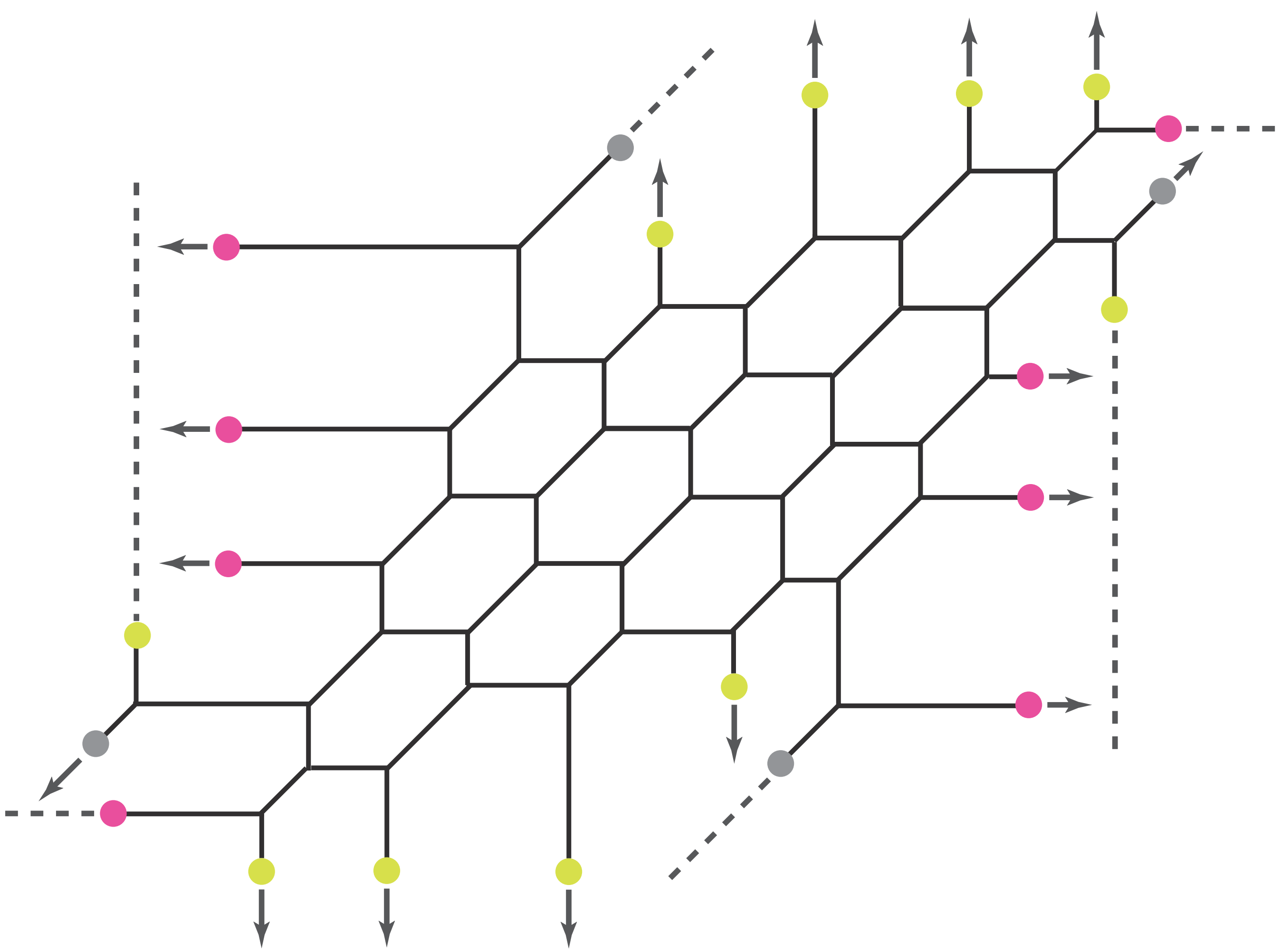
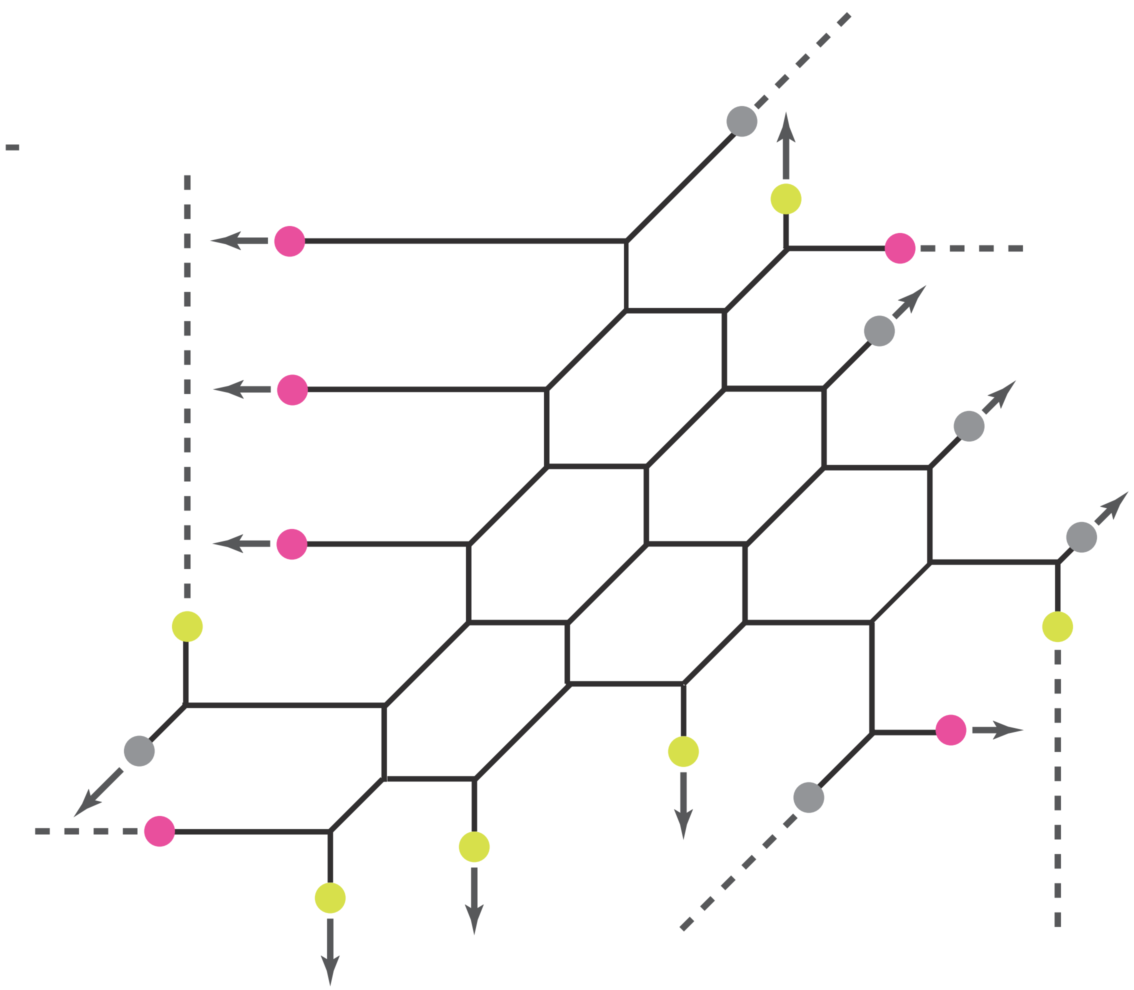
$N = 4$

…



$SU(4), N_f = 12$

Quiver type

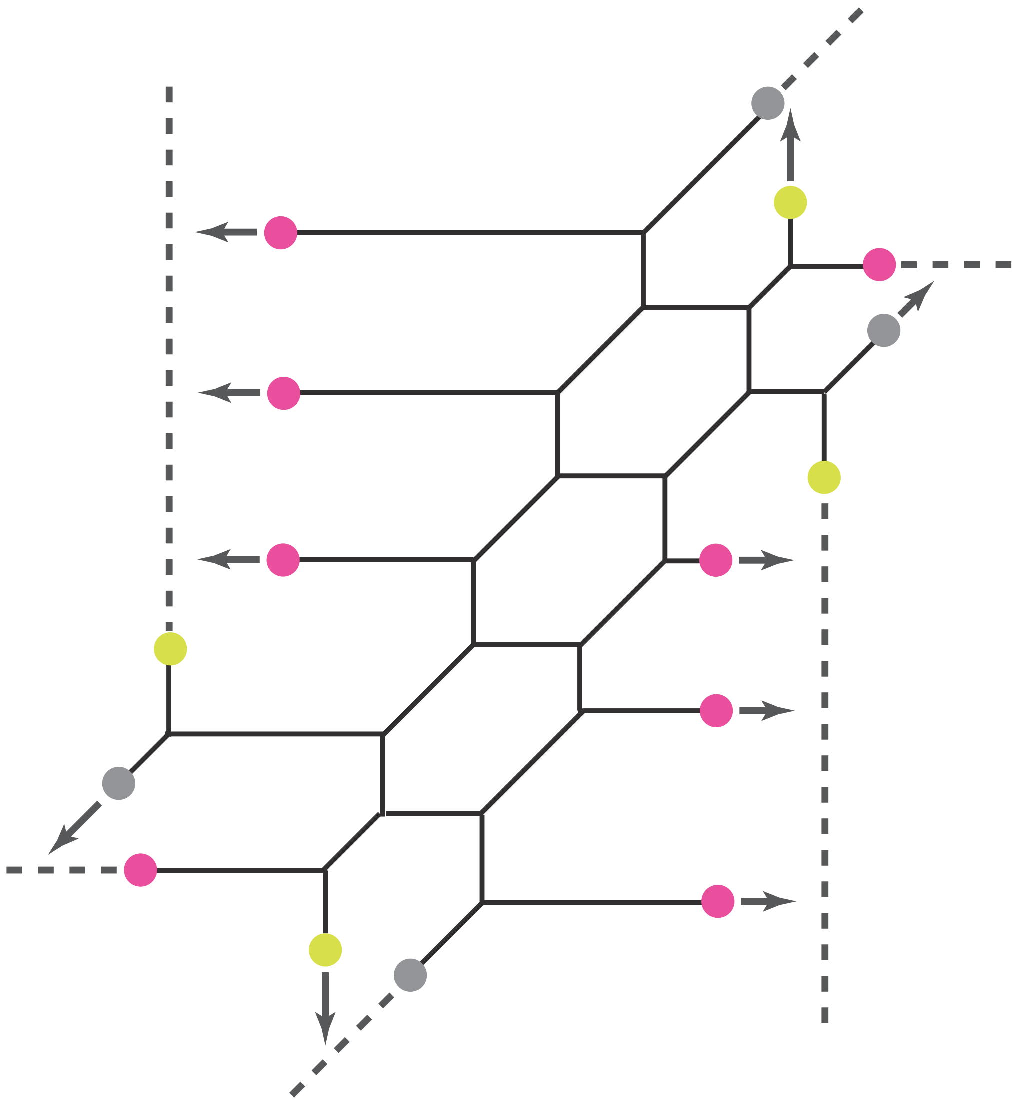
2. 6d Conformal matter

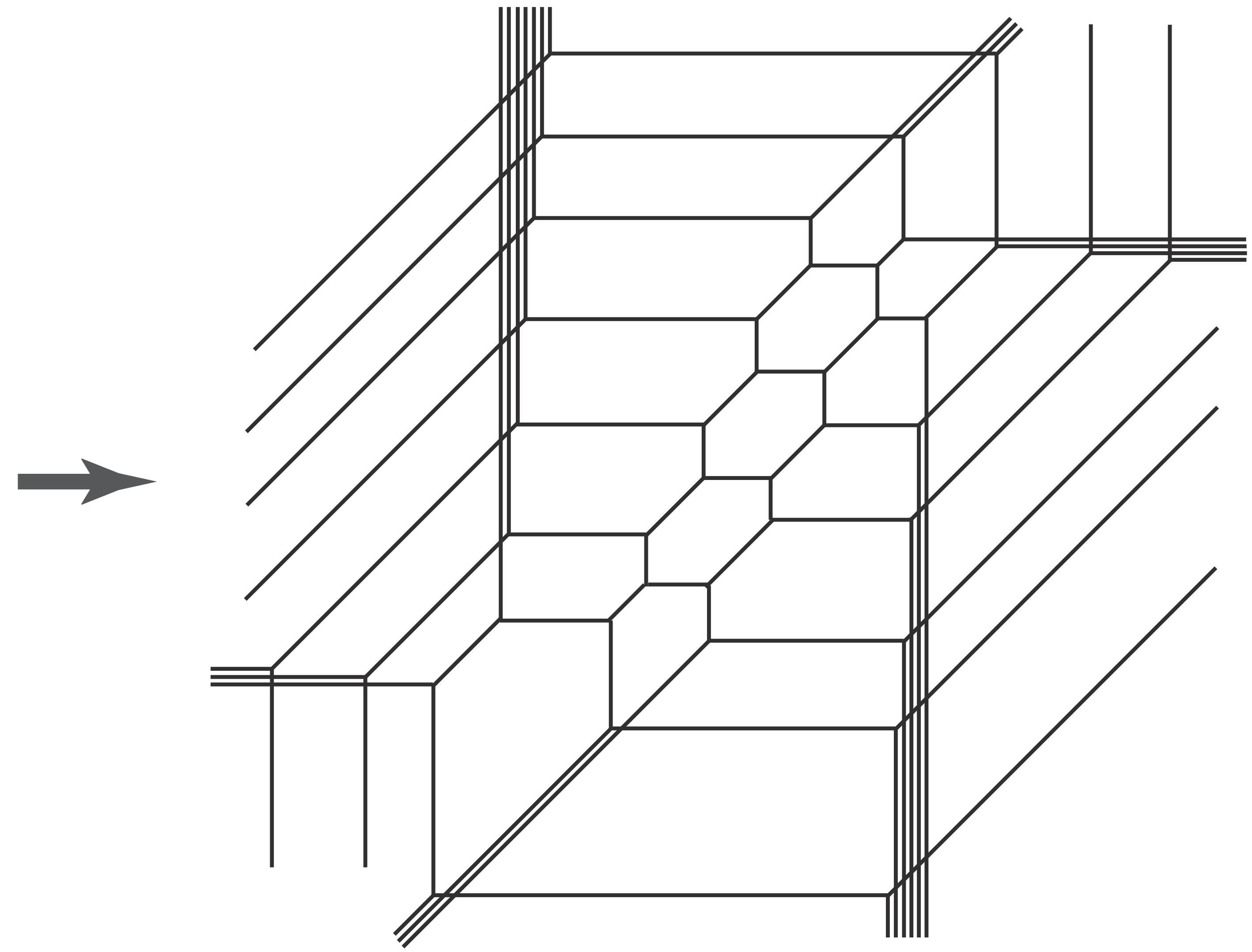
its 5d descriptions and dualities

What is 6d SCFT for this Tao?

5d SU(N) **N_f=2N+4**

arXiv:1505.04439





Conjecture

5d $\mathcal{N}$=1 SU(N) w/ Nf=2N+4 has 6d UV fixed point

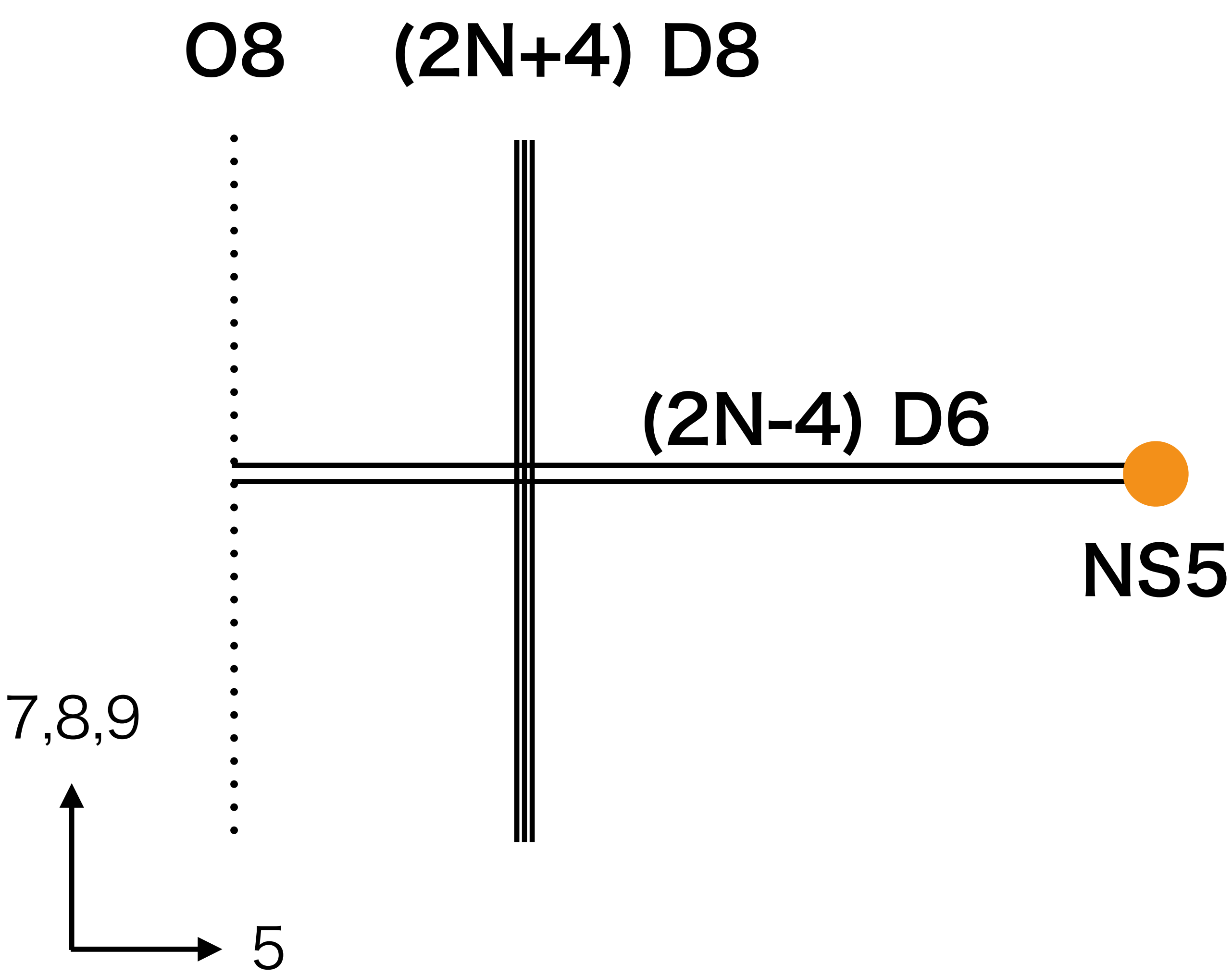
M5-brane probing D_{N+2} singularity
"(D_{N+2}, D_{N+2}) conformal matter"

[Del Zotto - Heckman - Tomasiello - Vafa '14]

M5-brane probing D_{N+2} singularity

Tensor branch

$$6\text{d } \mathcal{N}=(1,0)\ Sp(N-2) \text{ gauge theory}$$
$$N_f = 2N+4, \text{ w/tensor multiplet}$$

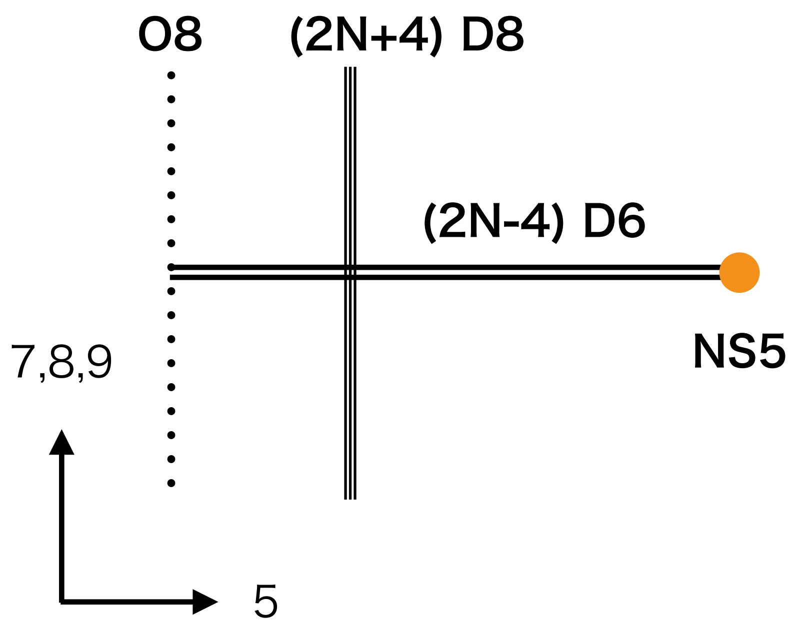


S^1

	0	1	2	3	4	5	6	7	8	9
D6-brane	×	×	×	×	×	×	×			
NS5-brane	×	×	×	×	×		×			
D8-brane	×	×	×	×	×		×	×	×	×
O8-plane	×	×	×	×	×		×	×	×	×

[Brunner, Karch ’97, Hanany, Zaffaroni ’97]

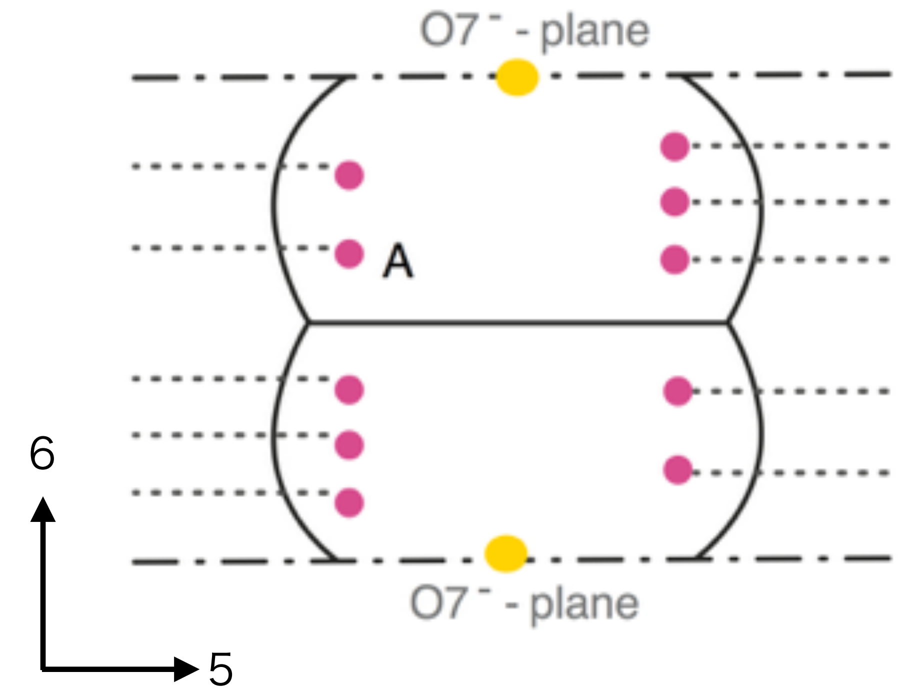
Diagrammatic "Derivation"



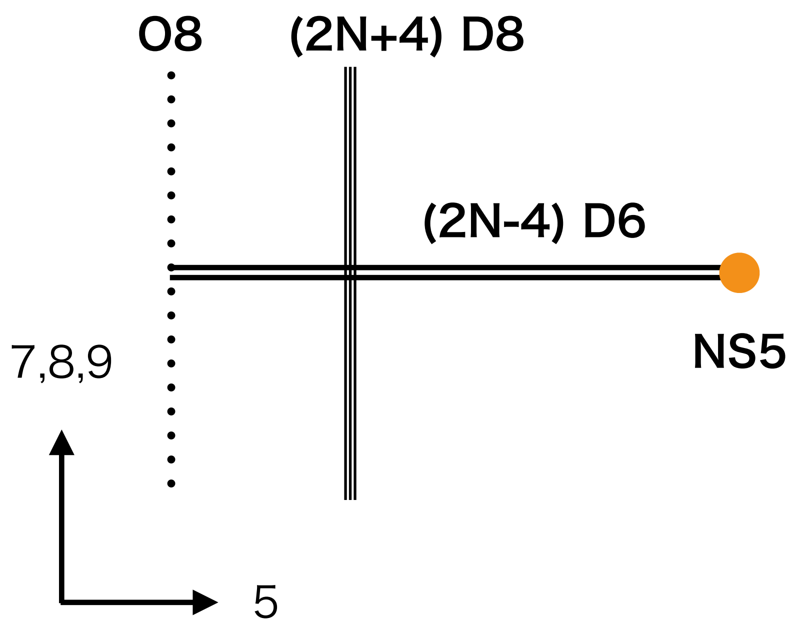
T-duality

(N=3)

$$6\text{d}\ Sp(1)\ N_f = 10$$



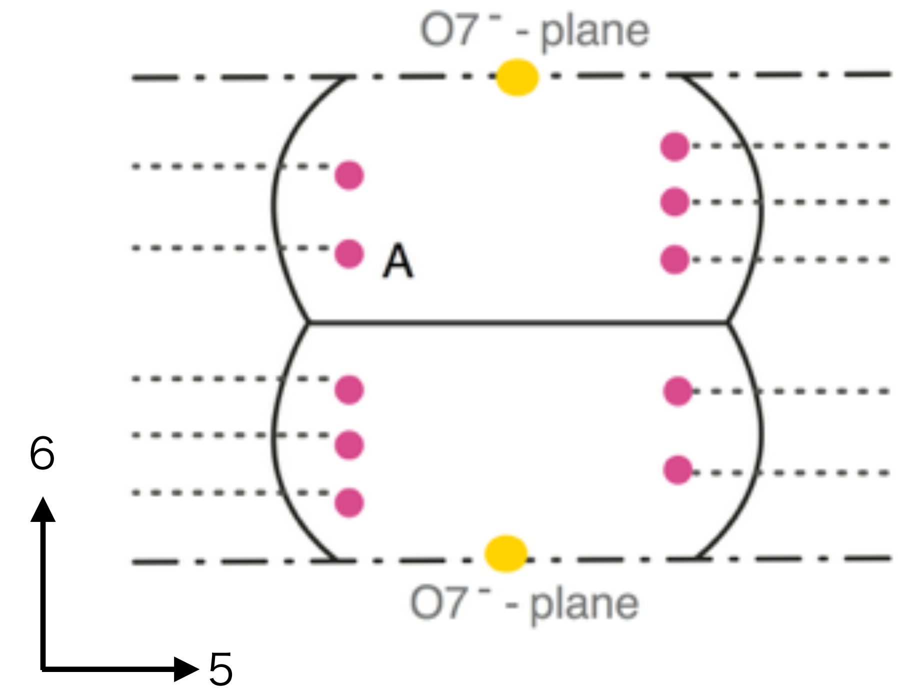
Diagrammatic “Derivation”



T-duality

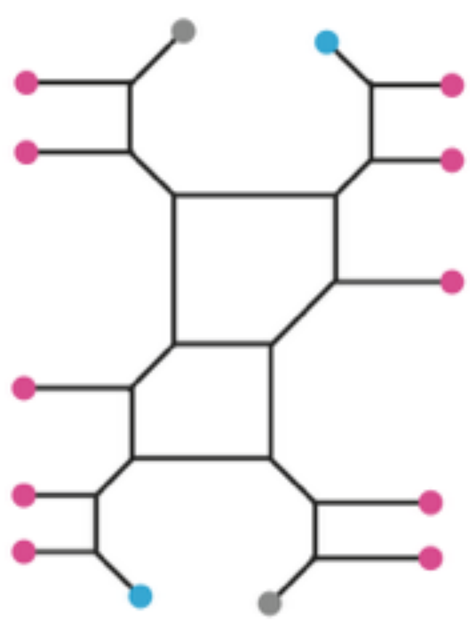
(N=3)

6d $Sp(1)$ $N_f = 10$

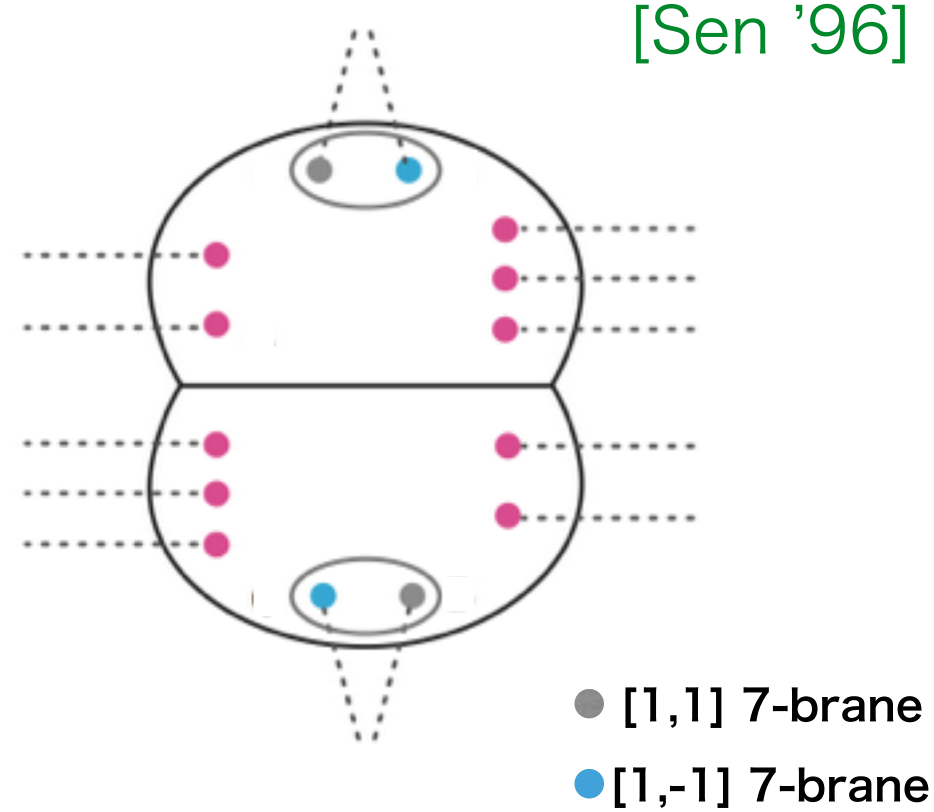


O7$^-$ -plane
= [1,1] 7-brane
+ [1,-1] 7-brane
[Sen ’96]

5d $SU(3)_0$ $N_f = 10$



Hanany-Witten transition

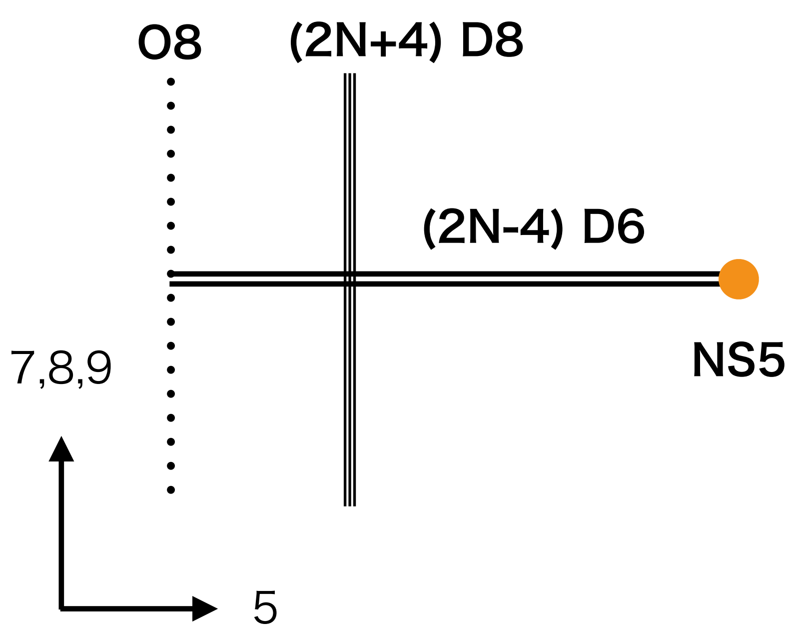


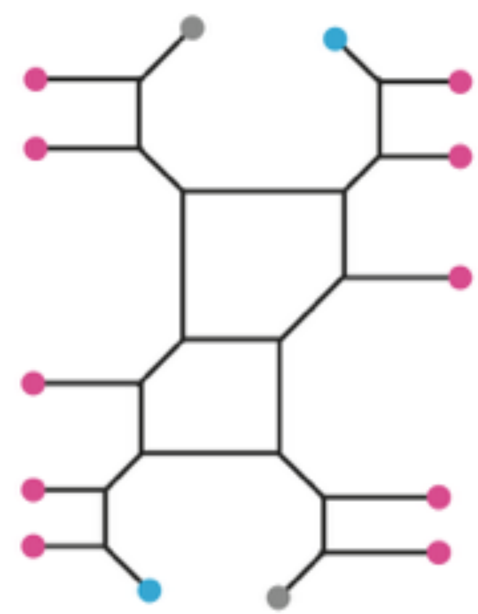
M5-brane probing
D_{N+2} singularity

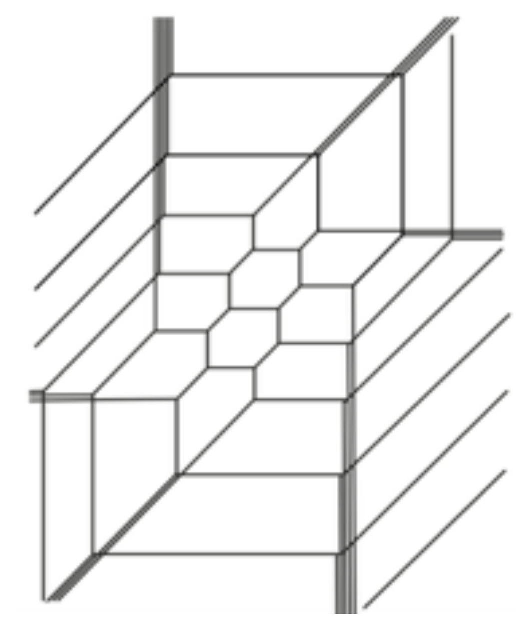
6d $Sp(N-2)$
$N_f = 2N+4,\ T$

5d $SU(N)_0\ N_f = 2N+4$

Tao diagrams





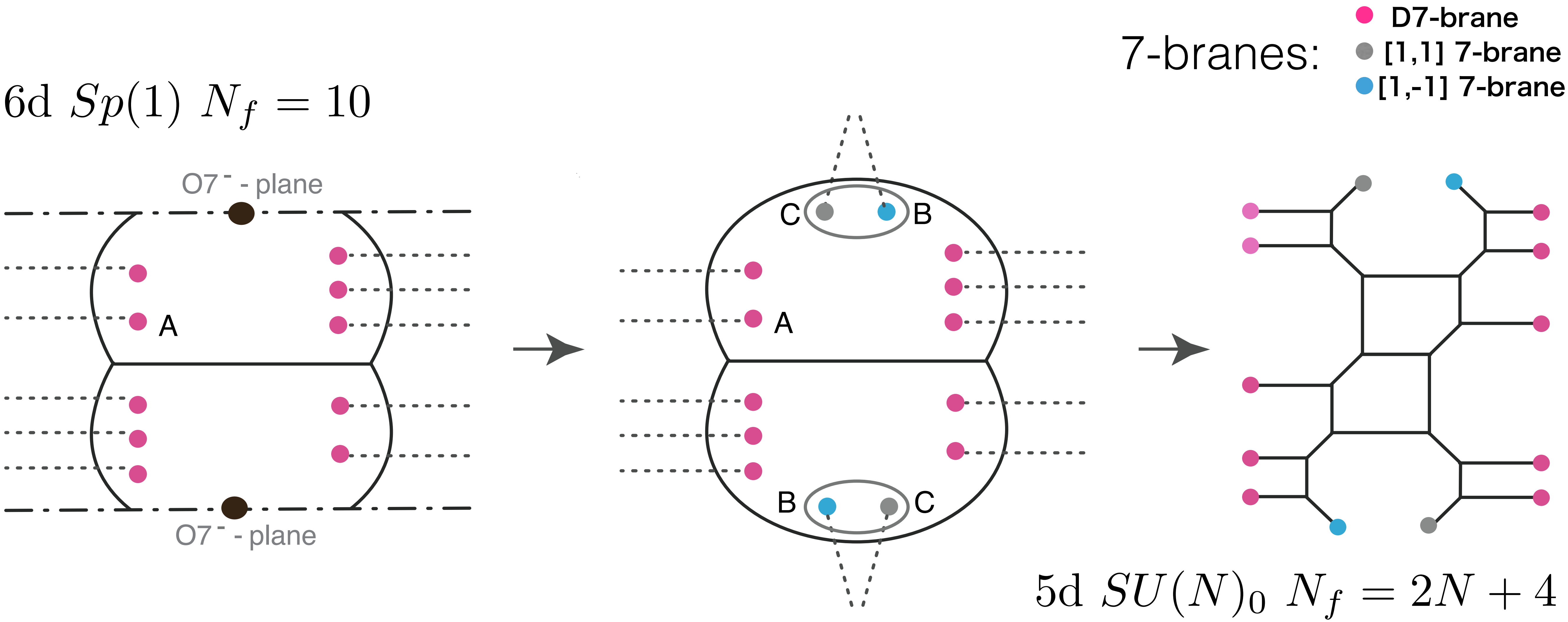


“SU-Sp duality”

6d Sp(N-2) theory with **N_f= 2N+4, a tensor**

resolve **two O7⁻'s**

5d $SU(N)_0$ theory with **N_f= 2N+4**



[Hayashi-SSK-Lee-Taki-Yagi '15]

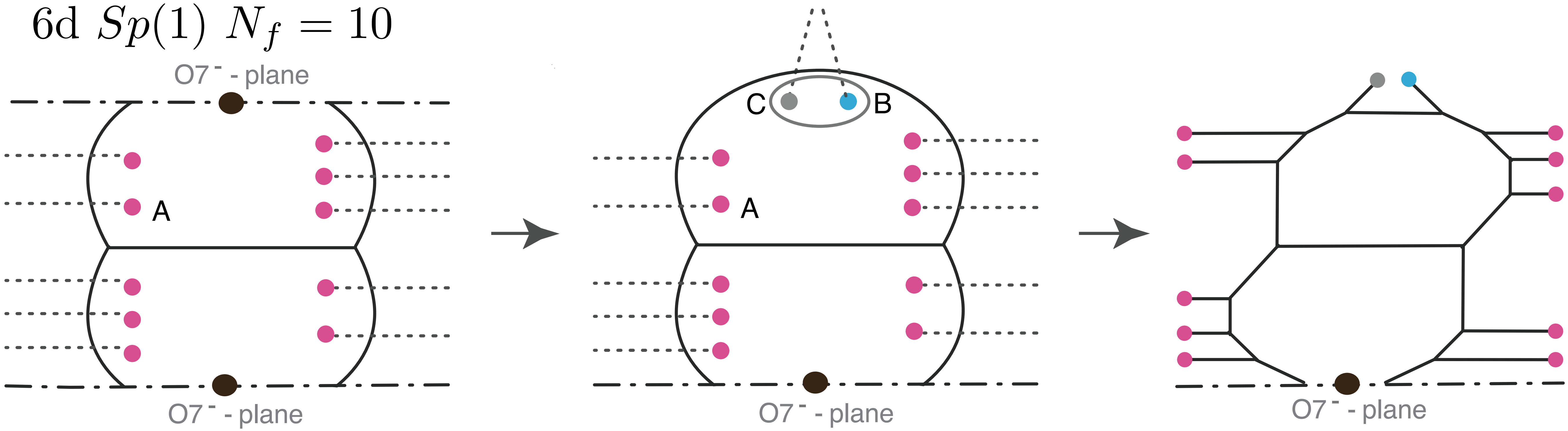
[Yonekura '15]

5d Sp(N-1) theory with **N_f= 2N+4**

Resolving **only one O7⁻**:

[Hayashi-SSK-Lee-Yagi '15]

6d $Sp(1)\ N_f = 10$



5d $Sp(2)\ N_f = 10$

We thus have **SU-Sp duality**

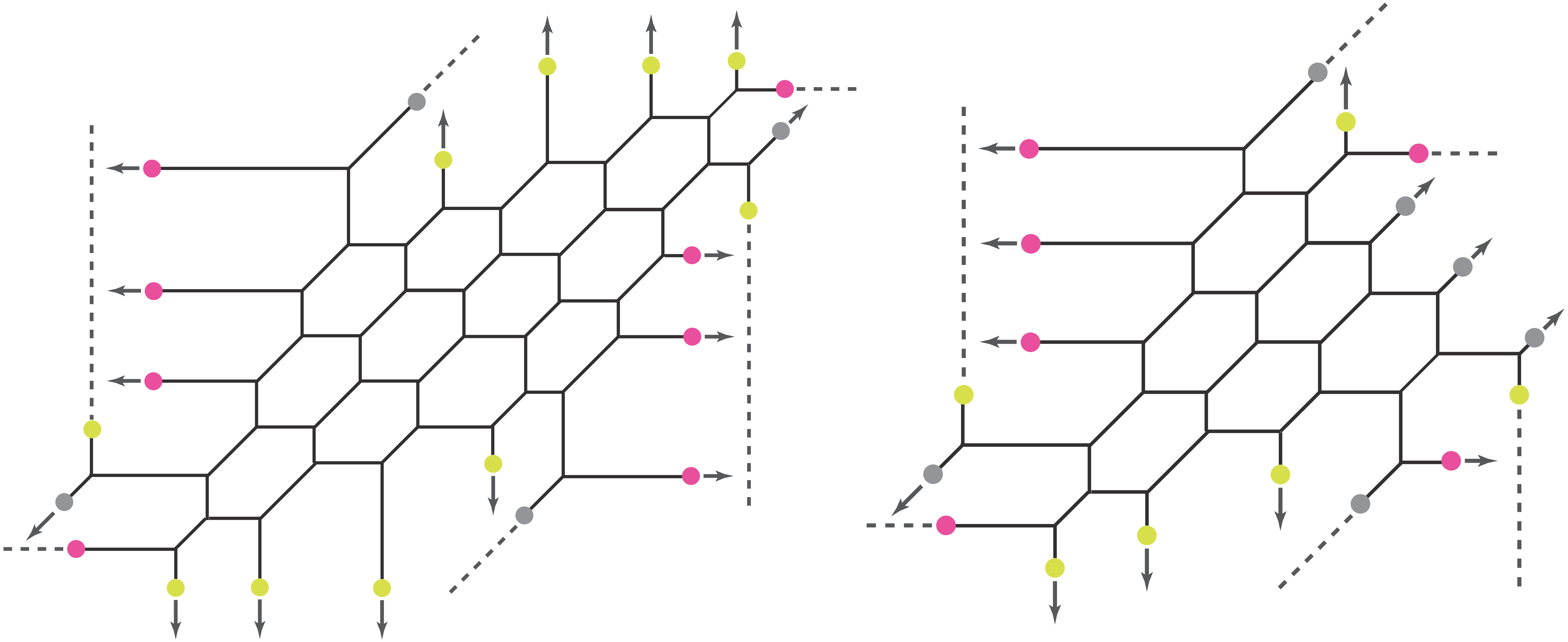
5d SU(N) theory ⟷ **5d Sp(N-1) theory**

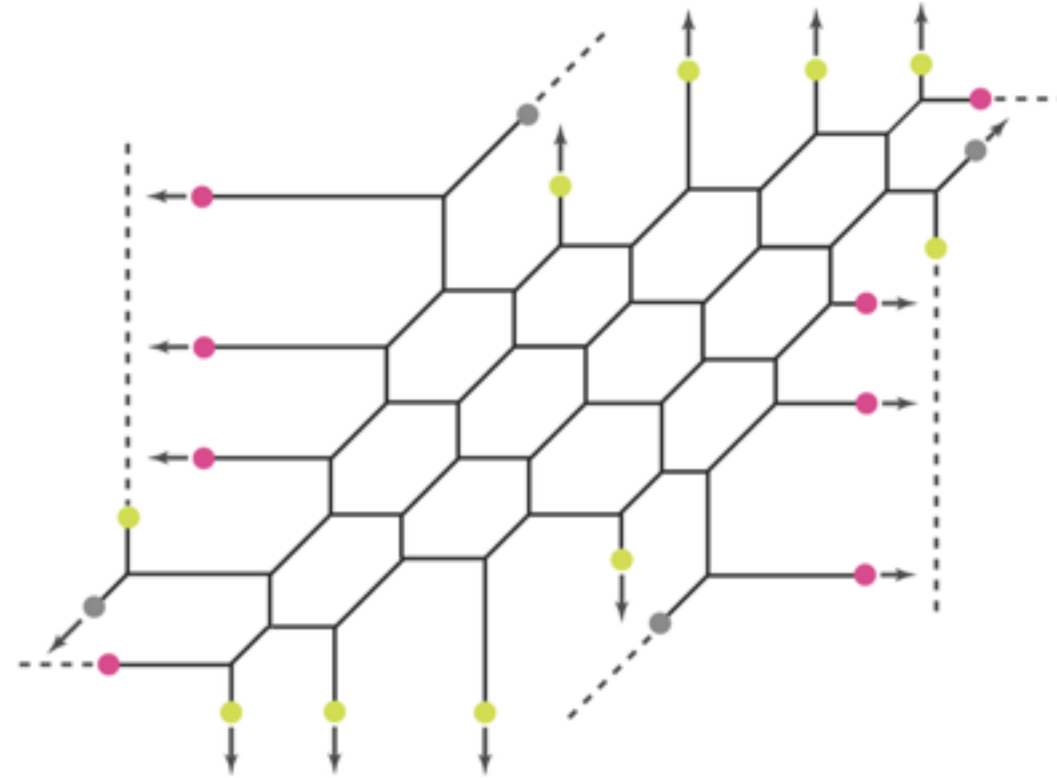
N_f= 2N+4 ⟷ **N_f= 2N+4**

both O7s — only one O7

Flavor decoupling —> 5d dualities [Gaiotto-Kim '15]

Quiver type?





$$\text{5d } [N+2] - \overbrace{SU(N) - \cdots - SU(N)}^{k} - [N+2]$$

['15 Zafrir]

['15 Yonekura]

$k = 2n+1$

$$\text{6d } Sp(N') - SU(2N'+8) - SU(2N'+16) - \cdots - SU(2N'+8(n-1)) - [2N'+8n]$$

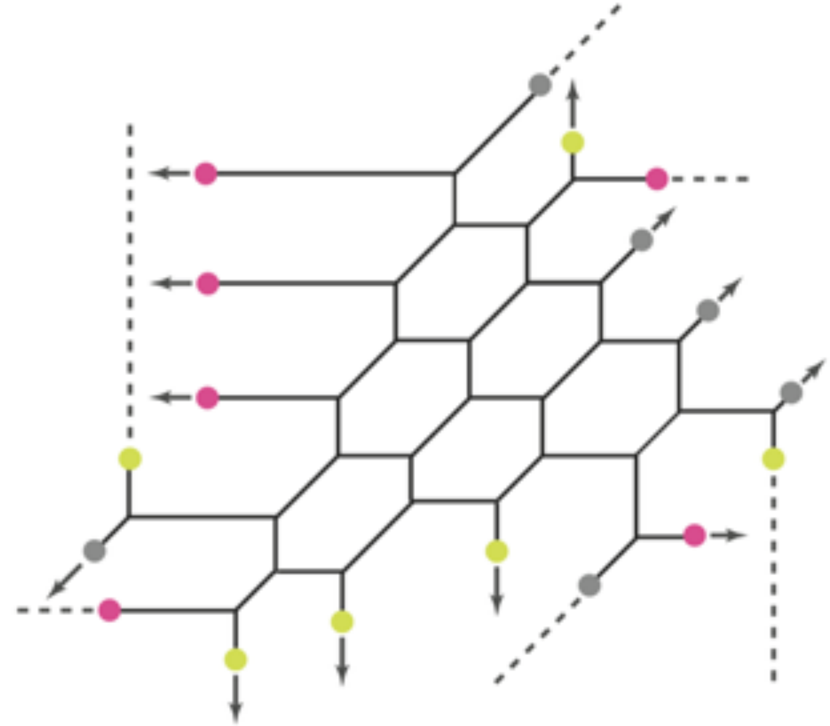
$$N' = N - 2n$$

$k = 2n$

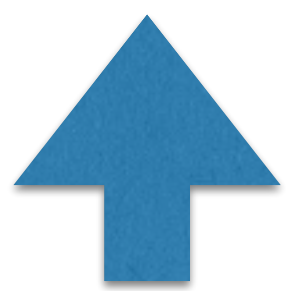
$$\text{6d } [A] - SU(N') - SU(N'+8) - SU(N'+16) - \cdots SU(N'+8(n-1) - [N'+8n+8]$$

$$N' = 2(N-2n+1)$$

hypermultiplet in
antisymmetric representation



$$\text{5d } [N+3] - SU(N) - SU(N-1) - SU(N-2) - \cdots - SU(3) - SU(2) - [3]$$



['15 Zafrir]

['15 Ohmori, Shimizu]

$$N = 3n: \quad \text{6d } SU(0) - SU(9) - \cdots - SU(9n) - [9n+9]$$

$$N = 3n+1: \quad \text{6d } SU(3) - SU(12) - \cdots - SU(3+9(n-1)) - [3+9n]$$

$$N = 3n+2: \quad \text{6d } \left[\tfrac{1}{2}\right]_{\Lambda^3} - SU(6) - SU(15) - \cdots - SU(6+9(n-1)) - [6+9n]$$

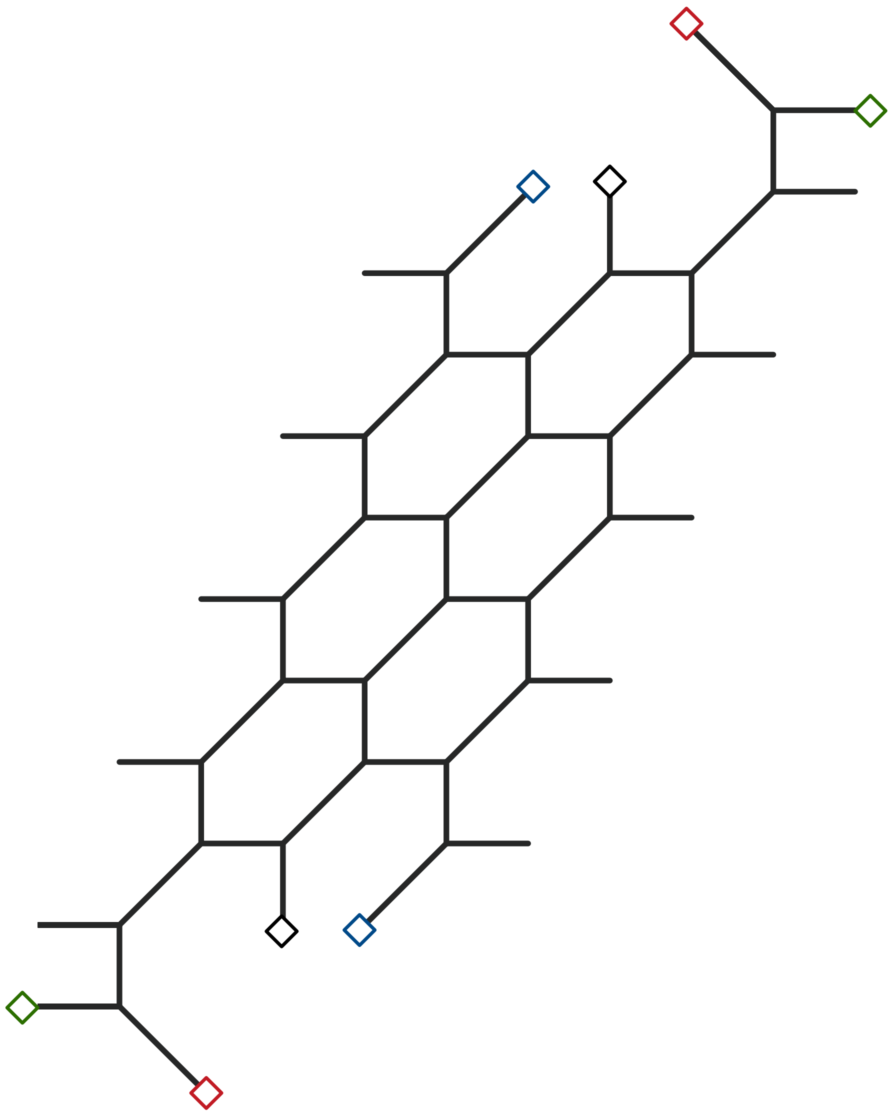
UV dualities

Multiple 5d gauge theories

have

an **identical** 6d UV fixed point

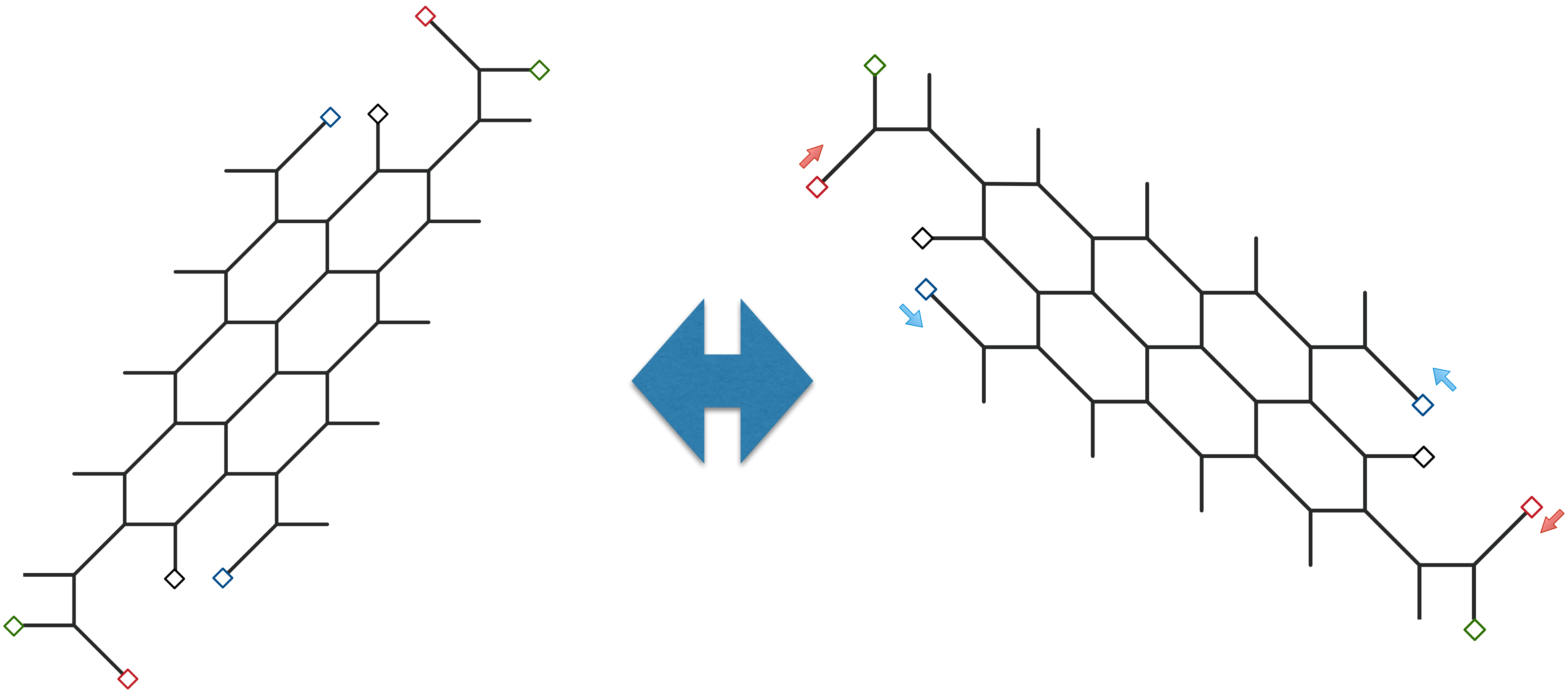
UV Dualities



5d [6]-SU(4)-SU(4)-[6]

6d [A]-SU(6)-[14]

S-duality

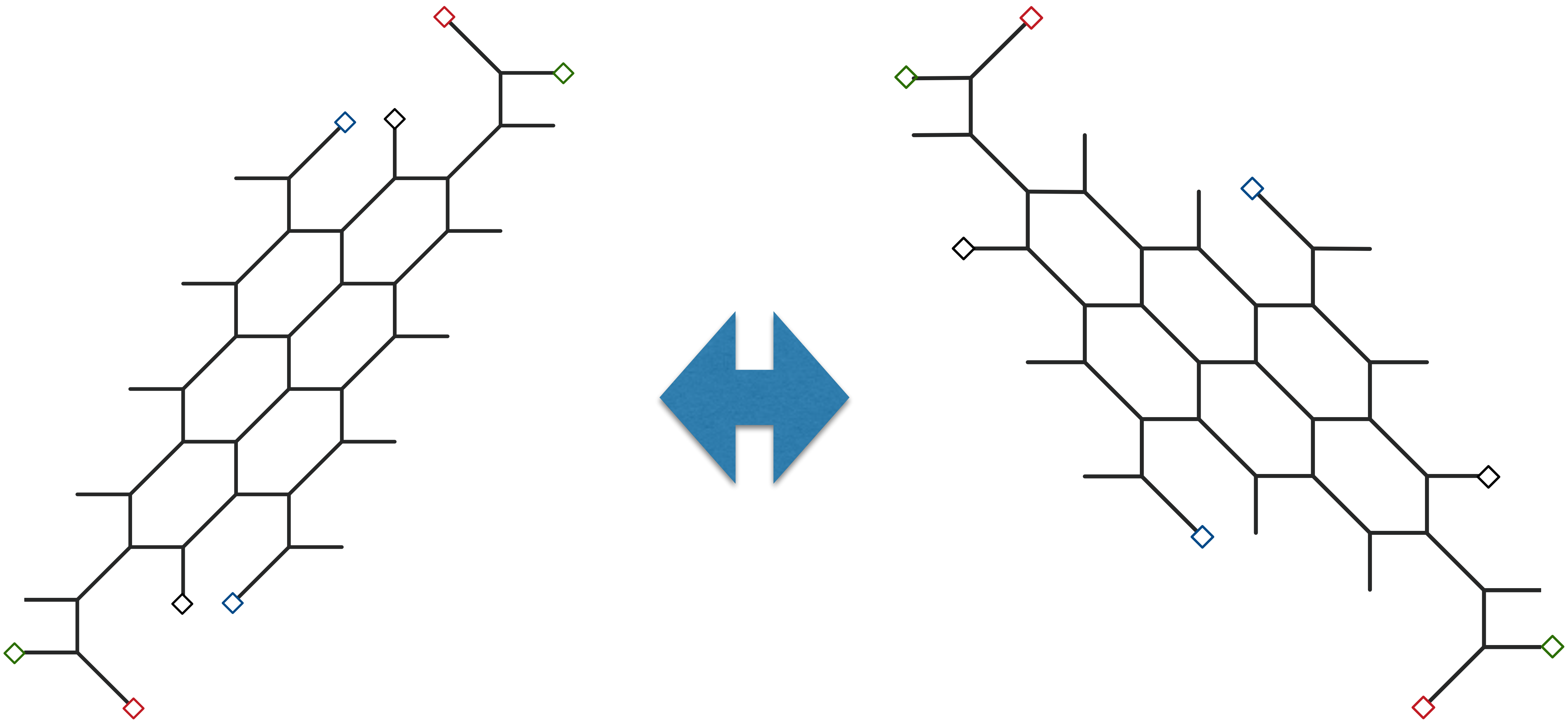


5d [6]-SU(4)-SU(4)-[6]

5d (?)-SU(3)-SU(3)-SU(3)-(?)

6d [A]-SU(6)-[14]

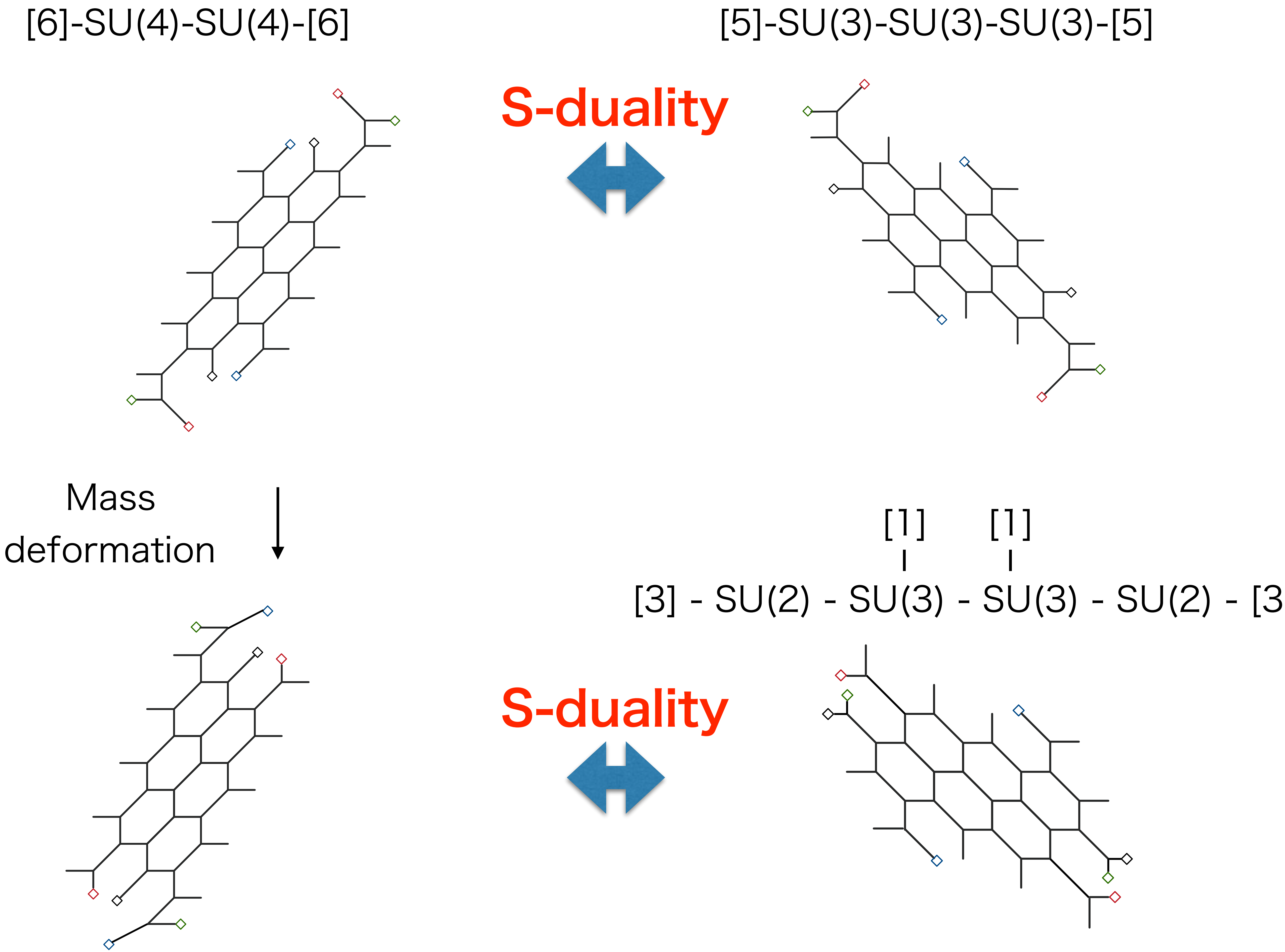
S-duality



5d [6]-SU(4)-SU(4)-[6]

5d [5]-SU(3)-SU(3)-SU(3)-[5]

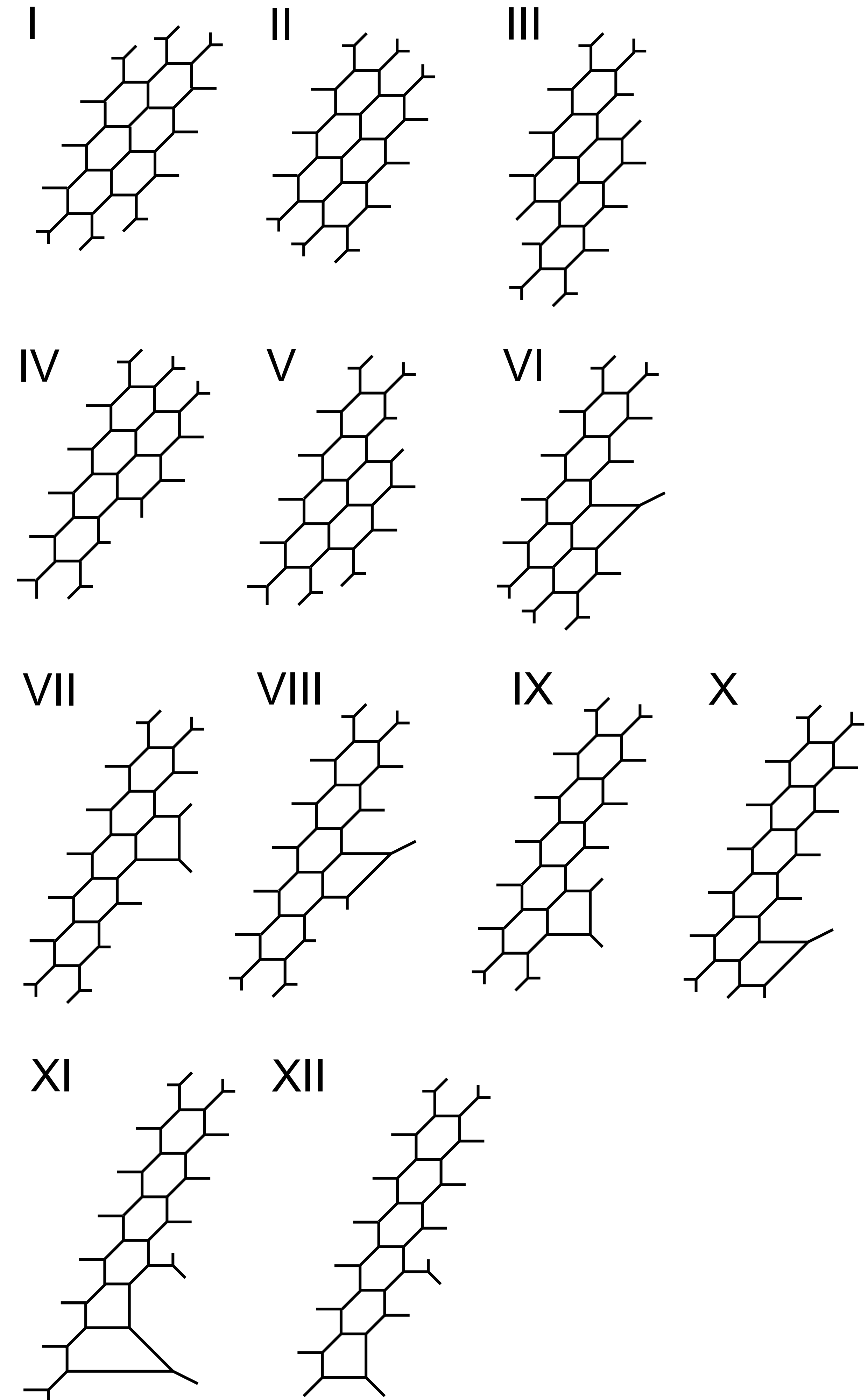
6d [A]-SU(6)-[14]



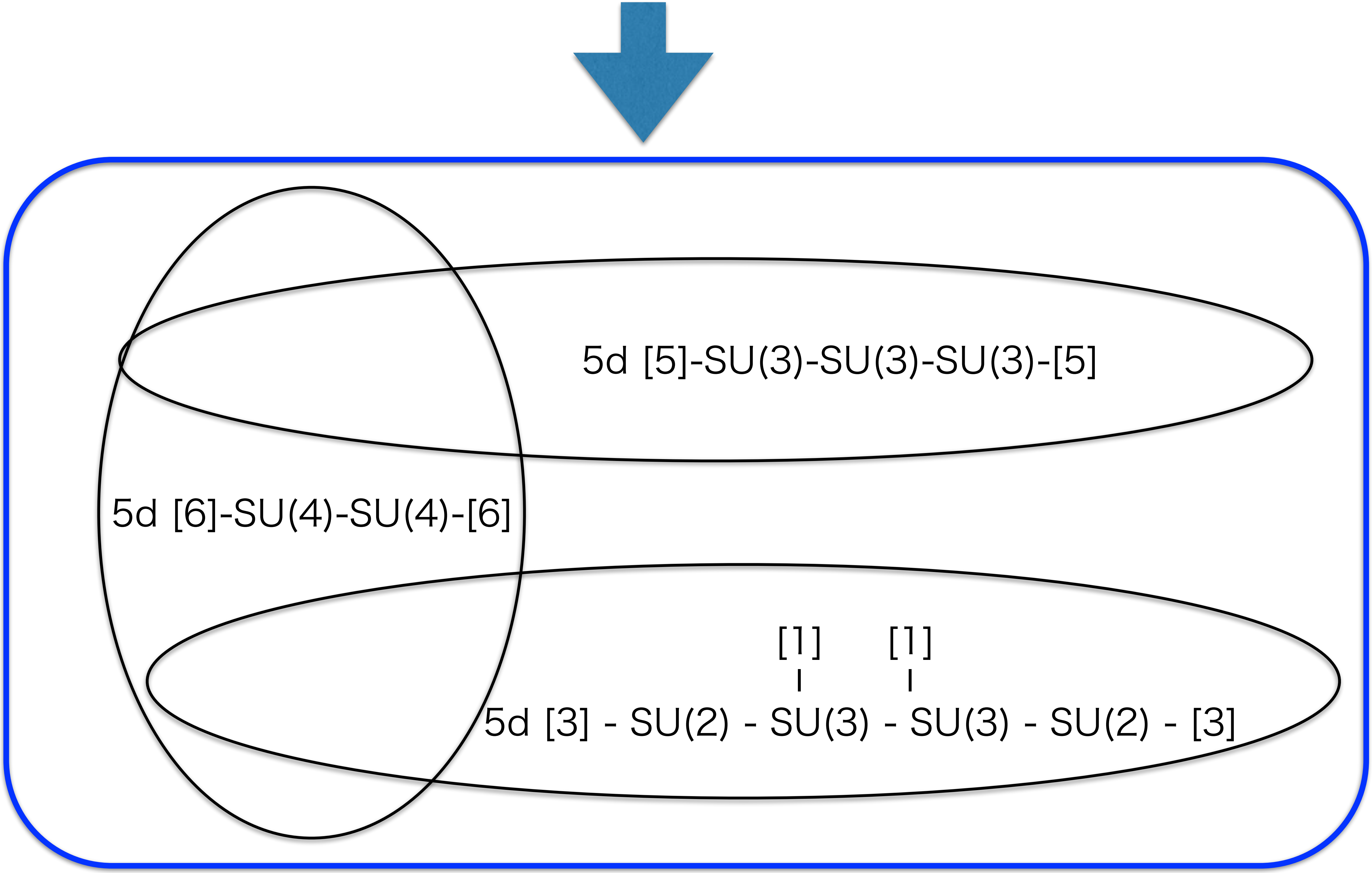
[6]-SU(4)-SU(4)-[6]
[5]-SU(3)-SU(3)-SU(3)-[5]
S-duality
Mass deformation
[1]
[1]
[3] - SU(2) - SU(3) - SU(3) - SU(2) - [3]
S-duality

6d SU(6) theory
with Nf=14, Na=1

various 5d quivers
(depending on
D5, D7 distributions)
Wilson lines



6d SU(6) 14 flavors, antisym. tensor + tensor mult.



All possible values of “gauge theory parameters”

Conclusion

6d SCFTs

5d descriptions

IIA brane setup with $O8^-$

ON^0 - $O6^{-/+}$ - $O8^-$

5d UV dualities

Tao diagram

new perspective

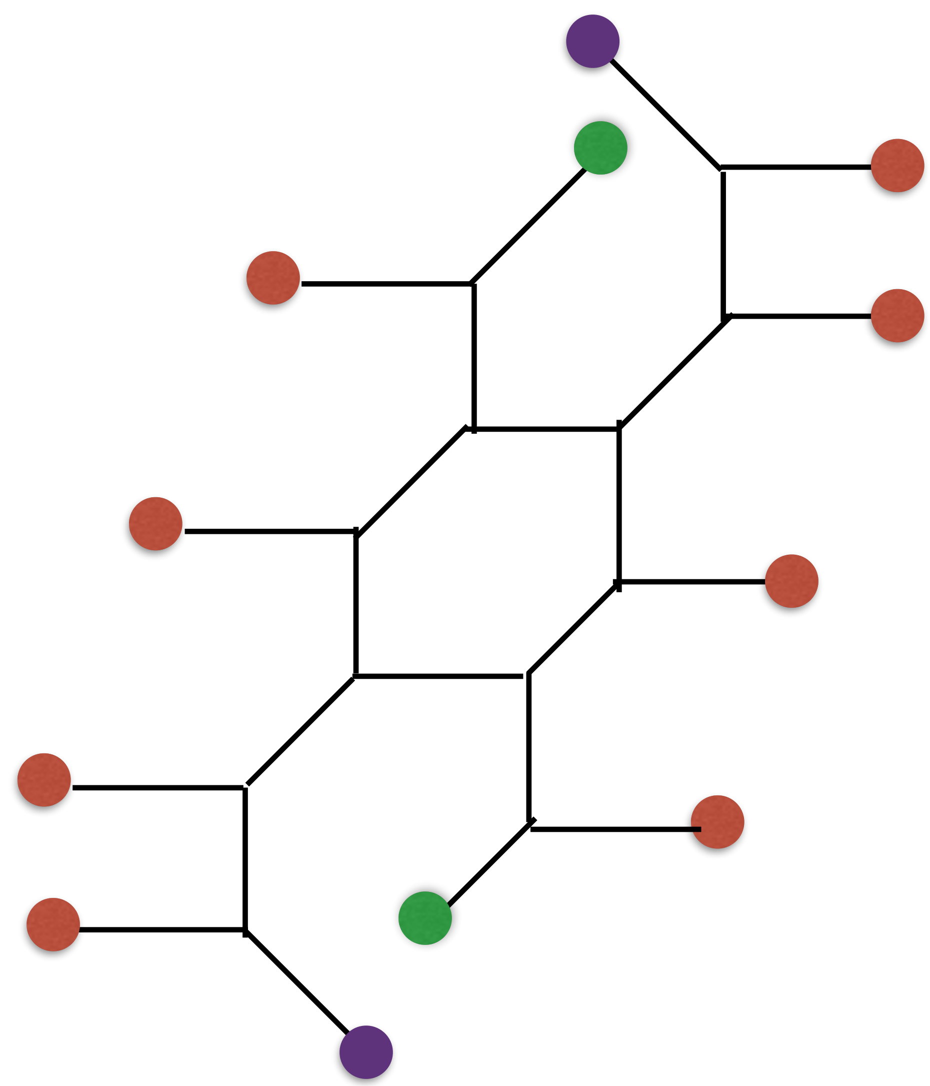
O7 into two 7-branes

S-duality

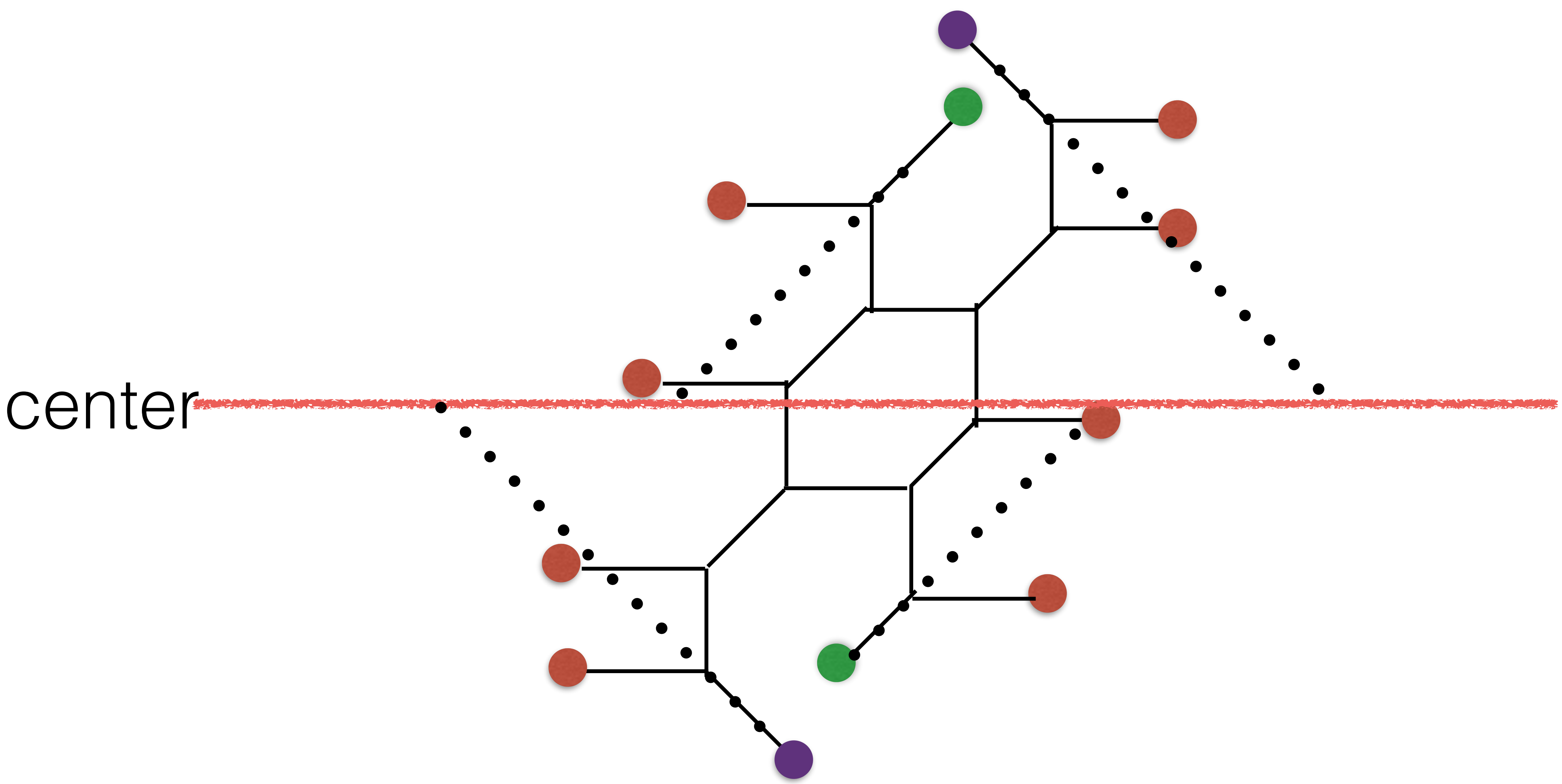
Wilson lines

Extra slides

N_f=8 : eight D5 branes or D7 branes (red dots)

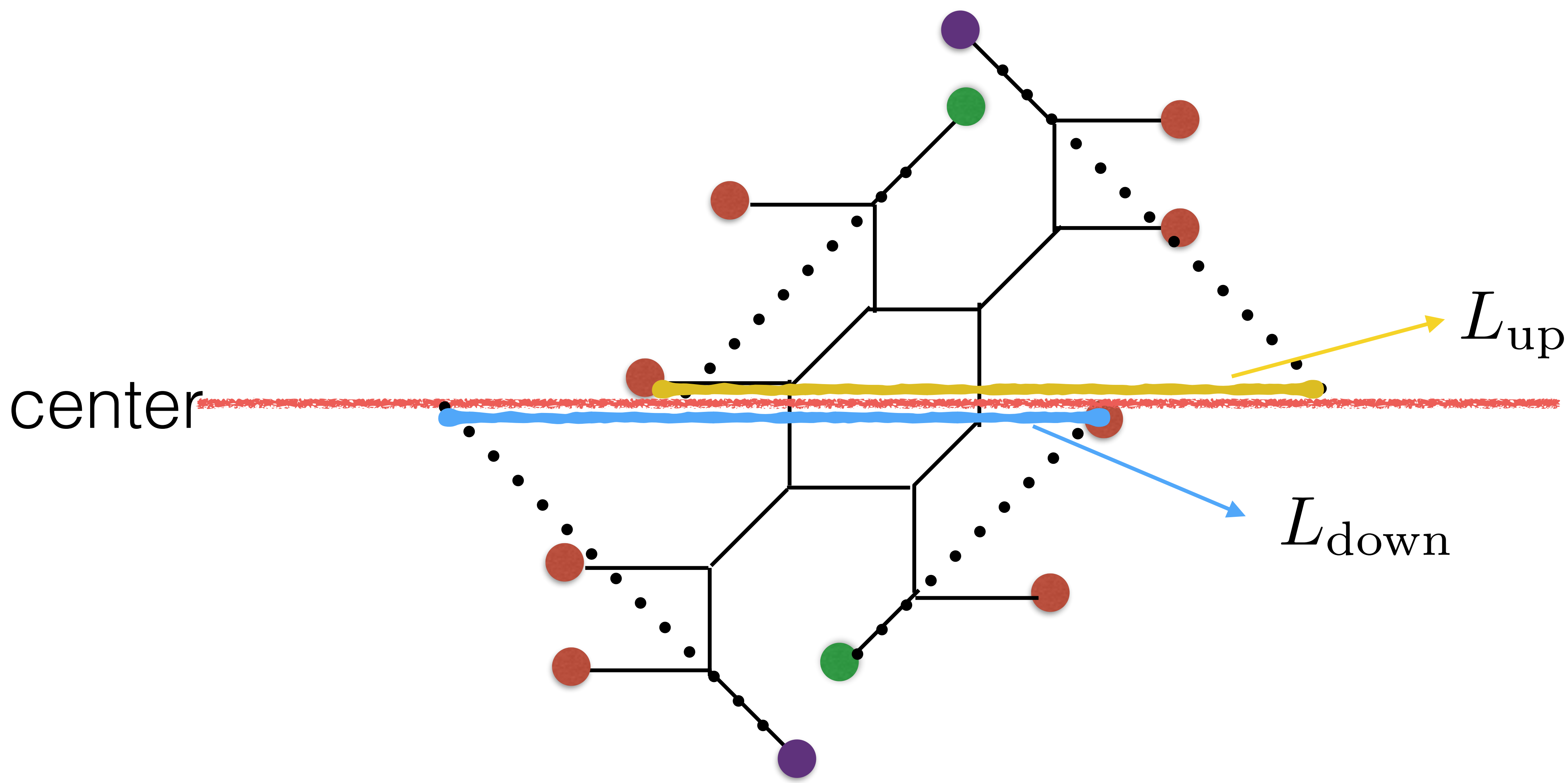


N_f=8 : eight D5 branes or D7 branes (red dots)



Instanton factor

Nf=8 : eight D5 branes or D7 branes (red dots)

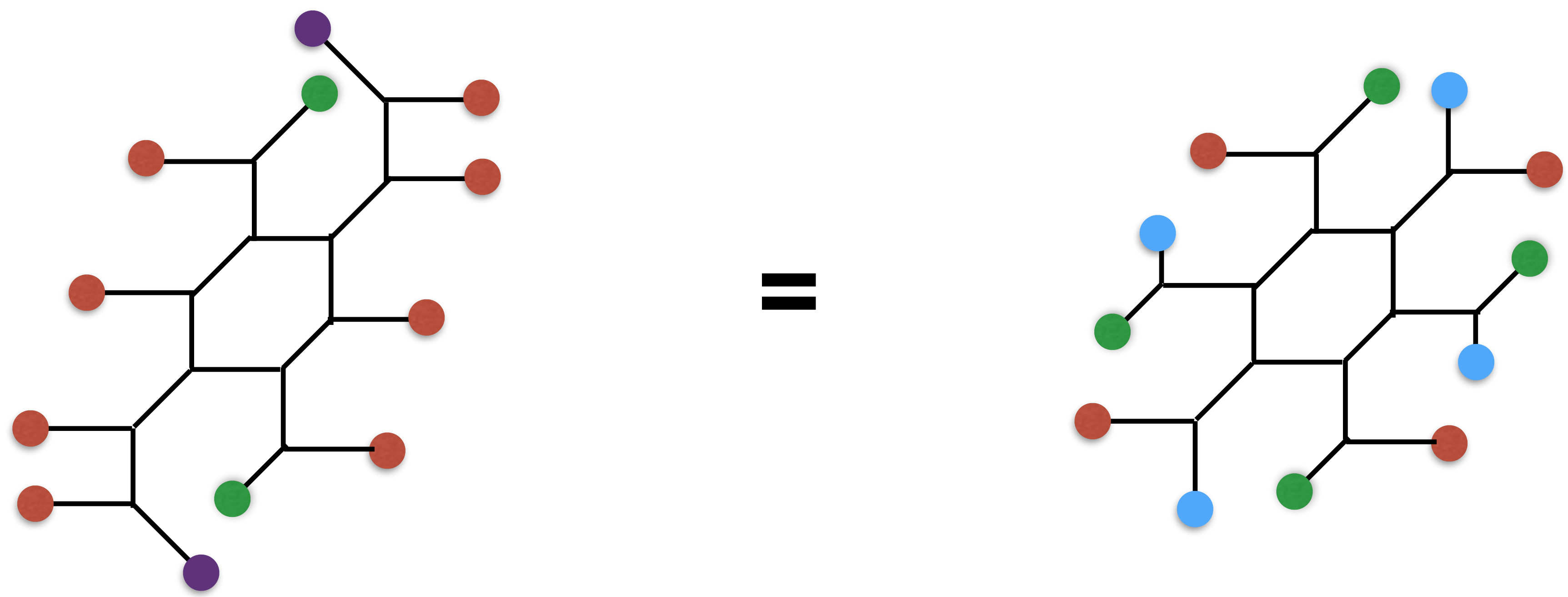


Instanton factor

$$q = e^{-size} = e^{-\frac{1}{g^2}} = (L_{\rm up}L_{\rm down})^{\frac{1}{2}}$$

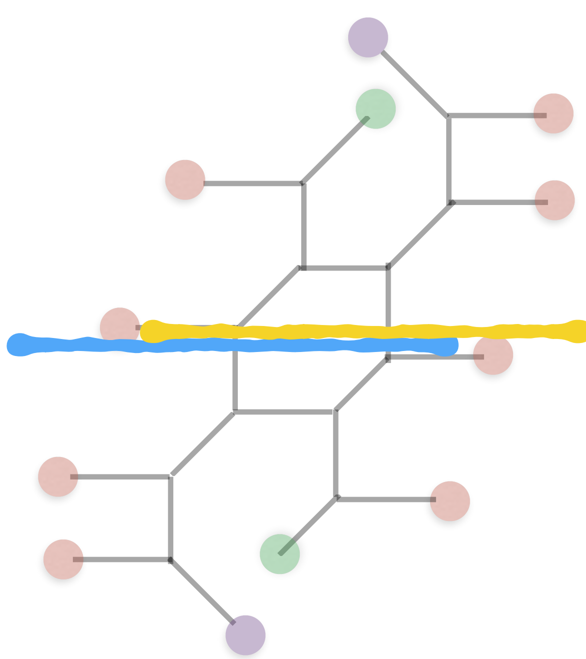
Another Tao diagram for SU(2) gauge theory with **8 flavors**

From Hanany-Witten transition



Another Tao diagram for SU(2) gauge theory with **8 flavors**

Instanton factor



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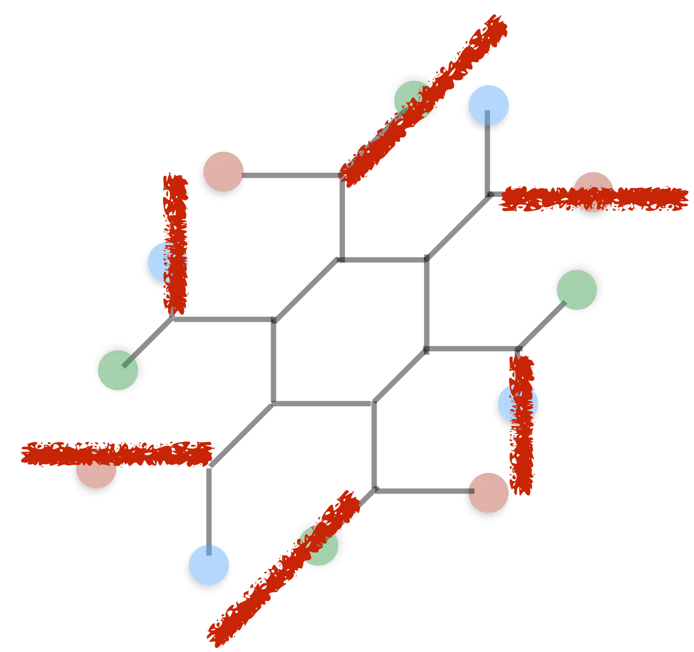

$$\boldsymbol{q = e^{-size} = e^{-\frac{1}{g^2}}}$$

$$= (L_{\mathrm{up}} L_{\mathrm{down}})^{\frac{1}{2}}$$

$$q^2$$

period = instanton

$$\prod_{i=1}^{6} \boldsymbol{\Delta}_i = q^2$$

$$q = e^{-size} = e^{-\frac{1}{g^2}}$$