제 10 회 고등과학원 기하학 겨울 학교

2015년 2월 1일 ~ 6일

2월 2일 월요일

양 성 덕 (고려대학교)

미분 기하의 계산에 Mathematica 활용하기

미분기하에서 발생하는 긴 계산들을 Mathematica의 Symbolic Computing 함수꾸러미를 이용하여 수행하는 방법에 대하여 설명합니다.

구체적인 예로는, 일반 상대론에 등장하는 여러 가지 우주 모델들(Schwarzschild metric, de Sitter metric, anti de Sitter metric, Kerr metric, 등등)이 아인슈타인의 방정식을 만족함을 Symbolic Computing 함수꾸러미를 통하여 살펴봅니다.

실습을 하고자 하는 분은 Mathematica와 Symbolic Computing 함수꾸러미 둘 다를 쓰실 수 있어야 합니다. Mathematica는 다음과 같이 두 가지 방법으로 체험판을 사용할 수 있습니다.

(1) https://www.wolfram.com/mathematica/trial/ 에서 15일 체험판 내려 받아 사용하는 방법. 이름과 이메일을 등록해야 하고 파일 불러오고 저장하는 것, Symbolic Computing 함수꾸러미를 사용하는 것은 가능하나 데이터를 불러오는 것은 불가능합니다.

(2) 다한테크에서 제공하는 30일 사용권한을 이용하는 방법.

먼저 다음 사이트에서 파일을 내려 받아 설치합니다.

http://dahanhard.iptime.org/MathematicaFiles/mathinstall/Mathematica10WIN.exe

http://dahanhard.iptime.org/MathematicaFiles/mathinstall/Mathematica10MAC.dmg

설치 마지막 단계에서 Activation Key를 넣으라고 할 텐데, 2월 2일 등록현장(이나 또는 강의현장)에서 Activation Key를 나눠드리겠습니다. 이는 2015년 2월 12일까지 사용 가능한 Key입니다. 사용시 사용자 등록을 하라는 문구가 뜰 지도 모르는데 등록을 안 하셔도 사용하는 데는 문제가 없을 거라고 합니다.

마지막으로, Symbolic Computing 함수꾸러미는 https://symbcomp.gist.ac.kr/ 에서 내려 받아 설치하시면 됩니다. 설치방법은 그 사이트에 자세히 나와 있습니다.

최 영 준 (고등과학원)

An introduction to the Kähler-Einstein metric

The Kähler-Einstein metric is the Kahler metric whose Ricci curvature is parallel to the metric. This metric was first constructed on canonically polarized compact Kähler manifolds by Audin. After that, Yau constructed the Kähler-Einstein metric on Kähler manifolds whose first Chern class vanishes.

In this talks, we discuss about the basic properties and the constructions of Kähler-Einstein metric. And we also discuss about the variations of Kähler-Einstein metrics.

황 승 수 (중앙대학교)

Linearization of the scalar curvature

On a compact n-dimensional manifold, it has been conjectured that a critical point metric of the total scalar curvature, restricted to the space of metrics with constant scalar curvature of unit volume, will be Einstein. This conjecture was proposed in 1984 by Besse, but has yet to be proved. In this talk, we survey the recent results on this conjecture. Moreover, we discuss the possible breakthrough of the conjecture through the related problems.

김 동 수 (전남대학교)

Geodesic chord property and isoparametric hypersurfaces of space forms

It is well-known that a circle is characterized as a closed plane curve such that the chord joining any two points on it meets the curve at the same angle at the two points. Generalizing this property, we study hypersurfaces in non-flat space forms satisfying the so-called *geodesic chord property*. As a result, we completely classify such hypersurfaces.

2월 3일 화요일

표 준 철 (부산대학교)

Minimal surfaces in the sphere

Minimal surfaces in the sphere are quite different from ones in Euclidean space or the hyperbolic space. First, we introduce some of the classical minimal surfaces in the sphere. Then, we introduce uniqueness theorems for minimal surfaces in the sphere of genus 0 and 1 such as the work of Almgren and Brendle.

한 강 진 (DGIST)

Geometry of Symmetric tensors and secant varieties of Veronese embeddings

Let V_i be a finite-dimensional vector space. The tensor product of d vector spaces $V_1, \ldots, V_d$ is a basic mathematical object, which appears in most of all the fields both in mathematics and sciences and is extremely useful in many applications.

We call an element of this tensor product a tensor. After basis have been fixed, a tensor can be represented by a multidimensional matrix, with d modes (or d ways). The case $d=2$ correspond to usual matrices. The case $d>2$ has some analogies and many differences with the case of matrices. The rank of a given tensor is defined as the minimum number of summands needed to express it as the sum of decomposable ones. The decomposable tensors naturally correspond to the Segre variety and so do those who are symmetric to the Veronese variety. Thus, the study of symmetric tensor rank could have a geometrical counterpart in the study of secant varieties to the Veronese variety, which is in the realm of Algebraic Geometry.

신 형 석 (서울대학교)

Holomorphic orbi-spheres in elliptic $\mathbb{P}^1$ orbifolds

The Gromov-Witten theory of orbifolds was introduced in the symplectic setting by Weimin Chen and Yongbin Ruan. In this talk, I will start with a brief introduction to orbifolds and orbifold Gromov-Witten invariants. Then, I will compute 3-point genus-0 Gromov-Witten invariants of elliptic $\mathbb{P}^1$ orbifolds by directly counting holomorphic orbi-spheres. This is a joint work with Hansol Hong.

최 인 송 (건국대학교)

Chasles 정리와 타원곡선의 군 구조

Chasles 정리는 두 3차 곡선이 만나는 9개 교점의 성질에 대한 정리로서, 고전 사영기하학의 Pappus 정리와 Pascal 정리를 새로운 방식으로 일반화한다. 또한 Chasles 정리를 이용하면 타원곡선 상의 덧셈 연산의 결합법칙을 쉽게 설명할 수 있다. 이 강의에서는 이러한 내용들을 살펴보고 보다 일반화된 Cayley-Bacharach 정리를 설명하고자 한다.

고 성 은 (건국대학교)

An elementary proof of the Poncelet Closure Theorem

An elementary proof of the Poncelet Closure Theorem will be given.

2월 4일 수요일

이 재 혁 (이화여자대학교)

Minimal Lagrangian Submanifolds I

In the first talk of two talks, we introduce the first variation formula of volume and apply it for minimal Lagrangian submanifolds. Moreover, we characterize so called hamiltonian minimal Lagrangians.

윤 갑 진 (명지대학교)

Harmonic Morphisms between Riemannian Manifolds

In this talk, we will introduce basic definitions and present fundamental properties on harmonic morphisms between Riemannian manifolds. The energy for a C^1 map $\varphi:(M^n,g)\to(N^m,h)$ between Riemannian manifolds is defined by

$$E(\varphi)=\frac{1}{2}\int_M |d\varphi|^2 dv_g,$$

where $|d\varphi|$ is the Hilbert-Schmidt norm of the differential $d\varphi$. When M is non-compact, the energy of φ can be defined locally on each bounded domain. We say a C^1 map $\varphi:(M^n,g)\to(N^m,h)$ is *harmonic* if it is a critical point of the energy functional $E(\varphi)$. The Euler-Lagrange equation for the energy functional is given by

$$\tau(\varphi):=-\delta d\varphi = tr\nabla d\varphi = 0.$$

Introducing local coordinates system, this can be written in the following form:

$$\tau(\varphi)^\alpha = \Delta\varphi^\alpha + \sum_{i,j=1}^{n}\sum_{\beta,\gamma=1}^{m} g^{ij},(\Gamma_N)^\alpha_{\beta\gamma}\frac{\partial\varphi^\beta}{\partial x^i}\frac{\partial\varphi^\gamma}{\partial x^j}=0.$$

A C^1 map $\varphi:(M^n,g)\to(N^m,h)$ is called a *harmonic morphism* if it is harmonic and preserves harmonic structures on (M,g) and (N,h). More precisely, φ is harmonic morphism if it is harmonic and satisfies the following property: for any harmonic function f on a set $V \subset N$ with $\varphi^{-1}(V) \neq \varnothing$, the composition $f \circ \varphi : \varphi^{-1}(V) \rightarrow \mathbb{R}$ is also harmonic. It is well-known that the composition $\psi \circ \varphi$ of two harmonic maps $\varphi : (M^n, g) \rightarrow (N^m, h)$ and $\psi : (N^m, h) \rightarrow (P^l, k)$ is not necessarily harmonic, but if both φ and ψ are harmonic morphisms, so is the composition $\psi \circ \varphi$. In this talk, we will present some fundamental properties, and show some rigidity or vanishing results on harmonic morphisms between Riemannian manifolds. Finally, we will introduce the notion of p-harmonic morphism as a generalization.

배 영 진 (기초과학연구원 기하학수리물리연구단 IBS CGP)

Legendrian singular links and singular connected sum

We study Legendrian singular links up to contact isotopy. By the special property of the singular point, we can define a singular connected sum of Legendrian singular links. This concept is a generalization of the connected sum and can be interpreted as the tangle replacement. This method provides a way to classify the Legendrian singular links.

2월 5일 목요일

이 재 혁 (이화여자대학교)

Minimal Lagrangian Submanifolds II

In the second talk of two talks, we introduce the second variation formula of volume and apply it for minimal Lagrangian submanifolds.

박 경 동 (고등과학원)

Deformation rigidity of odd Lagrangian Grassmannians

A study on deformations of the complex structure of complex manifolds goes back to Riemann. After one hundred years from Riemann, Kodaira and Spencer developed the deformation theory of higher dimensional compact complex manifolds. They showed that an infinitesimal deformation of a compact complex manifold should be represented by the Kodaira-Spencer class, which is an element of the first cohomology group with coefficients in the sheaf of germs of holomorphic vector fields. For a rational homogeneous manifold G/P for a complex simple Lie group G and a parabolic subgroup $P \subset G$, the Bott-Borel-Weil theorem and Kodaira-Spencer deformation theory imply the local deformation rigidity of rational homogeneous manifolds. Furthermore, Hwang and Mok have proved the global deformation rigidity of a rational homogeneous manifold of Picard number 1 different from the orthogonal Grassmannian $Gr_q(2,7)$. It is then natural to have questions about the deformation rigidity of quasi-homogeneous manifolds. Let V be a complex vector space endowed with a skew-symmetric bilinear form ω of maximal rank. When $\dim V$ is odd, say, $2n+1$, we call the variety $Gr_\omega(k,2n+1)$ of all k-dimensional isotropic subspaces of V as the odd symplectic Grassmannian, which is not homogeneous and has two orbits under the action of its automorphism group if $2 \leq k \leq n$. I explain the rigidity under Kähler deformation of the complex structure of odd Lagrangian Grassmannians, i.e., the Lagrangian case $Gr_{\omega}(n,2n+1)$ of odd symplectic Grassmannians. To obtain the global deformation rigidity of the odd Lagrangian Grassmannian, we use results about the automorphism group of this manifold, the Lie algebra of infinitesimal automorphisms of the affine cone of the variety of minimal rational tangents and its prolongations.

조 철 현 (서울대학교)

다항식을 행렬 인수 분해하는 기하학적 방법

다항식 f의 행렬 인수분해란, 다항식 곱하기 항등행렬을 두개의 행렬의 곱으로 분해하는 방법을 말하며, 이를 모으면 행렬 인수분해 범주를 만들 수 있다. 이러한 인수분해는 다항식이 정의하는 특이 구조와 관련이 있다. 평면위의 곡선으로부터, X 곱하기 항등행렬을 두 개의 다항식 행렬의 곱으로 나타내는 기하학적 방법을 알아보고, 이를 일반화하여, 기하학적 정보와 행렬인수분해 정보가 동치임을 제시하는 호몰로지 거울대칭 가설을 기하학적으로 이해하는 방법을 제시한다. (홍한솔, Siu-Cheong Lau와 공동연구)

김 호 일 (경북대학교)

Introduction to String theory

We want to introduce string theory from the elementary level. After explaining classical mechanics, quantum mechanics, classical field theory and quantum field theory, we will generalize to string involved theory. And then we will discuss on the underlying principle of this procedure from the viewpoints of physics and mathematics.

2월 6일 금요일

최 진 호 (동아대학교)

Associated curves of a Frenet curve in a 3-space

In this talk, we introduce some associated curves of a Frenet curve in a 3-dimensional Euclidean space. Indeed, we introduce the notions of Bertrand curves, Mannheim curves, principal-directional curves and principal-donor curves, and we give some advanced results.

변 태 창 (세종대학교)

Area and holonomy on the principal $U(n)$ bundles over Grassmannian manifolds

Consider the principal $U(n)$ bundles over Grassmann manifolds $U(n) \to U(n+m)/U(m) \xrightarrow{\pi} \mathrm{G}_{n,m}$. Given $X \in \mathrm{U}_{m,n}(\mathbb{C})$ and a 2-dimensional subspace $\boldsymbol{m}' \subset \boldsymbol{m} \subset \boldsymbol{u}(m+n)$, assume either $\boldsymbol{m}'$ is induced by $X, Y \in U_{m,n}(\mathbb{C})$ with $X^* Y = \mu I_n$ for some $\mu \in \mathbb{R}$ or by $X, iX \in U_{m,n}(\mathbb{C})$. Then $\boldsymbol{m}'$ gives rise to a complete totally geodesic surface S in the base space. Furthermore, let γ be a piecewise smooth, simple closed curve on S parametrized by $0 \le t \le 1$, and $\tilde{\gamma}$ its horizontal lift on the bundle $U(n) \to \pi^{-1}(\mathrm{S}) \xrightarrow{\pi} \mathrm{S}$, which is immersed in $U(n) \to U(n+m)/U(m) \xrightarrow{\pi} \mathrm{G}_{n,m}$. Then

$$\tilde{\gamma}(1) = \tilde{\gamma}(0) \cdot \left(e^{i\theta} I_n\right) \text{ or } \tilde{\gamma}(1) = \tilde{\gamma}(0)$$

depending on whether the immersed bundle is flat or not, where $A(\gamma)$ is the area of the region on the surface S surrounded by γ and $\theta = 2 \cdot \frac{n+m}{2n} A(\gamma)$.