Higgs Physics

Shinya KANEMURA

University of TOYAMA

Yangpyung, Particle Physics, School, December 17–20

Physics of Extended Higgs Sectors

Why one doublet?

- There is no principle for one doublet field
  - The minimal Higgs model For minimality
- VEV can be shared by multiple number of Higgs bosons
- Such multi-Higgs structure provides various new properties that the SM does not have
- In fact, many new physics models predict extended Higgs sectors

Strategy

- Although the 125 GeV Higgs boson was found , we do not know the structure of the Higgs sector yet
- Many new physics scenarios predict special non-minimal Higgs sectors
- Comprehensive study of various extended Higgs sectors is very important
- Reconstruction of the Higgs sector by future experiments at LHC, HL-LHC and future lepton colliders
- From the Higgs sector to new physics BSM!

Extended Higgs Sector

The "SM-like" does not necessarily mean the SM.
Every extended Higgs sector can contain the SM-like Higgs boson *h* in its decoupling regime.

General Extended Higgs models

Multiplet Structure

Φ_{SM}+Singlet, Φ_{SM}+Doublet (2HDM),

Φ_{SM}+Triplet, …

Additional Symmetry

Discrete or Continuous?

Exact or Softly broken?

Interaction

Weakly coupled or Strongly Coupled ?

Decoupling or Non-decoupling?

Multiplet Structure

If the Higgs sector contains more than one scalar bosons, possibility would be

- SM + extra Singlets (NMSSM, B-L Higgs, ...)
- SM + extra Doublets (MSSM, CPV, EW Baryogenesis, Neutrino mass, ...)
- SM + extra Triplets (Type II seesaw, LR models....)
- ....

Basic experimental quantities:

- Electroweak rho parameter
- Flavor Changing Neutral Current (FCNC)

Electroweak rho parameter

$\rho_{exp} = 1.0004^{+0.0003}_{-0.0004}$

$Q = I_3 + Y/2$

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_i \left[4T_i(T_i+1) - Y_i^2\right] |v_i|^2 c_i}{\sum_i 2Y_i^2 |v_i|^2}$$

T_i : SU(2)$_L$ isospin
Y_i : hypercharge
v_i : v.e.v.
c_i : 1 for complex representation
1/2 for real representation

N=1 SM Higgs doublet Φ (T=1/2, Y=1) $\rho = 1$!

N=2 What kind of (2 field) extended Higgs sector $\Phi + X(T_X, Y_X)$ can satisfy $\rho = 1$?

We solve the equation

$$4\,T_X(T_X+1) = 3Y_X^{\,2}$$

$(T_X,$	$Y_X)$	X
(0,	0)	Singlet
(1/2,	1)	Doublet
(3,	4)	Septet
(25/2,	15)	25-plet
….		….

Electroweak rho parameter

$\rho_{exp} = 1.0004^{+0.0003}_{-0.0004}$

$Q = I_3 + Y/2$

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = \frac{\sum_i \left[4T_i(T_i+1) - Y_i^2\right] |v_i|^2 c_i}{\sum_i 2Y_i^2 |v_i|^2}$$

T_i : $SU(2)_L$ isospin
Y_i : hypercharge
v_i : v.e.v.
c_i : 1 for complex representation
1/2 for real representation

Possibility

1. ρ=1 SM + doublets (ϕ) (+ singlets (S)), (Septet, ...)

2. ρ≈1 SM + Triplets (Δ)

$$\rho_{\text{tree}} = \frac{1 + \frac{2v_\Delta^2}{v_\Phi^2}}{1 + \frac{4v_\Delta^2}{v_\Phi^2}} \simeq 1 - \frac{2v_\Delta^2}{v_\Phi^2}$$

a) $v_\Delta << v_\varphi$

b) Combination of several representations

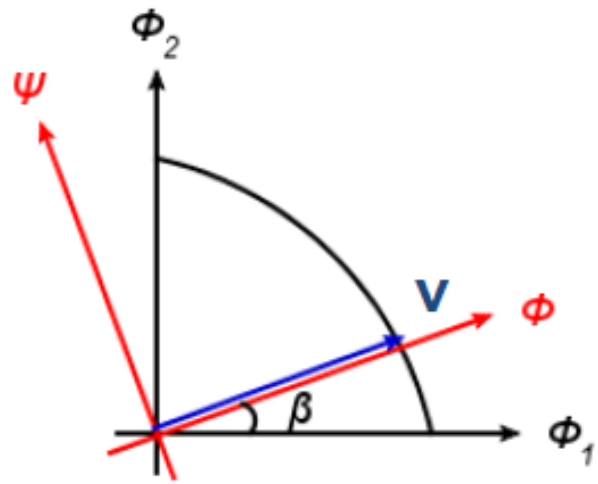
[(ex) Georgi-Machasek model] $v_\Delta \approx v_\varphi$

Multi-doublets (+singlets) seem the most natural choice?

2 Higgs doublet model

$\Phi_1(1/2, 1)$ $\Phi_2(1/2, 1)$

$\rho_{tree} = 1$ is satisfied



$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i = 1, 2)$$

β-Rotation: the Higgs basis

NG boson

Charged Higgs

$$\Phi = \begin{bmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h_1' + v + i G^0) \end{bmatrix} \qquad \Psi = \begin{bmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h_2' + i A) \end{bmatrix}$$

CP-even Higgs

CP-odd Higgs

(α-β)-Rotation for CP-even fields: → mass eigenstates (h, H)

$$\begin{pmatrix} h_1' \\ h_2' \end{pmatrix} \begin{pmatrix} \cos(\beta-\alpha) & \sin(\beta-\alpha) \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$\tan\beta = v_2/v_1$

SM-like Higgs

2 Higgs Doublet Model

$$V_{\text{THDM}} = +m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - \underline{m_3^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right)}$$
$$+\frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2$$
$$+\lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + (\text{h.c.})\right]$$

$$\Phi_i = \begin{bmatrix} w_i^+ \\ \frac{1}{\sqrt{2}}(h_i + v_i + i a_i) \end{bmatrix} \quad (i = 1, 2)$$

Φ_1 and $\Phi_2 \Rightarrow \underline{h,\ H,}\ A^0, H^\pm \oplus$ Goldstone bosons

↑ CPeven ↑ CPodd ↑charged

Diagonalization

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} H \\ h \end{bmatrix} \quad \begin{bmatrix} z_1^0 \\ z_2^0 \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} z^0 \\ A^0 \end{bmatrix}$$

$$\begin{bmatrix} w_1^\pm \\ w_2^\pm \end{bmatrix} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} w^\pm \\ H^\pm \end{bmatrix}$$

$$\frac{v_2}{v_1} \equiv \tan\beta$$

Masses

$$m_h^2 = v^2 \left(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + \frac{\lambda}{2}\sin^2 2\beta\right) + \mathcal{O}(\frac{v^2}{M_{\text{soft}}^2}),$$

$$m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda)\sin^2\beta\cos^2\beta + \mathcal{O}(\frac{v^2}{M_{\text{soft}}^2}),$$

$$m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,$$

$$m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.$$

M_{soft}: soft breaking scale

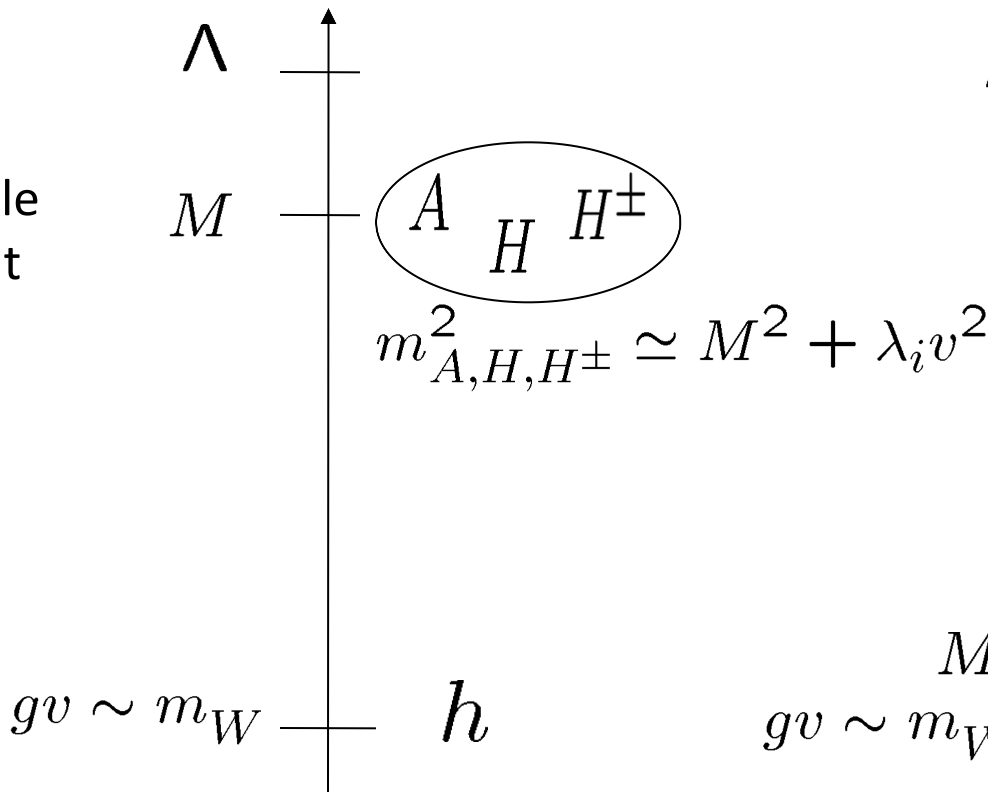
$M_{\text{soft}}\ \left(= \frac{m_3}{\sqrt{\cos\beta\sin\beta}}\right)$:

soft-breaking scale of the discrete symm.

Two Possibilities

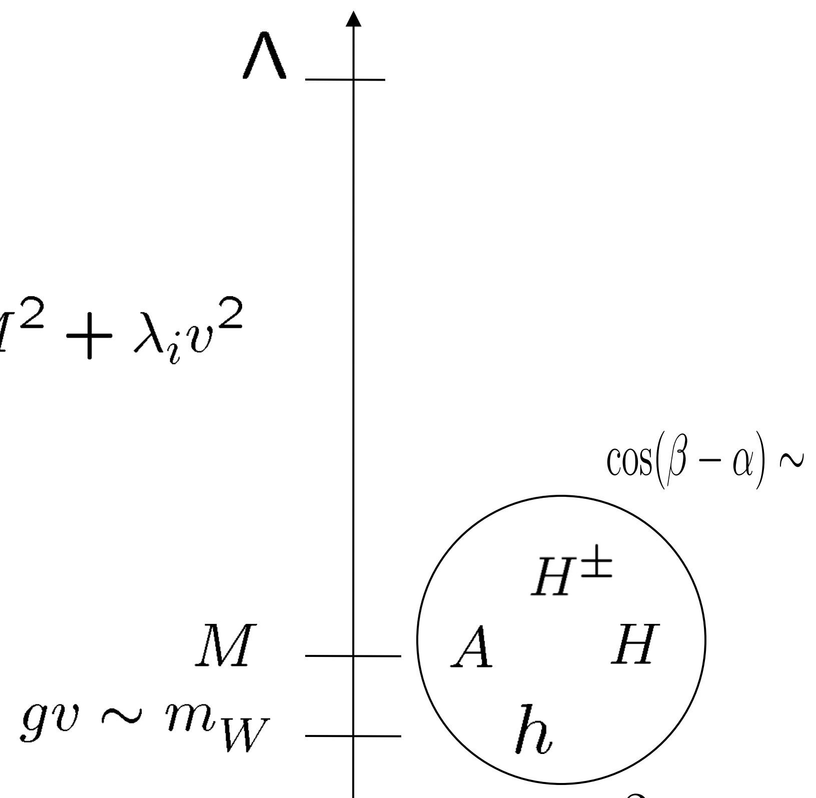
Λ: Cutoff

M: Mass scale irrelevant to VEV



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{v^2}{M^2}\mathcal{O}^{(6)}$$

Effective Theory is the SM



$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{nonSM}} + \frac{v^2}{\Lambda^2}\mathcal{O}^{(6)}$$

Effective Theory is an extended Higgs sector

Non-decoupling effect

Flavor Changing Neutral Currents

FCNC Suppression

Multi-Higgs model: FCNC appears via Higgs mediation

2 Higgs doublet models:

to avoid FCNC, give different charges to Φ_1 and Φ_2

Discrete sym. $\Phi_1 \rightarrow +\Phi_1, \quad \Phi_2 = -\Phi_2$

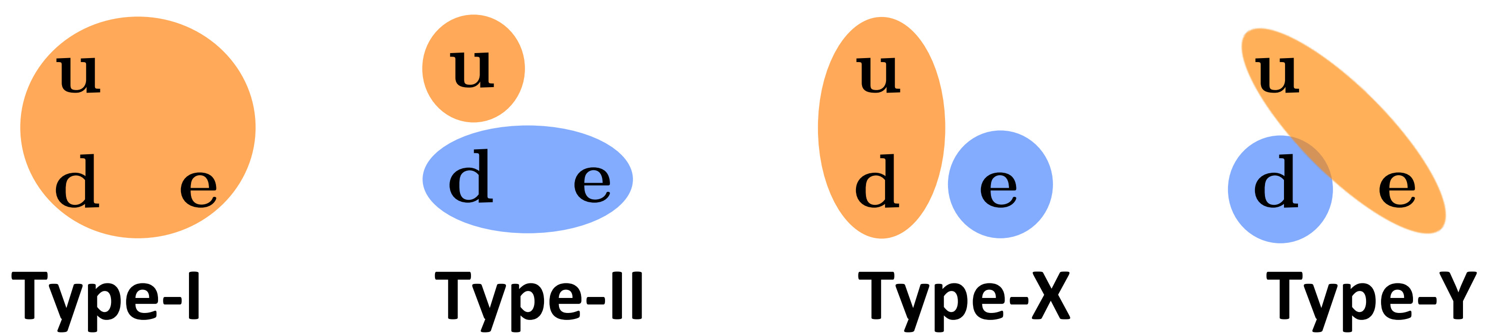
Each quark or lepton couples only one Higgs doublet

No FCNC at tree level

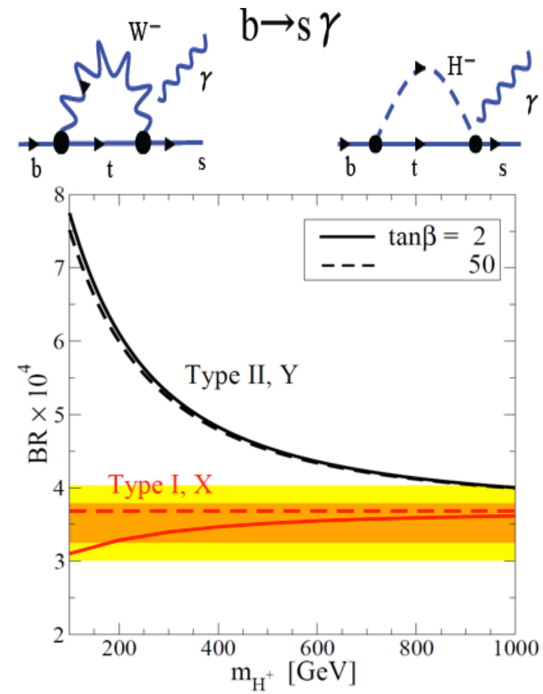
Barger, Hewett, Phillips

Classified by Z_2 charge assignment

Four Types of Yukawa coupling



Type of 2HDM

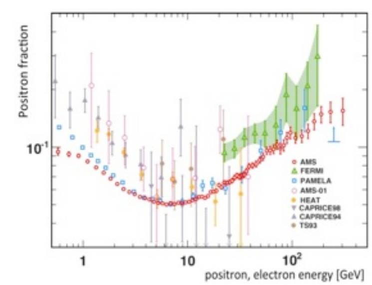


Aoki, SK, Tsumura, Yagyu (09)

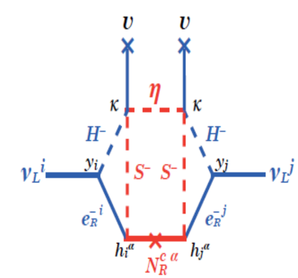
Type-I Fermiofobic 2HDM
Neutrinophillic 2HDM

Type-II MSSM, NMSSM, other
Extended SUSY Higgs models

Type-X Lepton-specific 2HDM
Radiative Neutrino mass
Positron Excess
H portal DM (tau spesific)

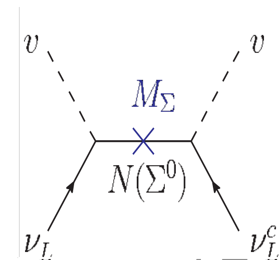


Goh, Hall, Kumar (09)



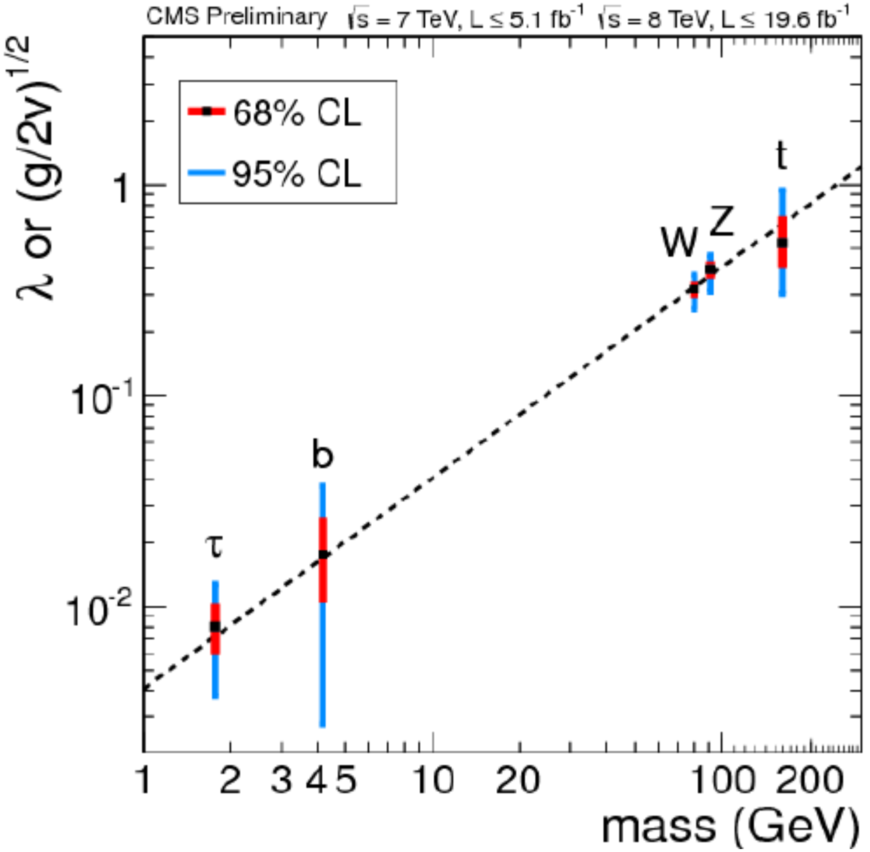
Aoki, SK, Seto (09)

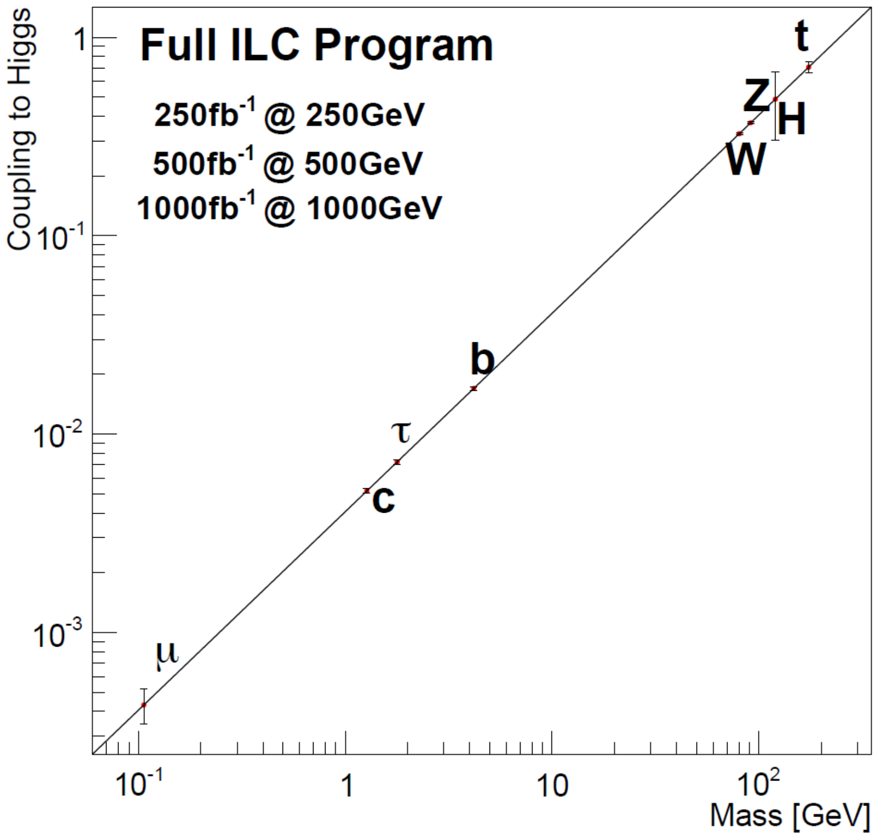
Type-Y Flipped 2HDM



Ma (02)

Current LHC data v.s. Full ILC





The precision must be improved in future at LHC 13-14 TeV and at the LC

The ILC is really needed!

Type2-2HDM (MSSM) Higgs couplings

VEV's : $v_1^2 + v_2^2 = v^2 \simeq (246\ \text{GeV})^2$

Higgs mixing

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$$\tan\beta = \frac{v_2}{v_1}$$

SM

2HDM Type2

Gauge coupling:

$\phi VV \quad (V = Z, W)$ $\Rightarrow$ hVV: $\sin(\beta - \alpha)$, HVV: $\cos(\beta - \alpha)$

Yukawa coupling:

$\phi b\bar{b}$ $\Rightarrow$ $hb\bar{b}$: $\frac{\sin\alpha}{\cos\beta}$, $Hb\bar{b}$: $\frac{\cos\alpha}{\cos\beta}$

$\phi t\bar{t}$ $\Rightarrow$ $ht\bar{t}$: $\frac{\cos\alpha}{\sin\beta}$, $Ht\bar{t}$: $\frac{\sin\alpha}{\sin\beta}$,

SM-like regime

hVV $\sin(\beta-\alpha)$ HVV $\cos(\beta-\alpha)$

$$\sin(\beta-\alpha)\simeq 1$$

Only the lightest Higgs *h* couples to weak gauge bosons

***h* behaves like the SM Higgs**

$$g_{hVV} \to g_{\phi VV}^{\text{SM}} \qquad g_{HVV} \to 0$$

$$y_{ht\bar{t}} \to y_{\phi t\bar{t}}^{\text{SM}} \qquad y_{Ht\bar{t}} \to y_{\phi t\bar{t}}^{\text{SM}} \cot\beta$$

$$y_{hb\bar{b}} \to y_{\phi b\bar{b}}^{\text{SM}} \qquad y_{Hb\bar{b}} \to y_{\phi b\bar{b}}^{\text{SM}} \tan\beta$$

$$y_{h\tau\tau} \to y_{\phi\tau\tau}^{\text{SM}} \qquad y_{H\tau\tau} \to y_{\phi\tau\tau}^{\text{SM}} \tan\beta$$

Type-II 2HDM

h(125) as a probe of extended Higgs sectors

How we experimentally study non-minimal Higgs sectors?

- **Direct Searches of additional Higgs bosons (H, A, H^+, H^{++}, …)**

- **Indirect Searches by detecting deviations in various quantities**

EW observables m_W, S, T, U, Zff, Wff', WWV, …

h(125) couplings hWW, hZZ, $h\gamma\gamma$, hff, hhh, …

They will be precisely measured at future experiments

Fingerprinting

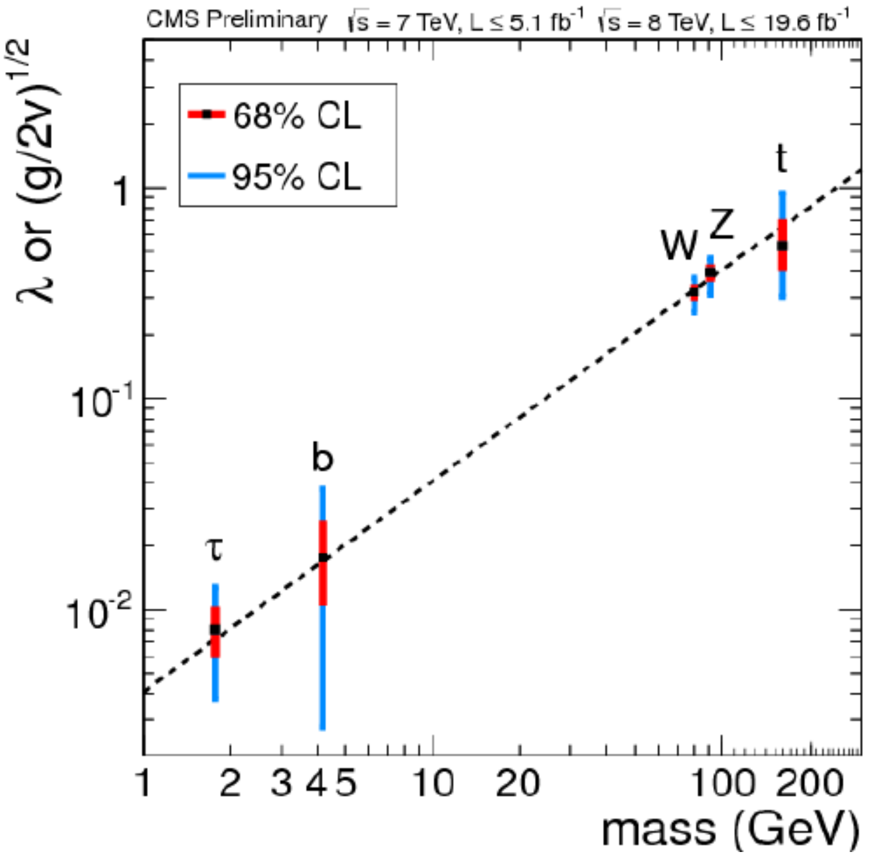
The ILC is an idealistic machine for precision measurement of Higgs boson couplings
Deviations with a pattern lead to identification of new physics

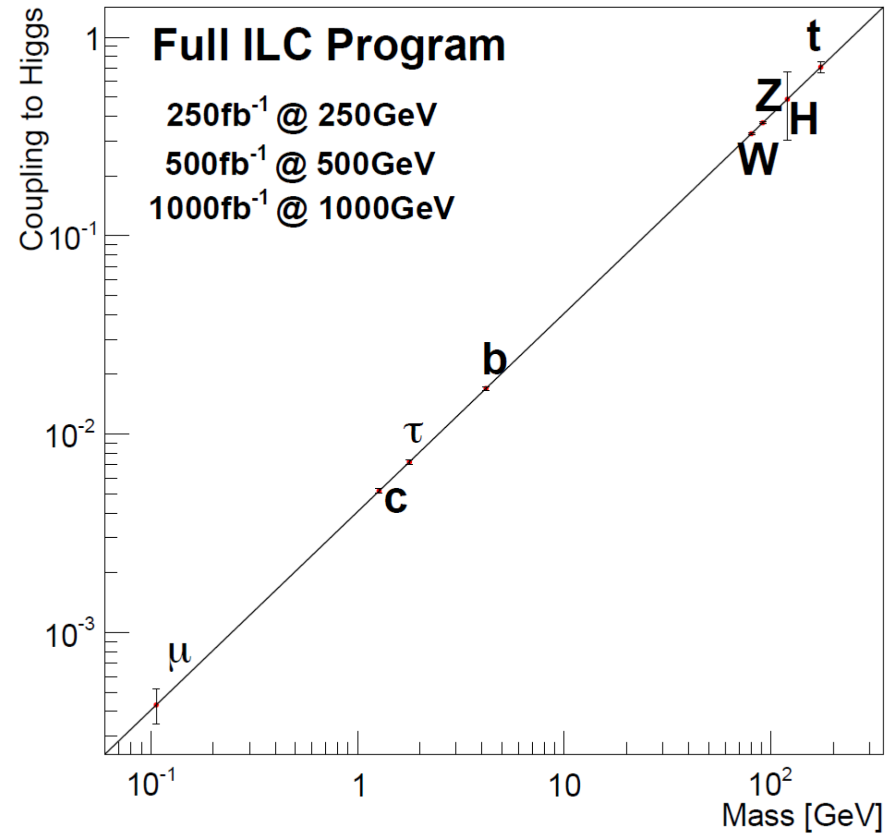
Future *h*(125)-coupling measurements

Facility	LHC	HL-LHC	ILC500	ILC500-up
$\sqrt{s}$ (GeV)	14,000	14,000	250/500	250/500
$\int \mathcal{L}dt$ (fb^{-1})	300/expt	3000/expt	250+500	1150+1600
κ_γ	$5-7\%$	$2-5\%$	8.3%	4.4%
κ_g	$6-8\%$	$3-5\%$	2.0%	1.1%
κ_W	$4-6\%$	$2-5\%$	0.39%	0.21%
κ_Z	$4-6\%$	$2-4\%$	0.49%	0.24%
κ_ℓ	$6-8\%$	$2-5\%$	1.9%	0.98%
$\kappa_d = \kappa_b$	$10-13\%$	$4-7\%$	0.93%	0.60%
$\kappa_u = \kappa_t$	$14-15\%$	$7-10\%$	2.5%	1.3%

Snowmass Higgs Working Group Report 1310.8361

Current LHC data v.s. Full ILC





The precision must be improved in future at LHC 13-14 TeV and at the LC

The ILC is really needed!

Yukawa Coupling in Extended Higgs Sectors

Multi-Higgs model: FCNC appears via Higgs mediation

2 Higgs doublet models:

to avoid FCNC, give different charges to Φ_1 and Φ_2

Discrete sym. $\Phi_1 \rightarrow +\Phi_1, \quad \Phi_2 = -\Phi_2$

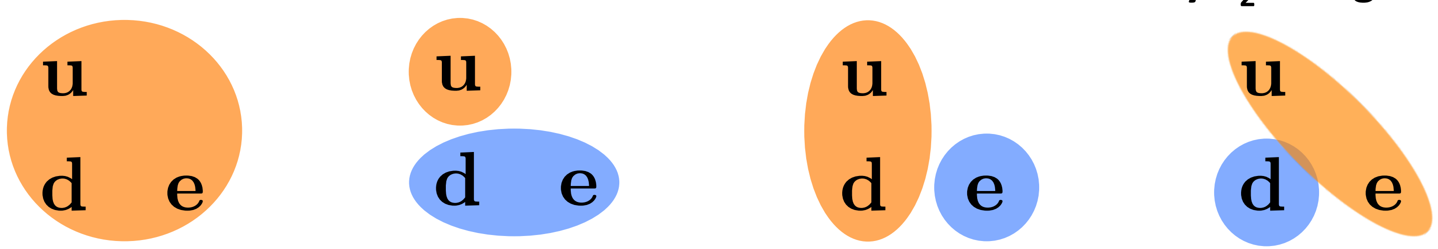
Each quark or lepton couples only one Higgs doublet

No FCNC at tree level

Four Types of Yukawa coupling

Barger, Hewett, Phillips

Classified by Z_2 charge assignment



Type-I — Neutrinophilic, Inert

Type-II — MSSM

Type-X — Radiative Seesaw, Lepton specific

Type-Y

Pattern in deviations of g_{hVV} and Y_{hff}

Model	μ	τ	b	c	t	g_V
Singlet mixing	↓	↓	↓	↓	↓	↓
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓

cos(β-α) < 0

Singlet can be distinguished from the Type-I 2HDM

Y_{hff}/g_V=1 in the singlet model but Y_{hff}/g_V ≠1 in the 2HDM-I

In the triplet model, quark-Yukawa couplings are universally smaller, Lepton-Yukawa deviate universal. **κ_V can be greater than 1**

κ_V > 1 is a signature of exotic Higgs (with higher representations)

Extended Higgs models are distinguishable by precisely measuring *hVV* and *hff*

Pattern in deviations of g_{hVV} and Y_{hff}

Model	μ	τ	b	c	t	g_V
Singlet mixing	↓	↓	↓	↓	↓	↓
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓

cos(β-α) < 0

Singlet can be distinguished from the Type-I 2HDM

Y_{hff}/g_V=1 in the singlet model but Y_{hff}/g_V ≠1 in the 2HDM-I

In the triplet model, quark-Yukawa couplings are universally smaller, Lepton-Yukawa deviate universal. **κ_V can be greater than 1**

κ_V > 1 is a signature of exotic Higgs (with higher representations)

Extended Higgs models are distinguishable by precisely measuring *hVV* and *hff*

Fingerprinting the 2HDM (tree level)

$$\kappa_V \equiv \frac{g_{hVV(2HDM)}}{g_{hVV(SM)}} = \sin(\beta - \alpha)$$

$x = \cos(\beta-\alpha)$ **SM-like: $x \ll 1$**

$$\kappa_V = 1 - (1/2)\,x^2 + \ldots$$

When a Fermion couples to Φ_1

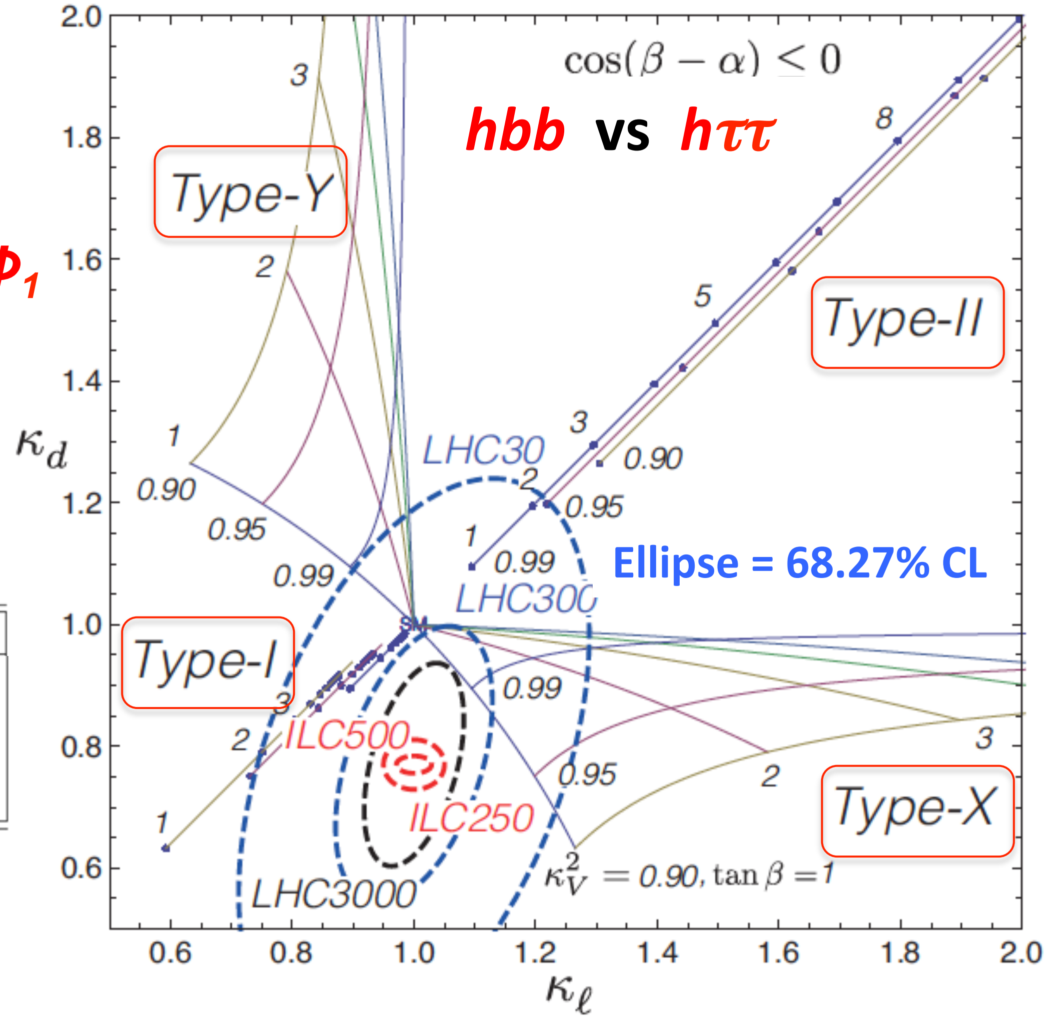
$$\kappa_f = 1 + \cot\beta\, x + \ldots$$

and if it couples to Φ_2

$$\kappa_f = 1 - \tan\beta\, x + \ldots$$

Model	μ	τ	b	c	t	g_V
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓

SK, K. Tsumura, K. Yagyu, H. Yokoya 2014
ILC Higgs White Paper 2013



How do this result change with radiative corrections?

Fingerptinting the model (Exotics)

SK, K. Tsumura, K. Yagyu, H. Yokoya 2014

Universal Fermion Coupling (κ_F)

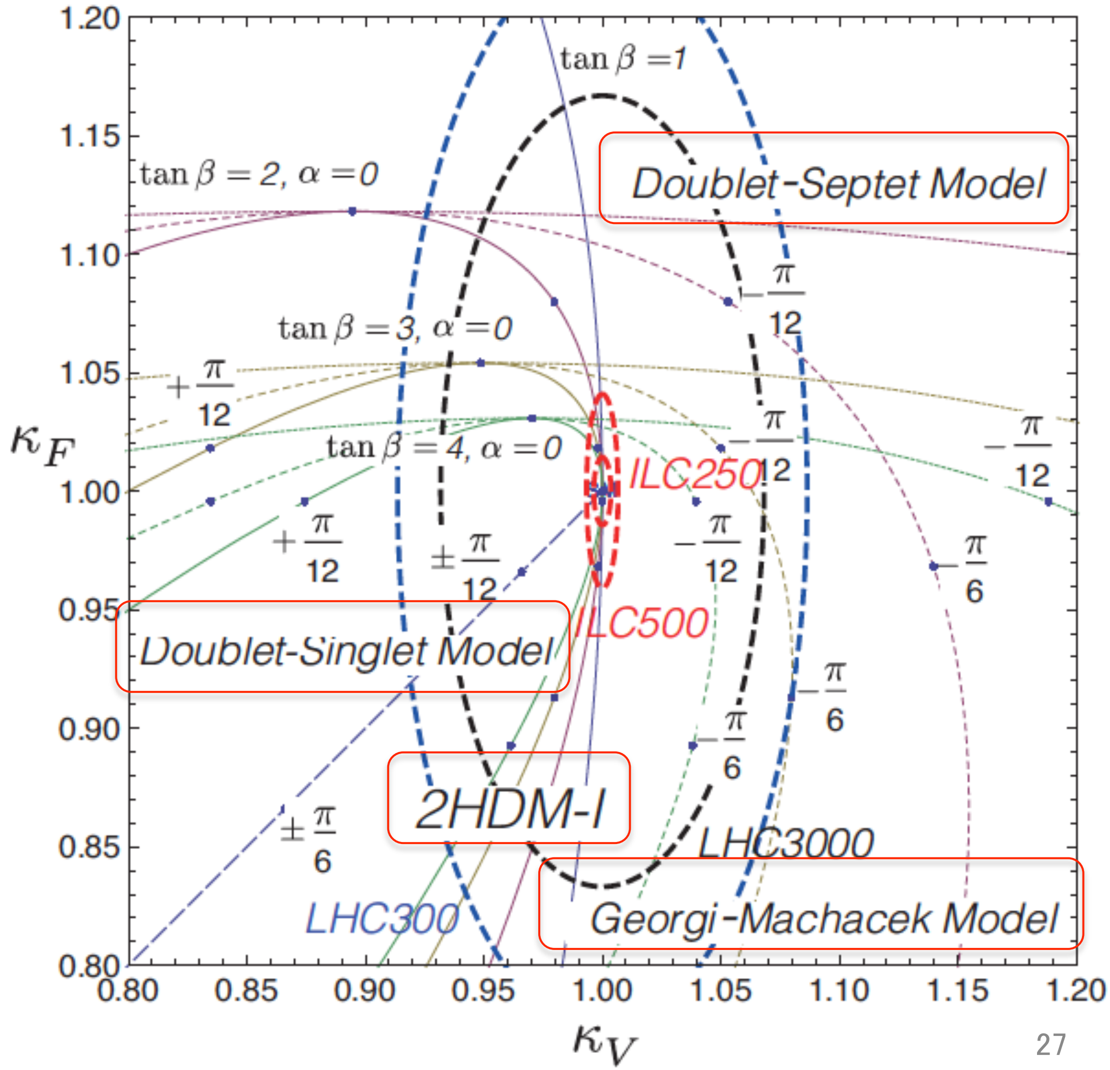
VS

***hVV* coupling (κ_V)**

Exotic models predict $\kappa_V > 1$

We can discriminate Exotic models

Ellipse = 68.27% CL



Deviation in *hff*

If $\Delta\kappa_V = 1\ \%$

Singlet, Exotics,

$\Delta\kappa_u = -(1/2)\,x^2$, $\Delta\kappa_d = -(1/2)\,x^2$, $\Delta\kappa_\tau = -(1/2)\,x^2$ — O(1) %

Type I 2HDM

$\Delta\kappa_u = +\cot\beta\, x$, $\Delta\kappa_d = +\cot\beta\, x$, $\Delta\kappa_\tau = +\cot\beta\, x$ — O(10) %

Type X (Lepton Specific) 2HDM

$\Delta\kappa_u = +\cot\beta\, x$, $\Delta\kappa_d = +\cot\beta\, x$, $\Delta\kappa_\tau = -\tan\beta\, x$ — O(10) %

MSSM (Type II 2HDM)

$\Delta\kappa_u = +\cot\beta\, x$, $\Delta\kappa_d = -\tan\beta\, x$, $\Delta\kappa_\tau = -\tan\beta\, x$ — O(10) %

MCHM4

$\Delta\kappa_u = -(1/2)\,x^2$, $\Delta\kappa_d = -(1/2)\,x^2$, $\Delta\kappa_\tau = -(1/2)\,x^2$ — O(1) %

MCHM5

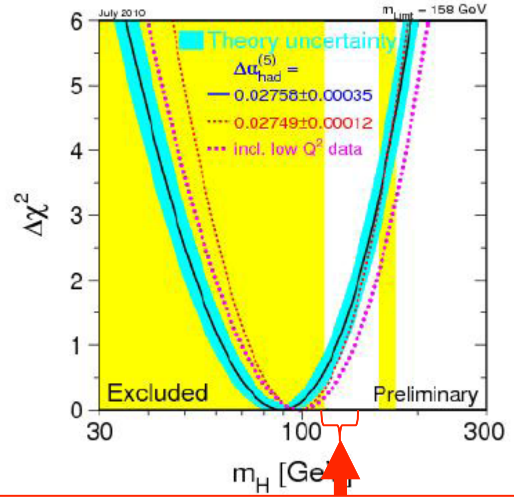
$\Delta\kappa_u = -(3/2)\,x^2$, $\Delta\kappa_d = -(3/2)\,x^2$, $\Delta\kappa_\tau = -(3/2)\,x^2$ — O(1) %

Summary of extended Higgs sector

- Various possibility of extended Higgs sector
- From the constraint from the rho parameter a multi-doublet (plus singlet) structure is favored   ex) 2HDM
- Other exotics can also possible if the VEV is small
- Mixing effect changes Higgs boson couplings from the SM
  - Gauge couplings *hVV* (*κ_V < 1 in multi-doublet, κ_V >1 in extotics*)
  - Yukawa couplings *hff* (deviations with a pattern)
- Future precision data can be used to test models

Higgs and Radiative Corrections

Higgs discovery in 2012



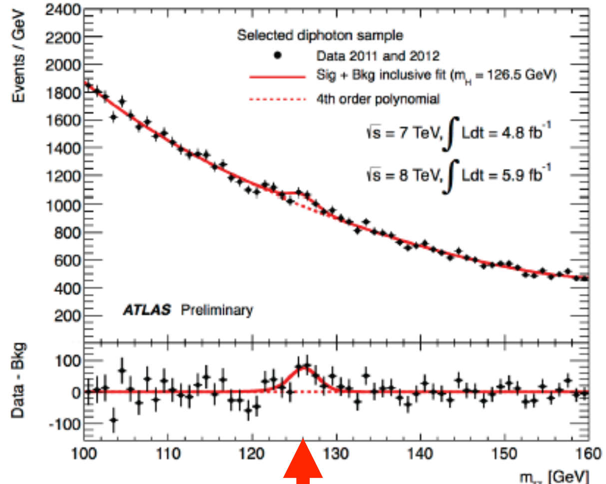
The mass is 125 GeV

Spin/Parity 0^+

It couples to ***γγ, ZZ, WW, bb, ττ, …***

Higgs Mass indicated by LEP/SLC

This is really a Higgs!



Measured couplings look consistent with the SM Higgs within the current errors

ATLAS/CMS July 2012

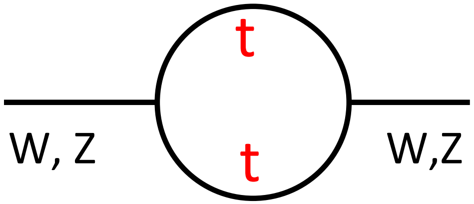
New Particle！

Radiative Corrections

Rho parameter（unity in the SM）

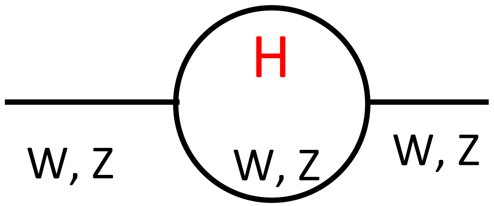
$\rho_{exp} = 1.0008\ ^{+0.0017}_{-0.0007}$

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \ (= 1)$$



Loop corrections

$$\Delta\rho = 4\sqrt{2}G_F\left[\Pi_T^{33}(p^2=0) - \Pi_T^{11}(p^2=0)\right]$$



Loop effect of *m*$_t$ and *m*$_H$

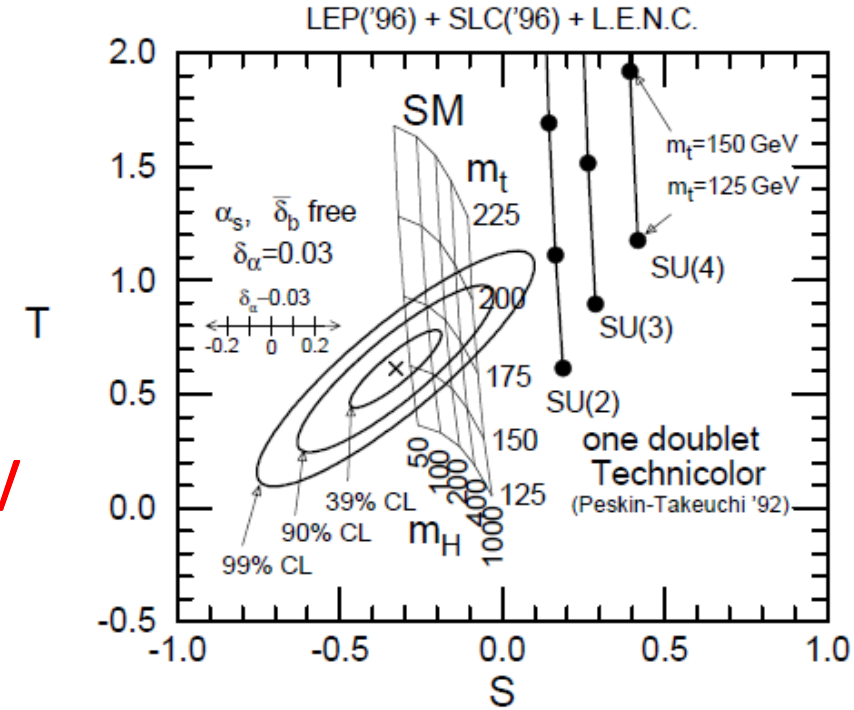
$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2}\left(m_t^2 - M_Z^2\sin^2\theta_W \ln\frac{m_H^2}{m_W^2}\right)$$

Quadratic **Logarithmic**

We knew the mass before discovery!

Case of the top quark

- Quadratic mass dep. in ρ parameter (T parameter)
- Forget about m_H because it is only logarismic
- LEP1 says m_t=150-200GeV
- Discovery at Tevatron （about 175GeV）

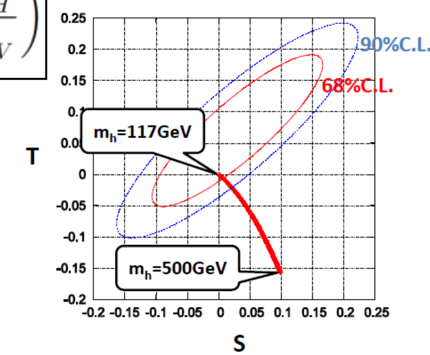


Hagiwara、et al

$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2}\left(m_t^2 - M_Z^2\sin^2\theta_W \ln\frac{m_H^2}{m_W^2}\right)$$

It was repeated for Higgs at LEP2

$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2}\left(m_t^2 - M_Z^2 \sin^2\theta_W \ln\frac{m_H^2}{m_W^2}\right)$$

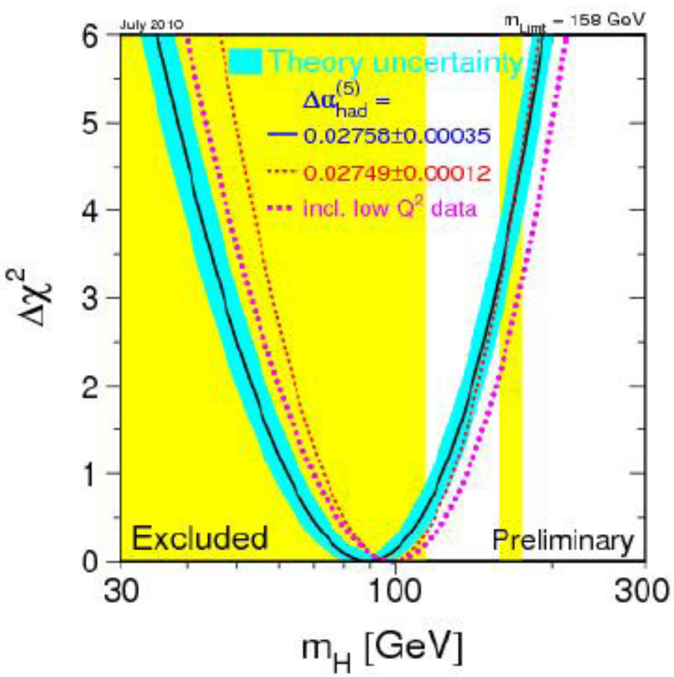


Case of Higgs boson

- Now we know top mass
- Rho is a funcution of only m_H
- Precision measurement at LEP2
- 114GeV< mH <150 GeV!
- LHC found new boson at 126GeV（Higgs boson!）

Victory of precision measurements and theory calculations

（VIVA! SM）



LEP Electroweak Working Group 2010

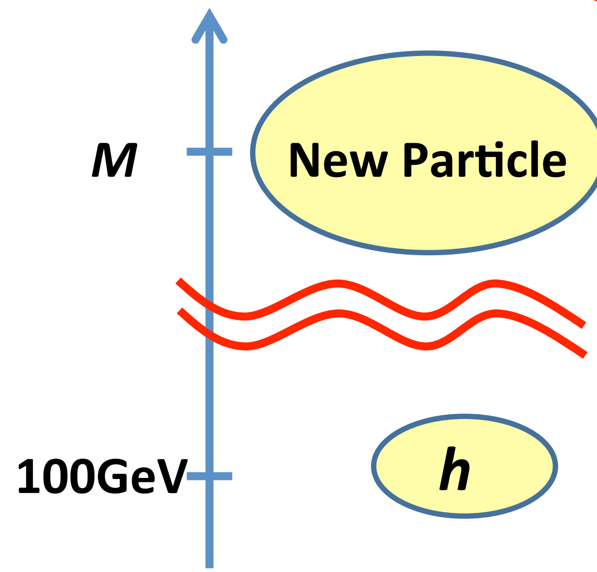
Decoupling Theorem and its breaking

Decoupling Theorem

Low energy observable $\mathcal{O}$

Renormalized quantity $\mathcal{O}$ is a function of *M* via loop contributions, but it decouples in the large mass limit

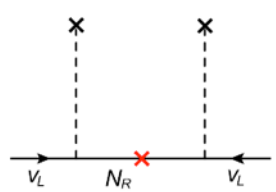
$$\mathcal{O}(M) \sim \frac{1}{M^n}$$



Ex) GUT scale (10^{16} GeV) physics does not affect TeV scale physics

Ex) Seesaw Mechanism (Dim 5) at the tree-level

$$\mathcal{L} = \frac{c}{\Lambda}(\Phi^T \overline{\nu_L^c})(\nu_L \Phi)$$



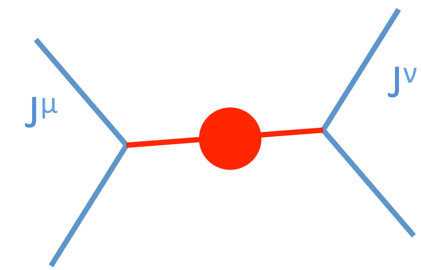
$$m_\nu \sim \frac{v^2}{M_{N_R}}$$

QED   Example of decoupling theorem

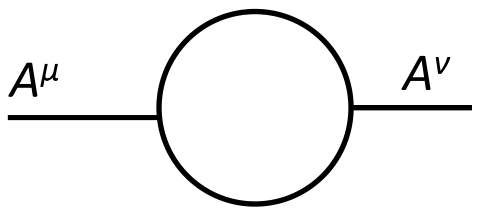
One-loop contributions to the two point functions

$$\mathcal{M}_{tree} \sim Qe\frac{1}{k^2}eQ' = \frac{QQ'}{\frac{1}{e^2}k^2}$$

$$\mathcal{M} \sim \frac{QQ'}{\frac{1}{e^2}k^2 - \Pi_{\rm new}(k^2)}$$



Self-Energy $\Pi_{new}(k^2)$ has dim. 2, so that it can have M^2 or $\ln M$ dependence from power counting (non-decoupling effects)



$$\Pi_{\rm new}(k^2) = \Pi_{\rm new}(0) + k^2\Pi'_{\rm new}(0) + \cdots$$

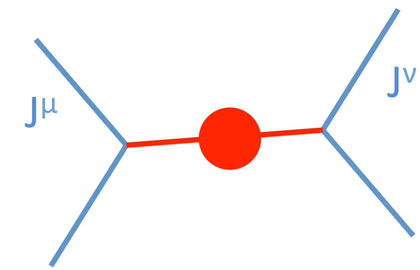
However from U(1) gauge symmetry $\Pi_{new}(0)=0$, and $\Pi'_{new}(0)$ is absorbed by renormalization

$$\mathcal{M} \sim \frac{QQ'}{\left(\frac{1}{e^2} - \Pi'(0)_{\rm New}\right)k^2 - \frac{(k^2)^2}{2}\Pi''_{\rm new}(k^2)} = \frac{QQ'}{\frac{1}{e_R^2}k^2 - \frac{(k^2)^2}{2}\Pi''_{\rm new}(0) + \cdots}$$

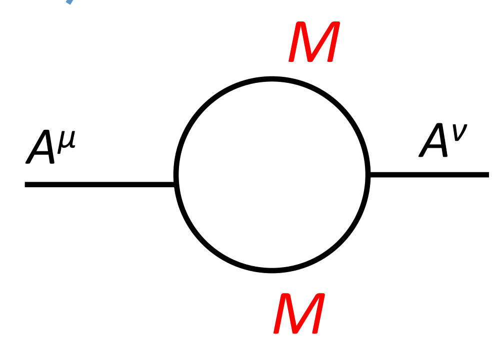
Remaining $\Pi''_{new}(0)$ is dim. −2, so that at most $1/M^2$ (Decouple!)

QED with spontaneously broken U(1)

$$\mathcal{M}_{tree} \sim Qe\frac{1}{k^2 - m_A^2}eQ' = \frac{QQ'}{\frac{1}{e^2}k^2 - v^2}$$



$$\mathcal{M} \sim \frac{QQ'}{\frac{1}{e^2}k^2 - v^2 - \Pi_{\rm new}(k^2)}$$



Self-Energy $\Pi_{new}(k^2)$ has dim. 2, so that it can have M^2 or ln$M$ dependence from power counting (non-decoupling effects)

$$\Pi_{\rm new}(k^2) = \Pi_{\rm new}(0) + k^2\Pi'_{\rm new}(0) + \cdots$$

This time, U(1) is spontaneously broken, so that $\Pi_{new}(0)$ is non-zero.
But this time, $\Pi_{new}(0)$ and $\Pi'_{new}(0)$ are absorbed by *v* (or m_A) and *e*

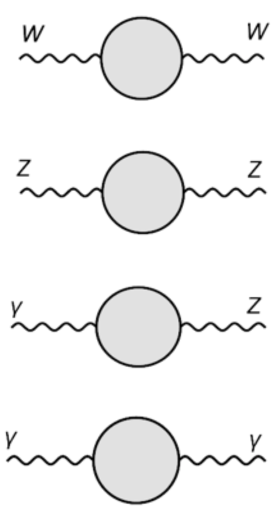
$$\mathcal{M} \sim \frac{QQ'}{\left(\frac{1}{e^2} - \Pi'_{\rm new}(0)\right)k^2 - (v^2 + \Pi_{\rm new}(0)) - \frac{(k^2)^2}{2}\Pi''_{\rm new}(0) + \cdots} = \frac{QQ'}{\frac{1}{e_R^2}k^2 - v_R^2 - \frac{(k^2)^2}{2}\Pi''_{\rm new}(0) + \cdots}$$

Remaining $\Pi''_{new}(0)$ is dim-(-2), so that at most $1/M^2$ (Decouple!)

Non-vanishing non-decoupling effect

Electroweak Theory SU(2) × U(1) with SSB

Two point functions 6 nondec. d.o.f.



$$= M_{\rm New}^2 + p^2 \ln \frac{M_{\rm New}^2}{p^2} + \cdots$$

$$= M_{\rm New}^2 + p^2 \ln \frac{M_{\rm New}^2}{p^2} + \cdots$$

$$= \cancel{M_{\rm New}^2} + p^2 \ln \frac{M_{\rm New}^2}{p^2} + \cdots$$

$$= \cancel{M_{\rm New}^2} + p^2 \ln \frac{M_{\rm New}^2}{p^2} + \cdots$$

$$\Pi_T^{\gamma\gamma}(p^2) = e^2 \Pi_T^{QQ}(p^2),$$
$$\Pi_T^{\gamma Z}(p^2) = e g_Z \left[\Pi_T^{3Q}(p^2) - s_W^2 \Pi_T^{QQ}(p^2)\right],$$
$$\Pi_T^{ZZ}(p^2) = g_Z^2 \left[\Pi_T^{33}(p^2) - 2 s_W^2 \Pi_T^{3Q}(p^2) + s_W^4 \Pi_T^{QQ}(p^2)\right],$$
$$\Pi_T^{WW}(p^2) = g^2 \Pi_T^{11}(p^2).$$

Input parameters（α, GF, MZ）can absorb 3 of 6 non-decoupling effects.

Still, there are 3 non-vanishing non-decoupling effects

3 non-decoupling parameters:
S,T,U (Peskin-Takeuchi)

$$S = 16\pi \left[\overline{\Pi}_T^{3Q\prime}(p^2=0) - \overline{\Pi}_T^{33\prime}(p^2=0)\right],$$

$$T = \frac{4\sqrt{2}G_F}{\alpha_{\rm EM}} \left[\overline{\Pi}_T^{33}(p^2=0) - \overline{\Pi}_T^{11}(p^2=0)\right],$$

$$U = 16\pi \left[\overline{\Pi}_T^{33\prime}(p^2=0) - \overline{\Pi}_T^{11\prime}(p^2=0)\right],$$

Non-decoupling effects

Non-decoupling effects on various electroweak parameters

Γ_Z, $\sin\theta_W$, m_W, ρ, …

are all described by S, T, U (at the leading level)

$$S = 16\pi\left[\overline{\Pi}_T^{3Q\prime}(p^2=0) - \overline{\Pi}_T^{33\prime}(p^2=0)\right],$$

$$T = \frac{4\sqrt{2}G_F}{\alpha_{\rm EM}}\left[\overline{\Pi}_T^{33}(p^2=0) - \overline{\Pi}_T^{11}(p^2=0)\right],$$

$$U = 16\pi\left[\overline{\Pi}_T^{33\prime}(p^2=0) - \overline{\Pi}_T^{11\prime}(p^2=0)\right],$$

$$\Delta\rho \equiv \rho - 1 = \alpha T$$

$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2}\left(m_t^2 - M_Z^2\sin^2\theta_W \ln\frac{m_H^2}{m_W^2}\right)$$

Non-decoupling effects

What kind of new physics can produce non-decoupling effects?

- Chiral Fermion Loop

  $m_f = 0 \quad \rightarrow \quad m_f = y_f\, v$

- Higgs Loop

  $m_h^2 = 2\, \lambda\, v^2$

- Scalar Loop

  $m_S^2 = \lambda\, v^2 + M_{inv}^2$

Custodial Symmetry

SM Higgs Potential has the Global Symmetry after EWSB (Custodial Symmetry)

$$V(\Phi) = +\mu^2|\Phi|^2 + \lambda|\Phi|^4$$

Define a bi-dobulet field $\mathcal{M} \equiv (\tilde{\Phi}, \Phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}$

Transformations

$$\mathcal{M} \to g_L \mathcal{M} \, (g_L \in SU(2)_L)$$

$$\mathcal{M} \to \mathcal{M} g_R^{-1} \, (g_R \in SU(2)_R)$$

The Higgs potential is invariant under $SU(2)_L \times SU(2)_R$ (= O(4)) transformations

$$V(\Phi) = V(\mathcal{M}) = +\frac{\mu^2}{2}\mathrm{Tr}[\mathcal{M}^\dagger \mathcal{M}] + \frac{\lambda}{4}\left(\mathrm{Tr}[\mathcal{M}^\dagger \mathcal{M}]\right)^2$$

By EWSB, $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$ Custodial symmetry

Note that the custodial symmetry is broken in general extended Higgs sectors

Yukawa sector does not respect $SU(2)_R$

Yukawa interaction in the SM can be written as

$$\mathcal{L} \sim (\bar{t}, \bar{b})_L (y_t \tilde{\Phi}, y_b \Phi) \begin{pmatrix} t \\ b \end{pmatrix}_R$$

$$= \frac{y_t + y_b}{2} (\bar{t}, \bar{b})_L \mathcal{M} \begin{pmatrix} t \\ b \end{pmatrix}_R + \underline{\frac{y_t - y_b}{2} (\bar{t}, \bar{b})_L \mathcal{M} \tau_3 \begin{pmatrix} t \\ b \end{pmatrix}_R}$$

$SU(2)_R$ is broken $\qquad g_L \tau_3 g_R^{-1} \neq \tau_3$

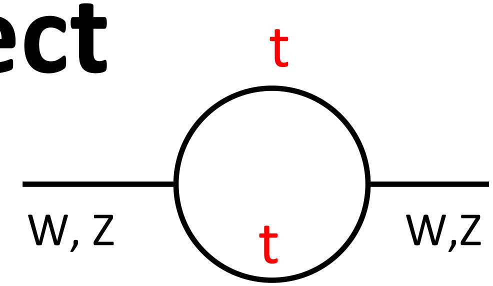
Only when $y_t = y_b$ (namely when $m_t = m_b$),
the Yukawa sector is invariant under $SU(2)_L \times SU(2)_R$

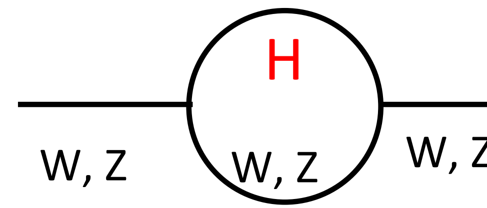
$$T \sim (m_t - m_b)^2$$

$$\Rightarrow \quad \Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2} \left(m_t^2 - M_Z^2 \sin^2\theta_W \ln \frac{m_H^2}{m_W^2} \right)$$

Non-decoupling effect

Example (Electroweak *T* parameter)





$$\rho = \frac{m_W}{m_Z \cos\theta_W}, \quad \Delta\rho = \rho - 1 = \alpha T$$

Data $|T| < 0.1$

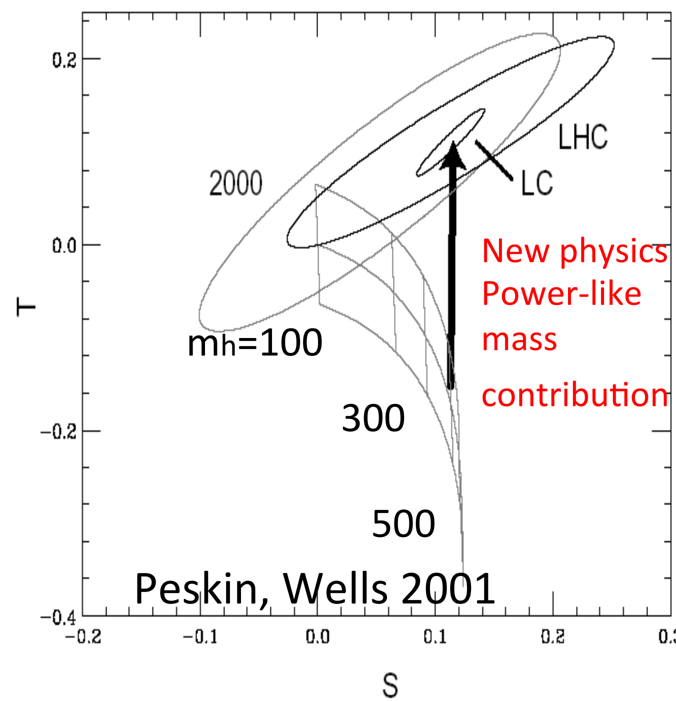
$$\Delta T_{\text{top}} \propto \frac{m_t^2}{M_W^2}$$

$$\Delta T_{\text{Higgs}} \simeq -\ln \frac{m_H^2}{M_W^2} \quad \text{(SM)}$$

$$\Delta T_{\text{Higgs}} \sim -\ln \frac{m_h^2}{M_W^2} + \frac{(m_A^2 - m_{H\pm}^2)^2}{M_W^2 m_A^2}$$

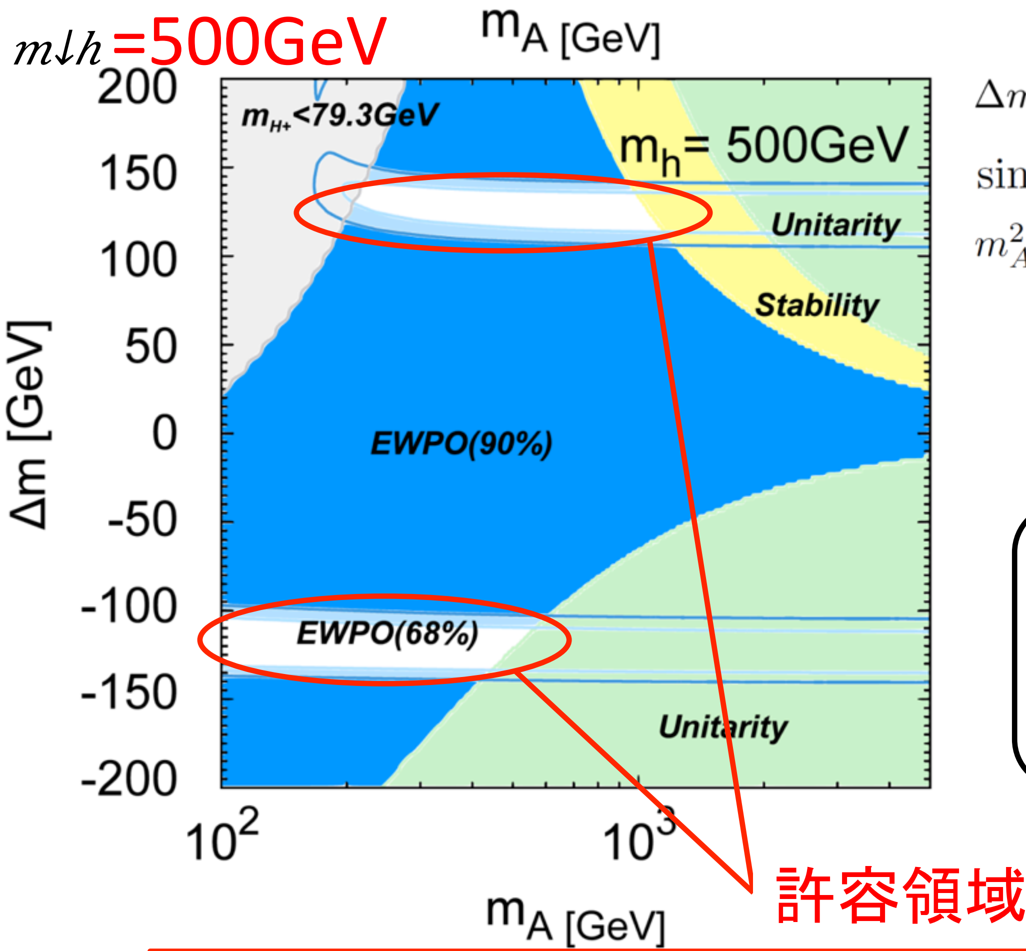
(2HDM)

Quadratic mass contribution (non-decoupling effect)



拡張すれば重いヒッグスは可能

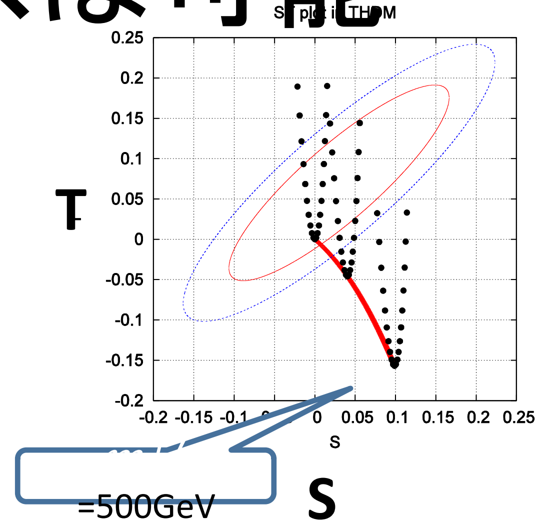
m_h=500GeV



$$\Delta m = m_A - m_{H^\pm}$$

$$\sin(\beta - \alpha) = 1$$

$$m_A^2 = m_H^2 = M^2$$



質量差 Δm（カストディアル対称性の破れ）の効果で、SM的ヒッグスが重い場合でも電弱精密データを説明できる

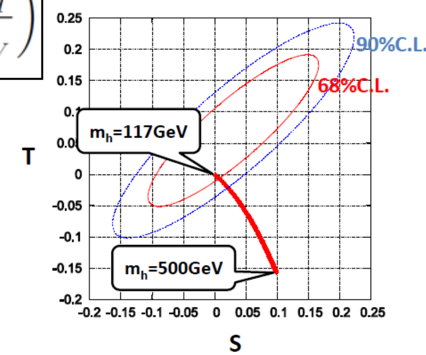
SK, Okada, Taniguchi, Tsumura, 2011

重いSM的ヒッグス場は可能

- 大きなH+とAの質量差が要求される
- 付加的なヒッグス場（A等）に質量の上限つく

It was repeated for Higgs at LEP2

$$\Delta\rho \simeq \frac{3G_F}{8\sqrt{2}\pi^2}\left(m_t^2 - M_Z^2 \sin^2\theta_W \ln\frac{m_H^2}{m_W^2}\right)$$

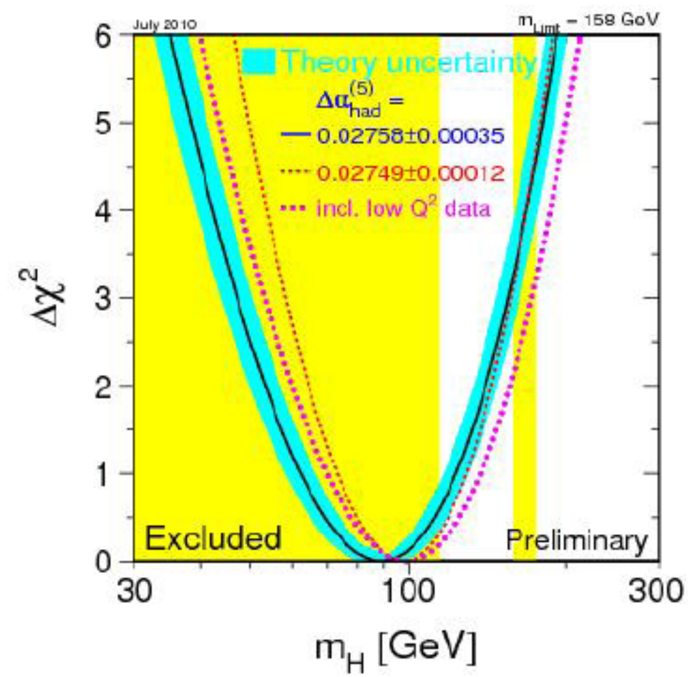


Case of Higgs boson

- **Now we know top mass**
- **Rho is a funcution of only m_H**
- **Precision measurement at LEP2**
- **114GeV< mH <150 GeV!**
- **LHC found new boson at 126 GeV（Higgs boson!）**

Victory of precision measurements and theory calculations

（VIVA! SM）



LEP Electroweak Working Group 2010

All SM parameters are found

Next target is new physics!

- **Importance of Radiative Correction calculation**
- **Future precision measurements**
  - ***S, T, U*** **(Giga Z, Mega W)**
  - **Top (e.g. ttZ) couplings**
  - **Couplings of the discovered Higgs**

hgg, hγγ, hWW, hZZ, htt, hbb, hττ, hμμ, hcc, …, hhh

At ILC, we may be able to distinguish models by detecting a pattern of deviations in the *h* couplings from the SM values!

Fingerprinting new physics models

Non-decoupling effect on the Higgs couplings

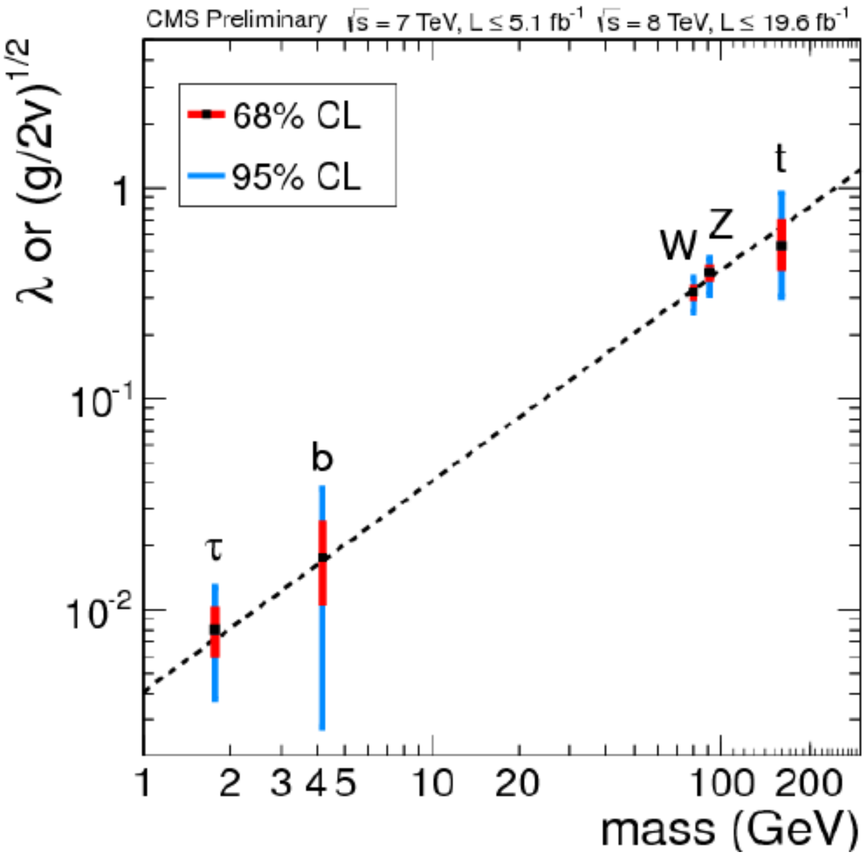
Top-loop contribution
in the SM

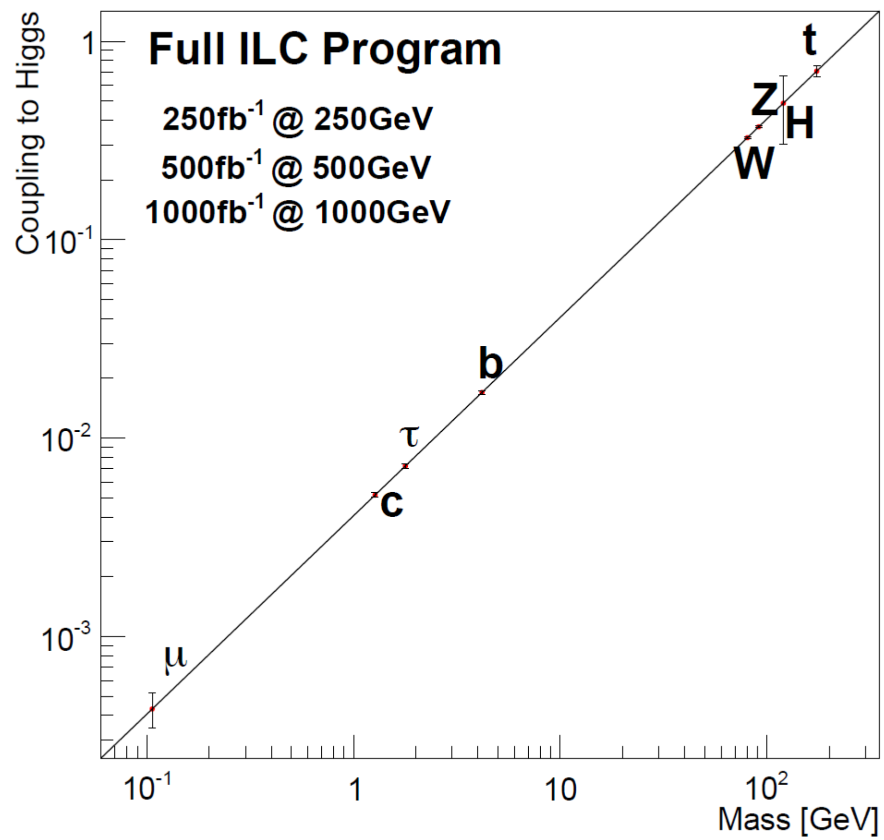
$$g^R_{hWW} \sim \frac{2m_W^2}{v}\left(1 - \frac{5N_c}{96\pi^2}\frac{m_t^2}{v^2}\right)$$

$$y^R_{hff} \sim \frac{\sqrt{2}m_f}{v}\left(1 - \frac{N_c}{12\pi^2}\frac{m_t^2}{v^2}\right)$$

How about the new physics loop contributions?

Current LHC data v.s. Full ILC



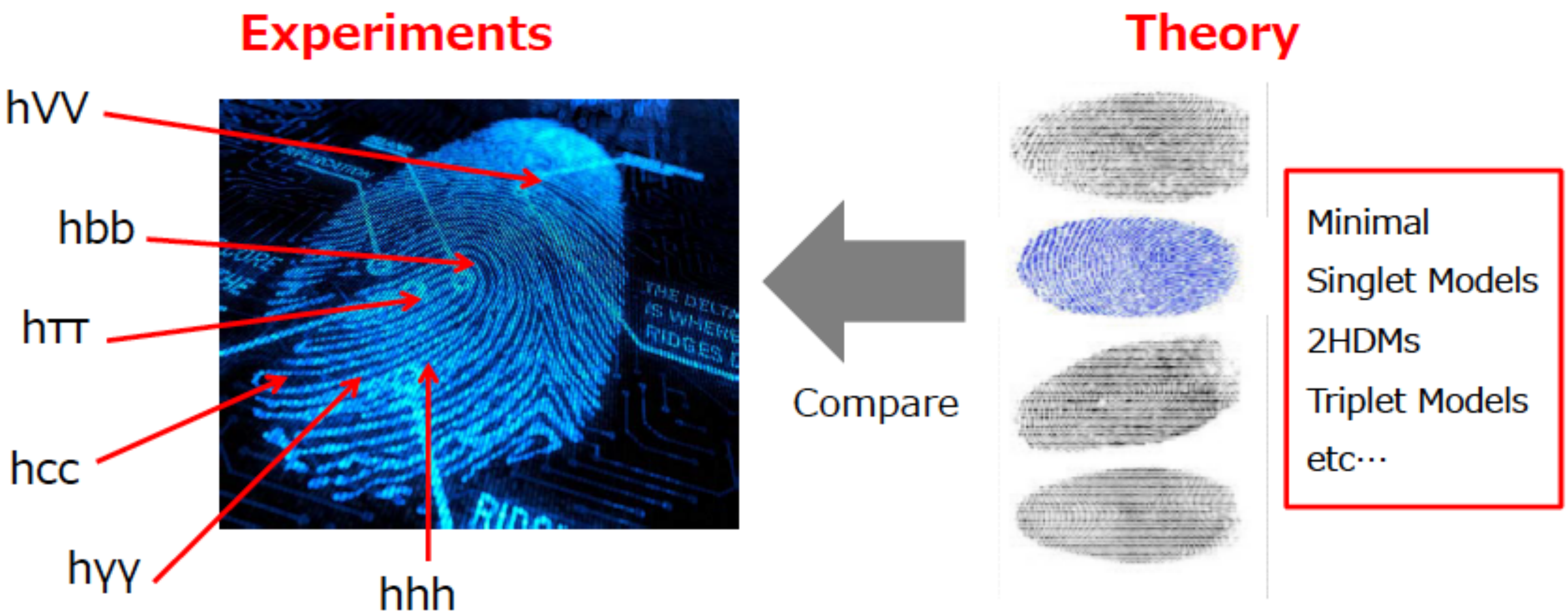


The precision must be improved in future at LHC 13-14 TeV and at the LC

The ILC is really needed!

All SM parameters are found

Next target is new physics!



Fingerprinting new physics models

Radiative Corrections

In future, the Higgs couplings will be measured with much better accuracies

Clearly, tree level analyses are not enough

Analysis with Radiative Corrections (including quantum effect of the 2^{nd} Higgs/BSM particles) is necessary

Theoretical predictions at loop levels × **Precision measurements at future colliders**

⇓

New Physics !

Scale Factors (1-loop level) in 2HDM

Mixing parameter x = cos(β−α) $\left[\sin(\beta-\alpha) = 1 - \frac{x^2}{2}\right]$ **SM-like x << 1**

Scale Factor of the *hVV* Couplings

$\Delta\kappa_X = \kappa_X - 1$

$$\Delta\hat{\kappa}_V \simeq -\frac{1}{2}x^2 - \underline{A(m_\Phi^2, M^2)}$$

mixing loop

Loop Effect

$$A(m_\Phi, M) = \frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi c_\Phi \frac{m_\Phi^2}{v^2}\left(1 - \frac{M^2}{m_\Phi^2}\right)^2$$

$m_\Phi^2 = M^2 + \lambda_i v^2$

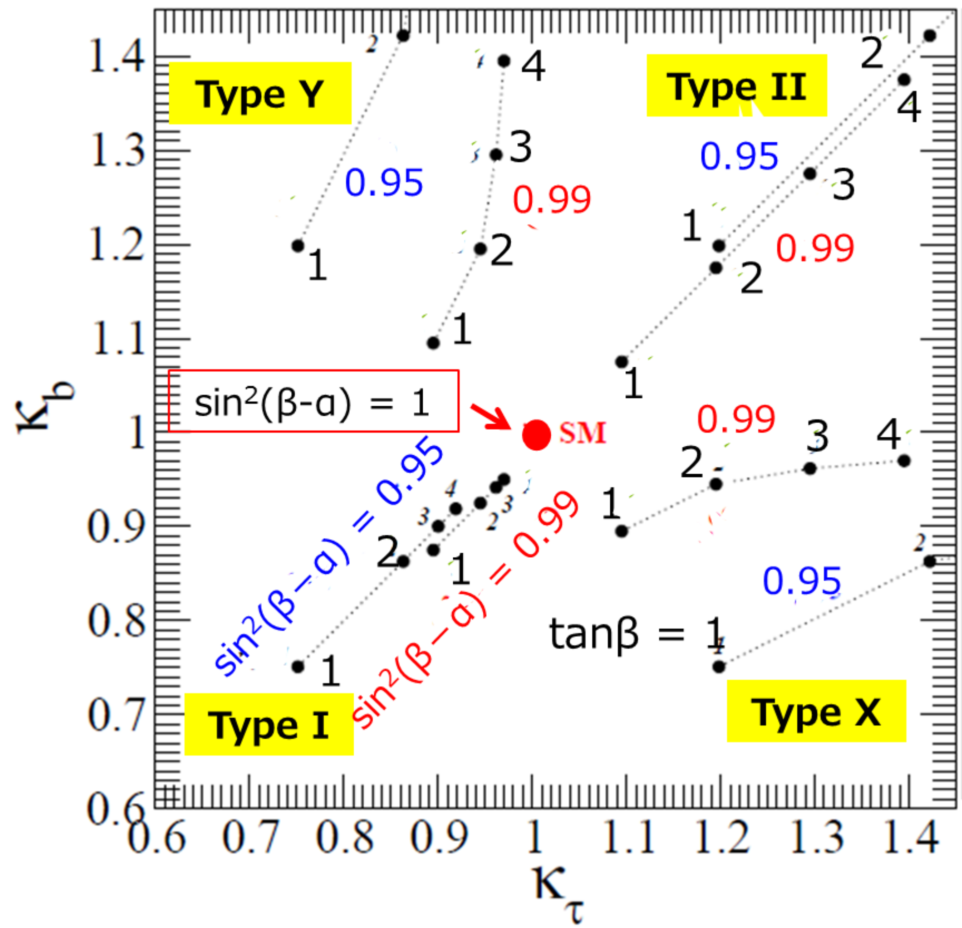
$(\Phi = H^\pm, A, H)$

where

$$m_\Phi^2\left(1 - \frac{M^2}{m_\Phi^2}\right)^2 \begin{cases} \propto \frac{1}{m_\Phi^2} & (M >> v) \quad \textbf{Decoupling!} \\ \propto m_\Phi^2 & (M \sim v) \quad \textbf{Non-decoupling!} \end{cases}$$

Which Yukawa Type ? (tree)

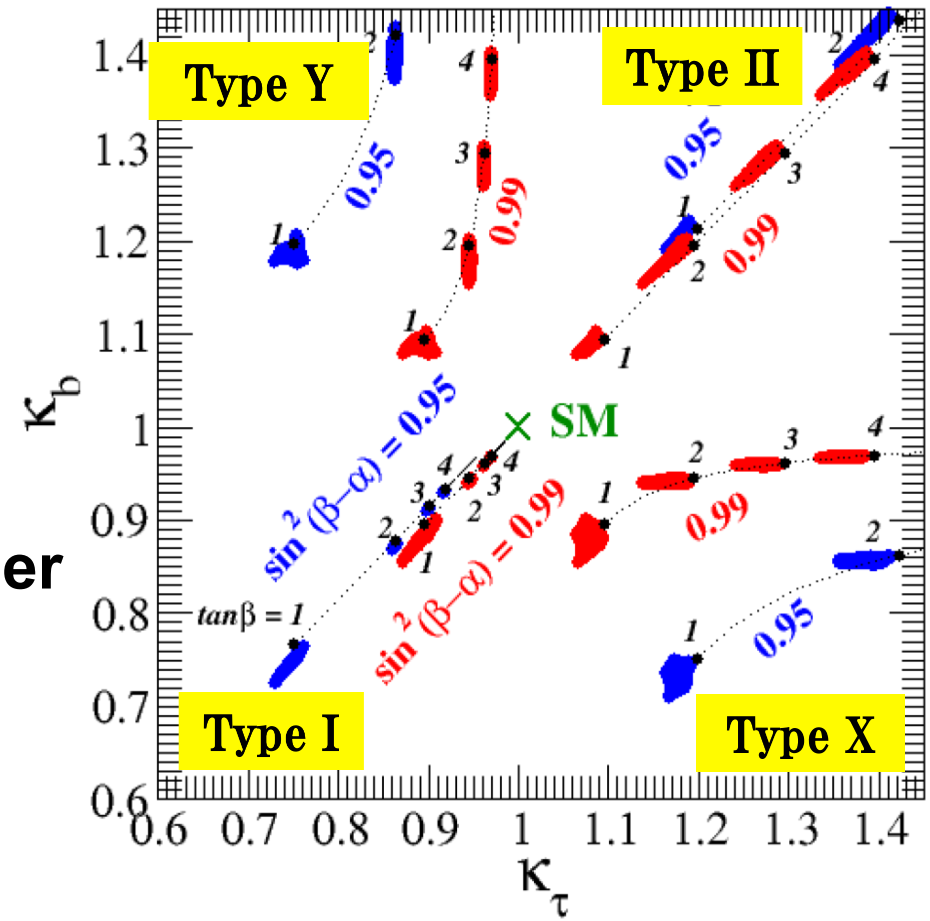
Model	μ	τ	b	c	t	g_V
Singlet mixing	↓	↓	↓	↓	↓	↓
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓



What is going on with radiative correction?

Which Yukawa Type ? (loop)

Model	μ	τ	b	c	t	g_V
Singlet mixing	↓	↓	↓	↓	↓	↓
2HDM-I	↓	↓	↓	↓	↓	↓
2HDM-II (SUSY)	↑	↑	↑	↓	↓	↓
2HDM-X (Lepton-specific)	↑	↑	↓	↓	↓	↓
2HDM-Y (Flipped)	↓	↓	↑	↓	↓	↓



<u>**Evaluation at one-loop**</u>

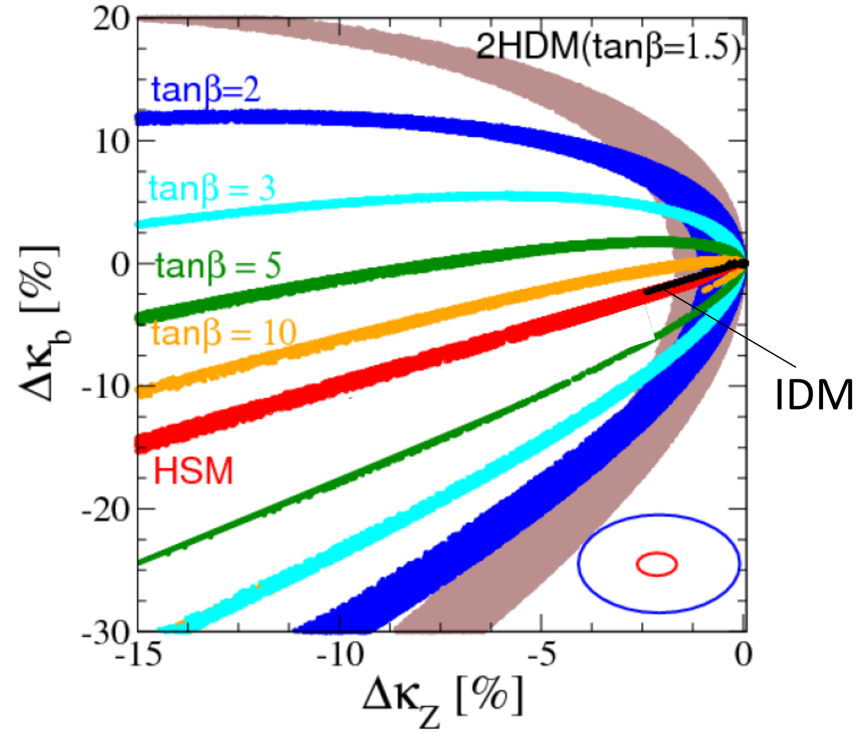
Scan of inner parameters under theoretical and experimental constraints (for each tanβ)

The separation of type can also be done at loop level !

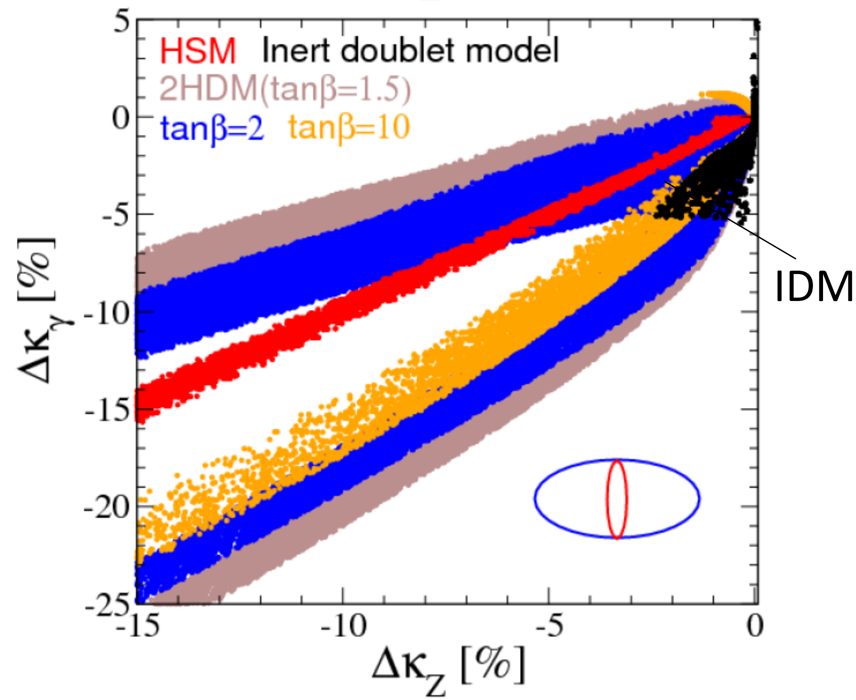
S.K., M. Kikuchi, K. Yagyu, PLB731 (2014) 27

Comparison of
1. 2HDM-I
2. Doublet-Singlet Model (HSM)
3. Inert Doublet Model (IDM)

Scan of inner parameters (mass, mixing angles) under the theoretical conditions of
Perturbative unitarity
Vacuum stability
Condition for avoiding wrong vacuum (HSM)



These models may be distinguished, as long as a deviation in κ_Z is detected



Ellipse, ±1σ at LHC3000 and ILC500

Extraction of parameters

Slide by
Mariko Kikuchi

In the future,
how much precise can we extract values of inner parameters by using LHC3000 and ILC500 data ?

Case A

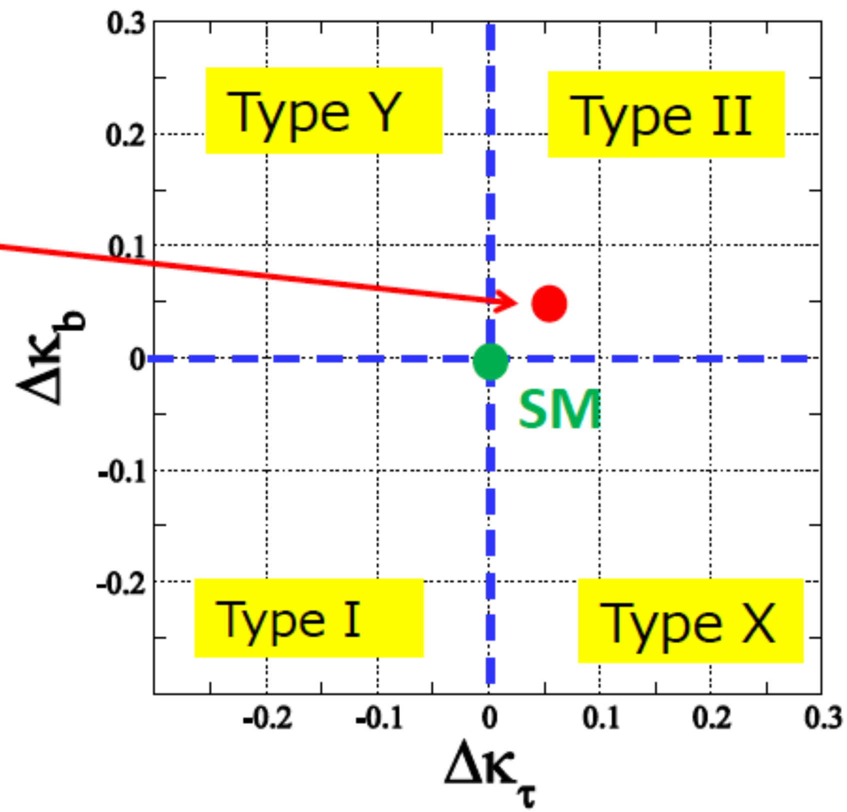
	LHC3000	ILC500	1σ
$\Delta\hat{\kappa}_V = -2.0$	± 2.0	$\pm 0.4\%$	
$\Delta\hat{\kappa}_\tau = +5$	± 2.0	$\pm 1.9\%$	
$\Delta\hat{\kappa}_b = +5$	± 4.0	$\pm 0.9\%$	

Errors are from ILC(500) *in Snowmass 2014 Rep.*

Type-II

We survey parameter regions by scanning inner parameters

$x, \tan\beta, m_\Phi, M$



$\Phi = H^+, H, A$

Extraction of parameters

$$x = \cos(\beta - \alpha)$$

$$\Delta\hat{\kappa}_\tau - \Delta\hat{\kappa}_V \simeq -\tan\beta\, x$$

Input

Errors are from
Snowmass 2014 Rep.

Case A

	LHC3000	ILC500
$\Delta\hat{\kappa}_V = -2.0$	± 2.0	$\pm 0.4\%$
$\Delta\hat{\kappa}_\tau = +5$	± 2.0	$\pm 1.9\%$
$\Delta\hat{\kappa}_b = +5$	± 4.0	$\pm 0.9\%$

Errors are from ILC(500)
in Snowmass 2014 Rep.

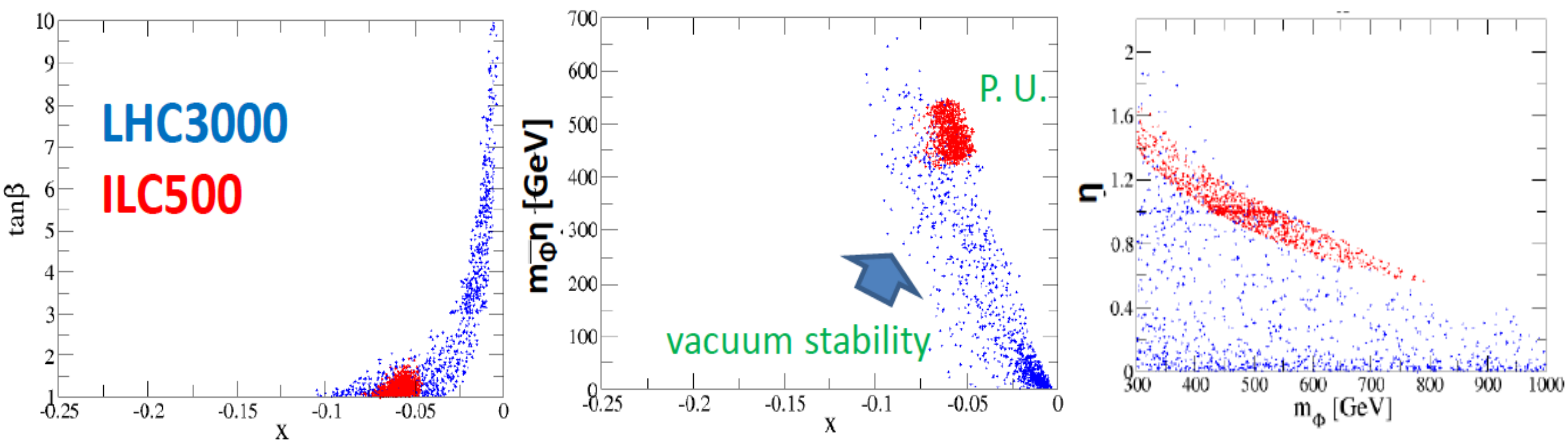
Type-II

$$\eta = 1 - \frac{M^2}{m_\Phi^2}$$

$$\Delta\hat{\kappa}_V \simeq -\frac{1}{2}x^2 - A(m_\Phi^2, M^2)$$

$$A(m_\Phi, M) = \frac{1}{16\pi^2}\frac{1}{6}\sum_\Phi c_\Phi \frac{1}{v^2}\left\{m_\Phi\left(1 - \frac{M^2}{m_\Phi^2}\right)\right\}^2$$

$$(\Phi = H^\pm, A, H)$$



New mass scale can be extracted! $m_\Phi < 800$ GeV

In addition to the type, parameters x and tanβ can be extracted !!

Another example of non-decoupling effects

Higgs potential

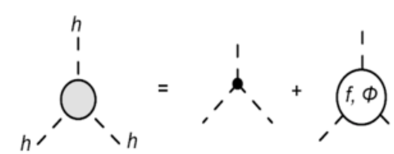
Self-Coupling Constant

It is very important to know *hhh* coupling to reconstruct the Higgs potential

$$V_{\text{Higgs}} = \frac{1}{2}\underline{m_h^2} h^2 + \frac{1}{3!}\underline{\lambda_{hhh}} h^3 + \frac{1}{4!}\lambda_{hhhh} h^4 + \cdots$$

Effective Potential $$V_{\text{eff}}(\varphi) = -\frac{\mu_0^2}{2}\varphi^2 + \frac{\lambda_0}{4}\varphi^4 + \sum_f \frac{(-1)^{2s_f} N_{C_f} N_{S_f}}{64\pi^2} m_f(\varphi)^4 \left[\ln\frac{m_f(\varphi)^2}{Q^2} - \frac{3}{2}\right]$$

Renoramalization $$\left.\frac{\partial V_{\text{eff}}}{\partial\varphi}\right|_{\varphi=v} = 0, \quad \left.\frac{\partial^2 V_{\text{eff}}}{\partial\varphi^2}\right|_{\varphi=v} = m_h^2, \quad \left.\frac{\partial^3 V_{\text{eff}}}{\partial\varphi^3}\right|_{\varphi=v} = \lambda_{hhh}$$



Top loop Effect in the SM

$$\lambda_{hhh}^{\text{SMloop}} \sim \frac{3m_h^2}{v}\left(1 - \frac{N_c m_t^4}{3\pi^2 v^2 m_h^2} + \cdots\right)$$

Non-decoupling effect

Tree level coupling $$\lambda_{hhh} = \frac{3m_h^2}{v_0}$$

Effective Potential

$$V_{\mathrm{eff}}(\varphi) = V_{\mathrm{tree}}(\varphi) + \frac{1}{64\pi^2} N_{c_i} N_{s_i} (-1)^{2s_i} (M_i(\varphi))^4 \left[\ln\left(\frac{(M_i(\varphi))^2}{Q^2} - \frac{3}{2} \right) \right]$$

Top quark effect $$M_\varphi = \frac{y_t \varphi}{\sqrt{2}}$$

Expand the V_{eff} by *h* $$\varphi = v_0 + h$$

$$V_{\mathrm{eff}} = -\frac{\mu^2}{2}(v_0 + h) + \frac{1}{4}\tilde{\lambda}(v_0 + h)^4 - \frac{N_c}{16\pi^2}\frac{y_t^4}{2} v_0^4 \left(\frac{h}{v_0} + \frac{7}{2}\frac{h^2}{v_0^2} + \frac{13}{3}\frac{h^3}{v_0^3} + \cdots \right)$$

$$\tilde{\lambda} = \lambda - \frac{N_c}{16\pi^2} y_t^4 \left(\ln \frac{y_t^2 v_0^2}{2Q^2} - \frac{3}{2} \right)$$

Renormalization conditions

$$\left.\frac{\partial V}{\partial \varphi}\right|_{\varphi=v} = 0, \qquad \left.\frac{\partial^2 V}{\partial h^2}\right|_{\varphi=v} = m_h^2, \qquad \left.\frac{\partial^3 V}{\partial h^3}\right|_{\varphi=v} = \lambda_{hhh}^R$$

$$\frac{\partial V_{\rm eff}}{\partial h} = -\mu^2 v_0 + \tilde{\lambda} v_0^3 - \frac{1}{2} A v_0^3 = 0,$$

$$\frac{\partial^2 V_{\rm eff}}{\partial^2 h} = -\mu^2 + 3\tilde{\lambda} v_0^2 - \frac{7}{2} A v_0^2 = m_h^2, \qquad A = \frac{N_c y_t^4}{16\pi^2}$$

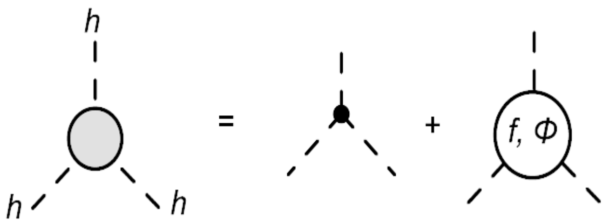
$$\frac{\partial^3 V_{\rm eff}}{\partial^3 h} = 6\tilde{\lambda} v_0 - 13 A v_0 = \lambda_{hhh}^R,$$

Eliminating μ^2 and $\tilde{\lambda}$, and using $y_t = \frac{\sqrt{2} m_t}{v_0}$

$$\lambda_{hhh}^R = \frac{3 m_h^2}{v_0}\left(1 - \frac{N_c}{3\pi^2}\frac{m_t^4}{v_0^2 m_h^2}\right)$$

Case of Non-SUSY 2HDM

- Consider when the lightest h is SM-lik [$\sin(\beta-\alpha)=1$]
- At tree, the *hhh* coupling takes the same form as in the SM
- At 1-loop, non-decoupling effect m_Φ^4 (If $M < v$)



Φ = H, A, H±

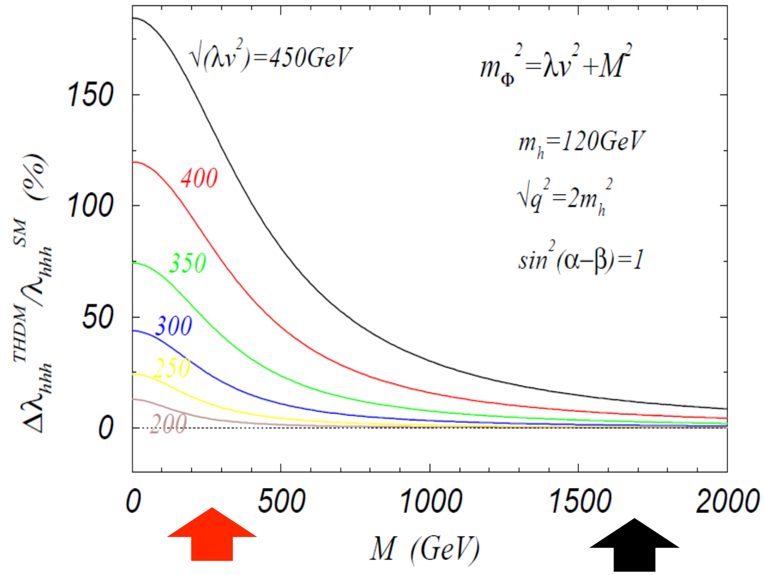
SK, Kiyoura, Okada, Senaha, Yuan, PLB558 (2003)

$$\lambda_{hhh}^{\rm 2HDM} \simeq \frac{3m_h^2}{v}\left[1+\frac{m_\Phi^4}{12\pi^2 m_h^2}\left(1-\frac{M^2}{m_\Phi^2}\right)^3-\frac{m_t^4}{\pi^2 v^2 m_h^2}\right]$$

Extra scalar loop   Top loop

$$m_\Phi^2 = M^2 + \lambda_i v^2$$

($\Phi = H, A, H^\pm$)



Non-decoupling effect   Decoupling

Correction can be huge ～ 100%

Summary of Radiative Corrections

- In order to really test theory calculation by using future precision data, evaluation with radiative corrections is inevitable

- Radiative correction to EW parameters (S, T, U) revealed mass of top and Higgs before their discovery!

- Radiative corrections to Higgs boson couplings in various new physics models make it possible to fingerprint models by future precision data

- The *hhh* coupling is essentially important to explore the Higgs potential. The coupling can deviate largely by the new physics loop effect