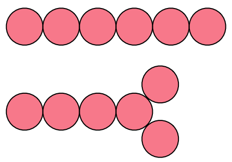
# M5-branes, Orientifolds, S-duality

Joonho Kim (KIAS)

Based on arXiv:1607.08557 with
Yoonseok Hwang (SNU) and Seok Kim (SNU)

# (2,0) theories

- They are first examples of 6d QFTs. [Witten '95].
- They are engineered from IIB string theory on ALE space. (2,0) theories follow the ADE classification.



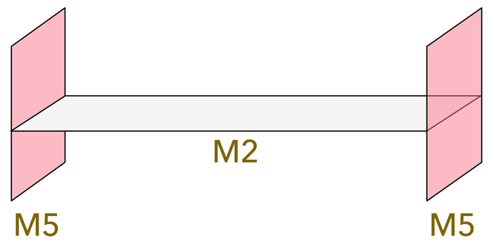
- Properties:
    1. (2,0) tensor multiplet: $(B_2, \Phi^1, \cdots, \Phi^5) + (\text{fermions})$

       self-duality: $H_3 = dB_2,\ H_3 = *H_3$ SO(5)$_R$ symmetry
    2. self-dual strings: source of the tensor multiplet

       tension $\sim (\text{VEV of } \Phi\text{'s})$ → tensionless at fixed point

- For AD-type, they describe the low-energy dynamics of M5's. (free energy) ~ $N^3$ for N M5-branes.

  [Strominger '95] [Klebanov, Tseytlin '96]



# 6d from 5d

- (2,0) theories have no known microscopic formulations.
  We study (2,0) theories using 5d maximal SYMs.

- 5d maximal SYMs are obtained by reducing (2,0) theories on $S^1$.

- Key claim: **5d MSYM + instanton solitons → 6d physics!**
  **Instantons are KK momentum modes along the 6th circle.**

[Douglas '10] [Lambert, Papageorgakis '10]

# 6d from 5d

- Some 6d BPS observables can be computed using 5d maximal SYMs with instanton corrections.
  - Witten index of (2,0) theory on $R^4$ X $T^2$
    [H.-C. Kim, S. Kim, Koh, K. Lee, S. Lee '11]
  - (2,0) superconformal index
    [Lockhart, Vafa; H.-C. Kim, JK, S. Kim, S.-S. Kim, Lee; Kallen, Qiu, Zabzine '12, '13]
  - (2,0) superconformal index with defects
    [H.-C. Kim, Bullimore '15]

- Most studies are focusing on A-type (2,0) theories. We want to
  1) study D-type (2,0) theories,
  2) study (2,0) theories on $S^1$ with an outer automorphism twist.

We study **5d maximal SYMs with classical gauge groups!**

# Outline

1) Introduction

   (2,0) theories. 5d maximal SYMs.

2) Instantons in 5d maximal SYM

   5d gauge theories, instanton partition functions,

   ADHM quantum mechanics, Witten index of ADHM QM

3) Applications

   - S-duality of 5d maximal SYM on $S^1$
   - (2,0) superconformal indices

# 5d maximal SYMs

- 5d N=2 SYM with G = SO(2N+1), Sp(N), SO(2N) gauge groups.
  - 16 SUSY preserved + $SO(5)_R$ R-symmetry.
  - N=2 vector multiplet: $(A_\mu,\ \phi^1, \cdots, \phi^5)$

- We turn on a real scalar VEV for only one scalar field.

$$\langle\phi^5\rangle = \alpha \neq 0,\ \langle\phi^1\rangle = \cdots = \langle\phi^4\rangle = 0 \longrightarrow G \to U(1)^N$$

  This can be called N=1 Coulomb branch.

# 5d maximal SYMs

- There are massive BPS particles in the Coulomb phase.
  - W-boson carries $U(1)^N$ electric charge, preserving 1/2 SUSY.
  - Instanton carries the topological U(1) charge

$$k = \frac{1}{8\pi^2}\int_{\mathbf{R}^4} \mathrm{Tr}\,(F\wedge F) \in \mathbb{Z}_+$$

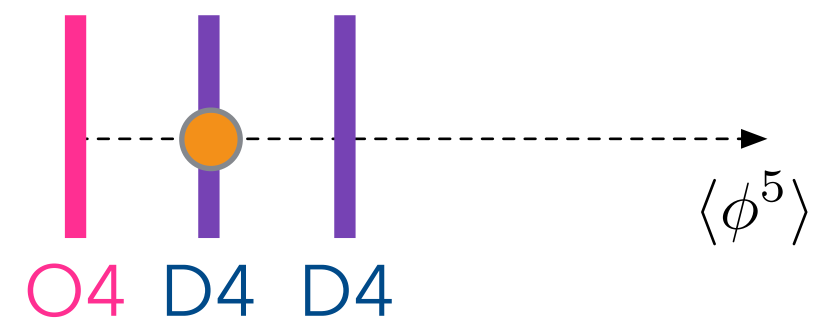
    preserving 1/2 SUSY.
  - They can form 1/4 BPS bound states, preserving 1/4 SUSY.

$$M = \frac{4\pi^2 k}{g_5^2} + \mathrm{Tr}\,(\alpha\cdot\Pi)$$

($\mathbf{\Pi}$ = $U(1)^N$ charge)

# 5d maximal SYMs

- IIA brane configuration
  - 5d N=2 SYMs live on O4-D4's.
  - Instantons are D0's bound to D4's.
  - W-bosons are F1's connecting D4's.




- Symmetries:

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| O4-D4 | ● | ● | ● | ● | ● | | | | | |
| F1 | ● | | | | | | | | | ● |
| D0 | ● | | | | | | | | | |

(Directions 1–4: $SO(4)_1$; directions 5–8: $SO(4)_2$)

  - $SO(4)_1 \times SO(4)_2 \rightarrow SU(2)_{1L} \times SU(2)_{1R} \times SU(2)_{2L} \times SU(2)_{2R}$

    ($\alpha$, $\dot{\alpha}$, $a$, $A$)

# Instanton partition function

- We study the instanton partition function [Nekrasov '02]

$$Z_{\text{inst}} = \mathrm{Tr}\Big[(-1)^F\, e^{-\beta\{Q,Q^\dagger\}}\, q^k\, t^{2(J_{1R}+J_{2R})} u^{2J_{1L}} v^{2J_{2L}} \prod_{a=1}^N w_a^{2\Pi_a}\Big] \qquad \begin{aligned} Q &= Q^{A=1}_{\dot\alpha=\dot1} \\ Q^\dagger &= Q^{A=2}_{\dot\alpha=\dot2} \end{aligned}$$

  - Witten index of 5d SYMs on $R^{1,4}$.
  - β is the regulator of Witten index, a circumference of the temporal circle in the path integral representation.
  - Chemical potentials $t = e^{-\epsilon_+},\ u = e^{-\epsilon_-},\ v = e^{-m},\ w_i = e^{-\alpha_i}$ are IR regulators, generating an effective mass gap.
  - q is the instanton number fugacity. $q = \exp(-\frac{4\pi^2\beta}{g_5^2})$ , $\frac{4\pi^2\beta}{g_5^2} \sim \frac{1}{R_M}$
  - Basic building block for curved-space partition functions.

[$S^5$: Lockhart, Vafa '12; H.-C. Kim, JK, S. Kim '12; Kallen, Qiu, Zabzine '12]
[$S^4$ X $S^1$: H.-C. Kim, S.-S. Kim, Lee '12] [$Y^{p,q}$: Qiu, Zabzine '13] …

# ADHM quantum mechanics

- To compute it, we introduce the ADHM quantum mechanics of the D0-D4-O4 system that realizes the 5d SYM and its instantons.
  - Gauged quantum mechanics

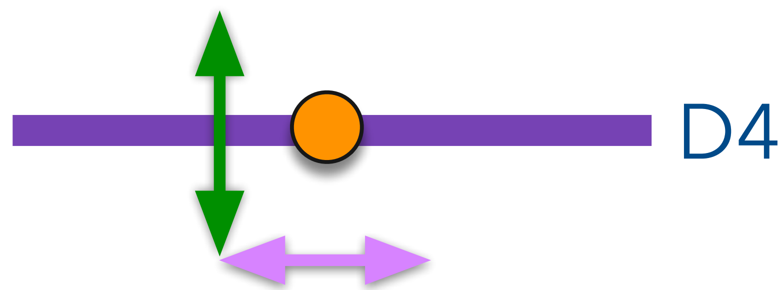
| Type of O4 | 5d group G | 1d group $\hat{G}$ | Rep. R |
|---|---|---|---|
| $O4^-$ | O(2N) | Sp(k) | antisym. |
| $O4^0$ | O(2N+1) | Sp(k) | antisym. |
| $O4^+$ | $Sp(N)_{\theta=0}$ | $O(k)_{\theta=0}$ | sym. |
| $\widetilde{O4}^+$ | $Sp(N)_{\theta=\pi}$ | $O(k)_{\theta=\pi}$ | sym. |

  - Has 8 supercharges $Q^A_{\dot{\alpha}}, Q^a_{\dot{\alpha}}$ → N=(4,4) SUSY
  - $SU(2)_{1L} \times SU(2)_{1R} \times SU(2)_{2L} \times SU(2)_{2R} \times U(1)^N$ global symm.

# ADHM quantum mechanics

- Matter contents are induced from open strings on D0-branes
  - vector multiplet $(A_0, \varphi, \varphi_{aA})$
  - hypermultiplet in R rep. $(a_{\alpha\dot{\alpha}})$
  - hypermultiplet in bif (G, Ĝ) $(q_{\dot{\alpha}})$

- Classical moduli space has two branches.




**These two branches are expected to get decoupled in infrared limit**: $g_1^2 \gg E^3$

1) Coulomb branch
2) Higgs branch
   = instanton moduli space

# Index of ADHM QM

- Consider the multi-particle index of D0-branes,

$$Z_{\rm ADHM} = 1 + \sum_{k=1}^{\infty} q^k \, I_k$$

whose coefficient $I_k$ is the Witten index of ADHM QM at given k.

- Due to IR decoupling of Higgs and Coulomb branches,

$$Z_{\rm ADHM} = Z_{\rm inst} \cdot Z_{\rm extra}$$

  - $Z_{\rm inst}$ : Higgs branch → 5d SYM instanton partition function
  - $Z_{\rm extra}$ : Coulomb branch → Bulk index irrelevant from 5d SYM

[Hayashi, H.-C. Kim, Nishinaka '13] [Bao, Mitev, Pomoni, Taki, Yagi '13]
[Bergman, Rodriguez-Gomez, Zafrir '13] [Hwang, JK, S. Kim, Park '14]

# Index of ADHM QM

- **Task 1.** Compute $I_k$ , Witten index of ADHM QM at given k.
  - Witten index of SUSY QM [Hwang, JK, S. Kim, Park '14] [Hori, Kim, Yi '14]
  - One can express the Witten index as the QM path integral.
  - Roughly speaking, SUSY localization reduces the path integral into the multi-dimensional contour integrals:

$$I_k = \sum_a \frac{1}{|W_a|} \oint \prod_{i=1}^{r_a} d\phi_i \cdot I^{(a)}_{\text{1-loop}}$$

space: eigenvalues of complexified gauge holonomies

$$\phi = i\beta A_t + \beta\varphi$$

integrand: Gaussian integral over massive fluctuations

contour: Jeffrey-Kirwan residue operation

$$\text{JK-Res}_{\phi_*}(\mathbf{Q}_*, \eta) \frac{d\phi_1 \wedge \cdots \wedge d\phi_r}{Q_{j_1}(\phi - \phi_*) \cdots Q_{j_r}(\phi - \phi_*)} = \begin{cases} |\det(Q_{j_1}, \cdots, Q_{j_r})|^{-1} & \text{if } \eta \in \text{Cone}(Q_{j_1}, \cdots, Q_{j_r}) \\ 0 & \text{otherwise} \end{cases} .$$

# Index of ADHM QM

- **Task 2.** Identify $Z_{extra}$ and divide it out.
  - $Z_{extra}$ captures D0-brane bound states irrelevant in 5d SYMs.
  - To identify it, we separately compute the D0-brane index in the pure orientifold background without D4-branes.
  - ADHM quantum mechanics has no bif. hypermultiplet.
    - vector multiplet $(A_0, \varphi, \varphi_{aA})$
    - hypermultiplet in R rep. $(a_{\alpha\dot{\alpha}})$

# Index of ADHM QM

- **Task 2.** Identify $Z_{\text{extra}}$ and divide it out.

$$Z_{\text{ADHM}}^{\text{D0-O4}^-} = \text{PE}\left[\frac{t^2(v+v^{-1}-u-u^{-1})(t+t^{-1})}{2(1-tu)(1-tu^{-1})(1+tv)(1+tv^{-1})}\frac{q}{1-q}\right]$$

$$Z_{\text{ADHM}}^{\text{D0-O4}^0} = \text{PE}\left[\frac{t^2(v+v^{-1}-u-u^{-1})(t+t^{-1})}{2(1-tu)(1-tu^{-1})(1+tv)(1+tv^{-1})}\frac{-q}{1+q}\right]$$

$$Z_{\text{ADHM}}^{\text{D0-O4}^+} = \text{PE}\left[\frac{t^2(v+v^{-1}-u-u^{-1})(t+t^{-1})}{2(1-tu)(1-tu^{-1})(1+tv)(1+tv^{-1})}\frac{q^2}{1-q^2}+\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-tu^{-1})}\frac{q}{1-q^2}\right]$$

$$Z_{\text{ADHM}}^{\text{D0-}\widetilde{\text{O4}}^+} = \text{PE}\left[\frac{t^2(v+v^{-1}-u-u^{-1})(t+t^{-1})}{2(1-tu)(1-tu^{-1})(1+tv)(1+tv^{-1})}\frac{-q^2}{1+q^2}+\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-tu^{-1})}\frac{q^2}{1-q^4}\right]$$

checked up to
$q^3$ (O4$^-$, O4$^0$),
$q^7$ (O4$^+$, $\widetilde{\text{O4}}^+$) order.

- They are reproduced from the trace over 11d SUGRA fields

$$\text{Tr}\left[(-1)^F\,\frac{\mathbf{1}+\mathcal{P}}{2}\,e^{-\beta\{Q,Q^\dagger\}}\,q^k\,t^{2(J_{1R}+J_{2R})}u^{2J_{1L}}v^{2J_{2L}}\right]$$

in $\mathbf{R}^{1,4}\times(\mathbf{R}^5/\mathbb{Z}_2)\times S^1$ and $\mathbf{R}^{1,4}\times(\mathbf{R}^5\times S^1)/\mathbb{Z}_2$ backgrounds.

11d uplifts of O4-planes: [Hori `98], [Gimon `98]

# Outline

1) Introduction

(2,0) theories. 5d maximal SYMs.

2) Instantons in 5d maximal SYM

5d gauge theories, instanton partition functions,

ADHM quantum mechanics, Witten index of ADHM QM

3) Applications

- S-duality of 5d maximal SYM on $S^1$
- (2,0) superconformal indices

# S-duality of 5d SYM

- There are D0-brane bound states with $O4^+$, $\widetilde{O4}^+$ planes.

[Keurentjes, Sethi '02]

$$Z_{\rm ADHM}^{\text{D0-O4}^+} = \text{PE}\left[\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-tu^{-1})}\frac{q}{1-q^2}\right]$$

$$Z_{\rm ADHM}^{\text{D0-}\widetilde{\text{O4}}^+} = \text{PE}\left[\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-tu^{-1})}\frac{q^2}{1-q^4}\right]$$

checked up to $q^7$ order.

- M-theory uplifts of $O4^+$, $\widetilde{O4}^+$ planes involve frozen M5-branes, wrapped on $R^4$ X $T^2$.

[Hori `98]

  - $T^2$ : temporal circle + M-theory circle. $q = e^{i\pi\tau}$ with $\tau = i\frac{R_t}{R_M}$

$O4^+ : \ \mathbf{R}^{1,4} \times (\mathbf{R}^5/\mathbb{Z}_2) \times S^1$

M5-brane at $Z_2$ fixed plane.

$\widetilde{O4}^+ : \ \mathbf{R}^{1,4} \times (\mathbf{R}^5 \times S^1)/\mathbb{Z}_2$

M5-brane wrapped on the shift orbifold, $X^{11} \rightarrow X^{11} + \pi R_M$, seeing the M-circle radius as halved.

$$q \rightarrow q^2 \quad (\tau \rightarrow 2\tau)$$

# S-duality of 5d SYM

- S-duality transformation swaps the temporal circle and M-circle.

$$\tau \to -\frac{1}{\tau},\ \epsilon_+ \to \frac{\epsilon_+}{\tau},\ \epsilon_- \to \frac{\epsilon_-}{\tau},\ m \to \frac{m}{\tau}$$

1) We trace how S-duality transforms the D0-O4⁺ index

$$\mathrm{PE}\left[\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-tu^{-1})}\frac{q}{1-q^2}\right] \xrightarrow{\ \mathrm{S}\ } \mathrm{PE}\left[-\frac{t(v+v^{-1}-u-u^{-1})}{(1-tu)(1-tu^{-1})}\frac{q}{1-q^2} - \frac{t^2(v+v^{-1}-u-u^{-1})(v+v^{-1}-t-t^{-1})}{(1-tu)(1-tu^{-1})(1+tu)(1+tu^{-1})}\frac{1}{1-q^2}\right]$$

using the S-dual invariance of Abelian M5-brane index.

[H.-C. Kim, S. Kim, S.-S. Kim, Lee `13]

2) $O4^+ \to O4^- + D4$, frozen at $O4^-$ due to $O(2)_-$ Wilson line.

[Gimon `98]

5d $O(2)_-$ instanton partition function shows the agreement.

# S-duality of 5d SYM

- Our result confirms **S-duality relation between O4-planes**.
- By adding N D4-branes, it is extended to **S-duality of 5d SYM**.

[Tachikawa '11]

1. SO(2N) SYM $\longleftrightarrow$ SO(2N) SYM
2. SO(2N+1) SYM $\longleftrightarrow$ SU(2N) SYM twisted by $Z_2$
3. Sp(N) SYM with θ = 0 $\longleftrightarrow$ SO(2N+2) SYM twisted by $Z_2$
4. Sp(N) SYM with θ = $\pi$ $\longleftrightarrow$ SU(2N+1) SYM twisted by $Z_2$

- Instanton partition functions can be S-dualized in the limit $m \to \epsilon_+$

$$Z_{\text{inst}}^{SO(2N)} = \text{PE}\left[\frac{N\,q}{1-q}\right] = \eta(\tau)^{-N} \qquad \longrightarrow \eta(\tau)^{-N}$$

**invariant**

$$Z_{\text{inst}}^{Sp(N)|_{\theta=0}} = \text{PE}\left[\frac{N\,q^2}{1-q^2} + \frac{q}{1-q^2}\right] = \eta(\tau)^{-N+1}\,\eta(\tfrac{\tau}{2})^{-1} \qquad \longrightarrow \eta(\tau)^{-N+1}\,\eta(2\tau)^{-1}$$

**in agreement with dual partition function**

$$Z_{\text{inst}}^{SO(2N+1)} = \text{PE}\left[\frac{N\,q}{1-q}\right] = \eta(\tau)^{-N} \qquad \longrightarrow \eta(\tau)^{-N}$$

$$Z_{\text{inst}}^{Sp(N)|_{\theta=\pi}} = \text{PE}\left[\frac{N\,q^2}{1-q^2} + \frac{q^2}{1-q^4}\right] = \eta(\tau)^{-N-1}\,\eta(2\tau)^{+1} \qquad \longrightarrow \eta(\tau)^{-N-1}\,\eta(\tfrac{\tau+1}{2})^{+1}$$

# (2,0) superconformal index

- It is the Witten index of the radially quantized (2,0) SCFT on $S^5$.

[Bhattacharya et al. '08]

$$\mathcal{I} = \mathrm{Tr}\left[(-1)^F e^{-\sigma\{Q,Q^\dagger\}} e^{-\sigma(E-\frac{R_1+R_2}{2})-\sigma(ah_1+bh_2+ch_3)-\sigma\mu\frac{R_2-R_1}{2}}\right] \text{ with } a+b+c=0$$

  - Expressed as SUSY path integral on $S^5 \times S^1$ ($\sigma \sim$ circumference of $S^1$)
  - Can be computed from 5d maximal SYMs on $S^5$.
  - In the special limit $a \to -b,\ c \to 0,\ \mu \to \frac{1}{2}$ → $\mathcal{I} = \mathrm{Tr}\left[(-1)^F e^{-\sigma\{Q,Q^\dagger\}} e^{-\sigma(E-R_1)}\right]$

  (E = scaling dimension, $R_{1,2}$ = $SO(5)_R$ charges)

- $S^5$ partition functions are computed by merging three copies of instanton partition functions.

[Lockhart, Vafa '12; H.-C. Kim, JK, S. Kim '12; Kallen, Qiu, Zabzine '12]

$$Z_{S^5}(\sigma,a,b,c,m) = \frac{1}{|W_G|}\int_{-\infty}^{\infty}\left[\prod_{i=1}^{N} d\lambda_i\right] e^{-\frac{2\pi^2\,\mathrm{Tr}\,\lambda^2}{\sigma(1+a)(1+b)(1+c)}} \cdot Z^{(1)}_{\rm pert}Z^{(1)}_{\rm inst}\cdot Z^{(2)}_{\rm pert}Z^{(2)}_{\rm inst}\cdot Z^{(3)}_{\rm pert}Z^{(3)}_{\rm inst}$$

# (2,0) superconformal index

- $A_{N-1}$ type SCI: $Z_{S^5}^{SU(N)} = e^{\frac{\sigma}{6}c_2|G|+\frac{\sigma(N-1)}{24}} \cdot \mathrm{PE}\left[\frac{\sum_{m=2}^{N} e^{-m\sigma}}{1-e^{-\sigma}}\right]$

[H.-C. Kim, JK, S. Kim '12]

  - Large N limit agrees with trace over $AdS^7$ X $S_4$ SUGRA.
  - Agrees with the vacuum character of $W_A$ algebra. [Beem et al. '14]
  - W-algebra conjecture: (2,0) theory of type G has a protected sector of operators which satisfy $W_G$ algebra. [Beem et al. '14]

- $D_N$ type SCI: $Z_{S^5}^{SO(2N)} = e^{\frac{\sigma}{6}c_2|G|+\frac{\sigma N}{24}} \cdot \mathrm{PE}\left[\frac{e^{-N\sigma} + \sum_{m=1}^{N-1} e^{-2m\sigma}}{1-e^{-\sigma}}\right]$

  - Large N limit agrees with trace over $AdS^7$ X $S_4$ / $Z_2$ SUGRA.

[H.-C. Kim, JK, S. Kim '12]

  - Agrees with **the vacuum character of $W_D$ algebra.**

# (2,0) superconformal index

- (2,0) theories on $S^1$ with outer automorphism twists are reduced to 5d maximal SYMs with non-simply-laced groups [Witten '09]
  1. SO(2N+1) SYM $\longleftrightarrow$ SU(2N)-type (2,0) theories w/ $Z_2$ twist
  2. Sp(N) SYM with θ = 0 $\longleftrightarrow$ SO(2N+2)-type (2,0) theories w/ $Z_2$ twist
  3. Sp(N) SYM with θ = $\pi$ $\longleftrightarrow$ SU(2N+1)-type (2,0) theories w/ $Z_2$ twist

- We obtain **SCI of (2,0) theory with outer automorphism twists**, computed from 5d non-simply-laced maximal SYMs on $S^5$.

$$Z_{S^5}^{SO(2N+1)} = e^{\frac{\sigma}{6}c_2|G|+\frac{\sigma N}{24}} \cdot \mathrm{PE}\left[\frac{-e^{-\sigma/2}+e^{-(N+\frac{1}{2})\sigma}+\sum_{m=1}^{N} e^{-(2m-1)\sigma}}{1-e^{-\sigma}}\right] \longrightarrow \mathrm{PE}\left[-\frac{e^{-\sigma/2}}{1-e^{-\sigma}}+\frac{e^{-\sigma}}{(1-e^{-\sigma})(1-e^{-2\sigma})}\right]$$

$$Z_{S^5}^{\mathrm{Sp(N)}|_{\theta=0}} = e^{\frac{\sigma}{6}c_2|G|+\frac{(N+1)\sigma}{48}} \cdot \mathrm{PE}\left[\frac{-e^{-\sigma/2}-e^{-\frac{N+1}{2}\sigma}-e^{-\frac{N+2}{2}\sigma}+\sum_{m=0}^{N}(e^{-\frac{2m+1}{2}\sigma}+e^{-\frac{2m+2}{2}\sigma})}{1-e^{-\sigma}}\right] \longrightarrow \mathrm{PE}\left[-\frac{e^{-\sigma/2}}{1-e^{-\sigma}}+\frac{e^{-\sigma/2}}{(1-e^{-\sigma/2})(1-e^{-\sigma})}\right]$$

$$Z_{S^5}^{\mathrm{Sp(N)}|_{\theta=\pi}} = e^{\frac{\sigma}{6}c_2|G|+\frac{(N+1/2)\sigma}{48}} \cdot \mathrm{PE}\left[\frac{-e^{-\sigma/4}-e^{-\frac{N+1}{2}\sigma}+\sum_{m=0}^{N} e^{-\frac{2m+1}{2}\sigma}}{1-e^{-\sigma/2}}\right] \longrightarrow \mathrm{PE}\left[-\frac{e^{-\sigma/4}}{1-e^{-\sigma/2}}+\frac{e^{-\sigma/2}}{(1-e^{-\sigma/2})(1-e^{-\sigma})}\right]$$

These indices have smooth large N limits.

# Summary

- We computed the instanton partition functions of 5d N=2 SYMs with SO(2N+1), Sp(N), SO(2N) gauge groups.

- These observables have various applications.

  1) We studied S-duality relation of orientifold backgrounds, extended to S-duality of 5d maximal SYM on $S^1$.

  2) We computed D-type (2,0) superconformal indices, displaying the BPS operator spectrum.
     They support the W-algebra conjecture.

  3) We proposed the superconformal indices of (2,0) theories on $S^1$ with outer automorphism twists.