String Junctions in $\mathcal{S}$-fold background & 4d $\mathcal{N} = 3$ SCFTs

Prarit Agarwal

Seoul National University, Seoul, Republic of Korea

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Outline

- Brief Summary of recent developments in understanding 4d $\mathcal{N} = 3$ SCFTs
- An explicit counting of degrees of freedom contributing to the central charges
- SUSY enhancement and $1/2$-BPS dyonic states
- 3-pronged strings in $\mathcal{S}$-fold background

Recent Developments

- 4d $\mathcal{N} = 3$ theories must necessarily be non-Lagrangian
- Aharony and Evtikhiev: Consider the constraints due to $\mathcal{N} = 2 \subset \mathcal{N} = 3$ superalgebra and compare with properties of $\mathcal{N} = 4$ theories (when thought of as trivial examples of $\mathcal{N} = 3$ theories)
  1. central charges are such that $a = c$
  2. no $\mathcal{N} = 3$ preserving deformations
  3. no non-R global symmetries
  4. dimension of Coulomb branch operators(CBO) must be ≥ 3
  5. dimension 2 CBO $\implies$ SUSY enhancement

F-theory construction

- By Inaki Garcia-Etxebarria and Diego Regalado
- k-fold generalization of the M-theory lift of orientifolds
- M-theory on $\mathbb{R}^{2,1} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$, $k = 1, 2, 3, 4, 6$
- $T^2 \to 0$ gives rise to a stack of N D3-branes with $\mathbb{C}^3/\mathbb{Z}_k$ as their transverse space
- k =1 gives $\mathcal{N} = 4$ $SU(N)$ SYM living on a stack of N D3 branes in flat background
- k=2 gives N D3-branes on top of an orientifold
- k=3,4,6 gives N D3-branes on top of an "$\mathcal{S}$-fold"

M-theory on $\mathbb{R}^{2,1} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$, $k = 3, 4, 6$

- T^2 : $\tau = e^{\frac{2\pi i}{k}}$
- Claim: This background generically preserves only 12 supercharges
- $\mathbb{Z}_k$ is embedded diagonally inside $SL(2,\mathbb{Z})_{em} \times SO(6)_R$
- $\mathbb{Z}_k \subset SL(2,\mathbb{Z})_{em}$:
  $(Q_1, Q_2, Q_3, Q_4) \to \gamma^{-\frac{1}{2}}(Q_1, Q_2, Q_3, Q_4),\ \gamma^k = 1$
- $\mathbb{Z}_k \subset SO(6)_R$:
  $(Q_1, Q_2, Q_3, Q_4) \to (\gamma^{\frac{1}{2}}Q_1, \gamma^{\frac{1}{2}}Q_2, \gamma^{\frac{1}{2}}Q_3, \gamma^{-\frac{3}{2}}Q_4)$
- Combined action therefore preserves only 3 sets of supercharges

Relationship with Complex Reflection Groups

- Aharony and Tachikawa: N D3-branes probing a $\mathbb{Z}_k$ twisted S-fold background are intimately related to the Complex Reflection Group $G(k, p, N)$
- Complex Reflection Groups generalize Euclidean reflections to the group of reflections in an N dimensional complex vector space with a Hermitian inner product
- For our purposes we can think of them as generalization of Weyl reflection groups of Lie algebras.

Complex Reflection Groups As Generalized Weyl Reflections

- Indeed

$$G(1,1,N) \equiv S_N, \text{ Weyl subgroup of } A_{N-1}$$
$$G(2,1,N) \equiv \text{Weyl subgroup of } B_N \text{ and } C_N$$
$$G(2,2,N) \equiv \text{Weyl subgroup of } D_N$$

- Weyl subgroup of the gauge symmetry associated to the $\mathcal{N}=4$ SYM theory living on the world volume of D3-branes probing $\mathbb{Z}_k$-twisted $\mathcal{S}$-fold for $k=1,2$
- Claim: $G(k,p,N)$ is the equivalent of "Weyl subgroup" of the gauge symmetry enjoyed by the world-volume theory of D3-branes probing a $\mathbb{Z}_k$-twisted $\mathcal{S}$-fold

- p corresponds to discrete torsion.
- N D3-branes transverse to $\mathbb{C}^3/\mathbb{Z}_k$ imply that vacuum moduli space of 4d theory is $(\mathbb{C}^3/\mathbb{Z}_k)^N/S_N$
- For k=2, 4d $\mathcal{N}=4$ theory with $SO(2N+1)$, $Sp(N)$ or $SO(2N)$ gauge symmetry
- Use R-symmetry and gauge transformations to make all the branes coplanar in the transverse space with positions $z_i,\ i=1,\ldots,N$
- Vector multiplet scalars: $\langle\varphi_1\rangle = \mathrm{diag}(z_1,\ldots,z_N)$ and $\langle\varphi_2\rangle = \langle\varphi_3\rangle = 0$

- Weyl subgroup of $Sp(N)$:

$$\begin{aligned} z_i \leftrightarrow z_j, &\quad \text{all others fixed,} \\ z_i \to -z_i, &\quad \text{all other } z_a \text{ fixed} \end{aligned}$$

- Gives $(\mathbb{C}/\mathbb{Z}_2)^N/S_N$, by rotating the 3 scalars into each other using R-symmetry, we get $(\mathbb{C}^3/\mathbb{Z}_2)^N/S_N$

- Weyl subgroup of $SO(2N)$:

$$\begin{aligned} z_i \leftrightarrow z_j, &\quad \text{all others fixed,} \\ (z_i, z_j) \to (-z_i, -z_j) &\quad \text{all other } z_a \text{ fixed} \end{aligned}$$

- Clearly, will not give the same moduli space

- Resolution: Introduce a $\mathbb{Z}_2$ discrete symmetry in the disconnected part of the gauge symmetry

- The gauge symmetry is $O(N)$ rather than $SO(N)$

- This action of $G(k, 1, N)$ on z_i is given by

$$
\begin{aligned}
&z_i \leftrightarrow z_j, && \text{all others fixed,} \\
&z_i \to \gamma z_i,\ \gamma^k = 1 && \text{all other } z_a \text{ fixed}
\end{aligned}
$$

- Orbifolding $\mathbb{C}^N$ by $G(k, 1, N)$ gives $(\mathbb{C}/\mathbb{Z}_k)^N / S_N$
- R-symmetry implies that the vacuum manifold is $(\mathbb{C}^3/\mathbb{Z}_k)^N / S_N$
- $\mathbb{Z}_k$ twisted $\mathcal{S}$-fold
- Refining the $\mathcal{S}$-fold classification by discrete torsion is equivalent to realizing a $\mathbb{Z}_p$ subgroup of $G(k, 1, N)$ as a discrete gauge group in the disconnected part of the gauge symmetry

- Gives $G(k, p, N) \times \mathbb{Z}_p$
- Action of $G(k, p, N)$

$$
\begin{aligned}
&z_i \leftrightarrow z_j, && \text{all others fixed,} \\
&(z_i, z_j) \to (\gamma z_i, \gamma^{-1} z_j) && \text{all other } z_a \text{ fixed} \\
&z_i \to \gamma^p z_i,\ \gamma^k = 1 && \text{all other } z_a \text{ fixed}
\end{aligned}
$$

- A-T showed that for $k < 6$ only $p = 1, k$ are realizable in M-theory. For $k = 6$, only $p = 6$ is physical
- When $p > 1$, there is a additional $\mathbb{Z}_p$ gauge symmetry in a disconnected sector of the gauge group
- $\mathbb{Z}_p$ gauge symmetry is such that it only changes the global structure of the gauge symmetry and doe not change the dynamics on $\mathbb{R}^4$
- Can ungauge the $\mathbb{Z}_p$ symmetry for the purpose of computing central charges
- Ring of Coulomb branch operators is given by the ring of invariants of $G(k, p, n)$

$$\Delta(\mathcal{O}_i) = k, \ldots, (n-1)k; n\ell, \ \ell = \frac{k}{p}$$

- Substitute in

$$2a - c = \frac{1}{4}\sum_i (2\Delta(\mathcal{O}_i) - 1),$$
$$a = c$$

- We find $4a = 4c = kN^2 + N(2\ell - k - 1)$

Explicit counting of degrees of freedom

- In $\mathcal{N} = 2$ theories, define effective number of vector multiplets, n_v and effective number of hypers n_h, s.t.

$$c = \frac{2n_v + n_h}{12}, \quad a = \frac{5n_v + n_h}{24}$$

- For the present case this gives

$$n_v = n_h = 4a = kN^2 + N(2\ell - k - 1)$$

- An $\mathcal{N} = 2$ vector combines with a hyper to form an $\mathcal{N} = 3$ vector

- Define effective number of $\mathcal{N} = 3$ vectors to be

$$\widetilde{n}_v = 4a = kN^2 + N(2\ell - k - 1)$$

- For $k = 1, 2$, $\widetilde{n}_v =$ dim. of adjoint rep. of the respective gauge group
- For $k \geq 3$, no such interpretation of $\widetilde{n}_v$ is possible as there is no Lie algebra
- Can still interpret $\widetilde{n}_v$ as the number of fundamental strings stretched between the D3-branes

- The space transverse to the D3-branes is $\mathbb{C}^3/\mathbb{Z}_k$
- $\mathbb{C}^3/\mathbb{Z}_k$ is k-fold connected
- For any pair of branes, suspend a fundamental string along each of the k homotopically distinct paths between them
- Each such string gives rise to a state labeled by a 4-vector of electromagnetic charges $(n_e^{a_1}, n_m^{a_1}; n_e^{a_2}, n_m^{a_2})$ with respect to the $U(1)_{a_1} \times U(1)_{a_2}$ gauge symmetry associated to the pair of branes. A string going in the opposite direction, then gives rise to the a state with a conjugate charge vector. See figure 1

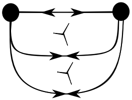


Figure 1: The fundamental string stretched between a pair of branes probing an $\mathcal{S}_3$-fold

- For each pair of branes we thus get $2k$ $\mathcal{N} = 3$ vector multiplets. In a stack of N branes, this gives $\binom{N}{2} \times 2k = kN(N-1)$ vector multiplets.

- We also have to include strings going from a brane to it self. We conjecture that when, $p = k$, the strings going from a brane to itself along non-contractible paths, do not give rise to any vector mutliplets and hence do not contribute to the central charges a, c. This gives rise to $2\ell - 1,\ \ell = \frac{k}{p}$ additional vector multiplets for each brane in the stack. See figure 2

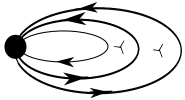


Figure 2: The fundamental string stretched from a brane to itself in an $\mathcal{S}_3$-fold background

- The total number of vector multiplets associated to strings suspended between the branes in the stack is therefore given by

$$\begin{aligned}\widetilde{n}_v &= kN(N-1) + (2\ell - 1)N \\ &= kN^2 + (2\ell - k - 1)N\end{aligned}$$

- An equivalent way of drawing the above cartoons is by considering the branes and their images in the $\mathcal{S}$-fold as shown in figure 3

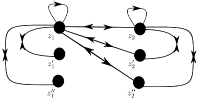


Figure 3: Strings suspended between branes and their images

- The state corresponding to a (p, q) string ending on the n-th image of a brane, acquires electromagnetic quantum numbers (n_e, n_m) with respect to the corresponding $U(1)$ gauge symmetry, such that

$$n_e + n_m\gamma = (p + q\gamma)\gamma^n, \ \gamma^k = 1$$

Rank-2 theories with SUSY enhancement

- Turns out that

$$G(3,3,2) \equiv S_3, \text{ Weyl subgroup of } SU(3)$$
$$G(4,4,2) \equiv \text{Weyl subgroup of } SO(5)$$
$$G(6,6,2) \equiv \text{Weyl subgroup of } G_2$$

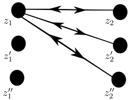
- In each case, the dimension of the CBO corresponding to the generalized pfaffian is 2
- A-T: When there are exactly two branes probing the $\mathcal{S}$-fold, there is SUSY enhancement to $\mathcal{N} = 4$ SYM with $SU(3)$, $SO(5)$ and G_2 gauge symmetry respectively
- The spectrum of CBO and central charges also match

Pair of D3-branes probing $\mathcal{S}_3$-fold

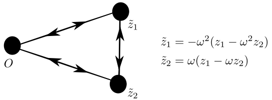
- A pair of branes probing $\mathcal{S}_3$-fold background with no discrete torsion i.e. $p = k = 3$
- Dual to 4d $\mathcal{N} = 4$ $SU(3)$ SYM
- The manifest $U(1)^2$ gauge symmetry arising from the branes is NOT isomorphic to the Cartan subgroup of $SU(3)$
- Admixture of the $SU(3)$ Cartans and their magnetic counterparts

- The 6 non-zero roots of $SU(3)$ are dual to

$$(n_e^1, n_m^1; n_e^2, n_m^2) = \pm(1,0;-1,0),\ \pm(0,1;1,1),\ \pm(1,1;0,1)$$



(a) $\mathcal{S}_3$-fold background



(b) Flat background

- Require that for every state arising from a (p, q)-strings suspended between the branes in the $\mathcal{S}_3$-fold background, there exists a corresponding dyon of the $\mathcal{N} = 4$ $SU(3)$ theory.

- This gives us the map between the theory in the $\mathcal{S}_3$-fold background and the flat background

$$\widetilde{z}_1 = -\omega^2(z_1 - \omega^2 z_2)$$
$$\widetilde{z}_2 = \omega(z_1 - \omega z_2)$$
$$\omega : \omega^3 = 1$$

and

$$n_e^1 = \widetilde{n}_e^1 - \widetilde{n}_m^1 - \widetilde{n}_m^2, \qquad n_m^1 = \widetilde{n}_e^1 + \widetilde{n}_e^2 - \widetilde{n}_m^2$$
$$n_e^2 = \widetilde{n}_e^2 - \widetilde{n}_m^1 - \widetilde{n}_m^2, \qquad n_m^2 = \widetilde{n}_e^1 + \widetilde{n}_e^2 - \widetilde{n}_m^1$$

$(n_e^1, n_m^1; n_e^2, n_m^2)$ and $(\widetilde{n}_e^1, \widetilde{n}_m^1; \widetilde{n}_e^2, \widetilde{n}_m^2)$ are the charge vectors in the $\mathcal{S}_3$-fold and flat background respectively.

String Junctions in $\mathcal{S}_3$-fold

- For simplicity, consider the 3-pronged string with prongs formed by $(1, 0)$, $(0, 1)$ and $(-1, -1)$ strings respectively
- In the $\mathcal{S}_3$-fold background, all three prongs have the same tension $\implies$ angle enclosed between any two prongs is $\frac{2\pi}{3}$
- Let $(1, 0)$ prong end of the first brane positioned at z_1, $(0, 1)$ prong end on the brane at z_2 and the $(-1, -1)$ prong end on the brane at the image of the second brane, at $z_2' = \omega z_2$

- only possible if,
    1. $z_1 = -\lambda\omega^2 z_2,\ \lambda > 0$ with the vertex of the string junction at $-\omega^2 z_2$
    2. $z_1 = \lambda\omega^2 z_2,\ \lambda > 0$ with the vertex of the string junction at the origin

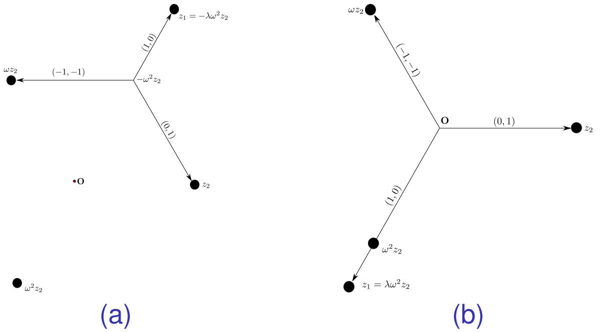


Figure 4: The two possible configurations for the 3-string

- Compute the mass of the above 3-string configurations and compare it the central charge of the corresponding state
- The mass of 3-string configuration in figure 4a matches perfectly with its central charge for all $\lambda > 0$. $1/3$-BPS state of the $\mathcal{N} = 3$ SUSY manifestly preserved by the background
- This configuration corresponds to a monopole of the $\mathcal{N} = 4$ $SU(3)$ SYM. More precisely, to a D1-string stretched from the brane at origin to the brane at $\widetilde{z}_1$
- The mass of the 3-string configuration in figure 4b is larger than its central charge for all $\lambda > 0$. Non-BPS state?

- The map between the electromagnetic charges suggests that this corresponds to a $1/2$-BPS magnetic monopole of the $\mathcal{N} = 4$ $SU(3)$ SYM. Such a state should therefore transform non-trivially under the action of at most two sets of supercharges.

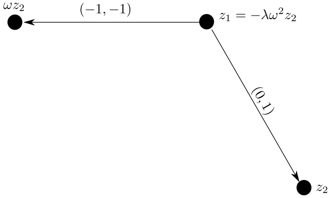
- The requirement that it be a non-BPS object of $\mathcal{N} = 3$ set-up suggest that it transforms non-trivially under all the three sets of $\mathcal{N} = 3$ supercharges

- Contradiction!

- Conjecture: Such 3-string configurations can not exist in $\mathcal{S}_3$-fold background

Walls of Marginal Stability

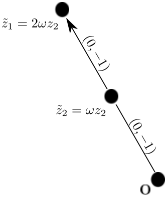
- Let $\lambda \to 1$ in figure 4a
- The $(1, 0)$ prong reduces to zero length



- The $(-1, -1)$ and $(0, 1)$ prongs can now move independent of each other

Corresponding phenomenon in Flat background

- $\lambda \to 1$ in $\mathcal{S}_3$-fold implies $z_1 \to -\omega^2 z_2$
- The position of the branes in the flat background are then given by: Origin, $\widetilde{z}_1 = 2\omega z_2$ and $\widetilde{z}_2 = \omega z_2$
- The D1-string between the branes at Origin and $\widetilde{z}_1$ can now break on the brane at $\widetilde{z}_2$



- In fact, $(-1,-1)$ prong of the 3-string in S-fold corresponds to a D1-string between the branes at Origin and $\widetilde{z}_2$
- $(0,1)$-prong corresponds to a D1-string between the branes at $\widetilde{z}_1$ and $\widetilde{z}_2$
- Wall crossing in $\mathcal{S}_3$-fold background gets mapped to wall-crossing in the flat background realizing $\mathcal{N}=4$ $SU(3)$ SYM

String Junctions in $\mathcal{N} = 4$ $SU(3)$ SYM

- 3-string with $(1, 0)$, $(0, 1)$ and $(-1, -1)$ prongs in flat background
- Let the $(-1, -1)$ prong terminate on the brane at the origin, the $(0, 1)$ prong terminate on the brane at $\tilde{z}_1$ and $(1, 0)$ prong terminate on the brane at $\tilde{z}_2$
- $1/4$-BPS state of $\mathcal{N} = 4$ $SU(3)$ SYM with

$$(\tilde{n}_e^1, \tilde{n}_m^1; \tilde{n}_e^2, \tilde{n}_m^2) = (0, 1; 1, 0)$$

- This maps to a $\mathcal{S}_3$-fold state with charges

$$(n_e^1, n_m^1; n_e^2, n_m^2) = (1, 1; 0, 0)$$

- This set of charges can only be generated by an F1-string stretched between the brane at z_1 and its image at $z_1' = \omega z_1$
- Consistent with the earlier assertion that in the absence of discrete torsion , a fundamental string stretched between a brane and its image does not give rise to an $\mathcal{N} = 3$ vector multiplet

Thank you!

- $T^2 \to 0$, N D3-branes transverse to $\mathbb{C}^3/\mathbb{Z}_k$
- Vacuum moduli space of 4d theory: $(\mathbb{C}^3/\mathbb{Z}_k)^N/S_N$
- For k=2, 4d $\mathcal{N} = 4$ theory with $SO(2N+1)$, $Sp(N)$ or $SO(2N)$ gauge symmetry
- Use R-symmetry and gauge transformations to make all the branes coplanar in the transverse space with positions $z_i,\ i = 1, \ldots, N$
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- Weyl subgroup of $Sp(N)$:

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\end{aligned}
$$

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\end{aligned}
$$

- Clearly, will not give the same moduli space
- Resolution: Introduce a $\mathbb{Z}_2$ discrete symmetry in the disconnected part of the gauge symmetry
- The gauge symmetry is $O(N)$ rather than $SO(N)$

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- Refining the $\mathcal{S}$-fold classification by discrete torsion is equivalent to realizing a $\mathbb{Z}_p$ subgroup of $G(k,1,N)$ as a discrete gauge group in the disconnected part of the gauge symmetry

- Gives $G(k, p, N) \times \mathbb{Z}_p$
- Action of $G(k, p, N)$

$$\begin{aligned} & z_i \leftrightarrow z_j, && \text{all others fixed,} \\ & (z_i, z_j) \to (\gamma z_i, \gamma^{-1} z_j) && \text{all other } z_a \text{ fixed} \\ & z_i \to \gamma^p z_i,\ \gamma^k = 1 && \text{all other } z_a \text{ fixed} \end{aligned}$$