5d descriptions of 6d SCFTs and their equivalences

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Autumn Symposium on String theories 2016-09-22

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arXiv:1607.07786 arXiv:1512.08239 arXiv:1509.03300 arXiv:1505.04439 arXiv:1504.03672

Introduction

- 6d **N=(1,0) SCFTs**

Talks by Seok Kim, Lockhart, Joonho Kim, Xie

M-theory constructions: M5 on ALE singularities

[Del Zotto-Heckman-Tomasiello-Vafa ’14]

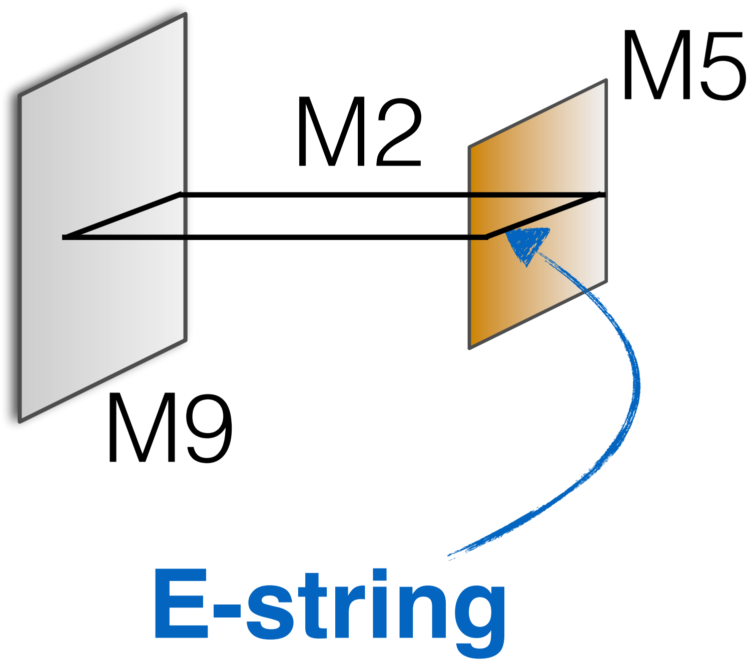
(1,0) theory: F-theory classifications [Heckman-Morrison-Rudleius-Vafa ’15]

- 6d theory on S^1 : 5d description

radius <—> gauge coupling

KK momentum <—> 5d instanton particle

6d N=(1,0) E-string theory

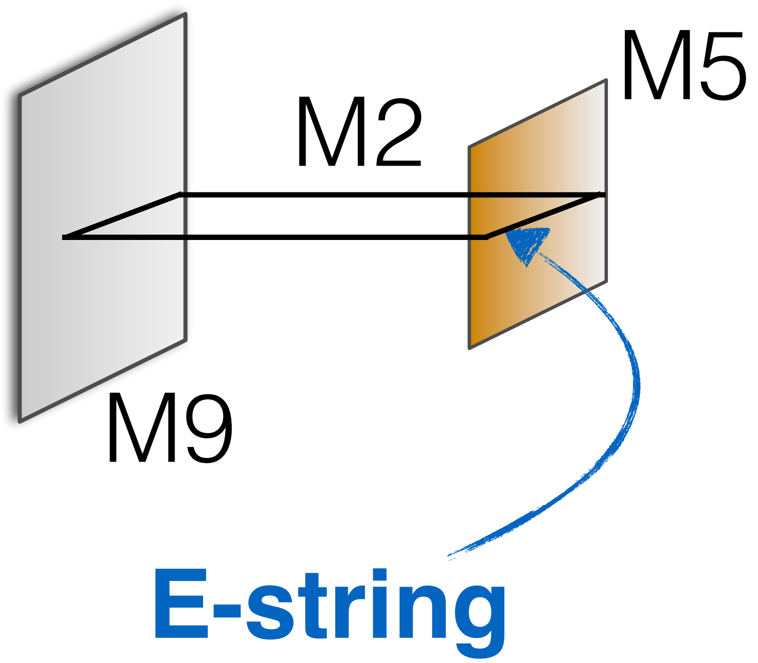


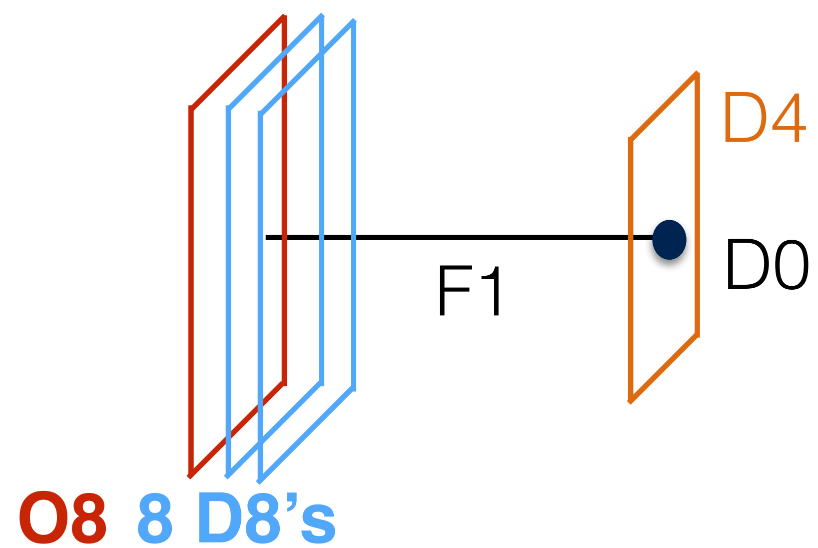
E-string partition function
(elliptic genus)

['14 Chiung Hwang, Joonho Kim, Seok Kim, Jaemo Park]

['14 Seok Kim, Joonho Kim, Kimyeong Lee, Jaemo Park, Vafa]

as the UV completion of 5d SU(2) w/ Nf=8 flavors





E-string theory on a circle = **5d SU(2) theory with Nf=8**

KK modes = **Instantons**

Like the E-string theory case,

Many other
N=(1,0) SCFTs

5d descriptions?

IIB (p,q) web

New perspectives on 6d and 5d SCFTs

5d dualities, Index functions

[Bergman-Zafrir ’13-‘15]

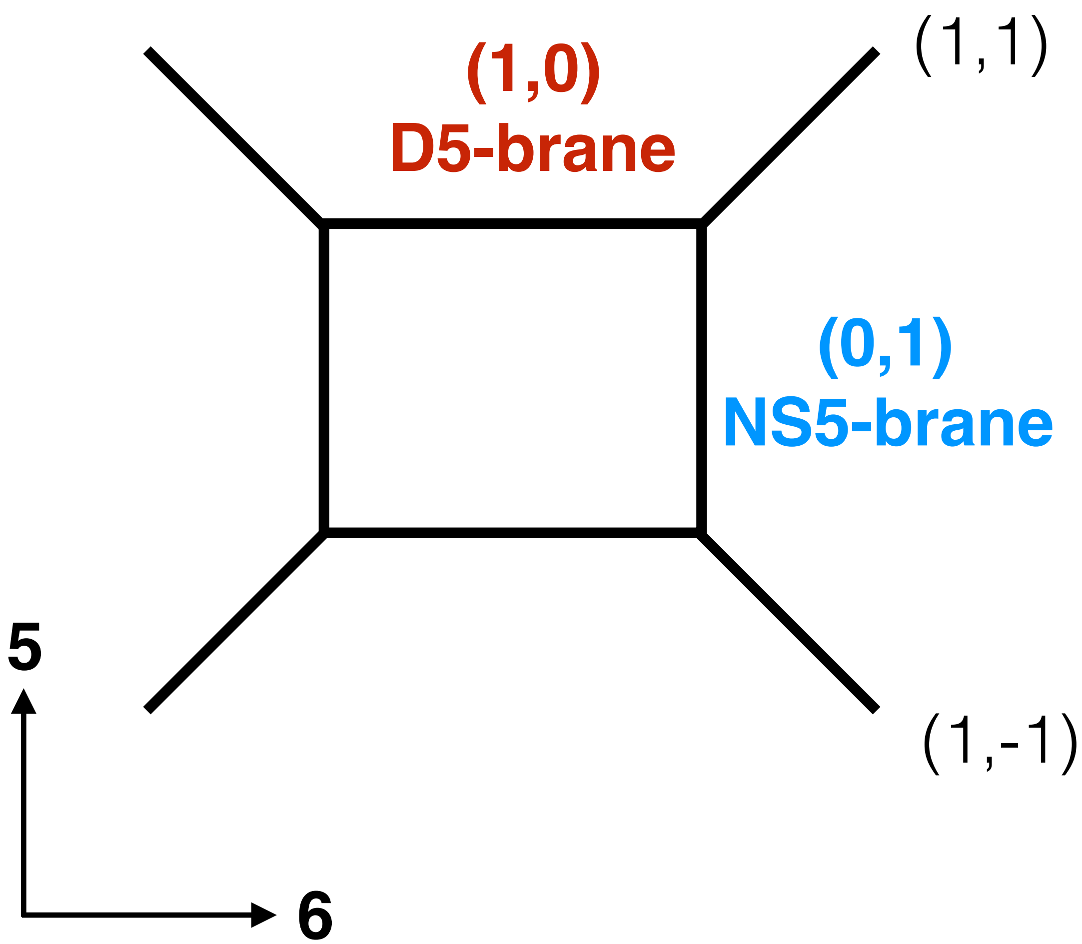
Contents

1. IIB brane configuration

5d SU(2) with 8 flavors v.s. E-string

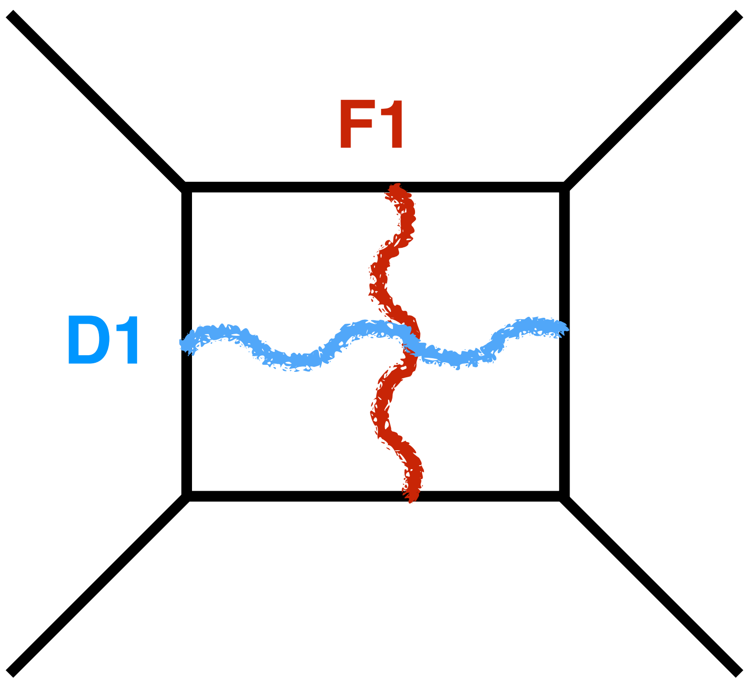
IIB brane picture of 5d pure SU(2) theory

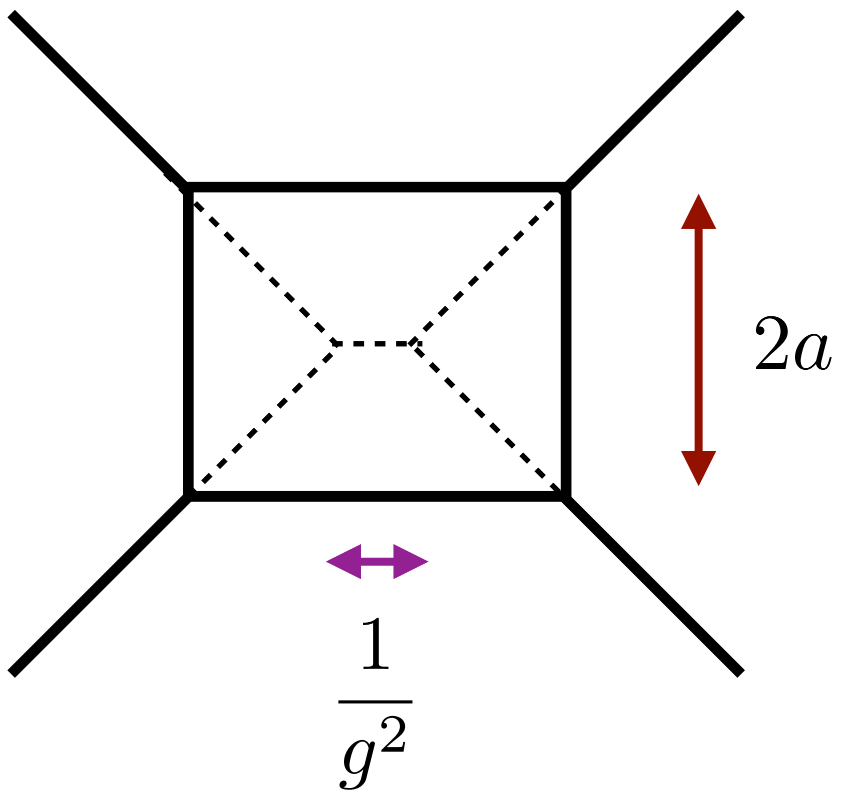
[Aharony-Hanany, ’97]



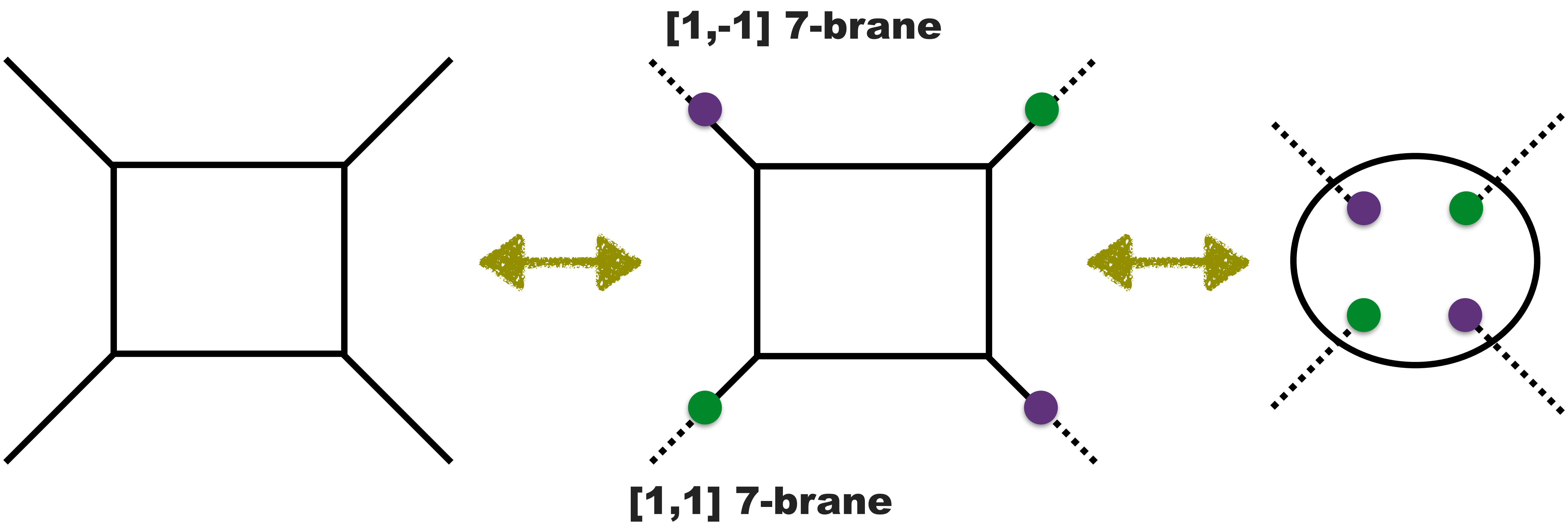
	0	1	2	3	4	5	6	7	8	9
NS5	—	—	—	—	—	—				
D5	—	—	—	—	—		—			
(1,1)	—	—	—	—	—	-	-			

Coulomb branch moduli and coupling





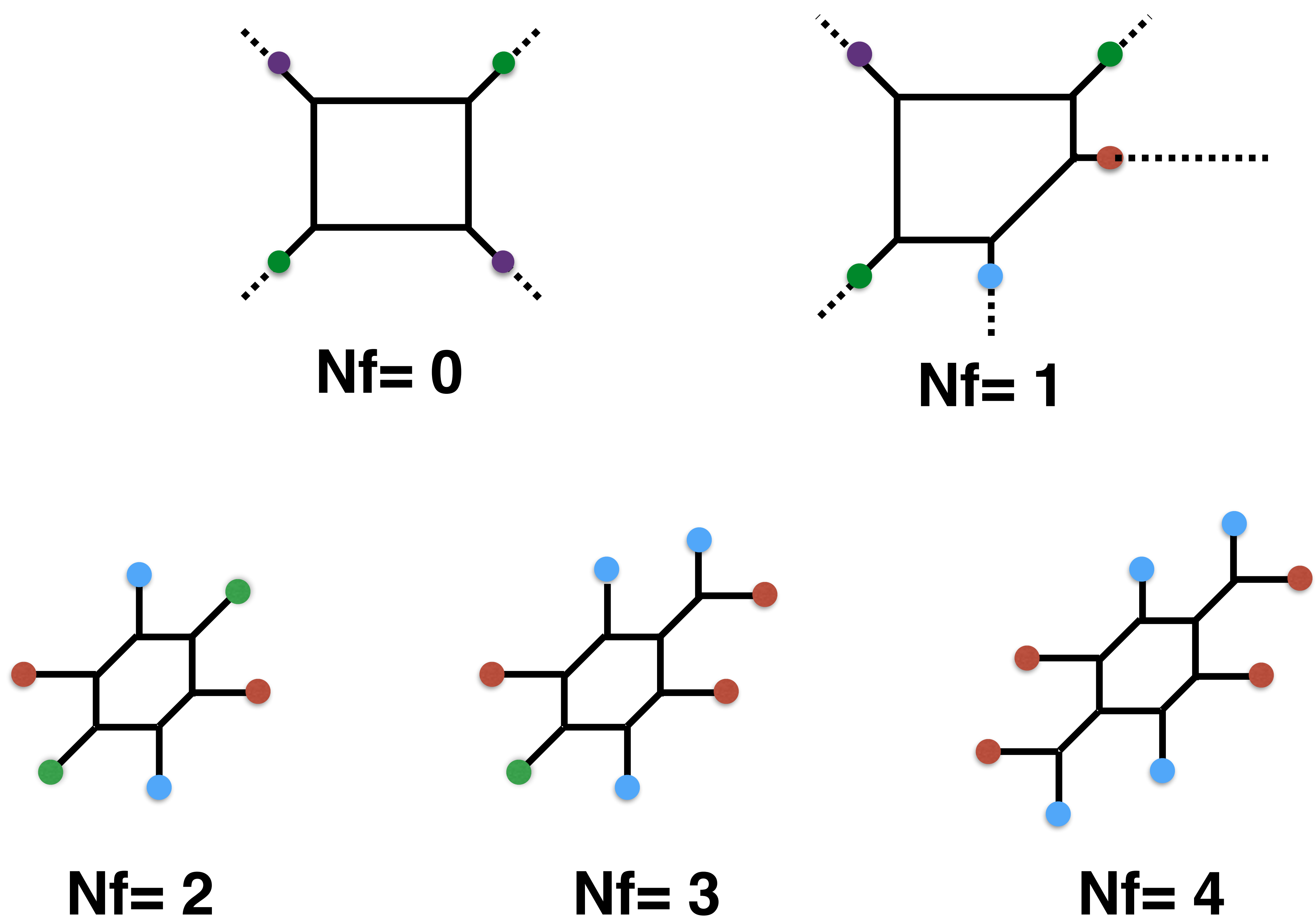
5d SU(2) theory and 7-branes



[DeWolfe et al. '99]

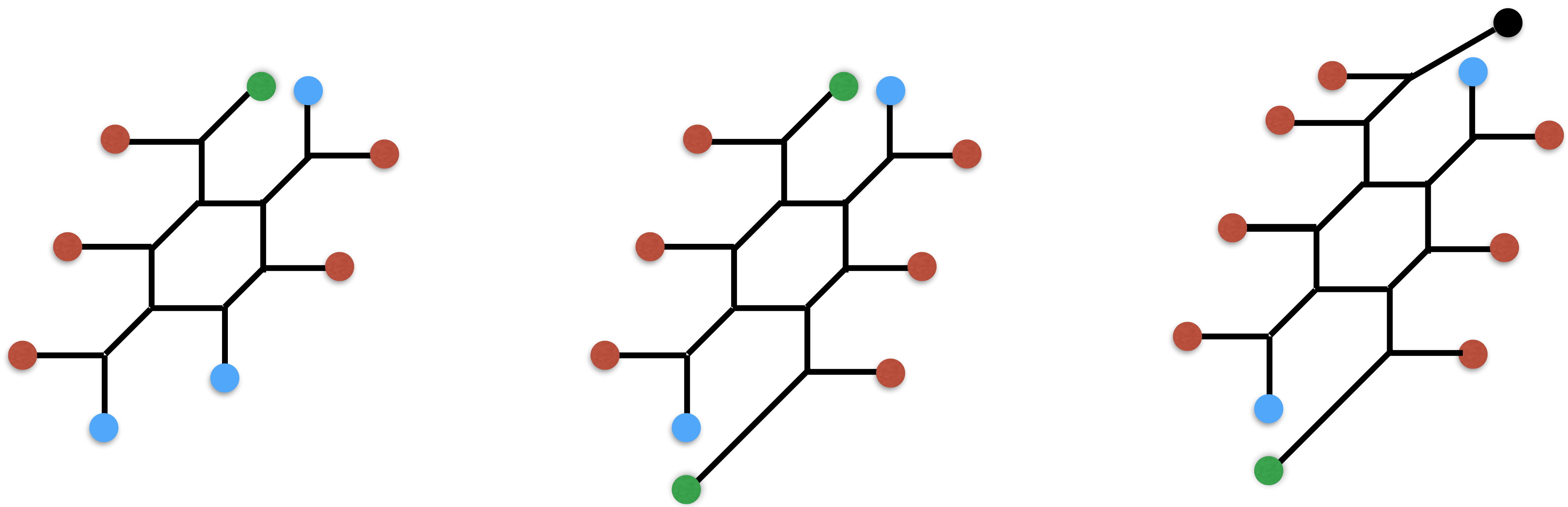
	0	1	2	3	4	5	6	7	8	9
NS5	—	—	—	—	—	—	.			
D5	—	—	—	—	—	.	—			
D7	—	—	—	—	—	.	.	—	—	—

Flavors = D7 brane



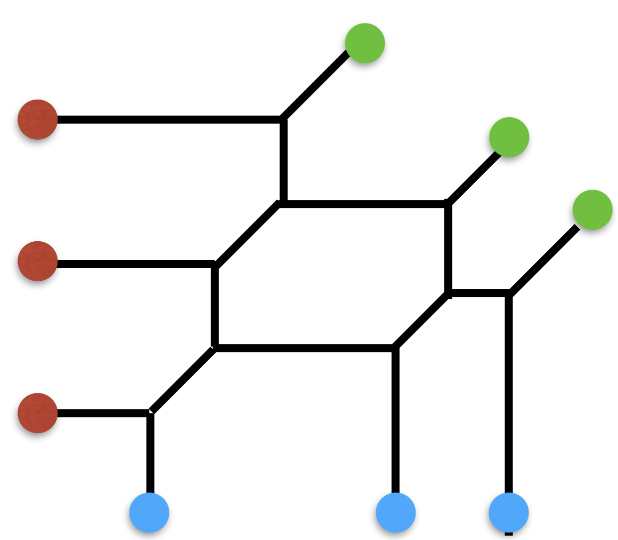
Five, six, seven flavors seem problematic,

$N_f = 5$ $N_f = 6$ $N_f = 7$



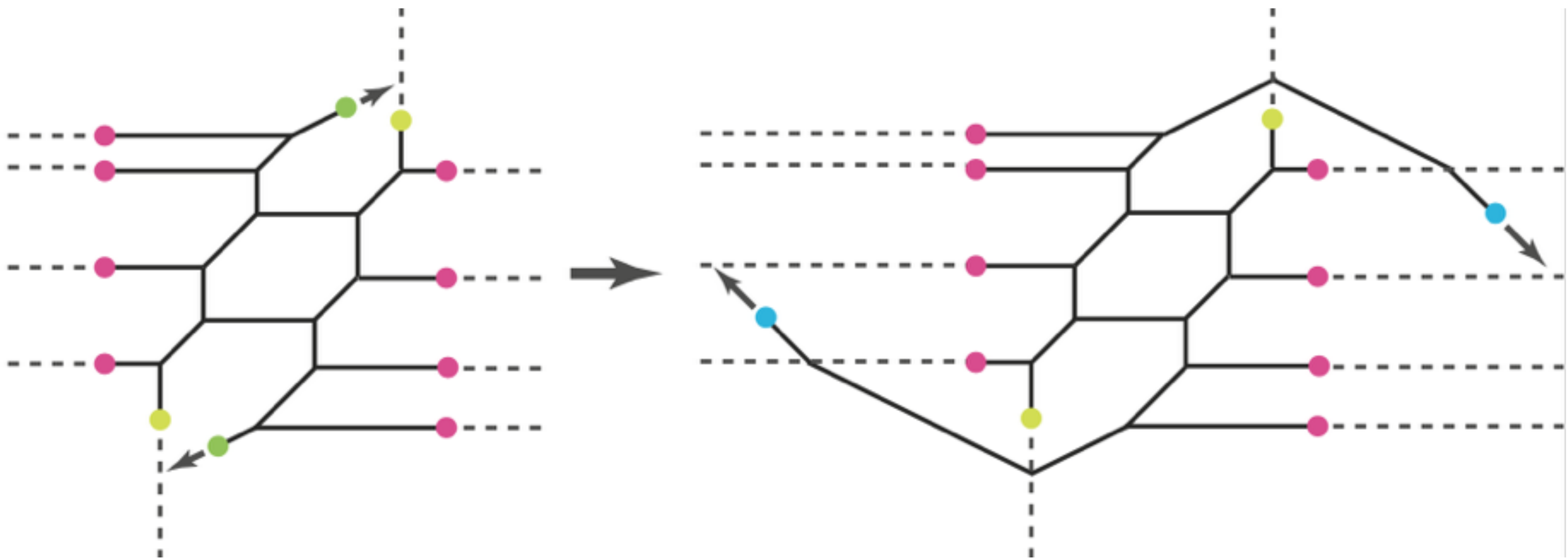
but possible to make sense using 7-brane monodromies

e.g., Nf=5

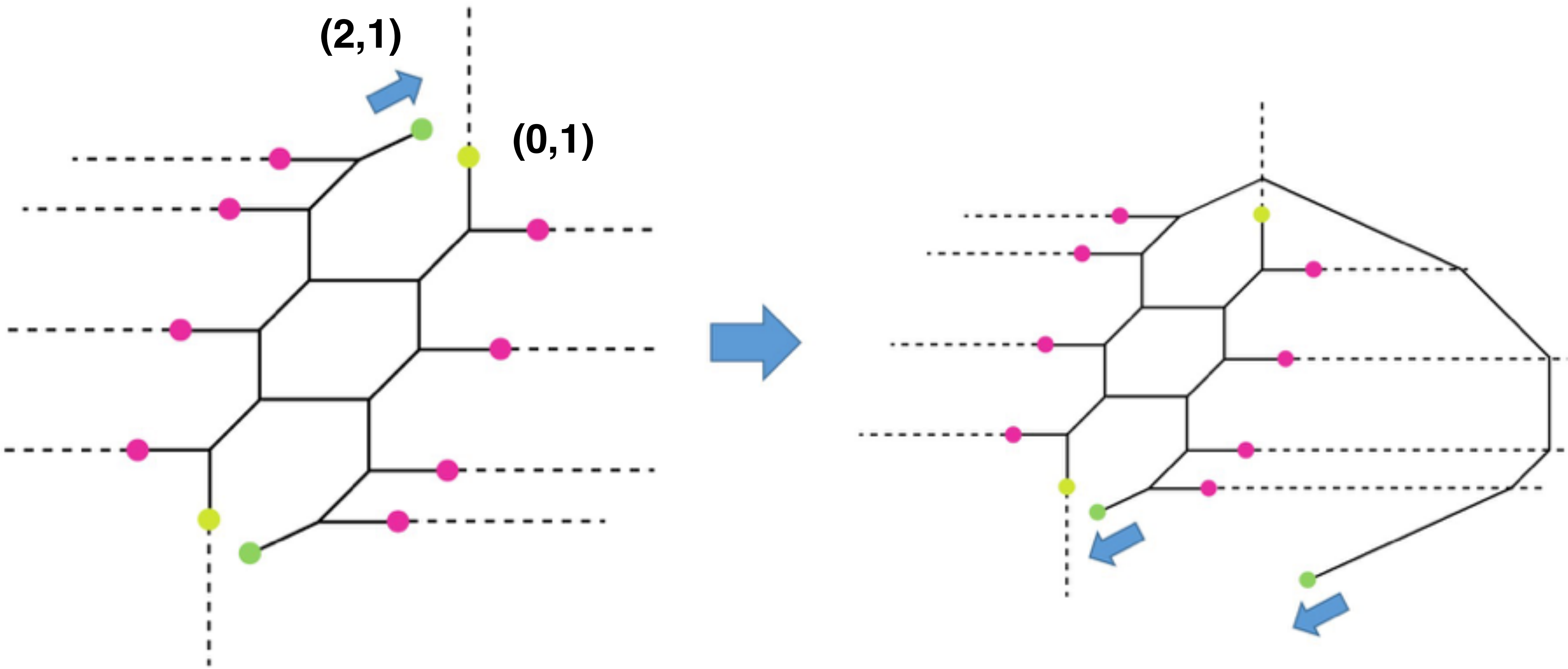


[Benini-Benvenuti-Tachikawa, '09]

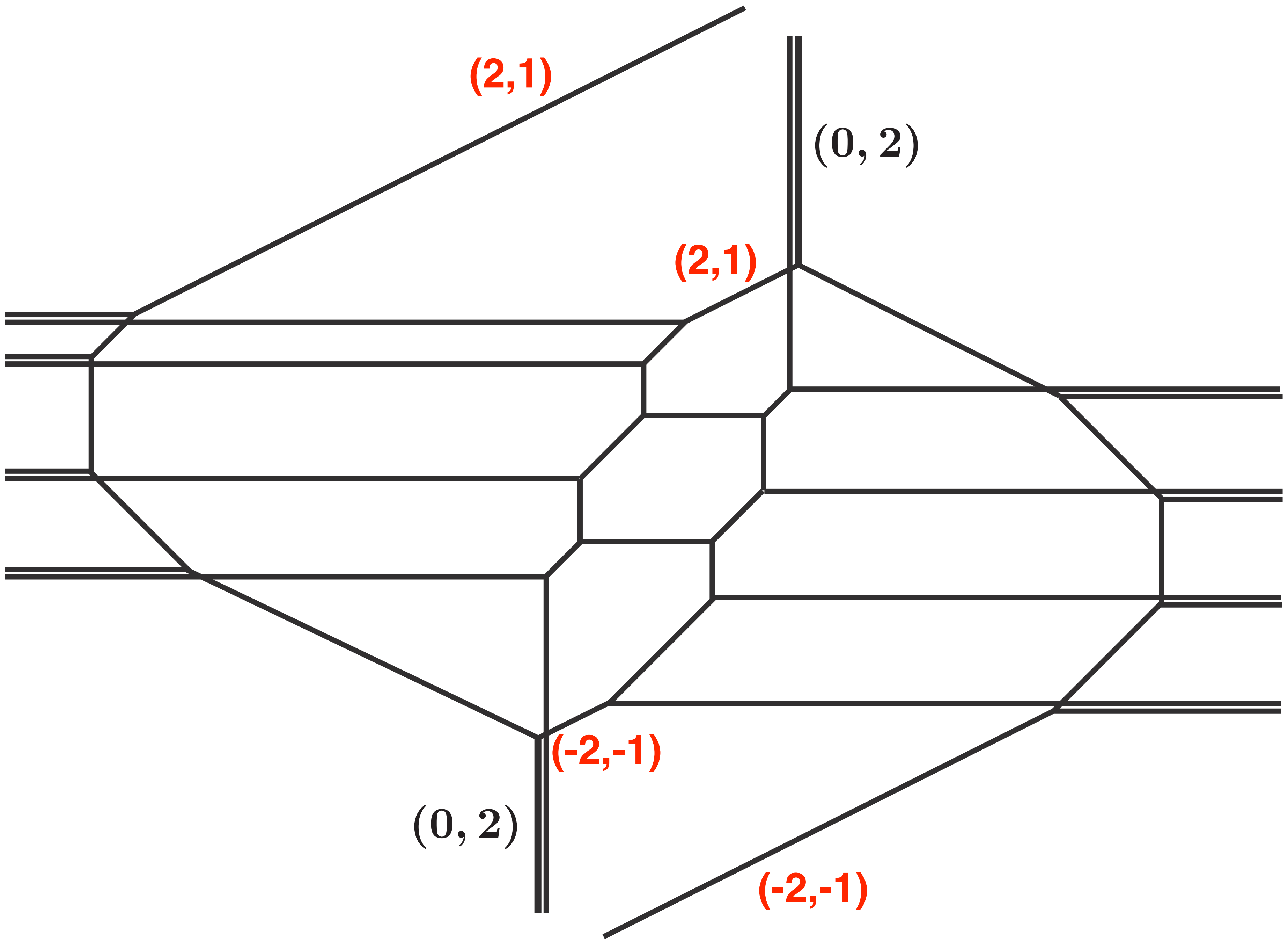
5d SU(2) theory with **Nf=8 flavors**



Spiral structure of the web diagram



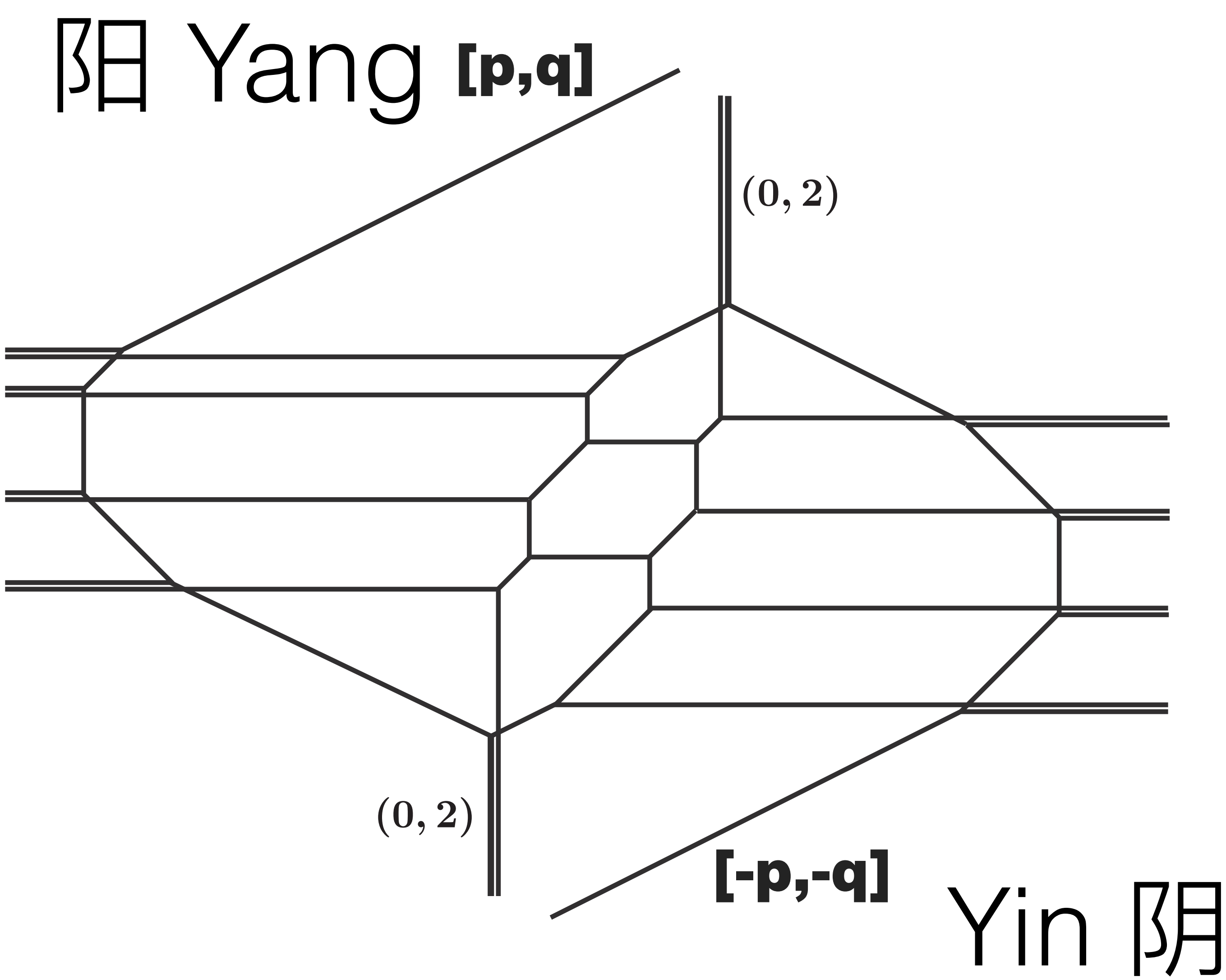
By pulling out 7-branes to infinity

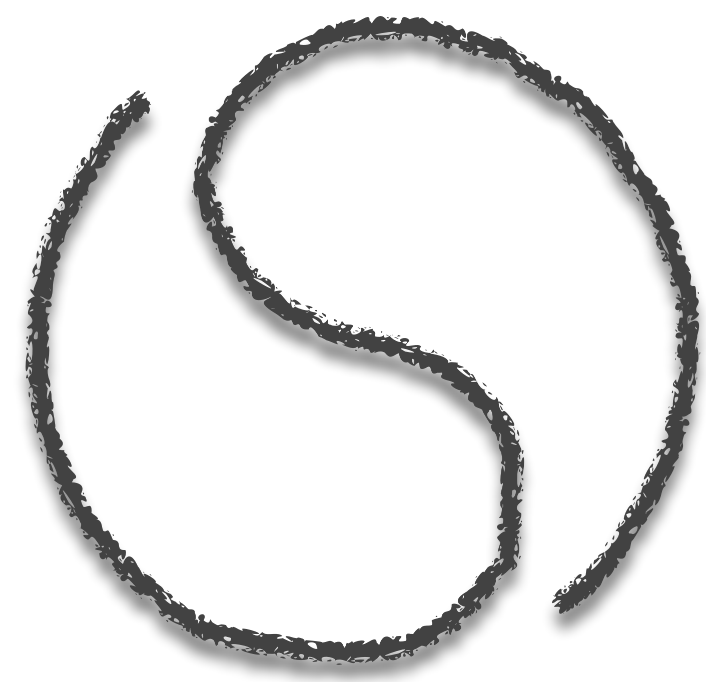


Spirally rotating! One revolution: charges remain the same.

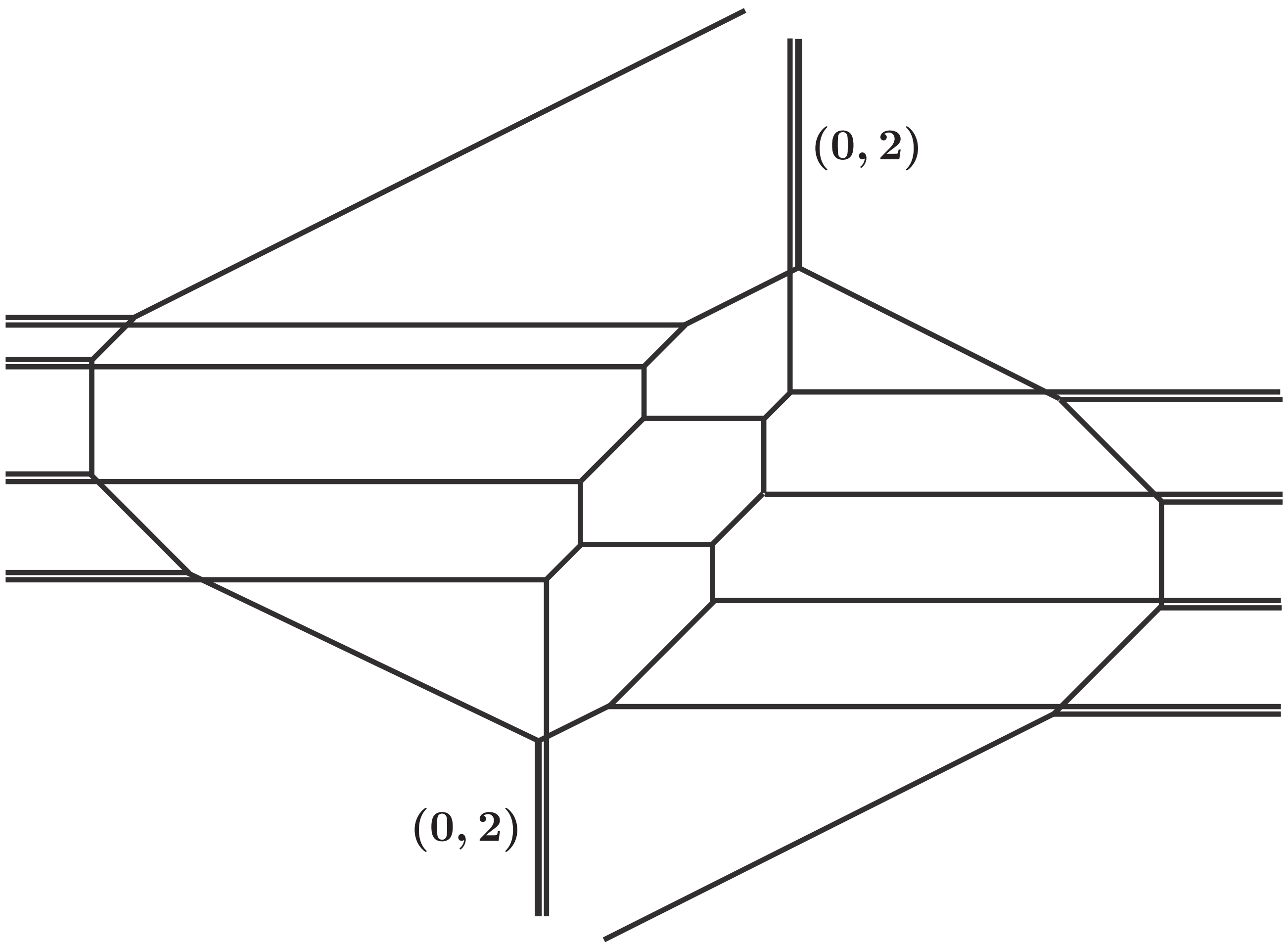
Infinitely rotating spiral diagram

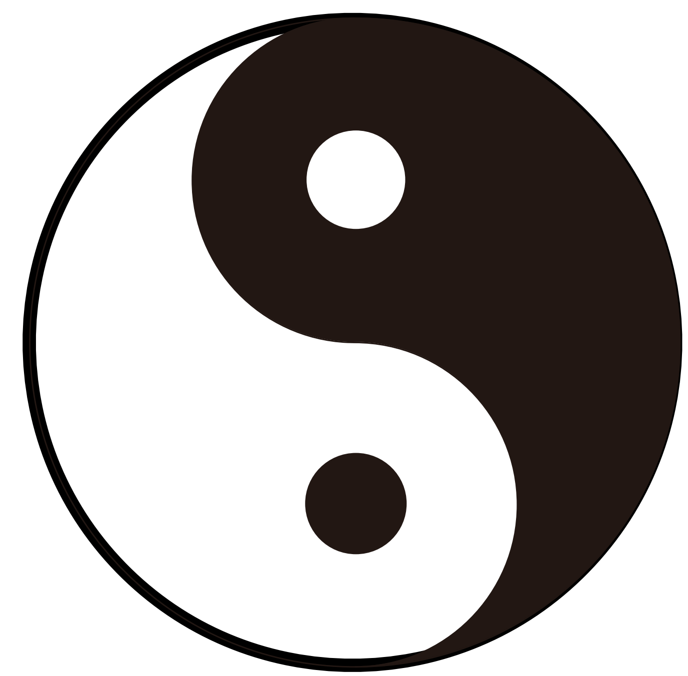
The shape looks like



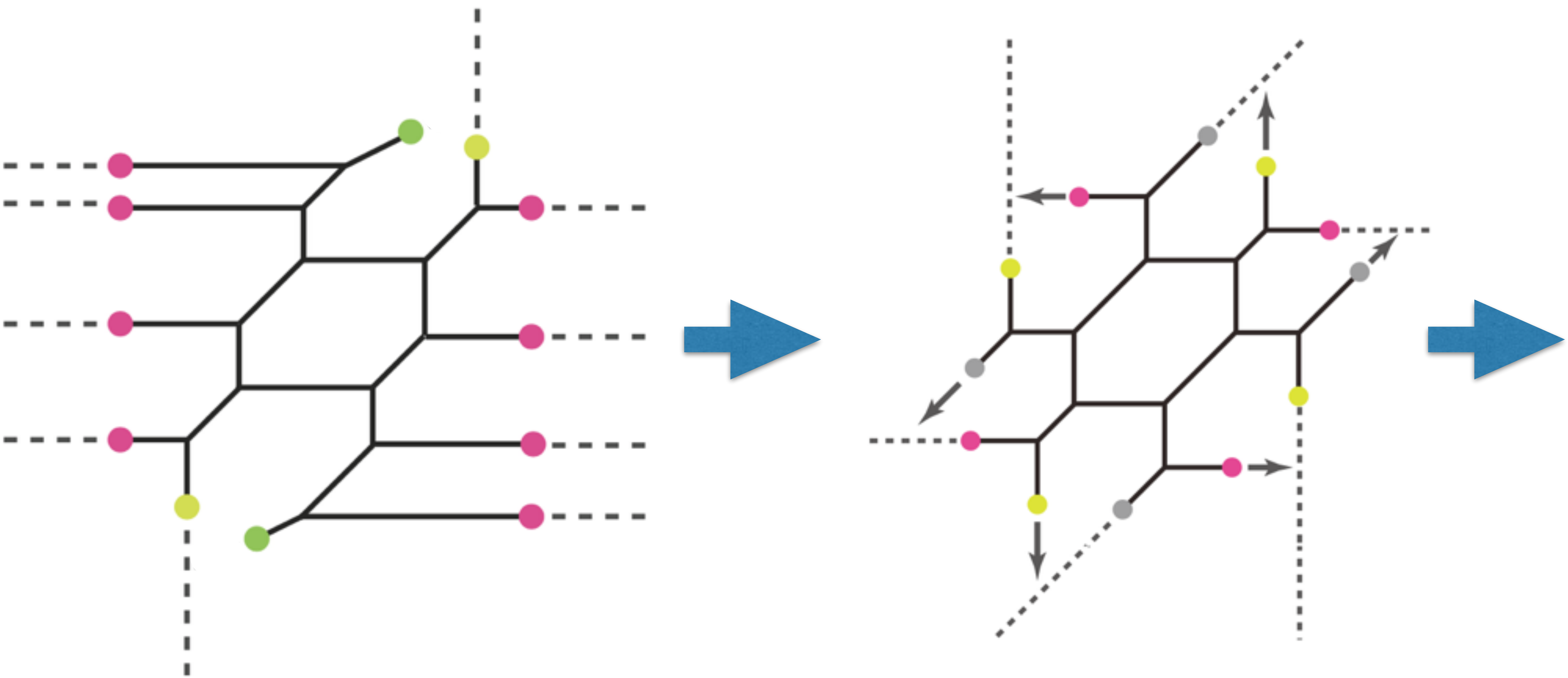


We call it **Tao diagram**…

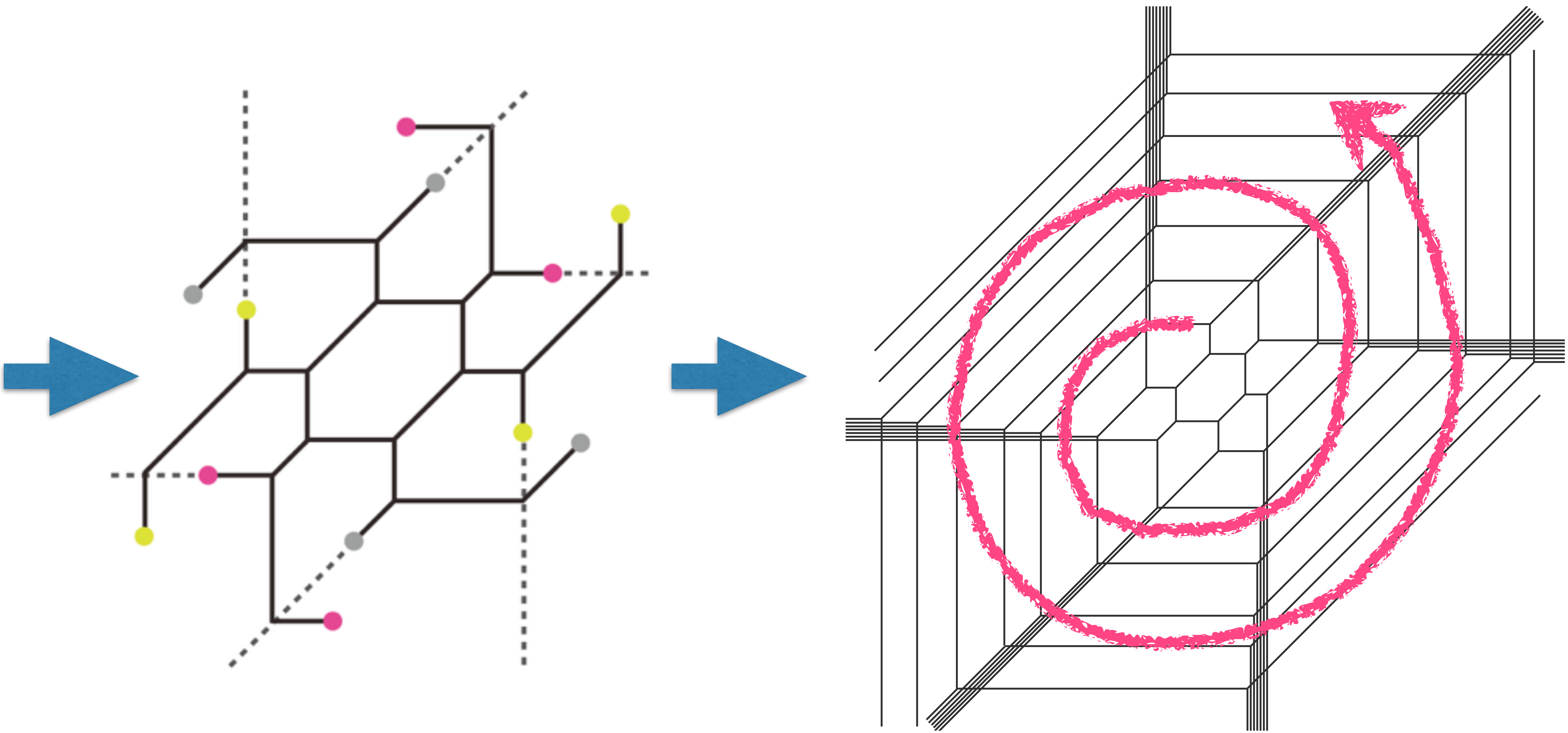




Equivalent Tao diagram

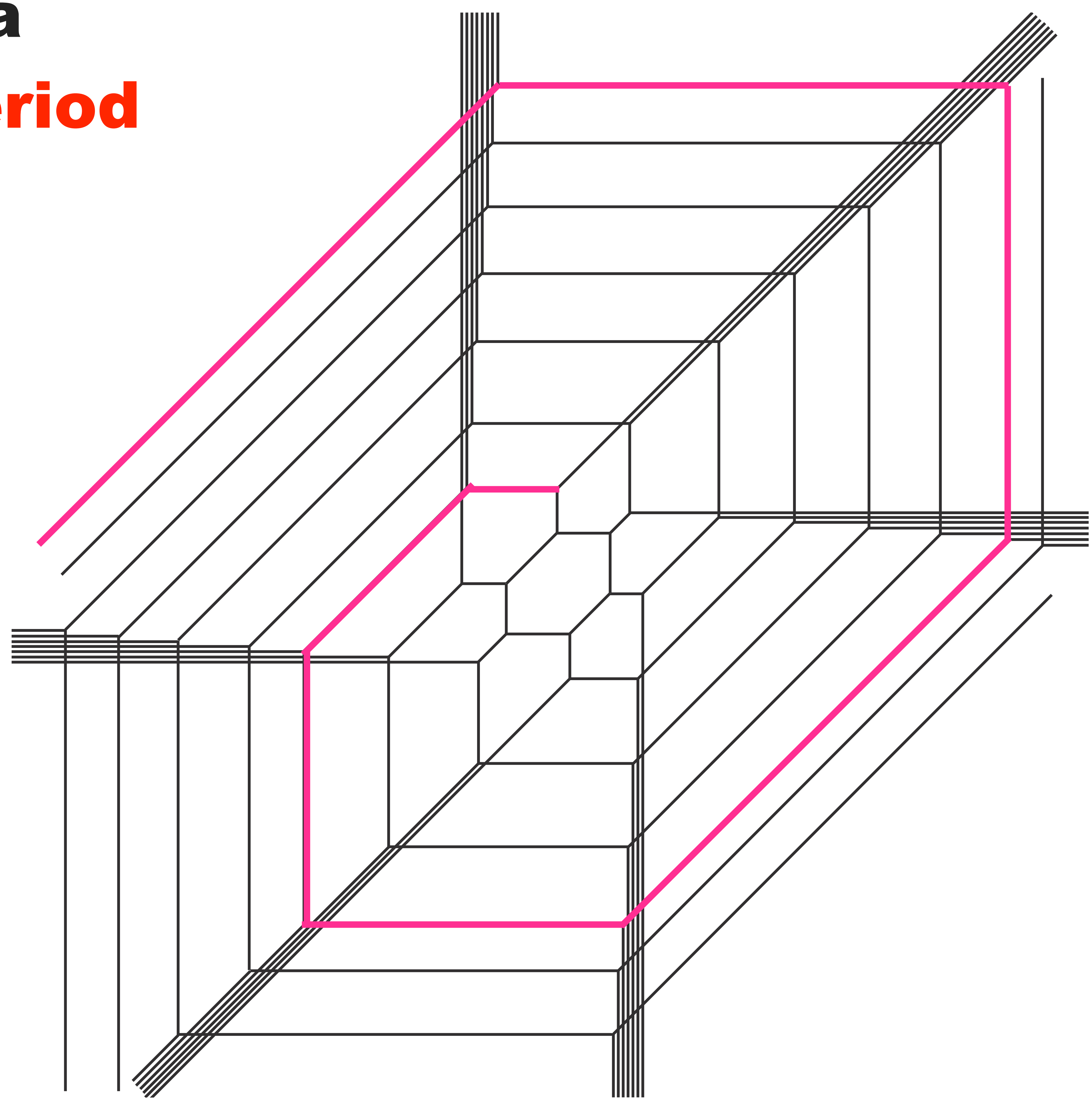


Equivalent Tao diagram



IIB brane realization of E-string theory on S^1?

spiral with a
constant period



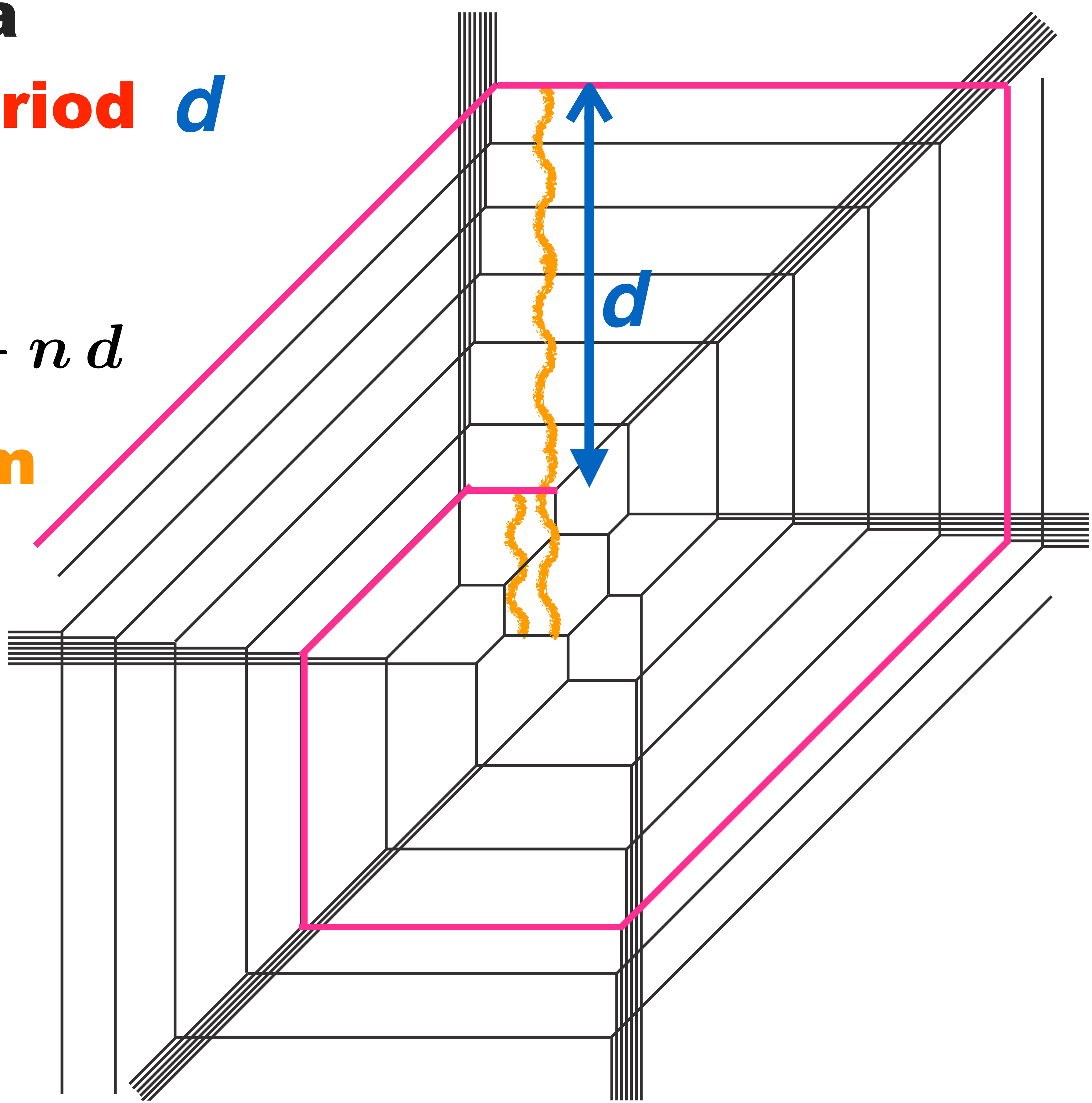
S¹ direction emerges?!

spiral with a

constant period d

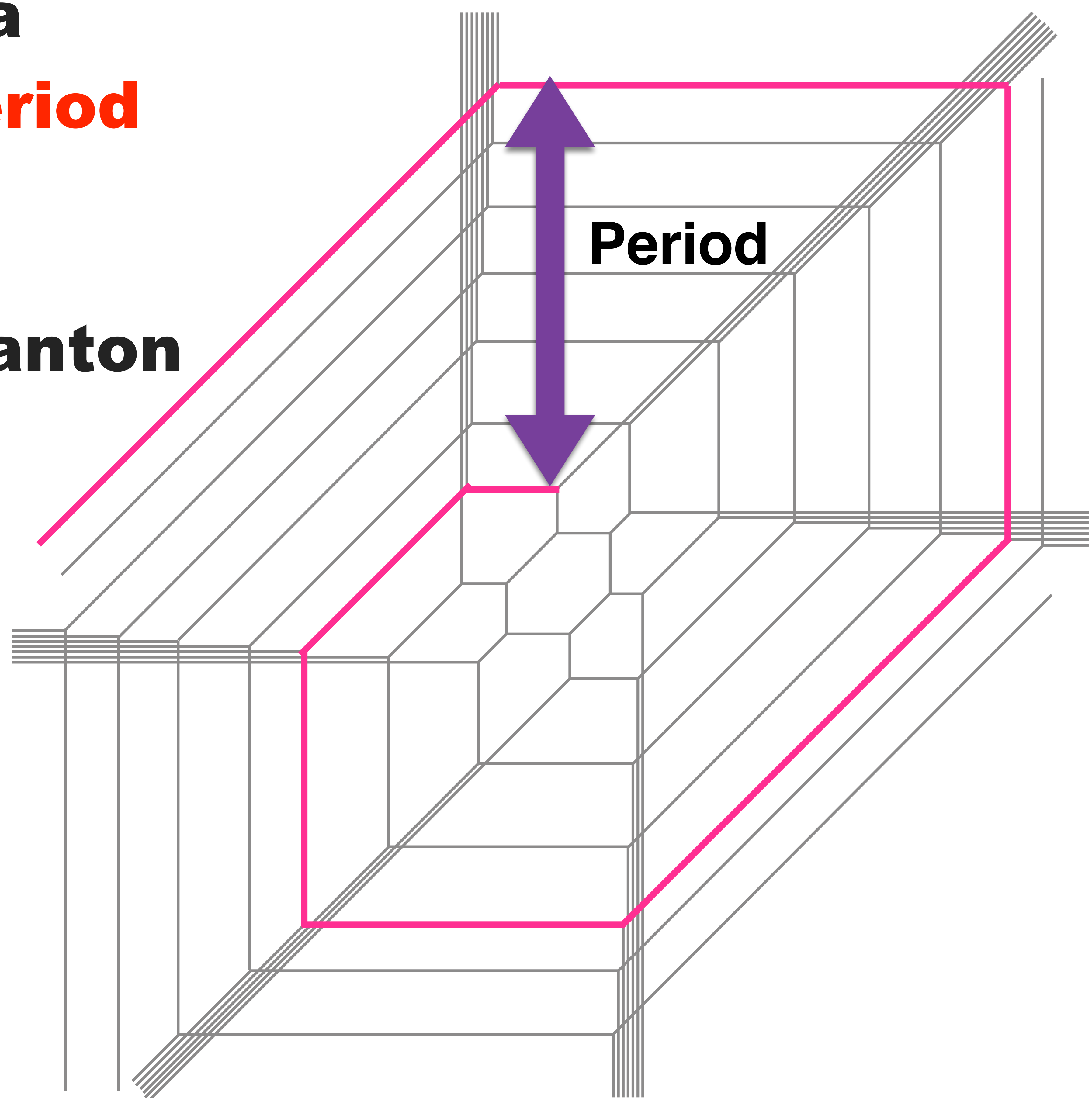
$$m_{(n)} = m_{(0)} + n\, d$$

KK spectrum



spiral with a
constant period

period = instanton
$\sim R^{-1}$



Tao diagrams

Infinite spirals (KK spectrum)
constant period (compactified radius)

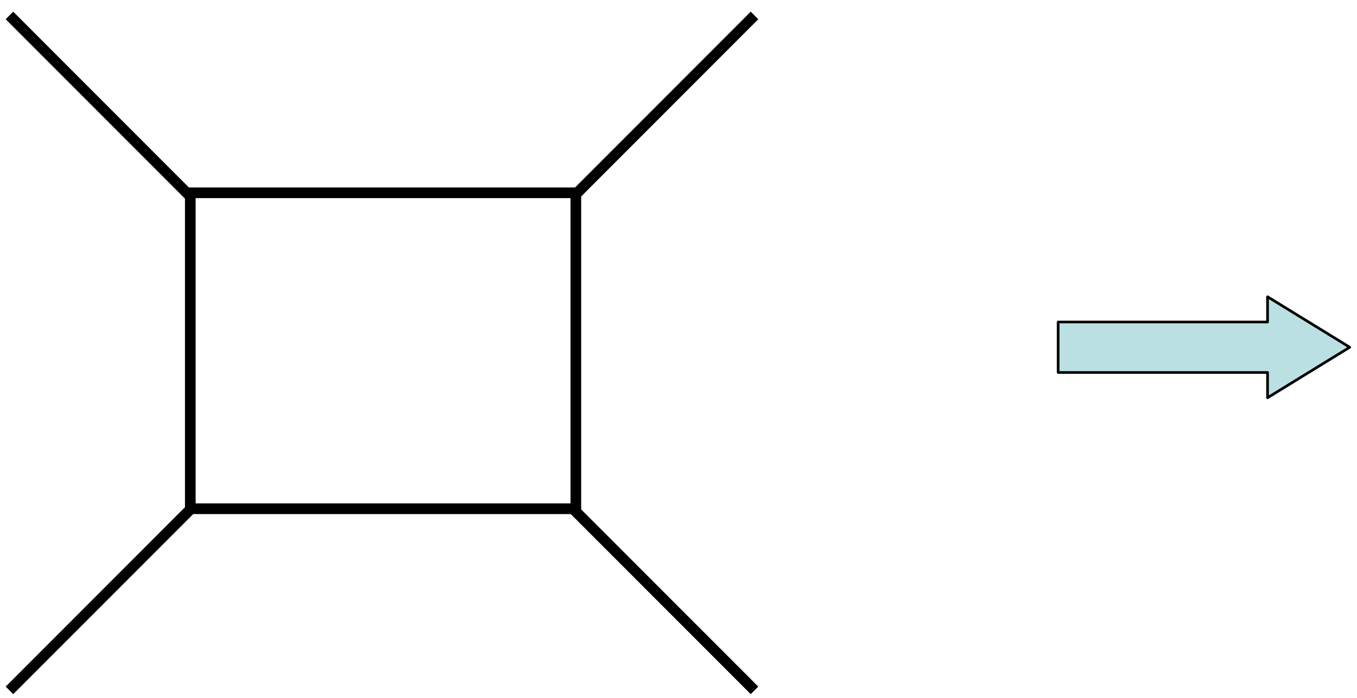
Naturally identified as **a 6d theory on a circle**
(compactification radius emerges as a spiral)

- Computational tool:
  Partition function

Topological Vertex formalism

[Vafa et al.]

web diagram

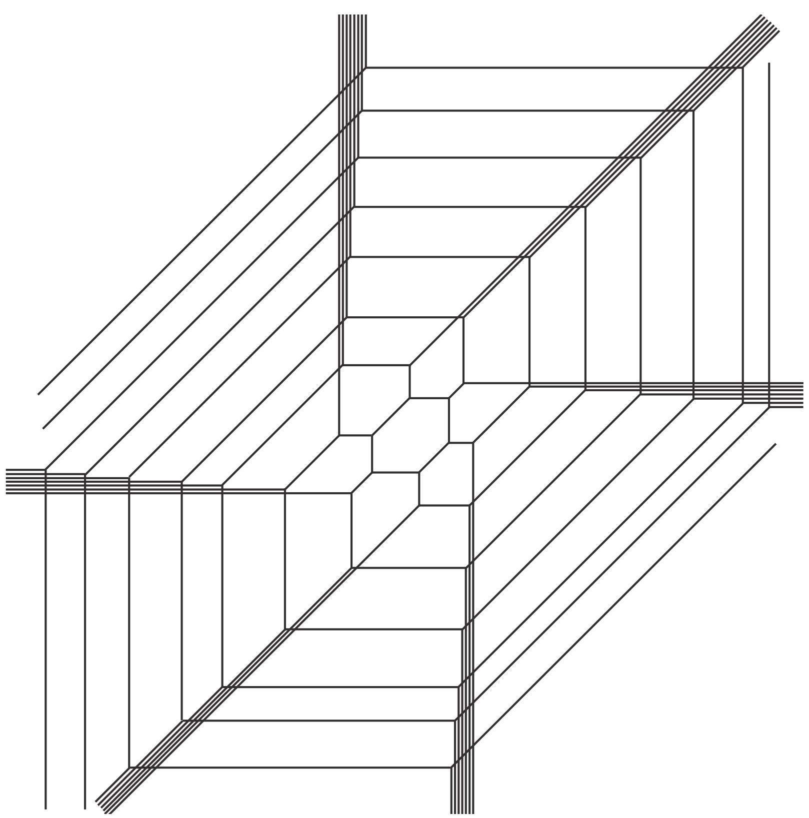


BPS partition function

$$Z = \cdots$$

Topological Vertex formalism

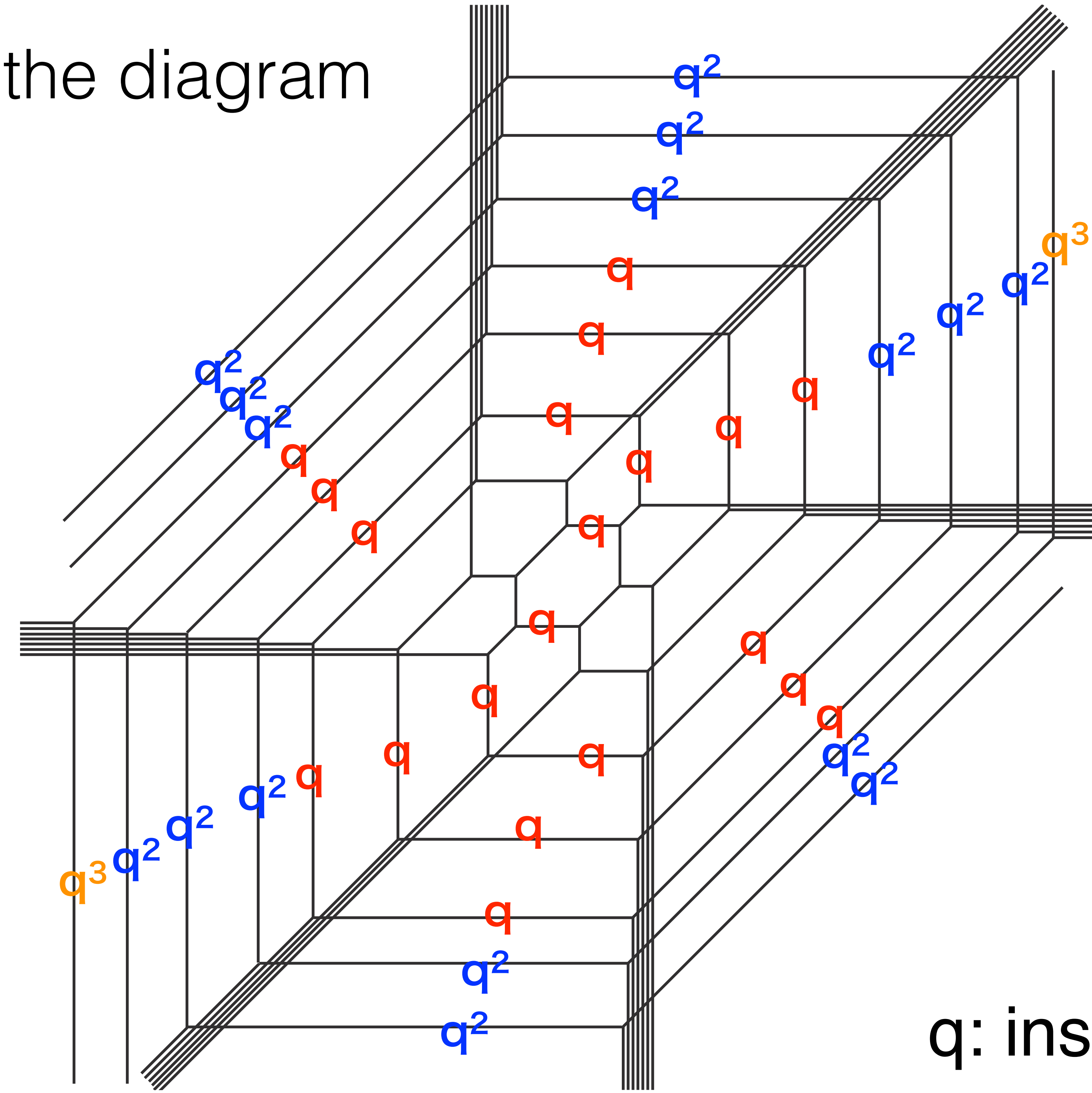
Tao web diagram



BPS partition function

$$Z \stackrel{?}{=} \cdots$$

Truncate the diagram

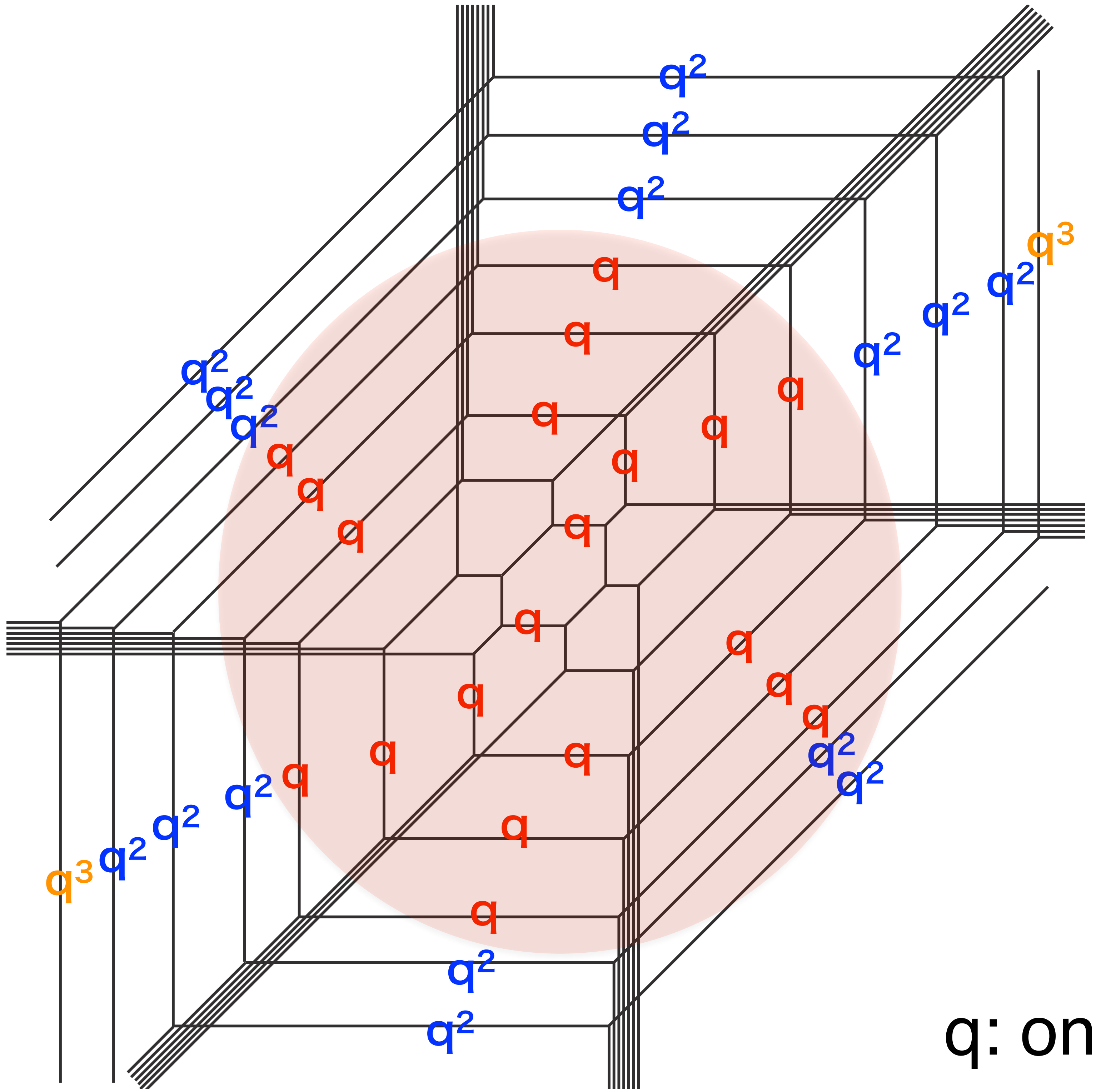


q: instanton fugacity

$$= \exp\left(-\frac{4\pi^2}{g^2}\right)$$

Compute order-by-order in **q**

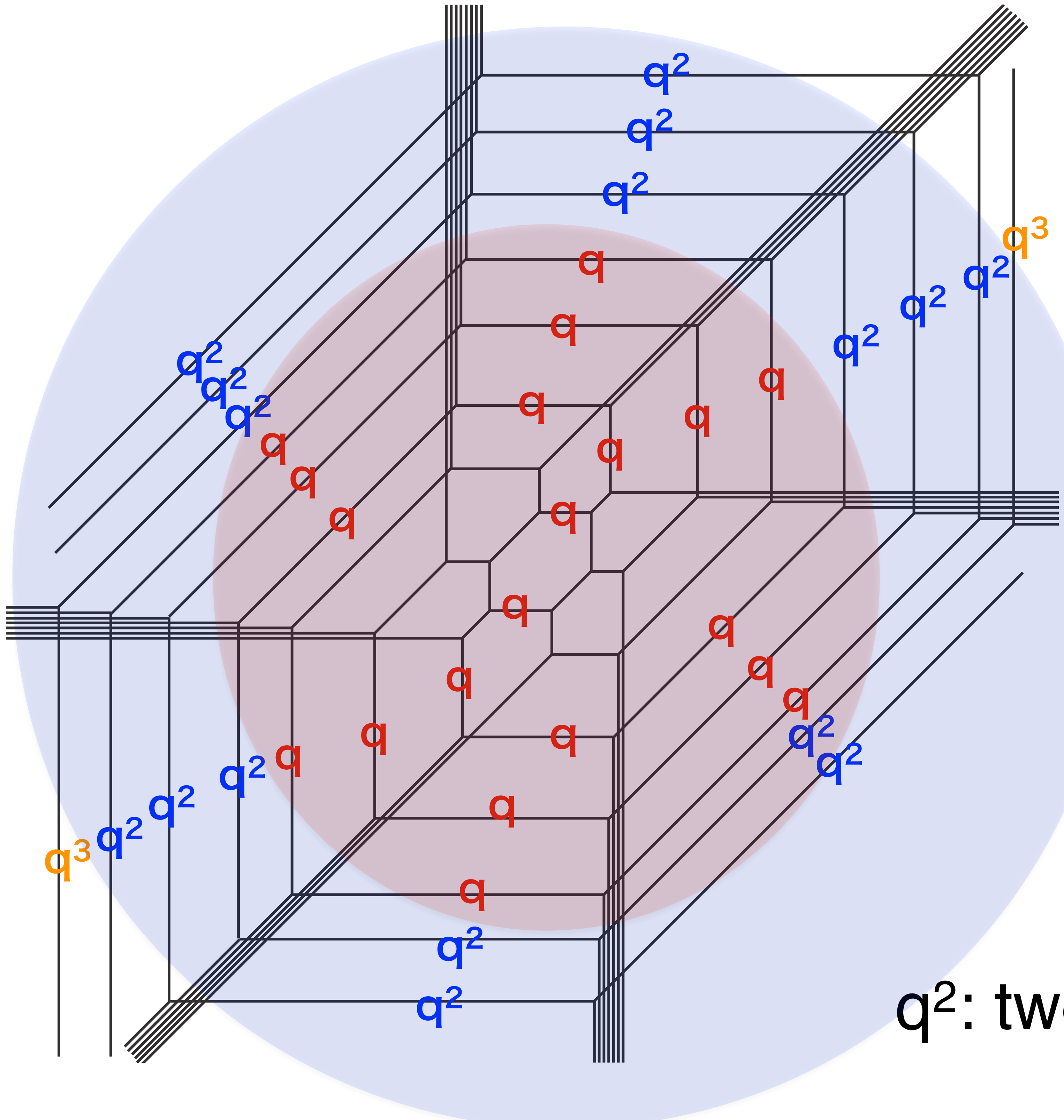
1-instanton, **2-instanton**, … , up to **q^k** order



q: one instanton

Compute order-by-order in **q**

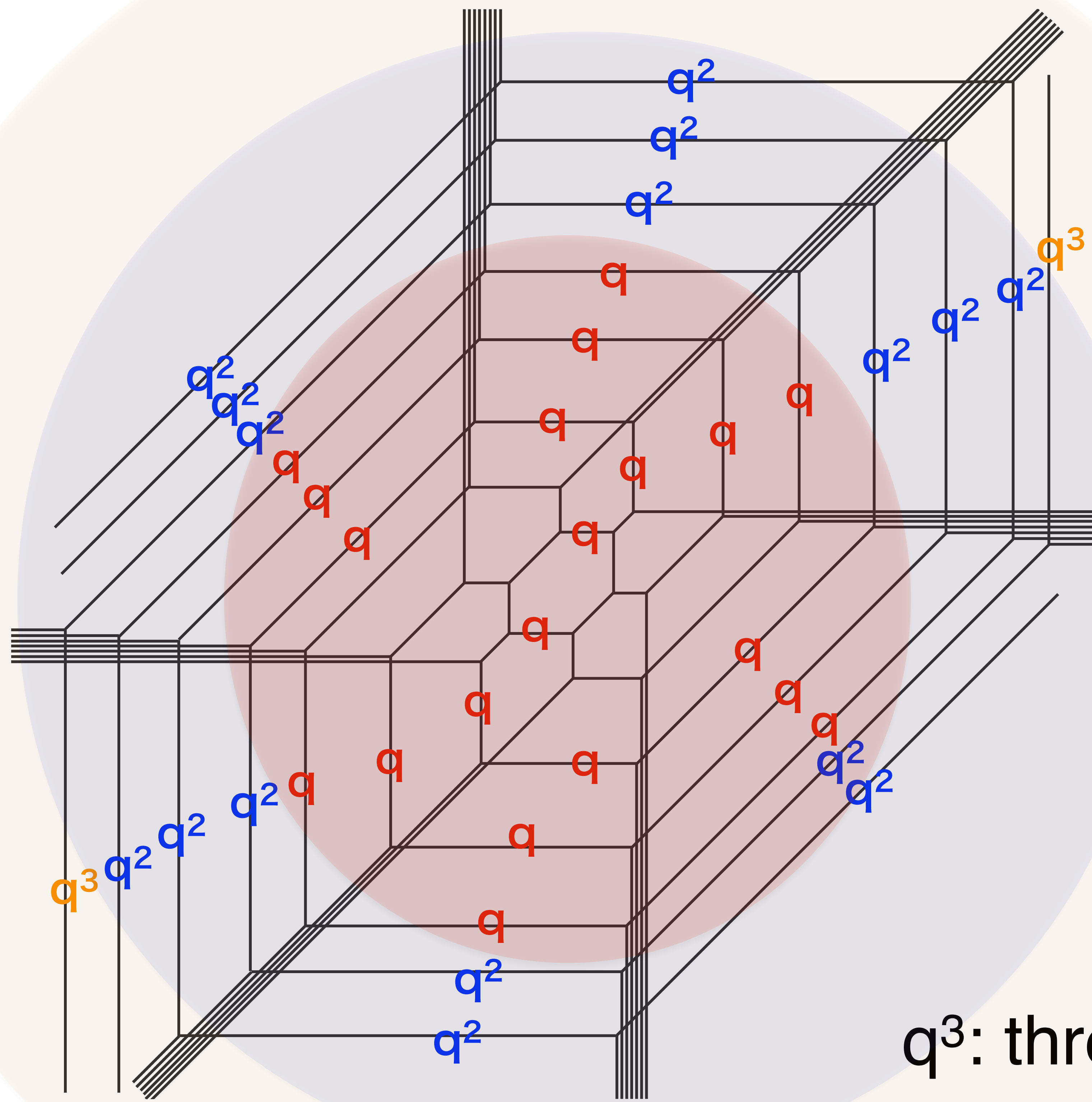
1-instanton, **2-instanton**, … , up to **q^k** order



q²: two instantons

Compute order-by-order in **q**

1-instanton, **2-instanton**, … , up to **q^k** order



q³: three instantons

Truncate up to q^k orders

Obtain the partition function, up to $\mathbf{q^k}$ orders

Partition function from Tao diagram

$$Z_{E\text{-}string} = \mathrm{PE}\Big[\sum_{m=0}^{\infty} \mathcal{F}_m(y, A, q)\mathfrak{q}^m\Big] = \mathrm{PE}\Big[\frac{1}{(1-q)(1-q^{-1})}\sum_{n=1}^{\infty} \tilde{f}_n A^n\Big]$$

$$\tilde{f}_1 = \chi^{(1)} + \chi_{\rm c}\,\mathfrak{q} + \Big(2\chi_{\mathbf{2}}(q)\chi^{(1)} + \chi^{(3)} + \chi^{(1)}\Big)\mathfrak{q}^2 + \Big(\chi^{(1)}\chi_{\rm s} + 2\chi_{\mathbf{2}}(q)\chi_{\rm c}\Big)\mathfrak{q}^3$$
$$+ \Big(3\chi_3(q) + 4\chi_2(q) + 2)\chi^{(1)} + 2\chi_2(q)\chi^{(3)} + \chi^{(5)} + \chi^{(1)}\chi^{(2)}\Big)\mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5), \tag{4.54}$$

$$\tilde{f}_2 = -2 - 2\chi_{\rm s}\,\mathfrak{q} - \Big(2\chi^{(4)} + (3\chi_{\mathbf{2}}(q) + 2)\chi^{(2)} + 4(\chi_{\mathbf{3}}(q) + \chi_{\mathbf{2}}(q) + 1)\Big)\mathfrak{q}^2$$
$$- \Big(2\chi^{(2)}\chi_{\rm s} + 3\chi_{\mathbf{2}}(q)\chi^{(1)}\chi_{\rm c} + 4(\chi_{\mathbf{3}}(q) + \chi_{\mathbf{2}}(q) + 1)\chi_{\rm s}\Big)\mathfrak{q}^3$$
$$+ \Big((5\chi_4(q) + 6\chi_3(q) + 11\chi_2(q) + 8)\chi^{(2)} + (4\chi_3(q) + 4\chi_2(q))\chi^{(4)} + (3\chi_2(q) - 2)\chi^{(6)}$$
$$+ (4\chi_3(q) + 3\chi_2(q) + 2)(\chi^{(1)})^2 + 3\chi_2(q)\chi^{(1)}\chi^{(3)} + 2\chi^{(1)}\chi^{(5)} + 2(\chi^{(2)})^2 + 2(\chi_s)^2$$
$$+ (6\chi_5(q) + 8\chi_4(q) + 16\chi_3(q) + 20\chi_2(q) + 10)\Big)\mathfrak{q}^4 + \mathcal{O}(\mathfrak{q}^5). \tag{4.55}$$

arXiv:1504.03672

reproduces the E-string partition function (elliptic genus)

by (up to 4 instantons) ['14 Kim, Kim, Lee, Park, Vafa]

Tao diagram indeed sees the E-string theory on a circle

Observation

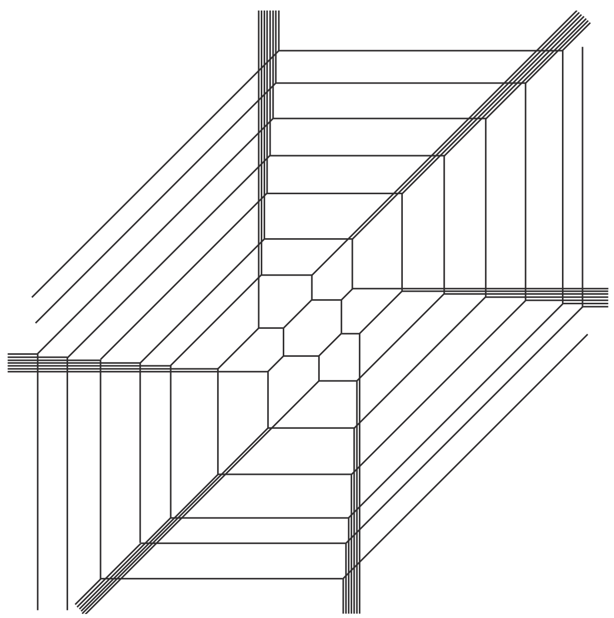
5d SU(2) theory with N_f flavors

$0 \leq N_f \leq 7$	5d UV fixed point	→	Finite diagram
$N_f = 8$	**6d UV fixed point**	→	**“Tao diagram”**
$N_f \geq 9$	No UV fixed point	→	No diagram

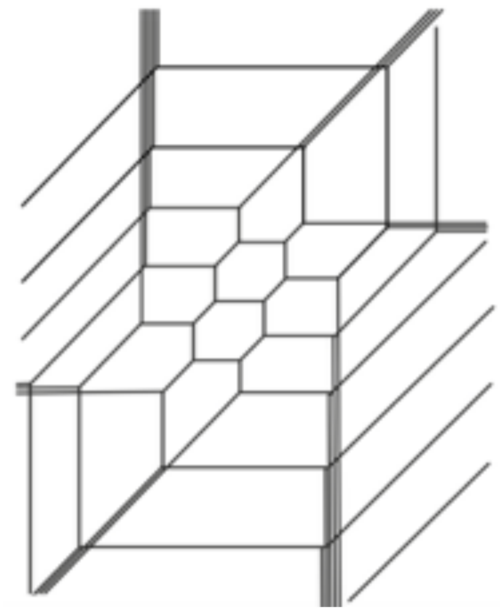
2. 6d Conformal matter

its 5d descriptions and dualities

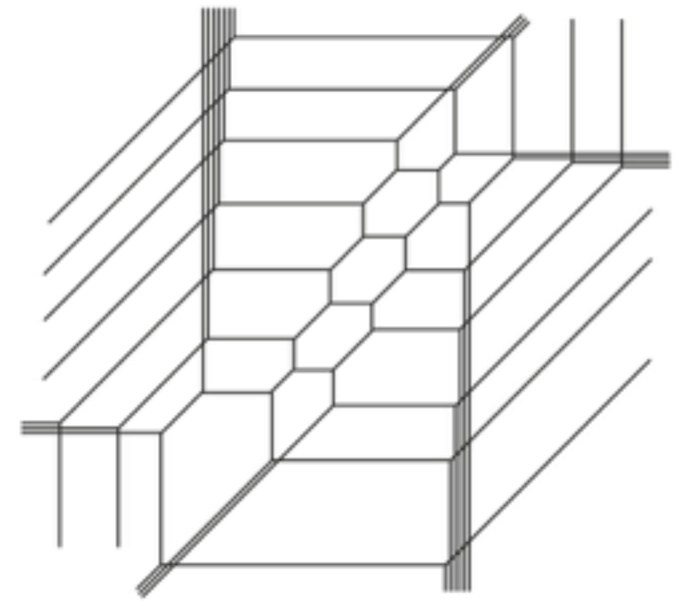
Many more Tao web diagrams



$N = 2$

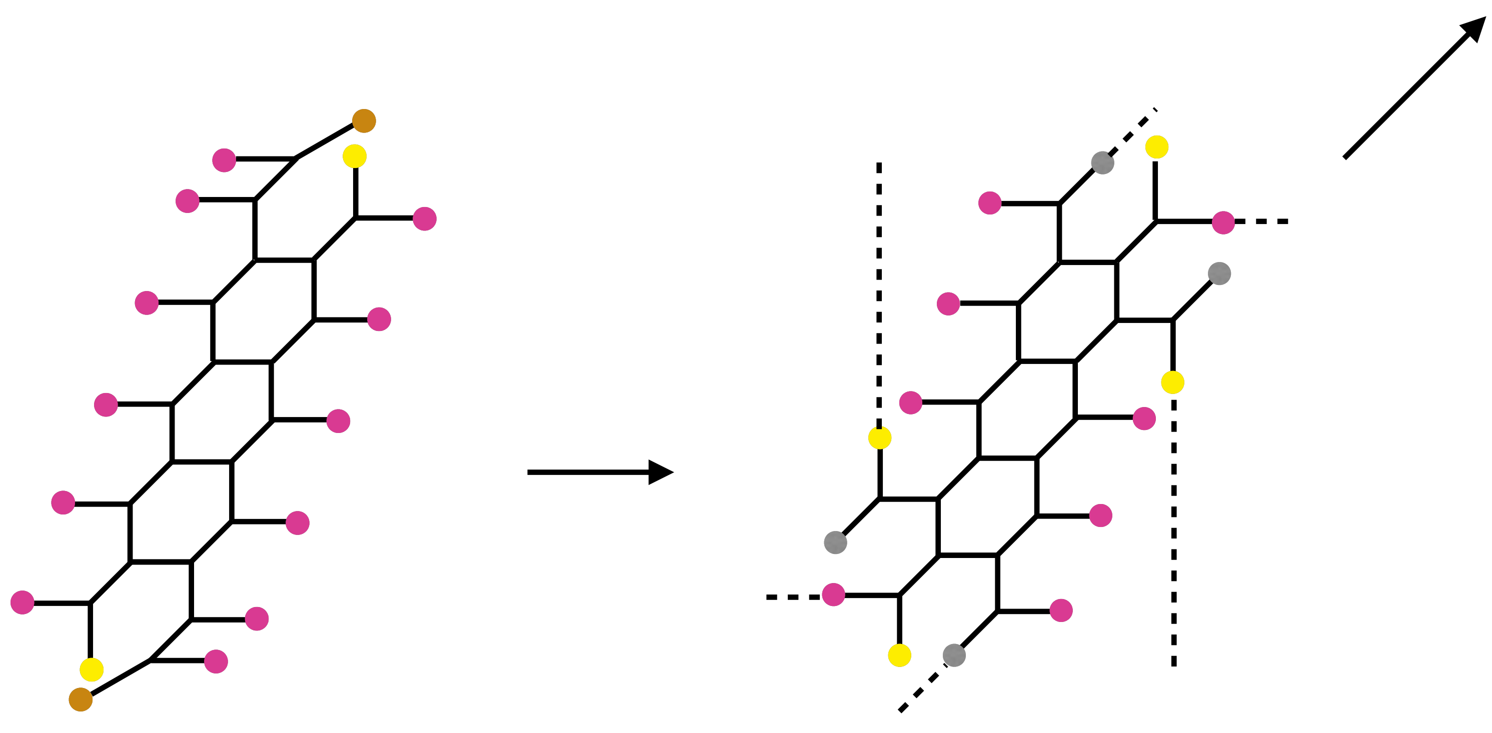


$N = 3$



$N = 4$

...



$SU(4), N_f = 12$

Claim:

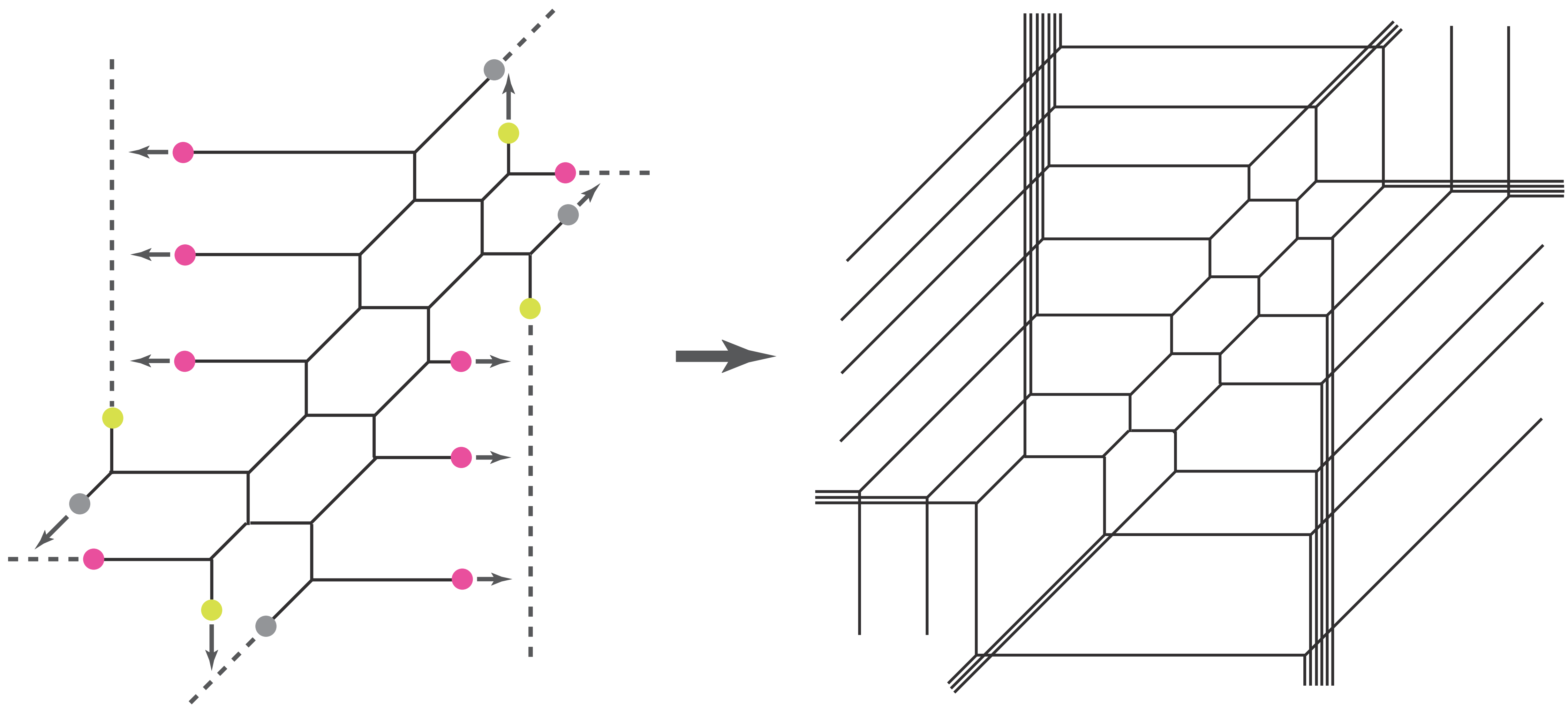
A **Tao web diagram** implies that
a 5d theory has UV completion as **6d SCFT**

A **Tao web diagram** is a 5d description of
a **6d SCFT** on a circle

What is 6d SCFT for this Tao?

5d SU(N) **N_f=2N+4**

arXiv:1505.04439



Conjecture

5d N=1 SU(N) w/ Nf=2N+4 has 6d UV fixed point

M5-brane probing D_{N+2} singularity
"(D_{N+2}, D_{N+2}) conformal matter"

[Del Zotto - Heckman - Tomasiello - Vafa '14]

Idea: **6d (1,0) SCFTs: Conformal matter**

↓ Tensor branch

IIA brane configurations

[Hanany, Zaffaroni '97, Brunner, Karch '97]

↓ on S^1 and T-dual

IIB (p,q) brane web diagrams

: read off 5d descriptions

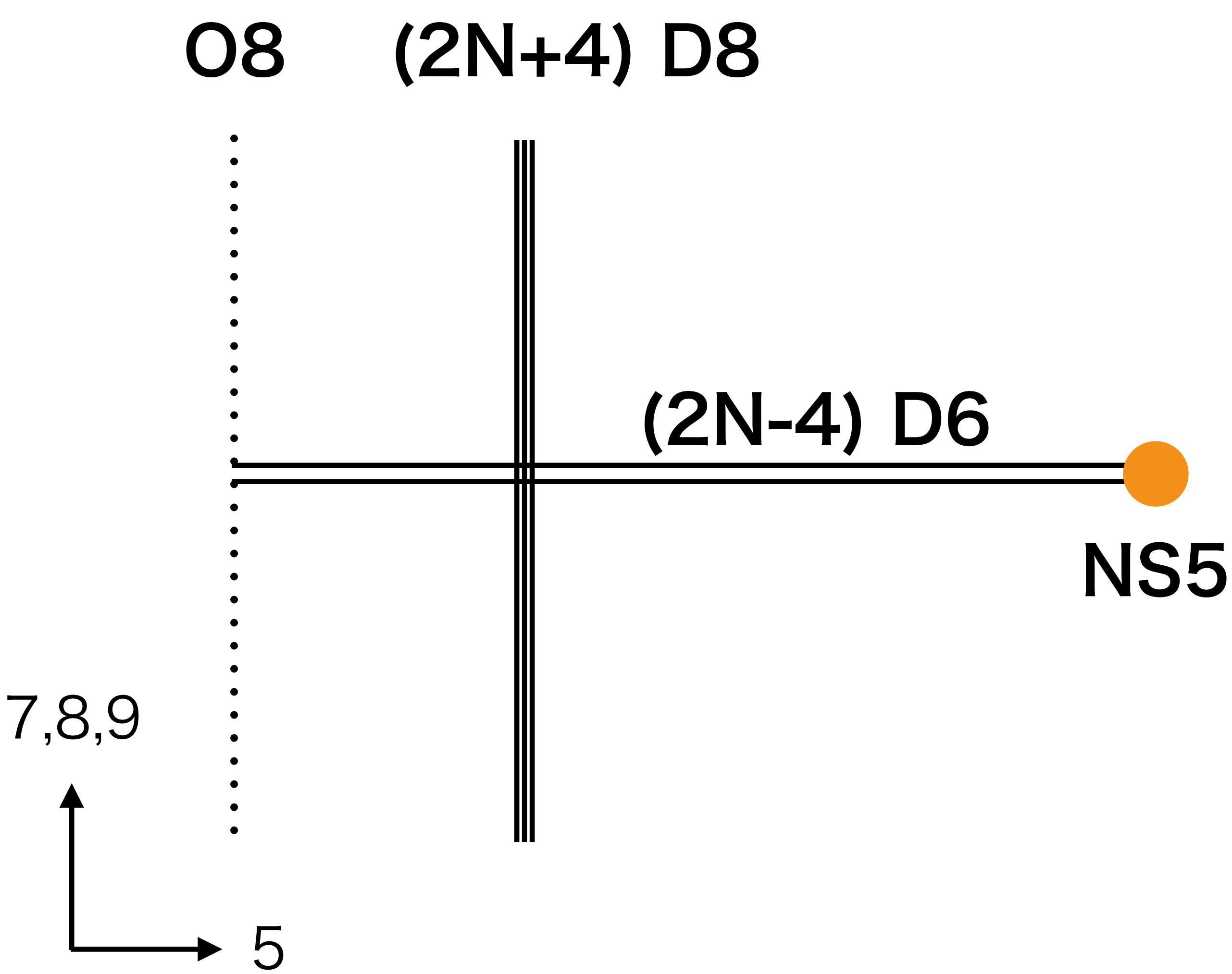
not just one but **many 5d theories**

(S-duality)

M5-brane probing D_{N+2} singularity

↓ Tensor branch

$$6\text{d } \mathcal{N} = (1,0)\ Sp(N-2) \text{ gauge theory}$$
$$N_f = 2N+4, \text{ w/tensor multiplet}$$

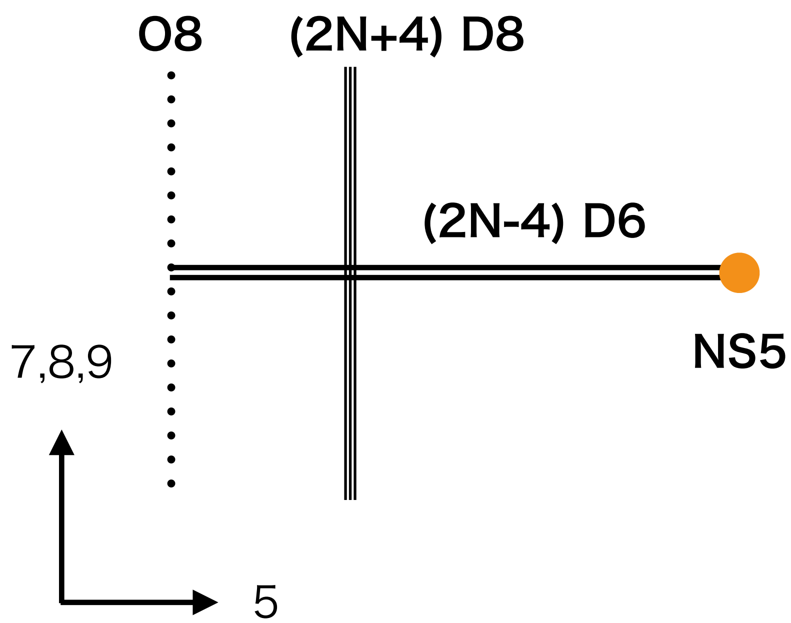


S^1 ↓ (direction 6)

	0	1	2	3	4	5	6	7	8	9
D6-brane	×	×	×	×	×	×	×			
NS5-brane	×	×	×	×	×		×			
D8-brane	×	×	×	×	×		×	×	×	×
O8-plane	×	×	×	×	×		×	×	×	×

[Hanany, Zaffaroni '97, Brunner, Karch '97]

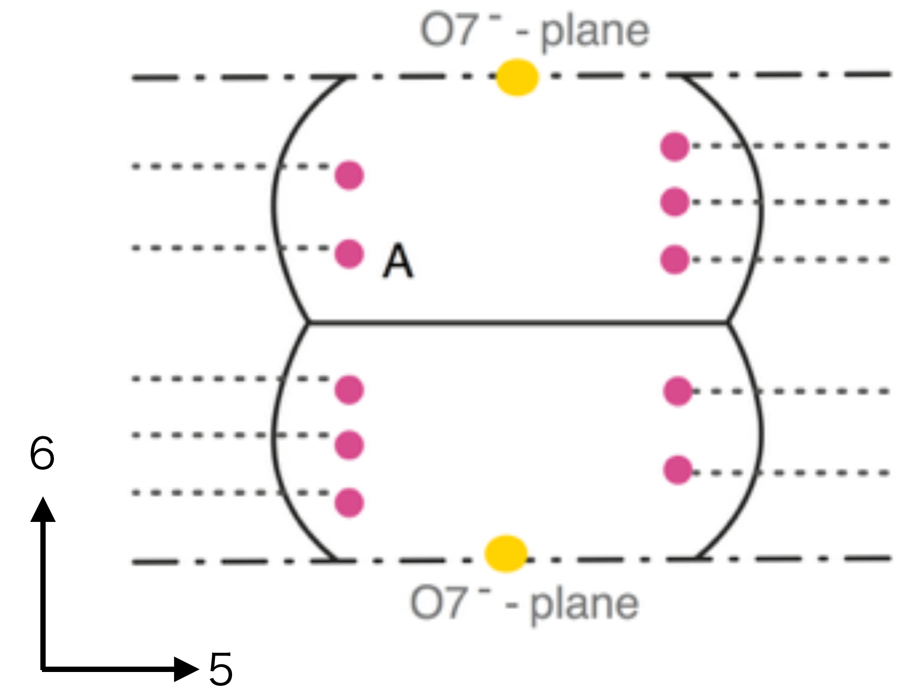
Diagrammatic “Derivation”



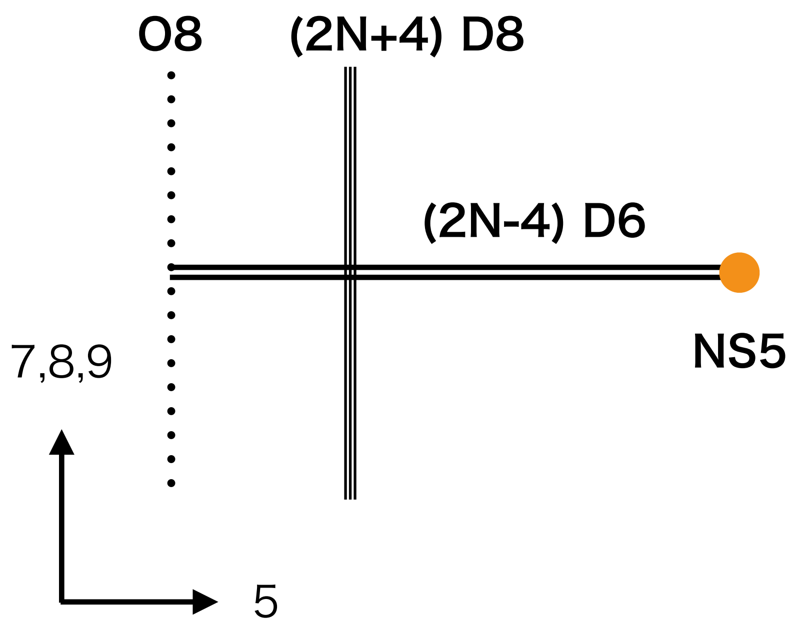
T-duality

(N=3)

$$6\text{d}\ Sp(1)\ N_f = 10$$



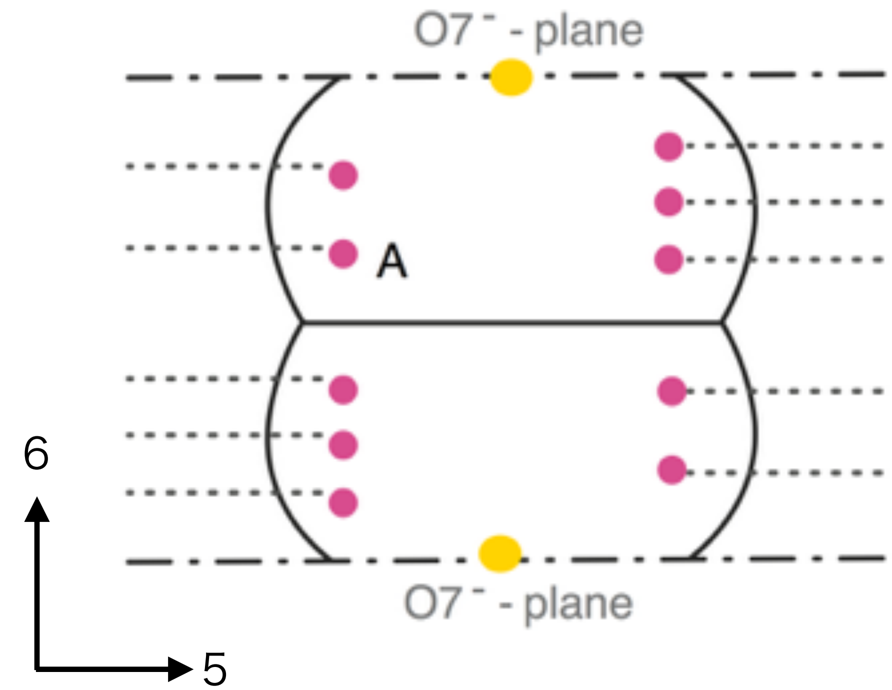
Diagrammatic "Derivation"



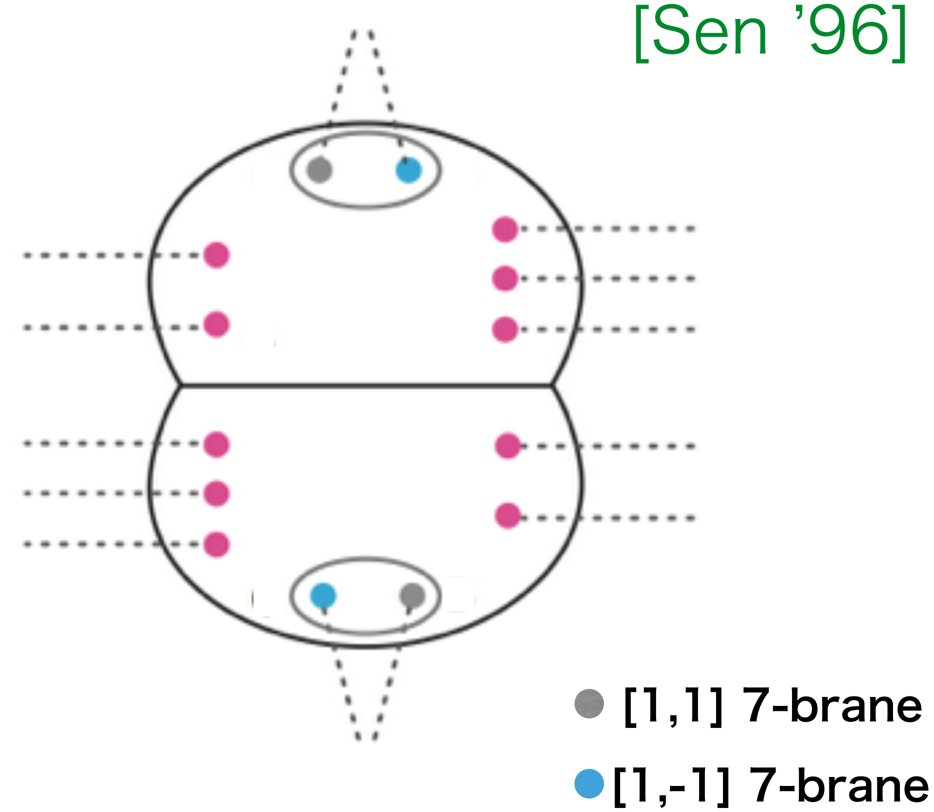
T-duality

(N=3)

$\text{6d } Sp(1)\ N_f = 10$

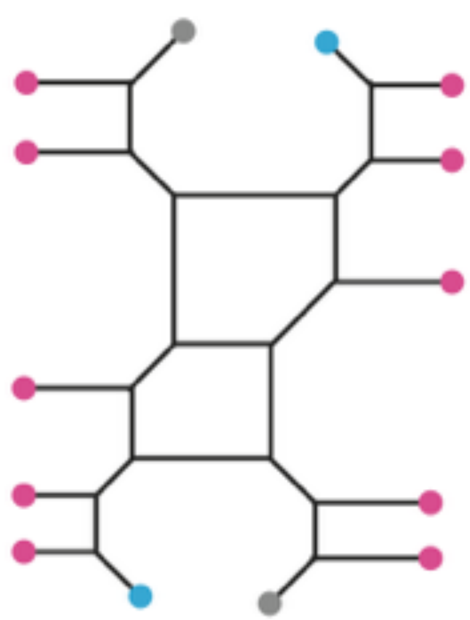


O7⁻ -plane
= [1,1] 7-brane
+ [1,-1] 7-brane
[Sen '96]



Hanany-Witten transition

$\text{5d } SU(3)_0\ N_f = 10$

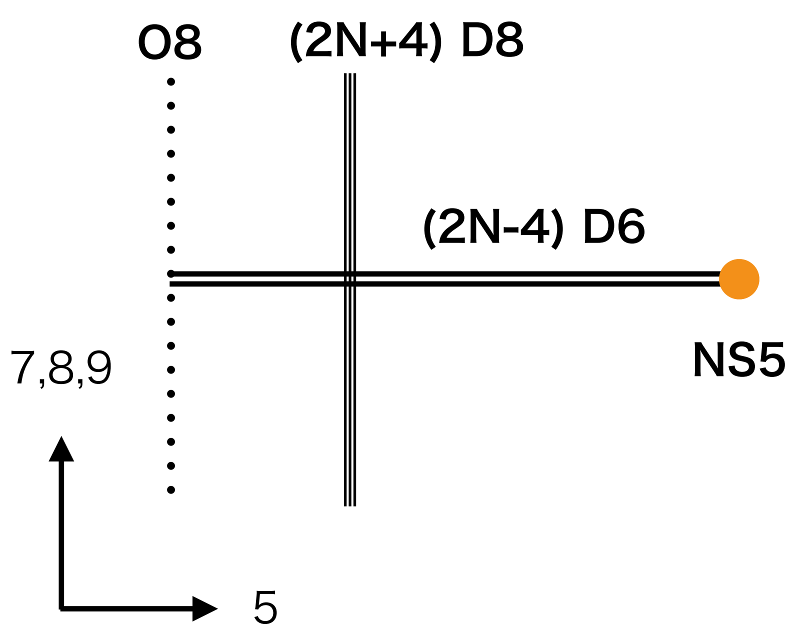


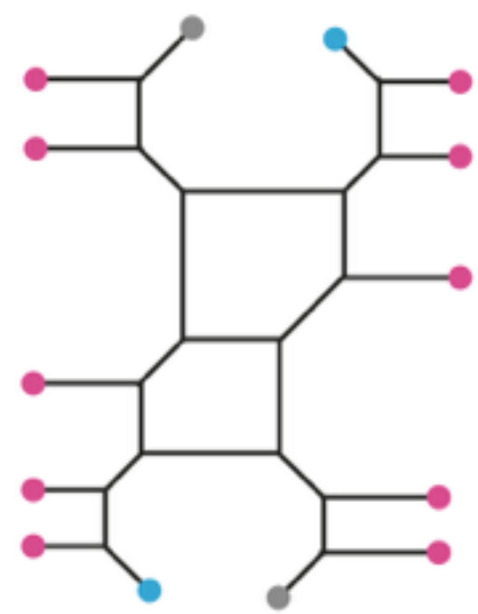
M5-brane probing
D_{N+2} singularity

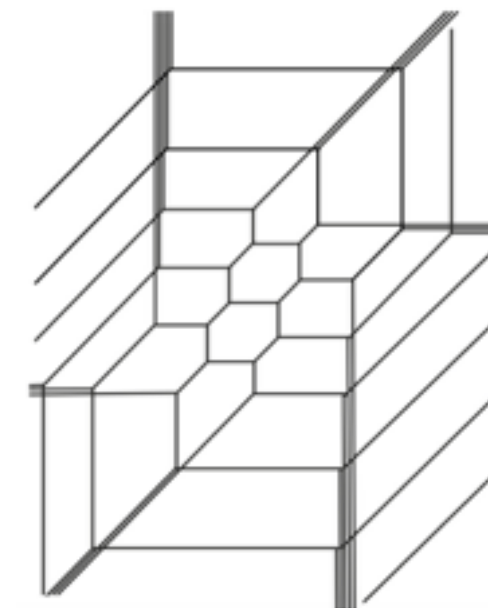
6d $Sp(N-2)$
$N_f = 2N+4, \ T$

5d $SU(N)_0 \ N_f = 2N+4$

Tao diagrams







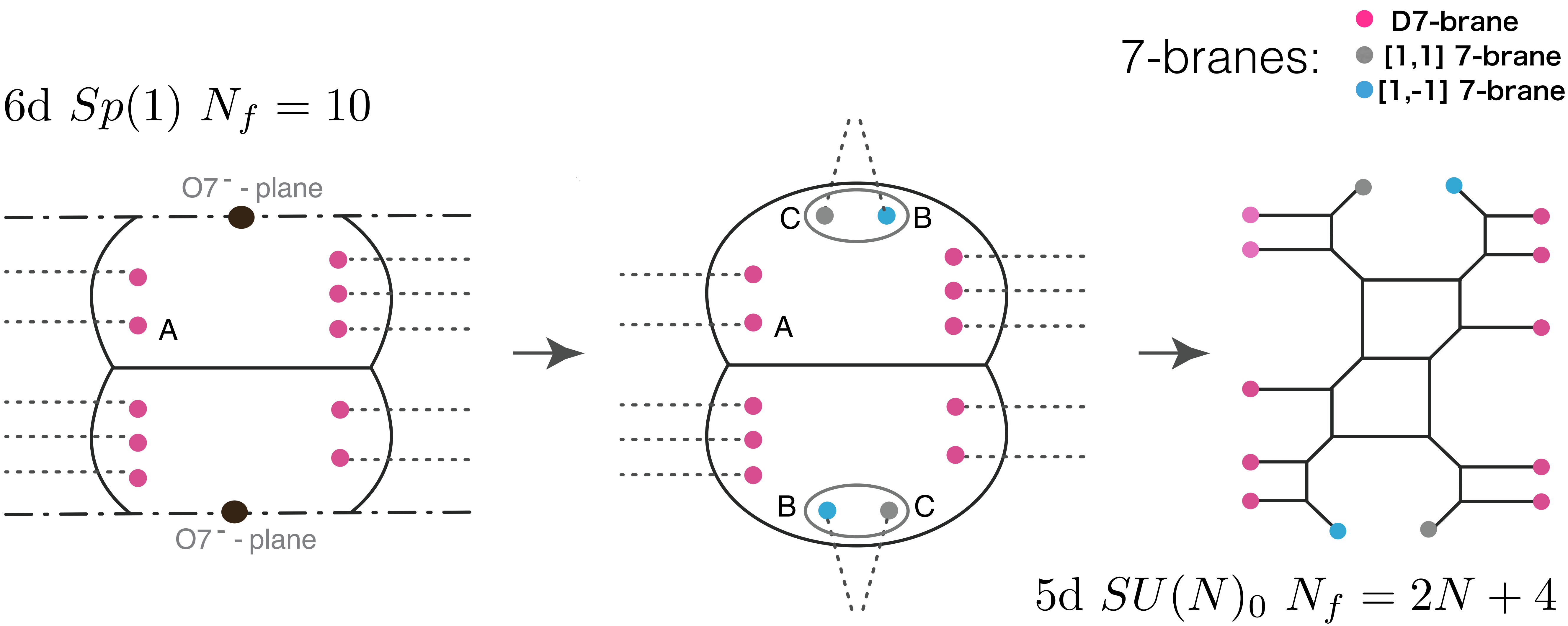
Note: N=2: 6d E-string —> 5d SU(2) with 8 flavors

Yet another 5d description: duality

6d Sp(N-2) theory with **N_f= 2N+4, a tensor**

resolve **two O7⁻'s**

5d $SU(N)_0$ theory with **N_f= 2N+4**



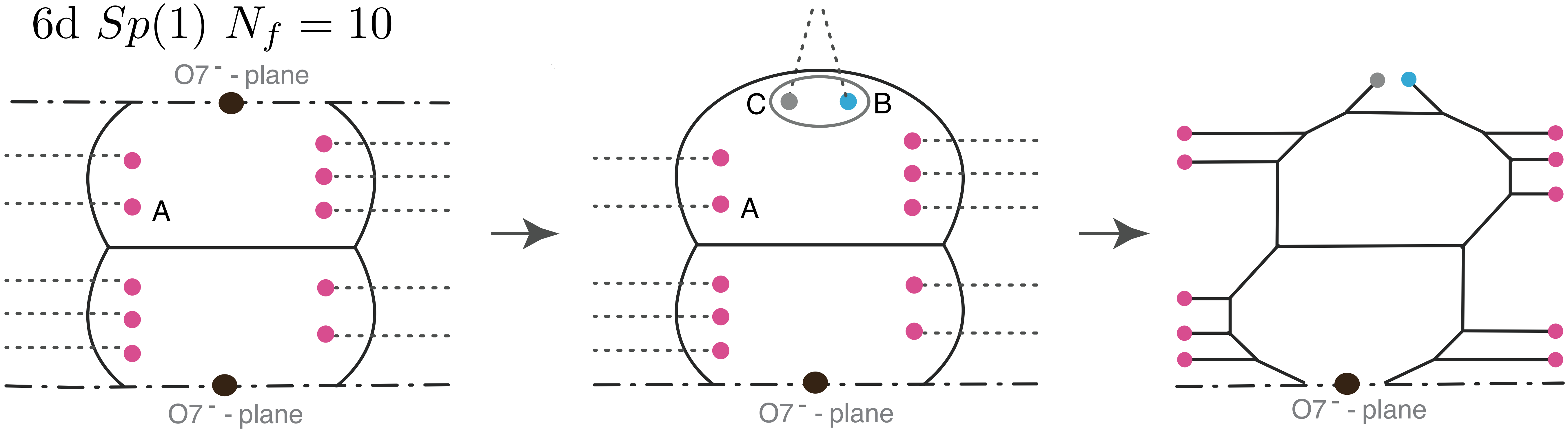
[Hayashi-SSK-Lee-Taki-Yagi '15]

[Yonekura '15]

5d Sp(N-1) theory with **N_f= 2N+4**

Resolving **only one O7⁻**: [Hayashi-SSK-Lee-Yagi '15]

6d $Sp(1)\ N_f = 10$



5d $Sp(2)\ N_f = 10$

SU-Sp duality (or “UV duality”)

[Gaiotto-Kim ’15]

5d SU(N) theory N_f= 2N+4	⟷	5d Sp(N-1) theory N_f= 2N+4
both O7s		only one O7

different descriptions but UV dual

6d Sp(N-2) theory
N_f= 2N+4, Tensor

one O8

conformal matter

5d SU(N) theory
N_f= 2N+4

5d Sp(N-1) theory
N_f= 2N+4

both O7s

only one O7

Explicit checks: Equivalence of the partition functions

6d **Sp(1)** theory
N_f=10, Tensor

Elliptic genus

['15 Kim, Kim, Lee]

Wilson lines

Wilson lines

5d **SU(3)** theory
N_f=10

['16 Hayashi-SSK-Lee-Yagi]

Map

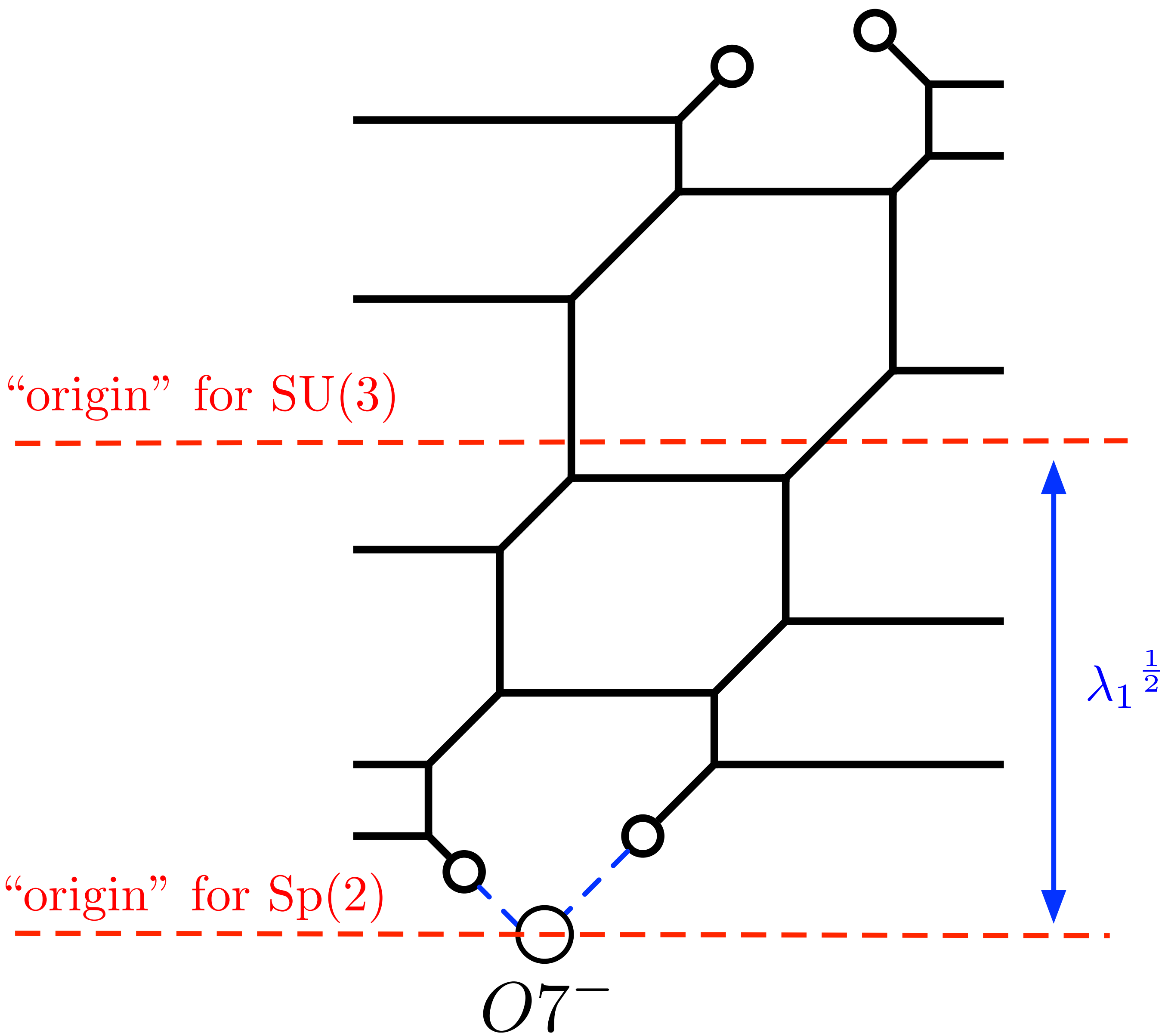
5d **Sp(2)** theory
N_f=10

['16 Hayashi-SSK-Lee-Yagi]

['16 Yun]

Map between **Sp(2)** and **SU(3)**

['15 Gaiotto-Kim]
['16 Hayashi-SSK-Lee-Yagi]



Sp(2) **SU(3)**

$$A'_i = \lambda_1^{\frac{1}{2}} A_i = \left(q^{\frac{1}{2}} \prod_{i=1}^{10} y_i^{-\frac{1}{4}} \right) A_i, \qquad (i = 1,2)$$

$$y'_i = \lambda_1^{\frac{1}{2}} y_i = \left(q^{\frac{1}{2}} \prod_{i=1}^{10} y_i^{-\frac{1}{4}} \right) y_i, \qquad (i = 1, \dots, 5)$$

$$y'^{-1}_i = \lambda_1^{\frac{1}{2}} y_i = \left(q^{\frac{1}{2}} \prod_{i=1}^{10} y_i^{-\frac{1}{4}} \right) y_i, \qquad (i = 6, \dots, 10)$$

Computations

['16 Hayashi-SSK-Lee-Yagi]

$$Z_{SU(3)} = Z_{\rm pert} Z_{\rm inst}$$

$$Z_{\rm inst} = {\rm PE}[\mathcal{F}_1(A,y)q + \mathcal{F}_2(A,y)q^2 + \cdots]$$

$$\mathcal{F}_1(A,y) = \frac{g}{(1-g)^2}\left[\frac{\sum_{n=0}^{10}\chi_n(-A_1)^{6-n}}{(A_1-A_2)^2(A_1-A_3)^2} + ({\rm cyclic})\right]$$

$$+ \frac{g}{(1-g)^2}\left[\sum_{n=0}^{2}(-1)^{n+1}\left({P_1}^{2-n}\chi_n + {P_{-1}}^{2-n}\chi_{10-n}\right)\right]$$

$$\mathcal{F}_2 = \cdots$$

Map

$$Z_{Sp(2)}(A', q', y')$$

Wilson line

$$\tilde{y}_{10} = y'_{10}q'^{-2}, \qquad \phi = A'_1 q' y'^{-1}_{10},$$

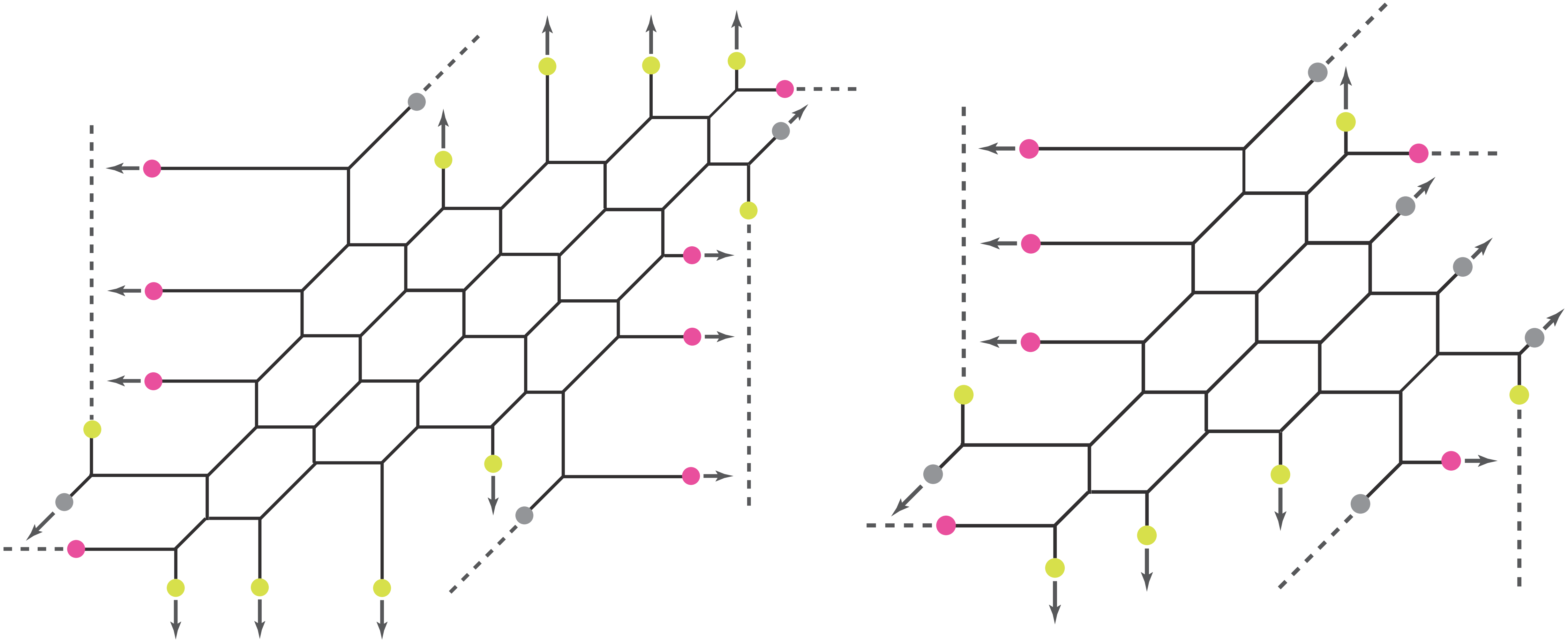
$$\tilde{q} = q'^2, \qquad \tilde{A} = A'_2, \qquad \tilde{y}_i = y'_i,$$

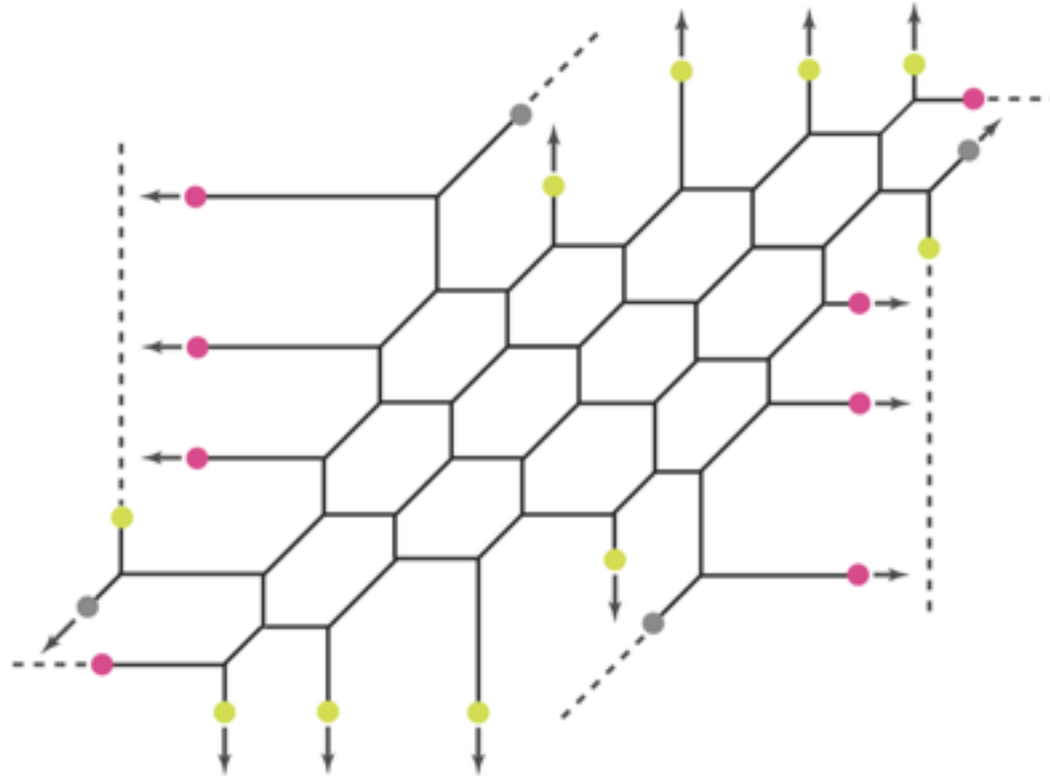
$$Z_{6d\ Sp(1)}(\tilde{A}, \tilde{q}, \tilde{y}, \phi)$$

['15 Joonho Kim-Seok Kim-Kimyeong Lee]

Other 5d/6d web diagrams?

Quiver type





$$5\text{d } [N+2] - \overbrace{SU(N) - \cdots - SU(N)}^{k} - [N+2]$$

['15 Zafrir]
['15 Yonekura]
['15 Hayashi-SSK-Lee-Yagi]

$k = 2n+1$

$$6\text{d } Sp(N') - SU(2N'+8) - SU(2N'+16) - \cdots - SU(2N'+8(n-1)) - [2N'+8n]$$

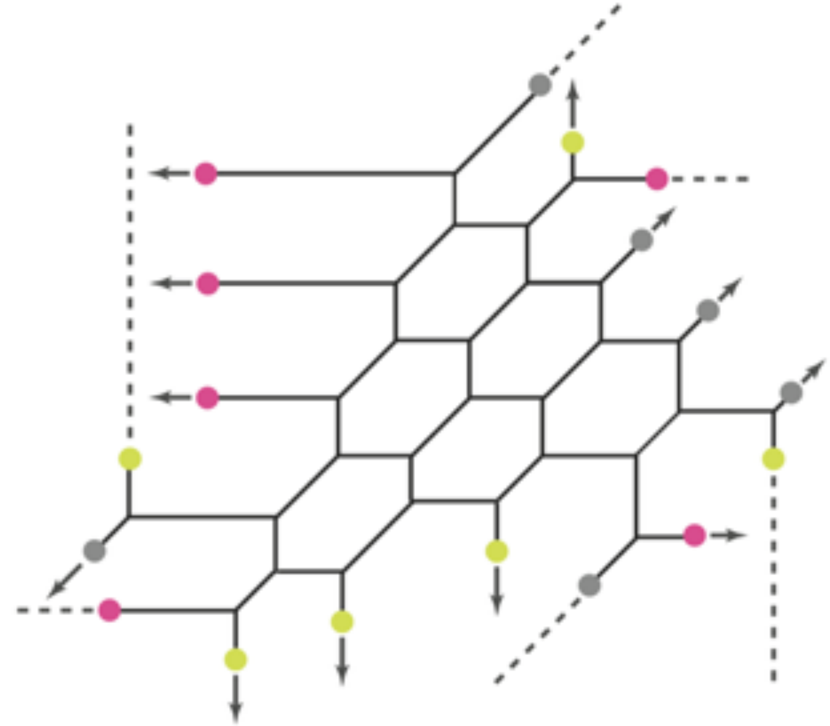
$$N' = N - 2n$$

$k = 2n$

$$6\text{d } [A] - SU(N') - SU(N'+8) - SU(N'+16) - \cdots SU(N'+8(n-1) - [N'+8n+8]$$

$$N' = 2(N-2n+1)$$

hypermultiplet in antisymmetric representation



$$\text{5d } [N+3] - SU(N) - SU(N-1) - SU(N-2) - \cdots - SU(3) - SU(2) - [3]$$

['15 Zafrir]
['15 Ohmori, Shimizu]
['15 Hayashi-SSK-Lee-Yagi]

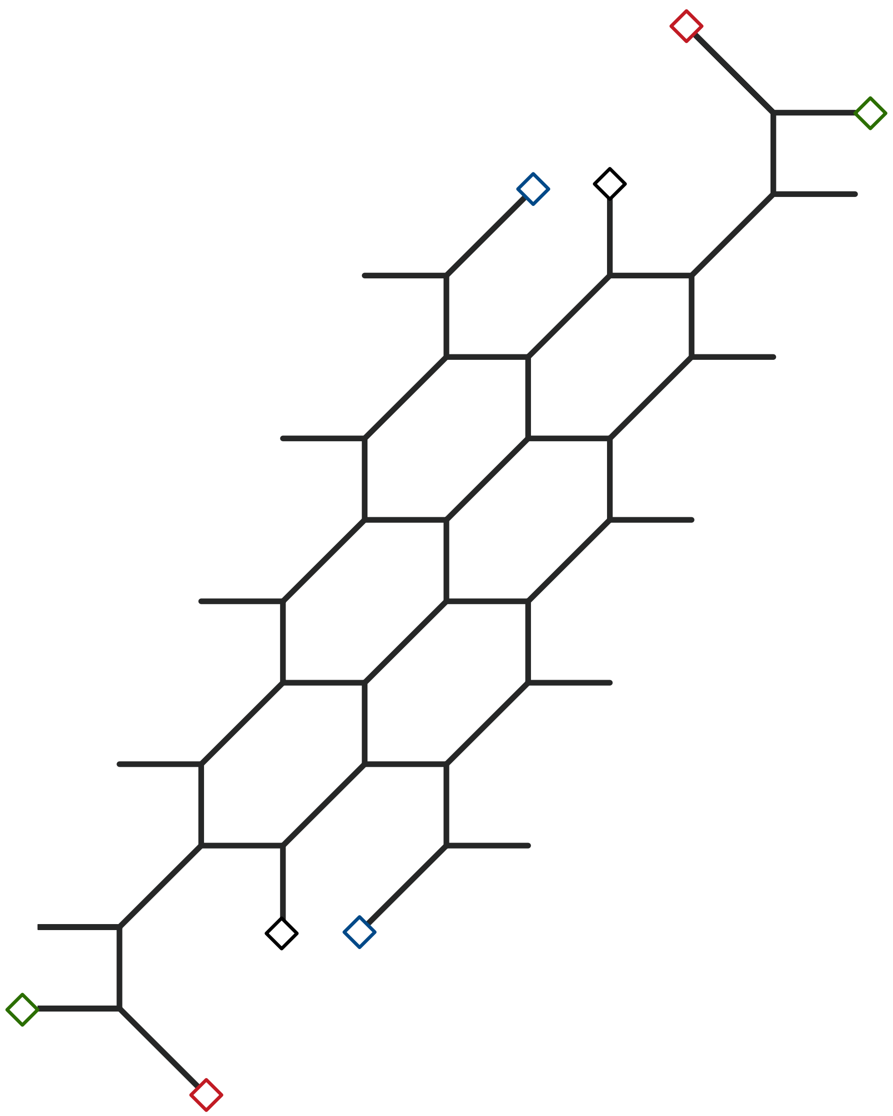
$$N = 3n: \quad \text{6d } SU(0) - SU(9) - \cdots - SU(9n) - [9n+9]$$

$$N = 3n+1: \quad \text{6d } SU(3) - SU(12) - \cdots - SU(3+9(n-1)) - [3+9n]$$

$$N = 3n+2: \quad \text{6d } \left[\tfrac{1}{2}\right]_{\Lambda^3} - SU(6) - SU(15) - \cdots - SU(6+9(n-1)) - [6+9n]$$

UV dualities:

different-looking 5d gauge theories
have
the **identical** 6d UV fixed point

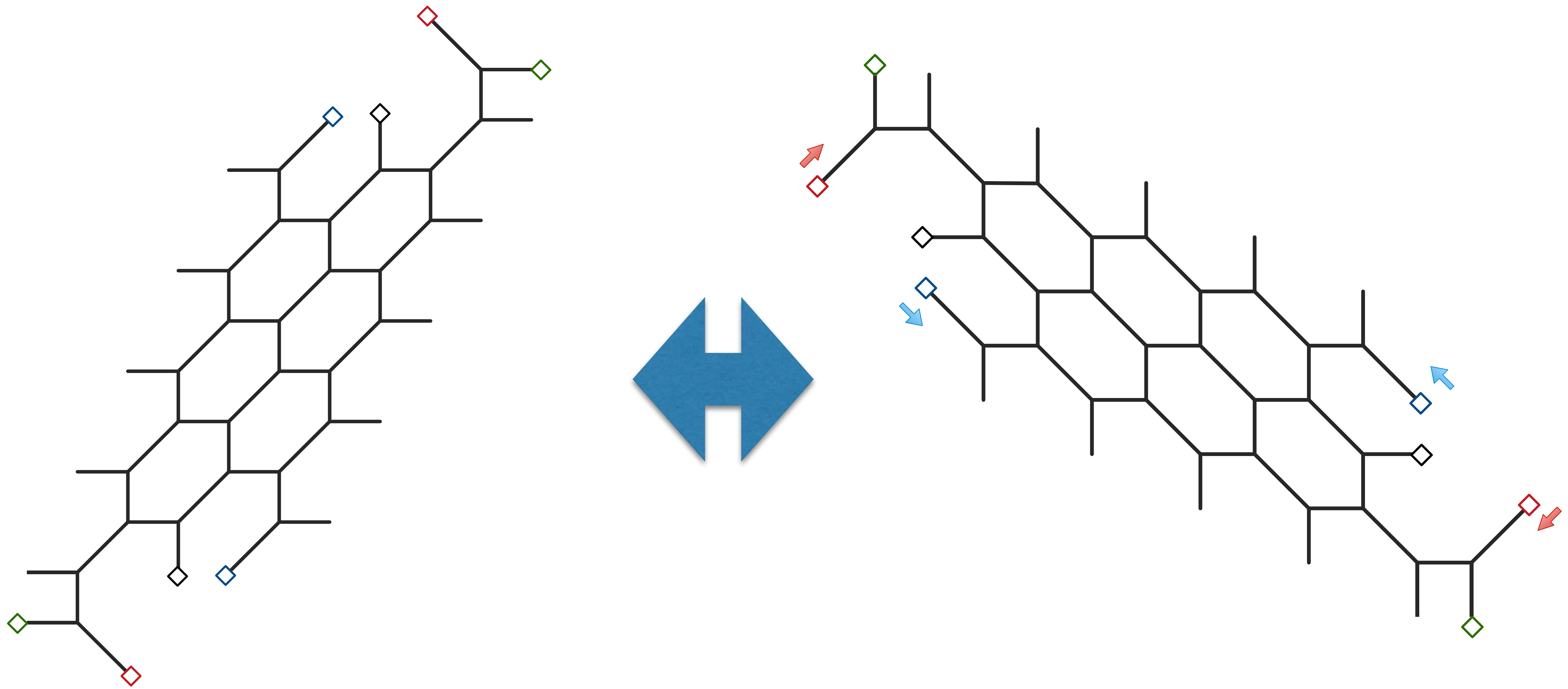


5d [6]-SU(4)-SU(4)-[6]

↑

6d [A]-SU(6)-[14]

S-duality

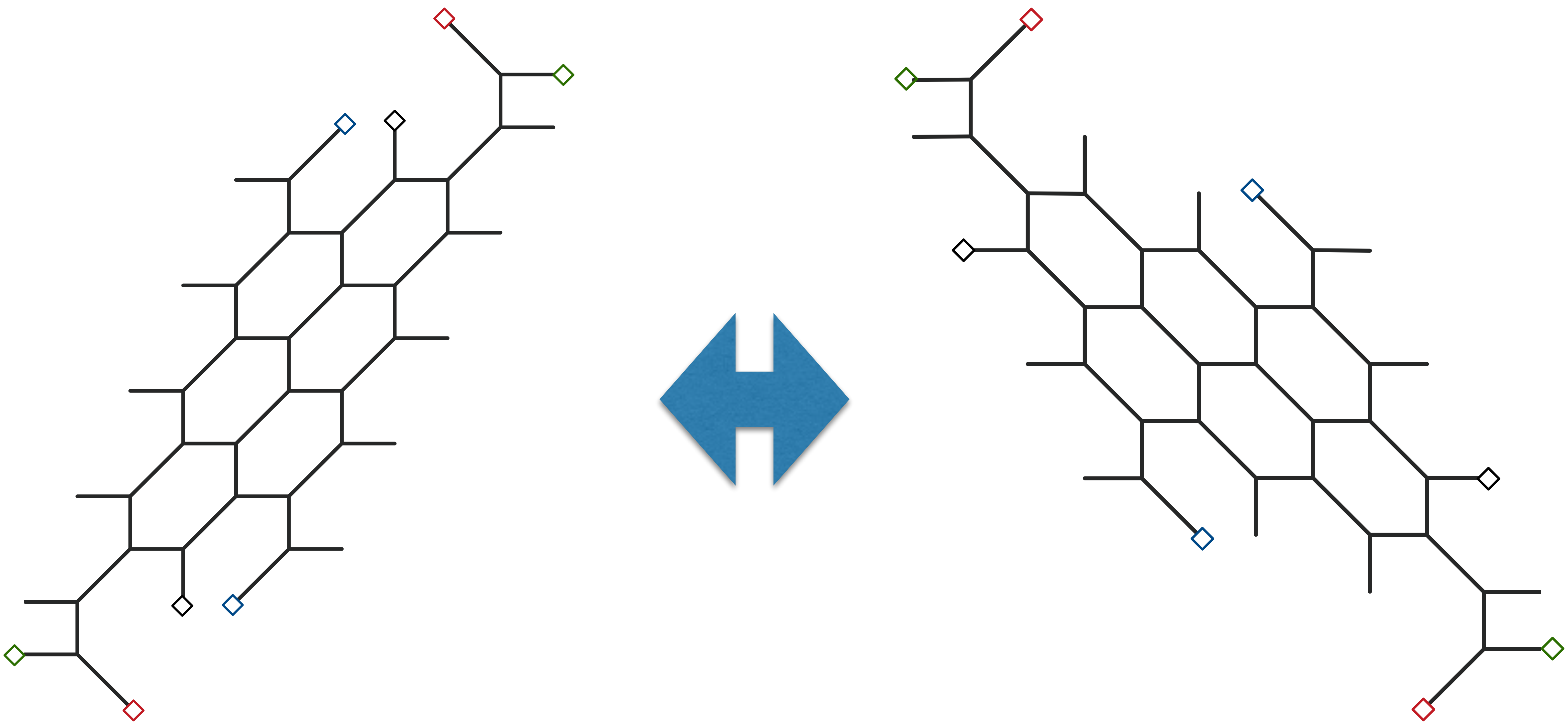


5d [6]-SU(4)-SU(4)-[6]

5d (?)-SU(3)-SU(3)-SU(3)-(?)

6d [A]-SU(6)-[14]

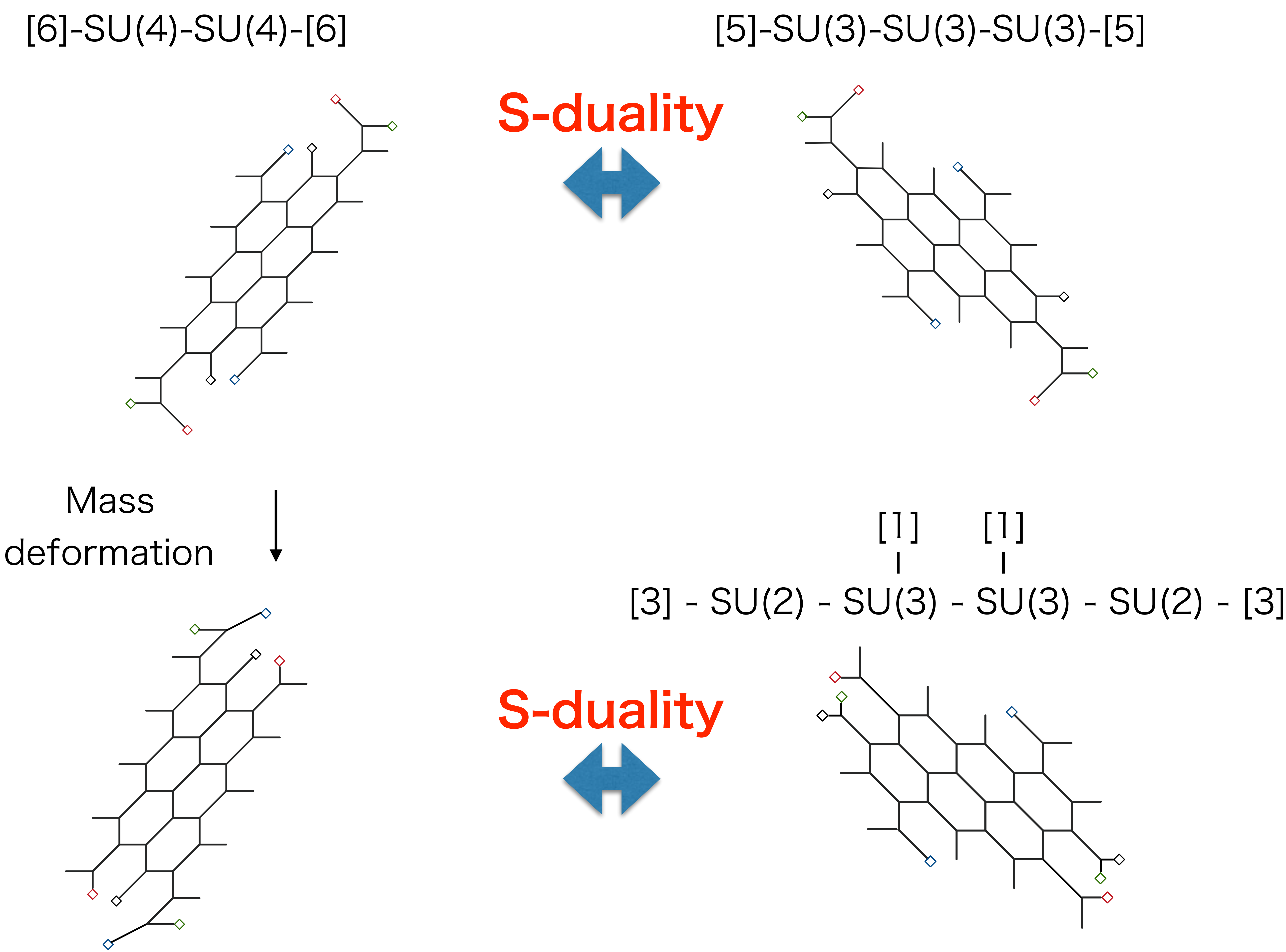
S-duality



5d [6]-SU(4)-SU(4)-[6]

5d [5]-SU(3)-SU(3)-SU(3)-[5]

6d [A]-SU(6)-[14]

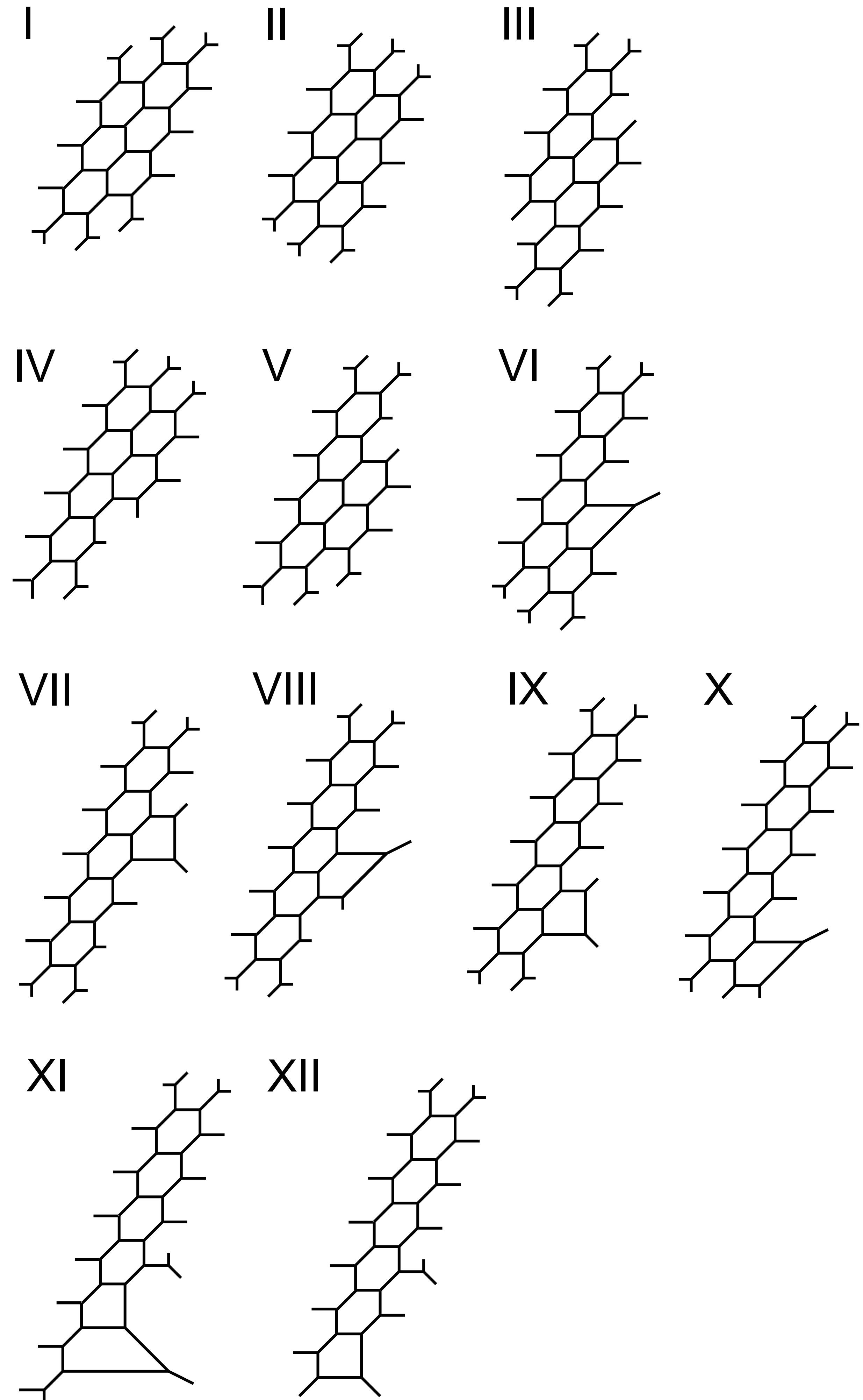


[6]-SU(4)-SU(4)-[6]
[5]-SU(3)-SU(3)-SU(3)-[5]
S-duality
Mass deformation
[1] [1]
[3] - SU(2) - SU(3) - SU(3) - SU(2) - [3]
S-duality

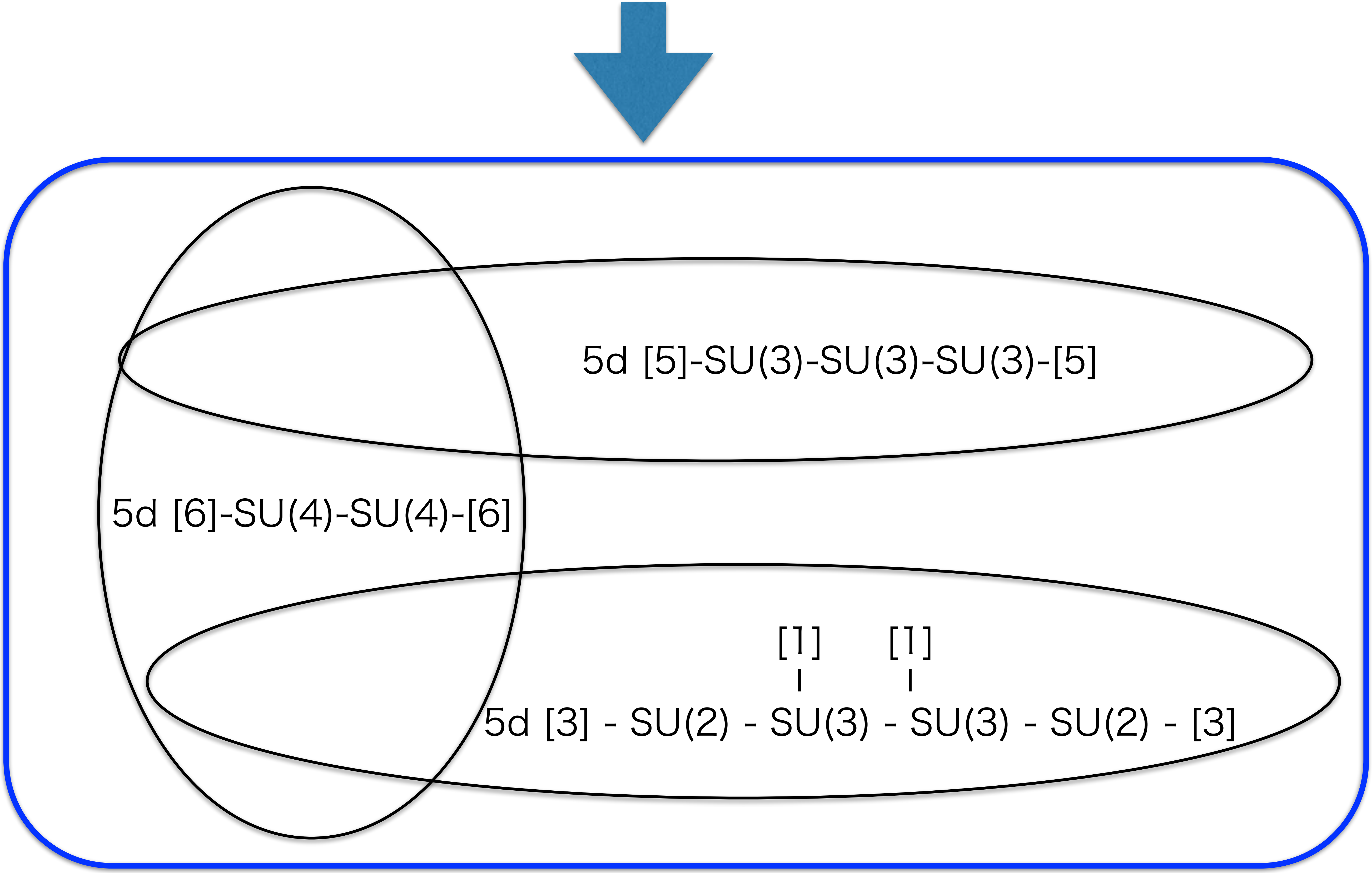
6d SU(6) theory
with Nf=14, Na=1

various 5d quivers
(depending on
D5, D7 distributions)

Wilson lines



6d SU(6) 14 flavors, antisym. tensor + tensor mult.



All possible values of “gauge theory parameters”

3. Generalization

6d SCFTs involving other O-planes

ON^0 - $O6^{-/+}$ - $O8^-$

- B_N, C_N, D_N gauge groups: other orientifolds

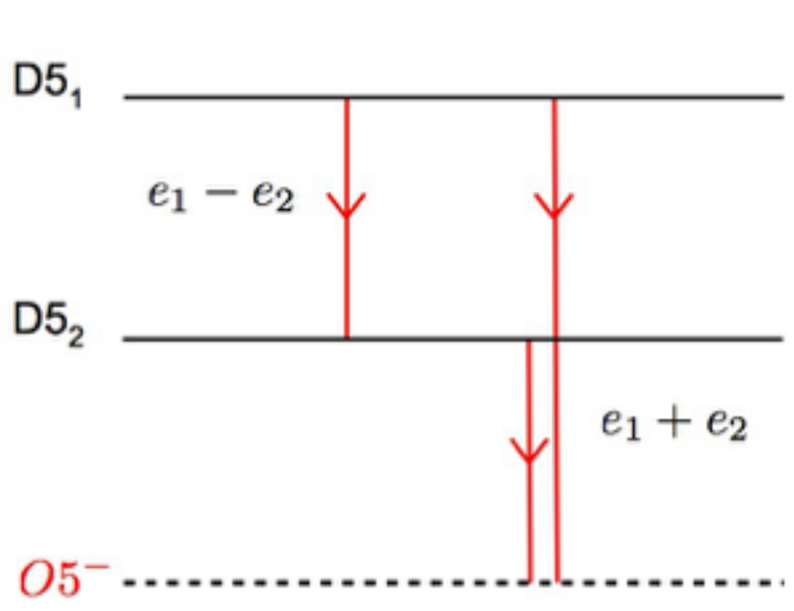
  IIA: $O6^+$, $O6^-$, $O8^+$ IIB: $O5^+$, $O5^-$, $O7^+$

- **ON^0** : S-dual of $O5^0$ (= $O5^-$+D5), NS-like orientifold

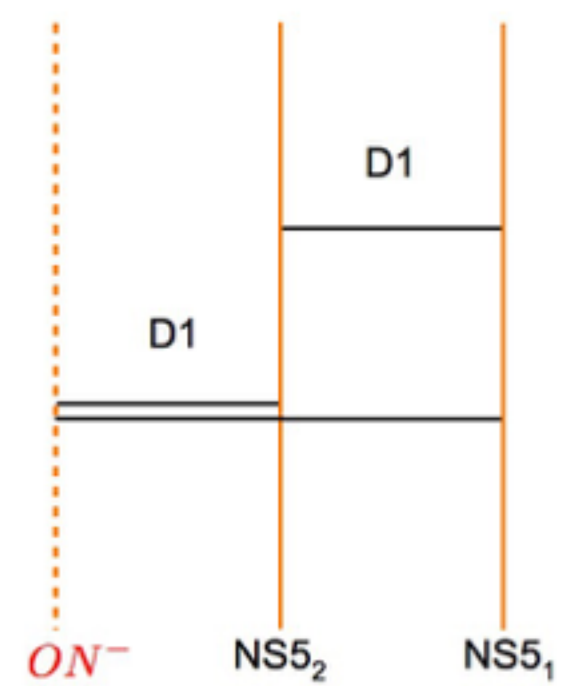
  ['95 Kutasov] ['96, 97, 98 Sen]

  Splitting of quiver gauge theories

  ['99 Hanany-Zaffaroni]



(a)



(b)

ON^0

Idea: **6d (1,0) SCFTs**

↓ Tensor branch

IIA brane configurations w/ O8 — **O6, ON^0**

on S^1 ↓

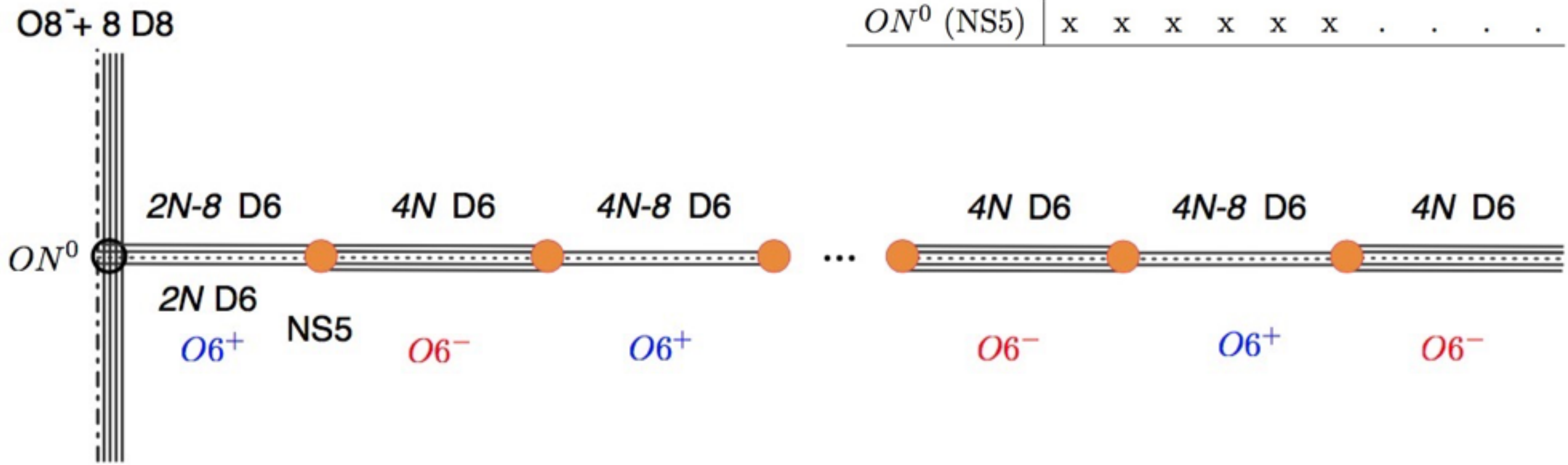
IIB (p,q) brane: web diagrams — **O5, ON^0**

O7- resolutions to two 7-branes

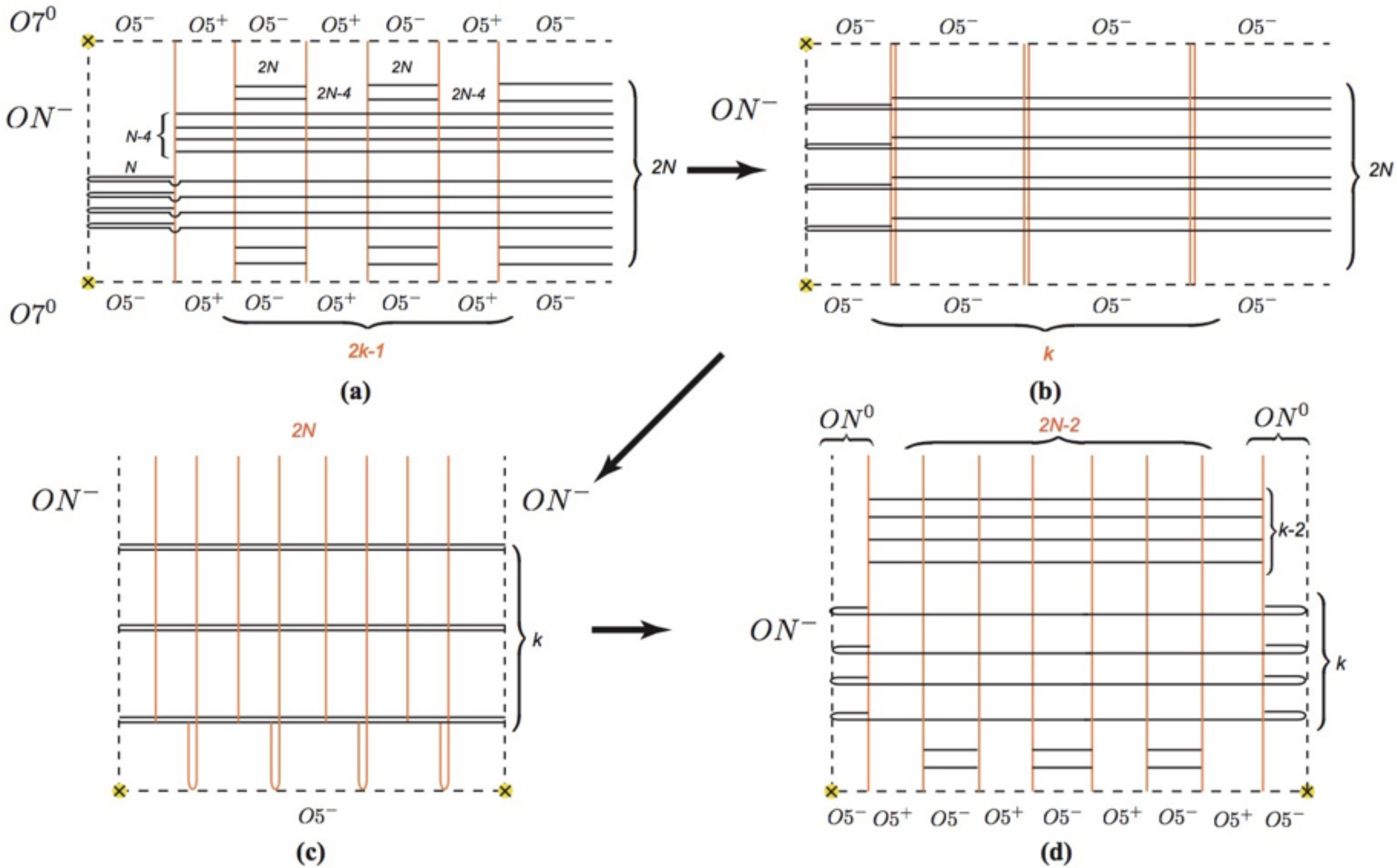
S-duality

Wilson lines

	0	1	2	3	4	5	6	7	8	9
$O8$/D8	x	x	x	x	x	x	.	x	x	x
$O6$/D6	x	x	x	x	x	x	x	.	.	.
ON^0 (NS5)	x	x	x	x	x	x	.	.	.	.

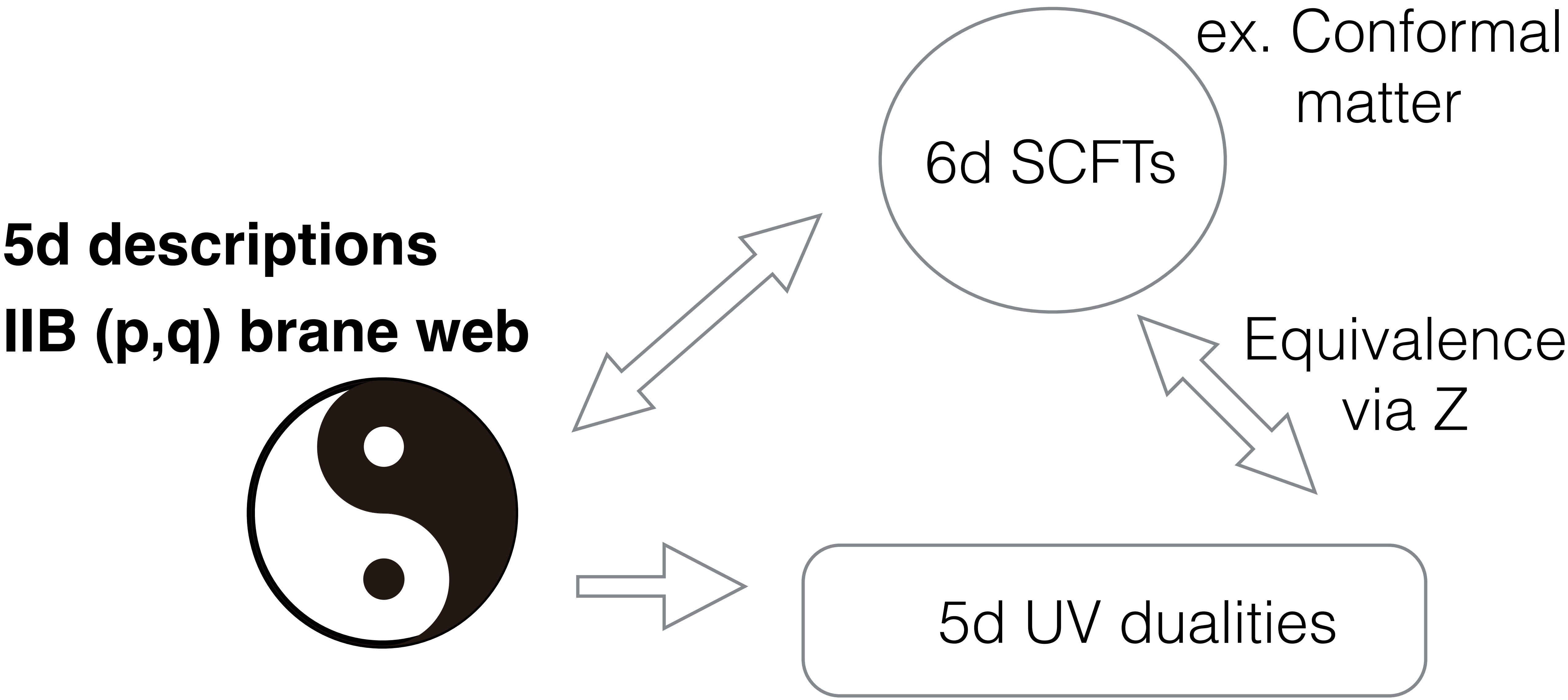


$$6d\ \ [8] - Sp(N) - \underbrace{\overset{\displaystyle Sp(N-4)}{\overset{|}{SO(4N)}} - Sp(2N-4) - SO(4N) - \cdots - Sp(2N-4)}_{2k-2} - [2N].$$



$$
\begin{array}{c}
\qquad\qquad\quad Sp(k-2) \qquad\qquad\qquad\qquad\qquad\quad Sp(k-2) \\
\qquad\qquad\quad | \qquad\qquad\qquad\qquad\qquad\qquad\quad | \\
5d \;\; [4] - Sp(k) - SO(4k) - Sp(2k-2) - \cdots - SO(4k) - Sp(k) - [4],
\end{array}
$$

Summary



new perspective on 5d and 6d SCFTs