

$N=1$ Deformations and RG Flows of $N=2$ SCFTs

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Motivation

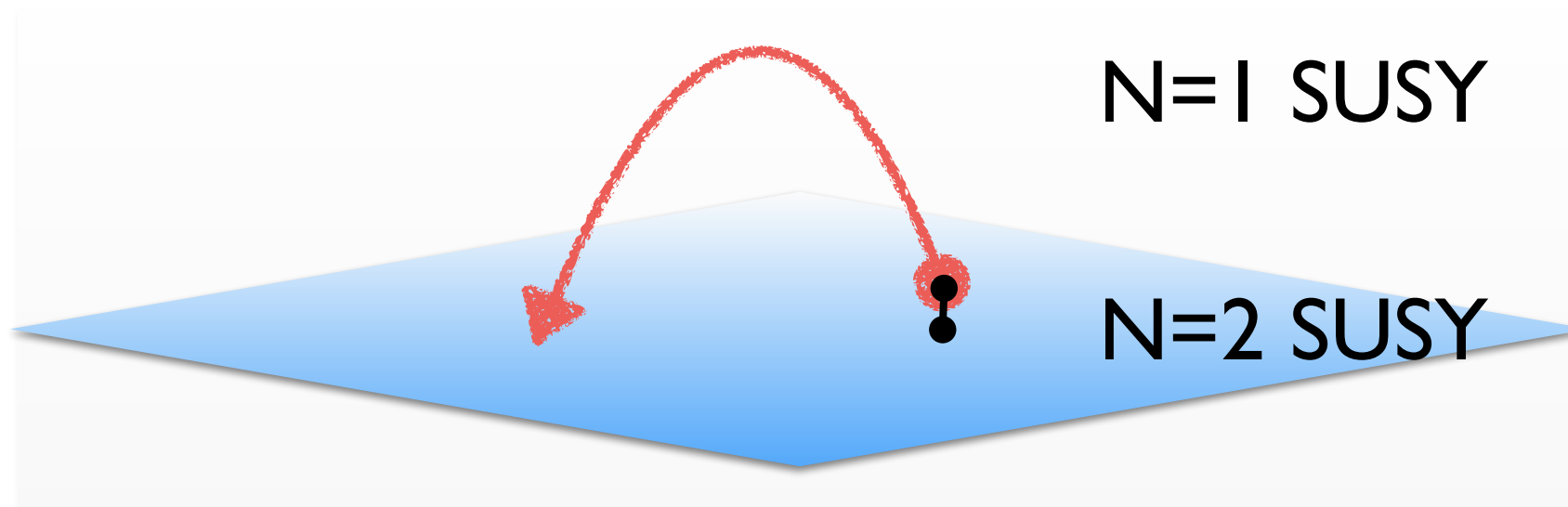
- Explore the landscape of $N=1$ SCFTs
- Restrict to the class S , class S_k [\[Bah-Beem-Bobev-Wecht\],\[Xie\],...](#)
[\[Gaiotto-Razamat\]\[Hanany-Maruyoshi\],...](#)
- (non-Lagrangian) $N=1$ SCFTs from deforming (non-Lagrangian) $N=2$ SCFT.
- Renormalization group (RG) flows between (non-Lagrangian) $N=2$ and $N=1$ theories.

Summary

- We study certain class of $N=1$ preserving deformation ρ of $N=2$ SCFT $\mathcal{T}$ with **non-abelian global symmetry** G .
- The deformation triggers a flow to new $N=1$ SCFT $\mathcal{T}_{\text{IR}}[\mathcal{T}, \rho]$.
- We study a number of non-Lagrangian/Lagrangian examples.
 - By deforming T_N , we obtain $N=1$ SCFT associated to the sphere with no puncture.
 - Argyres-Douglas theories, SQCDs

Summary - Surprise!

- Emergent $N=2$ supersymmetry:
- For a number of cases, SUSY enhances to $N=2$ at the fixed point.
- $N=1$ RG flows between (known) $N=2$ SCFTs



Summary - Surprise!

- Simple $N=1$ deformation of the **Lagrangian** $N=2$ SQCD flows to the “**non-Lagrangian**” Argyres-Douglas (AD) theories!
 - This enables us to compute the **full superconformal indices** of the AD theories.
 - One can use this “Lagrangian description” to compute any RG invariant quantities.
 - cf) $N=1$ gauge theory flowing to $N=2$ E_6 SCFT
- [Gadde-Razamat-Willett]

$N=1$ ‘Lagrangians’ for ‘non-Lagrangian’ $N=2$ SCFTs

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Outline

- ‘Lagrangian’ for the Argyres-Douglas theory H_0, H_1, H_2
- $N=1$ deformations of $N=2$ SCFT with non-abelian global symmetry
- Deforming $N=2$ SQCDs: Flows to the (generalized) Argyres-Douglas theories
- Flows between generalized AD theories, rank 1 SCFTs, Class S theories, ...

‘Lagrangian’ for the Argyres-Douglas theory

Argyres-Douglas theory

- Originally discovered by looking at a special point in the Coulomb branch of $N=2$ $SU(3)$ SYM [\[Argyres-Douglas\]](#) or $N=2$ $SU(2)$ SQCD [\[Argyres-Plesser-Seiberg-Witten\]](#).
- At this special point, mutually non-local massless particles appear. Upon appropriate scaling limit, one obtains interacting SCFT.
- It is a strongly-coupled theory with no tunable coupling.

Argyres-Douglas theory H_0

- Seiberg-Witten geometry is simple (A_2 singularity). The **BPS particle spectrum** in the Coulomb branch determined. [\[Shapere-Vafa\]](#)
- There is a chiral operator of **dimension $6/5$** .
- **Central charges** have been computed [\[Shapere-Tachikawa\]](#).

$$a = \frac{43}{120} , \quad c = \frac{11}{30}$$

- The **c** above is the **minimal** value of any interacting $N=2$ SCFT! [\[Liendo-Ramirez-Seo\]](#)

Generalizations

- One can obtain AD theories from higher-rank SYM or SQCD by going to a special point in the Coulomb branch.
- H_0 , H_1 , H_2 theories (subset of 4d $N=2$ SCFTs living on a single D3-brane probing 7-brane singularities.)
 - one-dimensional Coulomb branch
 - Chiral operator of dimension $6/5$, $4/3$, $3/2$ respectively.
- H_1 , H_2 has the Higgs branch given by 1-instanton moduli space of $SU(2)$ and $SU(3)$ respectively.

Even more generalizations

- $(A_k, A_n), (A_k, D_n), (A_k, E_n), \dots$: This nomenclature is from the shape of the **BPS quiver** [Cecotti-Neitzke-Vafa]
- $H_0=(A_1, A_2), H_1=(A_1, A_3)=(A_1, D_3), H_2=(A_1, D_4)$
- General Argyres-Douglas theories from **6d (2, 0) theory** wrapped on a sphere with one **irregular puncture** (and a regular puncture). [Xie][Xie-Zhao][Wang-Xie]

AD theory at conformal point

- Not much has been known about the **conformal point** of the AD theory.
- Dimension of the chiral operators = Coulomb branch operators
- Central charges [Shapere-Tachikawa '08]
- Supposedly the minimal model among the $N=2$ SCFTs, good target for a **conformal bootstrap**.

[Beem-Lemos-Liendo-Rastelli-van Rees] [Lemos-Liendo]

AD theory at conformal point

- **The chiral algebra** associated to the AD theory is given by **non-unitary minimal model**. The vacuum character gives the Schur index.
[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]
- **Schur index** has been computed using the relation between massive **BPS spectrum** on the Coulomb branch and the index.
[Cordova-Shao][Cecotti-JS-Vafa-Yan]
[Cordova-Gaiotto-Shao]
- **Macdonald index** was computed inspired by the M5-brane realization.
[Buican-Nishinaka][JS]
 - It is given by the **'refined' vacuum character** of the chiral algebra
[JS, to appear]

N=1 gauge theory flowing to $H_0=(A_1,A_2)$

Matter content

	q	q'	ϕ	M
$SU(2)$	2	2	adj	1

Superpotential

$$W = \phi q q + M \phi q' q'$$

This theory has a anomaly free $U(1)$ global symmetry that can be mixed with R-symmetry. (R-charges are not fixed)

a-maximization

[Intriligator-Wecht]

- UV R-symmetry may not be the same as the IR R-symmetry when there is non-baryonic $U(1)$ symmetry.

$$R_{IR} = R_{UV} + \sum_i \epsilon_i F_i$$

- The exact R-charge at the fixed point is determined by maximizing the trial a-function, which is given in terms of the $U(1)_R$ 't Hooft anomalies: [Anselmi-Freedman-Grisaru-Johansen]

$$a(\epsilon) = \frac{3}{32} (3 \text{Tr} R_\epsilon^3 - \text{Tr} R_\epsilon)$$

a-maximization and decoupling

- Upon determining exact R-charge, we can determine the operator dimension via $\Delta = 3/2 R$.
- There is a caveat here. We have to check whether all the gauge invariant operators satisfy the **unitarity bound**: $\Delta > 1$.
- If any operators hits the unitarity bound along the flow, it becomes free and get decoupled. We have to remove them and re-a-maximize.

[Kutasov]

RG Flow to the H_0 theory

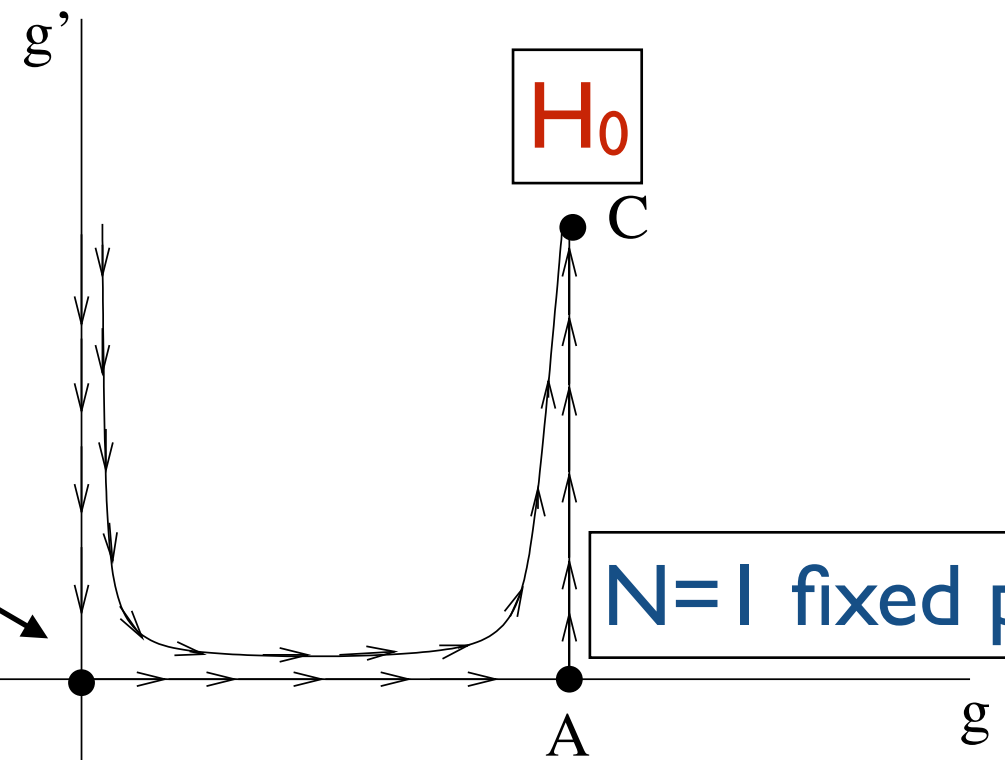
[Nardoni-Maruyoshi-JS, work in progress]

Adjoint SQCD $SU(2)$, $N_f=1$

$\text{Tr}\phi^2$ decouples

$W=0$ fixed point

$N=1$ fixed point



@C, we get: $a = \frac{43}{120}$, $c = \frac{11}{30}$ $\Delta(M) = \frac{6}{5}$

Coulomb branch emerges in the IR!

$N=1$ gauge theory flowing to $H_1=(A_1,A_3)=(A_1,D_3)$

- Matter contents:

	q	q'	ϕ	M
$SU(2)$	2	2	adj	1

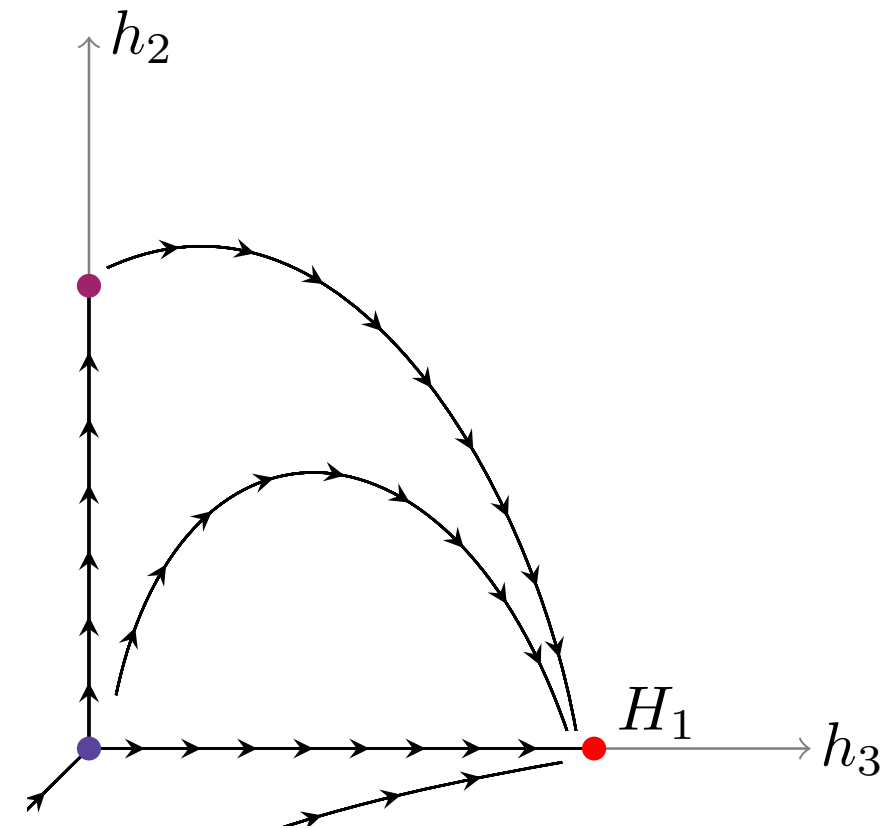
- Interaction:

$$W = Mqq'$$

This theory has a $U(1)\times U(1)$ global symmetry.
One of the $U(1)$ can be mixed with R-symmetry.

Flow to H_1 theory

- $\text{Tr}\phi^2$ operator decouples along the flow to $W=0$ fixed point.
- $M_{qq'}$ is relevant ($R < 2$) at $W=0$ fixed point.
- We obtain: $a = \frac{11}{24}$, $c = \frac{1}{2}$, $\Delta(M) = \frac{4}{3}$
- From the superconformal index, we find the **baryonic symmetry** $U(1)$ gets **enhanced** to $SU(2)$.



$N=1$ gauge theory flowing to $H_2=(A_1, D_4)$

- Matter contents:

	q_1, q_1'	q_2, q_2'	ϕ	M
$SU(2)$	2	2	adj	1

- Interaction:

$$W = \phi q_1 q_1' + M q_2 q_2'$$

This theory has a $U(1)^3$ global symmetry.
One of the $U(1)$ can be mixed with R-symmetry.

Flow to the H_2 theory

- $\text{Tr}\phi^2$ operator decouples along the flow to $W=0$ fixed point.
- Two terms in the superpotential are relevant at $W=0$ point.
- We obtain $a = \frac{7}{12}$, $c = \frac{2}{3}$, $\Delta(M) = \frac{3}{2}$
- From the superconformal index, we find the **baryonic symmetry** $U(1)^2$ gets **enhanced** to $SU(3)$.
- One should be able to compute the elliptic genus of the $SU(3)$ instanton strings by putting this theory on S^2 .

[cf. Talks by Seok Kim and Guglielmo Lockhart]

$N=1$ Deformations of $N=2$ SCFTs

N=1 Deformations of N=2 SCFT with global symmetry

- For an N=2 SCFT with **global symmetry** F , it has a **moment map** operator μ (dim=2) transforms as the adjoint of F .
- Add a chiral multiplet M transforming as the adjoint of F .
- Add the following superpotential:

$$W = \text{Tr}(M\mu)$$

- $SU(2) \times U(1)$ R-symmetry broken to $U(1)_R \times U(1)_F$

N=1 Deformation

- Nilpotent Higgsing

- Now, we give a *nilpotent vev* to M. [Gadde-Maruyoshi-Tachikawa-Yan]
[Agarwal-Bah-Maruyoshi-JS]
[Agarwal-Intriligator-JS]
- This triggers an renormalization group flow.
- Any nilpotent matrix is classified by SU(2) embedding ρ :
SU(2) $\rightarrow$ F. [Jacobson-Morozov]
$$\langle M \rangle = \rho(\sigma^+)$$
- Commutant becomes the global symmetry of the IR SCFT.
- It *preserves the U(1) symmetry* that can be mixed with R-symmetry.

Some group theory

- Under the embedding ρ , the adjoint representation of G decomposes into

$$\text{adj} \rightarrow \bigoplus_j V_j \otimes R_j$$

where V_j is the spin- j irrep. of $SU(2)$ and R_j is the rep. of the commutant.

- The **principal embedding** leaves no commutant. Under this, the adjoint decomposes into

$$\text{adj} \rightarrow \bigoplus_{i=1}^r V_{d_i-1}$$

where d_i are the degrees of Casimirs of F .

- For classical groups $SU(N)$, $SO(N)$, $Sp(N)$, the $SU(2)$ embeddings are classified by the (subset of) **partitions of N** .

Nilpotent Higgsing - general

- Upon Higgsing, the superpotential can be written as

$$W = \mu_{1,-1,1} + \sum_{j,j_3,f} M_{j,-j_3,f} \mu_{j,j_3,f}$$

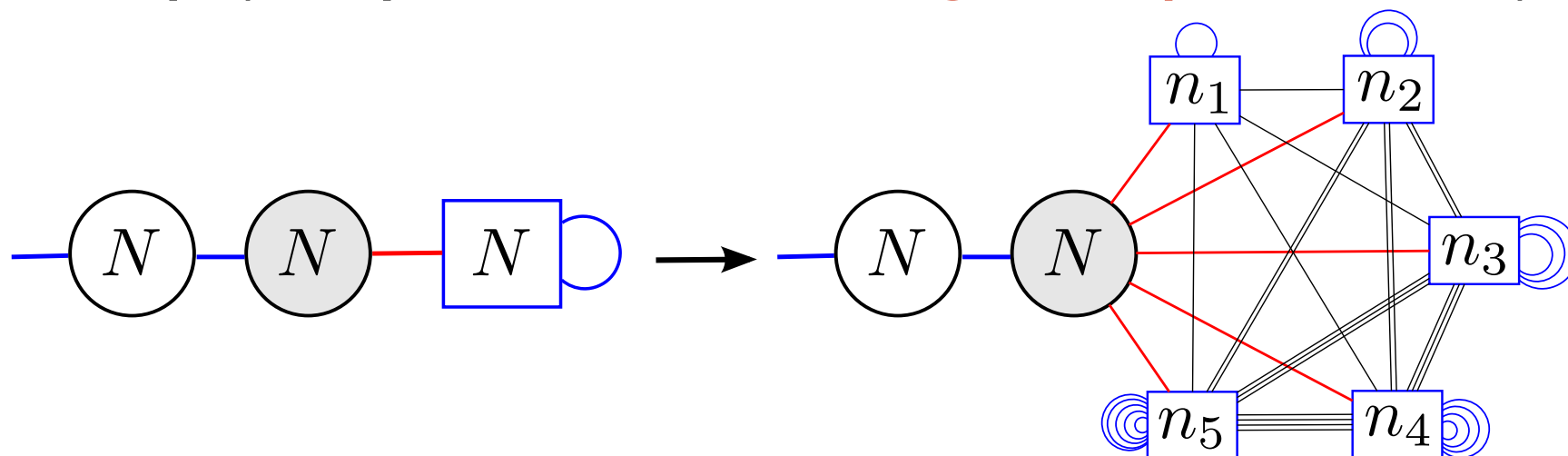
- $U(1)_F$ symmetry is preserved, upon appropriate shift.
- After Higgsing, the conserved current for F is no longer conserved:

$$(D^2 J_F)_{j,j_3,f} = \delta W = \mu_{j,j_3-1,f}$$

- The operator appear on the right-hand side is non-BPS.
All the *M components that couple to the ones on the RHS of the above equation decouples*. Therefore we're left with $M_{j,-j,f}$

Nilpotent Higgsing - Lagrangian theory

- For a Lagrangian theory, giving a vev to M is to add a **nilpotent mass** to the quarks.
- Quarks can be **integrated out**.
- Some components of M remain coupled.
- Write **all** possible superpotential terms consistent with the symmetry. (many of them are **dangerously irrelevant**)



SU(N) with $N_f=2N$, Principal

fields	$SU(N)$	$U(1)_B$	(J_+, J_-)
q	$\square$	1	$(1, -2N + 1)$
$\tilde{q}$	$\bar{\square}$	-1	$(1, -2N + 1)$
ϕ	adj	0	$(0, 2)$
$M_j, (j = 1, 2, \dots, 2N - 1)$	1	0	$(0, 2j + 2)$

$$W = \sum_{j=1}^{2N-1} M_j (\phi^{2N-1-j} q \tilde{q})$$

- Upon **Higgsing** $F=SU(2N)$ with the **principal nilpotent vev**, we obtain the above theory. It has $U(1)_B$ symmetry.
- Along the RG flow, $\text{Tr} \phi^k$ ($k=2, \dots, N$), M_j ($j=1, \dots, N$) decouples.
- Removing all the decoupled operators, we obtain:

$$a = \frac{12N^2 - 5N - 5}{24(N + 1)}, \quad c = \frac{3N^2 - N - 1}{6(N + 1)} \quad \Delta(M_j) = \frac{j + 1}{N + 1} \quad (A_1, A_{2N-1}) \text{ theory!}$$

SU(N) N_f=2N, Non-Principal [2N-1, 1]

fields	$SU(N)$	$U(1)_1$	$U(1)_2$	J_+	J_-
q_1	$\mathbf{N}$	1	$2N - 1$	1	0
$\tilde{q}_1$	$\bar{\mathbf{N}}$	-1	$-(2N - 1)$	1	0
q_2	$\mathbf{N}$	1	-1	1	$-2(N - 1)$
$\tilde{q}_2$	$\bar{\mathbf{N}}$	-1	1	1	$-2(N - 1)$
ϕ	adj	0	0	0	2
M_j , ($j = 0, \dots, 2N - 2$)	$\mathbf{1}$	0	0	0	$2j + 2$
$\widetilde{M}_{N-1}$	$\mathbf{1}$	0	$2N$	0	$2N$
$\dot{\widetilde{M}}_{N-1}$	$\mathbf{1}$	0	$-2N$	0	$2N$

$$\begin{aligned}
 W = & \text{Tr} \tilde{q}_1 \phi q_1 + \text{Tr} \dot{\widetilde{M}}_{N-1} q_1 \tilde{q}_2 + \text{Tr} \widetilde{M}_{N-1} \tilde{q}_1 q_2 + \text{Tr} q_1 \tilde{q}_1 M_0 + \text{Tr} q_2 \tilde{q}_2 \phi^{2N-1} + \\
 & \sum_{j=0}^{2N-2} \sum_{l=0}^{2N-2-j} \text{Tr} q_2 (M_0)^l M_j \tilde{q}_2 \phi^{2N-2-j-l} .
 \end{aligned}$$

SU(N) $N_f=2N, [2N-1, 1]$ after decoupling

fields	$SU(N)$	$U(1)_1$	$U(1)_2$	$R_{\mathcal{N}=1} = \frac{3N-1}{3N}J_+ + \frac{1}{3N}J_-$	$\mathcal{F} = \frac{1}{2}J_+ - \frac{1}{2}J_-$
q_1	$\mathbf{N}$	1	$2N - 1$	$\frac{3N-1}{3N}$	$\frac{1}{2}$
$\tilde{q}_1$	$\bar{\mathbf{N}}$	-1	$-(2N - 1)$	$\frac{3N-1}{3N}$	$\frac{1}{2}$
q_2	$\mathbf{N}$	1	-1	$\frac{N+1}{3N}$	$\frac{2N-1}{2}$
$\tilde{q}_2$	$\bar{\mathbf{N}}$	-1	1	$\frac{N+1}{3N}$	$\frac{2N-1}{2}$
ϕ	adj	0	0	$\frac{2}{3N}$	-1
$M_j, (N \leq j \leq 2N - 2)$	$\mathbf{1}$	0	0	$\frac{2j+2}{3N}$	$-(j + 1)$

$$W = \phi q_1 \tilde{q}_1 + M_j \phi^{2N-2-j} q_2 \tilde{q}_2$$

- We obtain the **(A₁, D_{2N}) theory**.

$$a = \frac{1}{12}(6N - 5), \quad c = \frac{1}{6}(3N - 2), \quad \Delta(M_j) = \frac{j + 1}{N}$$

- The **flavor symmetry enhanced** to SU(3) for N=2 and SU(2)xU(1) for (N>2).

$Sp(N)$ with $1/2 N_f=4N+4$: Principal

fields	$Sp(N)$	(J_+, J_-)	$(R_0, \mathcal{F})$
q	$\square$	$(1, 0)$	$(\frac{1}{2}, \frac{1}{2})$
q'	$\square$	$(1, -4N - 2)$	$(\frac{1}{2}(-4N - 1), \frac{1}{2}(4N + 3))$
ϕ	adj	$(0, 2)$	$(1, 1)$
$M_j, (j = 1, 3, \dots, 4N + 1)$	1	$(0, 2j + 2)$	$(j + 1, -j - 1)$
M'_{2N+1}	1	$(0, 4N + 4)$	$(2N + 2, -2N - 2)$

$$W = \phi q q + \sum_{i=1}^{2N+1} M_{2i-1} (\phi^{4N+3-2i} q' q') + M'_{2N+1} q q'$$

- Upon principal Higgsing $SO(4N+4)$ and integrating out massive quarks, we get the theory above.
- $\text{Tr} \phi^k$ ($k=2, 4, \dots, 2N$), M_j ($j=1, \dots, 2N+1$), M'_{2N+1} **decouples**.
- Removing all the decoupled operators, we obtain:

$$a = \frac{N(24N + 19)}{24(2N + 3)}, \quad c = \frac{N(6N + 5)}{6(2N + 3)}, \quad \Delta(M_j) = \frac{j + 1}{2N + 3} \quad (\mathbf{A_1, A_{2N}}) \text{ theory!}$$

Sp(N) with $1/2 N_f=4N+4$ Non-principal $[4N+1, 1, 1, 1]$

fields	$Sp(N)$	$SO(3)$	J_+	J_-
q_1	2N	3	1	0
q_2	2N	1	1	$-4N$
ϕ	adj	1	0	2
M_j , ($j = 2k + 1$, $0 \leq k \leq 2N - 1$)	1	1	0	$2j + 2$
M_0	1	3	0	2
M_{2N}	1	3	0	$4N + 2$

- $SO(3)=SU(2)$ global symmetry preserved.
- $\text{Tr}\phi^k$ ($k=2, 4, \dots, 2N$), M_j ($j < 2N+1$), decouples.
- We obtain: $a = \frac{N(8N+3)}{16N+8}$, $c = \frac{N}{2}$, $\Delta(M_j) = \frac{j+1}{2N+1}$

(A₁, D_{2N+1}) theory!

Deformations of $SU(N)$ $N_f=2N$

$SU(2N)$	$\rho : SU(2) \hookrightarrow SU(2N)$	a	c	4d $\mathcal{N} = 2$ SUSY
$SU(4)$	$[1^4]$	$\frac{23}{24}$	$\frac{7}{6}$	Yes; $N_c = 2, N_f = 4$
	$[3, 1]$	$\frac{7}{12}$	$\frac{2}{3}$	Yes; (A_1, D_4) AD th.
	$[4]$	$\frac{11}{24}$	$\frac{1}{2}$	Yes; (A_1, A_3) AD th.
$SU(6)$	$[1^6]$	$\frac{29}{12}$	$\frac{17}{6}$	Yes; $N_c = 3, N_f = 6$
	$[5, 1]$	$\frac{13}{12}$	$\frac{7}{6}$	Yes; (A_1, D_6) AD th.
	$[6]$	$\frac{11}{12}$	$\frac{23}{24}$	Yes; (A_1, A_5) AD th.
$SU(8)$	$[1^8]$	$\frac{107}{24}$	$\frac{31}{6}$	Yes; $N_c = 4, N_f = 8$
	$[2, 1^6]$	$\frac{73801}{17424}$	$\frac{43121}{8712}$	?
	$[4, 4]$	$\frac{9097}{3888}$	$\frac{5129}{1944}$	?
	$[7, 1]$	$\frac{19}{12}$	$\frac{5}{3}$	Yes; (A_1, D_8) AD th.
	$[8]$	$\frac{167}{120}$	$\frac{43}{30}$	Yes; (A_1, A_7) AD th.
$SU(10)$	$[1^{10}]$	$\frac{247}{24}$	$\frac{71}{6}$	Yes; $N_c = 5, N_f = 10$
	$[5, 1^5]$	$\frac{5553943}{1383123}$	$\frac{6257387}{1383123}$	?
	$[5, 3, 1^2]$	$\frac{92540867}{24401712}$	$\frac{52091009}{12200856}$	?
	$[9, 1]$	$\frac{25}{12}$	$\frac{13}{6}$	Yes; (A_1, D_{10}) AD th.
	$[10]$	$\frac{15}{8}$	$\frac{23}{12}$	Yes; (A_1, A_9) AD th.
$SU(12)$	$[1^{12}]$	$\frac{247}{24}$	$\frac{71}{6}$	Yes; $N_c = 6, N_f = 12$
	$[4^3]$	$\frac{754501}{138384}$	$\frac{424727}{69192}$	?
	$[11, 1]$	$\frac{31}{12}$	$\frac{8}{3}$	Yes; (A_1, D_{12}) AD th.
	$[12]$	$\frac{397}{168}$	$\frac{101}{42}$	Yes; (A_1, A_{11}) AD th.

Here we list some of the deformations that gives **rational** central charges.

Other deformations give **irrational** central charges, therefore they flow to $N=1$ theories.

Deformations of $Sp(N)$, $1/2 N_f=4N+4$

$SO(4N+4)$	$\rho : SU(2) \hookrightarrow SU(4N+4)$	a	c	4d $\mathcal{N} = 2$ SUSY
$SO(8)$	$[1^8]$	$\frac{23}{24}$	$\frac{7}{6}$	Yes; $N_c = 1, N_f = 8$
	$[3^2, 1^2]$	$\frac{7}{12}$	$\frac{2}{3}$	?
	$[4, 4] \equiv [5, 1^3]$	$\frac{11}{24}$	$\frac{1}{2}$	Yes; (A_1, D_3) AD th.
	$[5, 3]$	$\frac{6349}{13872}$	$\frac{3523}{6936}$	?
	$[7, 1]$	$\frac{43}{120}$	$\frac{11}{30}$	Yes; (A_1, A_2) AD th.
$SO(12)$	$[1^{12}]$	$\frac{37}{12}$	$\frac{11}{3}$	Yes; $N_c = 2, N_f = 12$
	$[4^2, 2^2]$	$\frac{105027}{59536}$	$\frac{61145}{29768}$	?
	$[9, 1^3]$	$\frac{19}{20}$	1	Yes; (A_1, D_5) AD th.
	$[11, 1]$	$\frac{67}{84}$	$\frac{17}{21}$	Yes; (A_1, A_4) AD th.
$SO(16)$	$[1^{16}]$	$\frac{51}{8}$	$\frac{15}{2}$	Yes; $N_c = 3, N_f = 16$
	$[5, 1^{11}]$	$\frac{109031}{27744}$	$\frac{123889}{27744}$	?
	$[5, 3^3, 1^2]$	$\frac{18250741}{5195568}$	$\frac{10440877}{2597784}$	?
	$[13, 1^3]$	$\frac{81}{56}$	$\frac{3}{2}$	Yes; (A_1, D_7) AD th.
	$[15, 1]$	$\frac{91}{72}$	$\frac{23}{18}$	Yes; (A_1, A_6) AD th.
$SO(20)$	$[1^{20}]$	$\frac{65}{6}$	$\frac{38}{3}$	Yes; $N_c = 4, N_f = 20$
	$[2^2, 1^{16}]$	$\frac{4181}{400}$	$\frac{2463}{200}$	?
	$[3^4, 2^4]$	$\frac{29}{4}$	$\frac{133}{16}$	?
	$[4^4, 2^2]$	$\frac{28361329}{4702512}$	$\frac{16338643}{2351256}$	?
	$[9, 5, 3, 1^3]$	$\frac{737}{192}$	$\frac{817}{192}$	?
	$[11, 1^9]$	$\frac{6638927}{1976856}$	$\frac{3700169}{988428}$	?
	$[11, 2^2, 1^5]$	$\frac{106413731}{31795224}$	$\frac{59339969}{15897612}$	?
	$[11, 2^4, 1] \equiv [11, 3, 1^6]$	$\frac{26650955}{7990296}$	$\frac{14869241}{3995148}$	?
	$[11, 3, 2^2, 1^2]$	$\frac{106793099}{32127576}$	$\frac{59613689}{16063788}$	?

Principal deformations

$\mathcal{T}$	F	$\mathcal{N} = 2$	Sugawara	k_F bound	$\mathcal{T}_{IR}[\mathcal{T}, \rho]$
$(A_1, D_k), (k \geq 4)$	$SU(2)$	yes	yes	no	(A_1, A_{k-1})
$(I_{N, Nm+1}, F)$	$SU(N)$	yes	yes	no	(A_{N-1}, A_{Nm+N})
H_1	$SU(2)$	yes	yes	yes	H_0
H_2	$SU(3)$	yes	yes	yes	H_0
D_4	$SO(8)$	yes	yes	yes	H_0
E_6	E_6	yes	yes	yes	H_0
E_7	E_7	yes	yes	yes	H_0
E_8	E_8	yes	yes	yes	H_0
$SU(N)$ SQCD	$SU(2N)$	yes	yes	yes	(A_1, A_{2N-1})
$Sp(N)$ SQCD	$SO(4N + 4)$	yes	yes	yes	(A_1, A_{2N})
$\mathcal{N} = 4$ $SU(2)$	$SU(2)$	no	yes	no	new
$[IV^*, Sp(2) \times U(1)]$	$Sp(2)$	no(?)	no	yes	new
$[III^*, SU(2) \times U(1)]$	$SU(2)$	no	no	no	new
$[III^*, Sp(3) \times SU(2)]$	$Sp(3) \times SU(2)$	no	no	yes	new
$[II^*, SU(3)]$	$SU(3)$	no(?)	no	no	new
$[II^*, SU(4)]$	$SU(4)$	no	no	no	new
$[II^*, Sp(5)]$	$Sp(5)$	no	no	yes	new
T_N	$SU(N)^3$	no	no	yes	new
$R_{0,N}$	$SU(2N)$	no	no	yes	new

When does SUSY gets enhanced to N=2?

- Any 4d N=2 SCFT can be associated with a **2d chiral algebra** with [\[Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees\]](#)

$$c_{2d} = -12c_{4d}, \quad k_{2d} = -\frac{1}{2}k_{4d}.$$

- If the chiral algebra is given by the affine Kac-Moody algebra of level k_{2d} , one can use **Sugawara construction** to realize the Virasoro algebra. Then the central charge is given by

$$c_{\text{Sugawara}} = \frac{k_{2d} \dim F}{k_{2d} + h^\vee}$$

- It seems **all** theories exhibiting **SUSY enhancement** under the principal deformation satisfies $c_{2d} = c_{\text{Sugawara}}$. Why? (except for N=4 SU(2) SYM, which has extended chiral algebra)

Summary

Summary

- To a given $N=2$ SCFT $\mathcal{T}$ with **non-abelian global symmetry** F , one can obtain $N=1$ SCFT $\mathcal{T}_{\text{IR}}[\mathcal{T}, \rho]$ labelled by $SU(2)$ embedding ρ of F .
- When $\mathcal{T} = \text{SQCD}$, and ρ is (nearly) principal, $\mathcal{T}_{\text{IR}}[\mathcal{T}, \rho]$ is Argyres-Douglas theory of type (A_1, A_n) and (A_1, D_n) .
 $N=1$ Lagrangian theory flowing to the **$N=2$ AD theory**.
- Experimentally, $N=2$ enhancement happens whenever **2d chiral algebra** for the $\mathcal{T}$ has the stress tensor given by the **Sugawara construction** and ρ is the **principal** embedding.

Future directions

- When and why $N=2$ enhancement happens?
 - Is ‘Sugawara condition’ necessary or sufficient? Why?
 - string/M-theory realizations?
- Can we find $N=1$ gauge theory flowing to more general AD theory?
- More supersymmetric partition functions
- Surface defects, line operators...

Thank you!