6d strings and exceptional instantons

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Talk based on:

Hee-Cheol Kim, SK, Jaemo Park,
"6d strings from new chiral gauge theories"
1608.03919.

Hee-Cheol Kim, Joonho Kim, SK, Jaemo Park,
work in progress.

See also the following recent papers, which partly overlap with ours:

Shimizu, Tachikawa,
"Anomaly of strings of 6d N=(1,0) theories"
1608.05894.

Del Zotto, Lockhart,
"On exceptional instanton strings"
1609.00310.

4d N=2 gauge theories & instantons

- Simplest: SU(2) without any matters [Seiberg, Witten] (1994)
- Seiberg-Witten effective action in the Coulomb branch:

$$\frac{1}{4\pi}\mathrm{Im}\left[\int d^4\theta \frac{\partial \mathcal{F}(A)}{\partial A}\overline{A} + \int d^2\theta \frac{1}{2}\frac{\partial^2 \mathcal{F}(A)}{\partial A^2} W_\alpha W^\alpha\right]$$

- Acquires non-perturbative contributions from Yang-Mills instantons

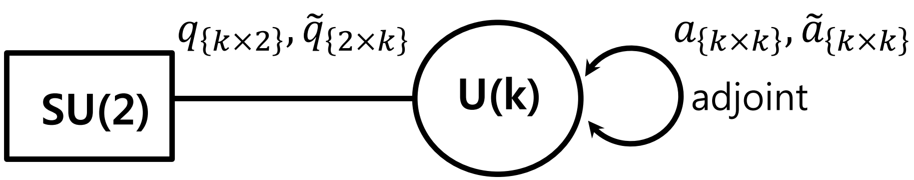
$$\mathcal{F}(v,q) = \mathcal{F}_{\rm pert}(v) + \sum_{k=1}^{\infty} \mathcal{F}_k(v) q^k$$

- Computed microscopically by Nekrasov's instanton calculus. [Nekrasov] (2002)

$$Z(v,q,\epsilon_1,\epsilon_2) = \sum_k Z_k(v,\epsilon_1,\epsilon_2) q^k \to \exp\left[-\frac{\mathcal{F}(v,q)}{\epsilon_1\epsilon_2} + \cdots\right] \quad \text{as} \quad \epsilon_1,\epsilon_2 \to 0$$

- The coefficients $Z_k(v)$ are computed from the following ADHM matrix model

[Atiyah, Drinfeld, Hitchin, Manin] [Christ, Weinberg, Stanton] (1978)



5d N=1

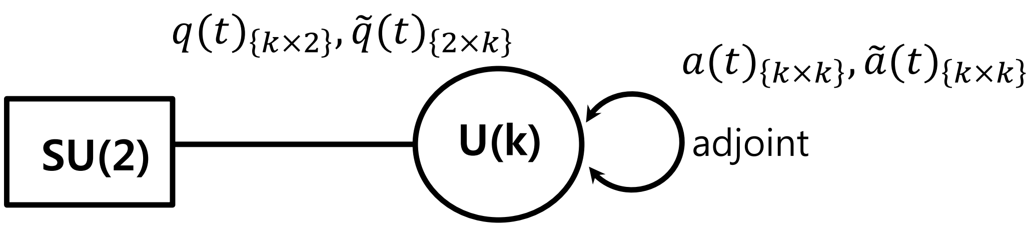
- R^4 x S^1: same problem of 4d effective action in Coulomb branch, acquires contribution from KK towers.

- This is actually where Nekrasov started his studies (then take S^1 to be small)

- Z_k is a Witten index of k instanton "particles" (& W-bosons in Coulomb branch).

$$F_{\mu\nu} = \star_4 F_{\mu\nu} \qquad k \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\,(F \wedge F) \in \mathbb{Z}$$

- Computed from the ADHM quantum mechanics



6d N=(1,0) … ?

- We try to ask the same question in 6d, on $R^4 \times T^2$: instanton strings
- To cancel 1-loop gauge anomaly, one couples 6d SYM to a tensor multiplet.

$$B_{\mu\nu} \text{ with } H = dB = \star dB \ , \quad \Psi^A \ , \quad \Phi$$

- Classical & 1-loop anomalies may cancel via the Green-Schwarz mechanism.

$$S^{\rm bos}_{\rm v+t} = \int \left[\frac{1}{2}d\Phi \wedge \star d\Phi + \frac{1}{2}H \wedge \star H\right] + \sqrt{c}\int \left[-\Phi {\rm tr}(F \wedge \star F) + B \wedge {\rm tr}(F \wedge F)\right]$$

$$H \equiv dB + \sqrt{c}\ {\rm tr}\left(AdA - \frac{2i}{3}A^3\right)$$

- Without matters, this mechanism works for SU(2), SU(3), G_2, SO(8), F_4, E_6, E_7, E_8

- The system further has to be free of the global anomaly. [Bershadsky, Vafa] (1997)

$$\pi_6(SU(2)) = \mathbf{Z_{12}} \qquad SU(2) \ : \ n_2 = 4, 10, \cdots \quad \in 4 + 6\mathbb{Z}_{\geq 0}$$

$$\pi_6(SU(3)) = \mathbf{Z_6} \qquad SU(3) \ : \ n_3 = 0, 6, 12, \cdots \quad \in 6\mathbb{Z}_{\geq 0}$$

$$\pi_6(G_2) = \mathbf{Z_3} \qquad G_2 \ : \ n_7 = 1, 4, 7, \cdots \quad \in 1 + 3\mathbb{Z}_{\geq 0}$$

- SU(2) SYM without matters is inconsistent. (Those with matters: Higgsable)
- In a sense, the simplest 6d SYM comes with SU(3) gauge group.

The 6d SCFTs and Yang-Mills

- SU(3) pure SYM theory is one of the important building blocks of the 6d SCFTs.
- "atoms" of 6d SCFTs [Morrison,Vafa] [Witten] (1996) [Morrison,Taylor] (2012)

  [Heckman, Morrison, Vafa] (2013) [Heckman, Morrison, Rudelius, Vafa] (2015)

n	1	2	3	4	5	6	7	8	12
gauge symmetry	-	-	$SU(3)$	$SO(8)$	F_4	E_6	E_7	E_7	E_8
global symmetry	E_8	-	-	-	-	-	-	-	-
matters	-	-	-	-	-	-	$\frac{1}{2}\mathbf{56}$	-	-

base	$3,2$	$3,2,2$	$2,3,2$
gauge symmetry	$G_2\times SU(2)$	$G_2\times Sp(1)\times\{0\}$	$SU(2)\times SO(7)\times SU(2)$
matters	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1})+\frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2})$

- make quivers: glue two CFTs using n=1 SCFT, gauging subgroups of E_8.
- "unHiggs" to bigger gauge groups w/ more hypermultiplet matters

- SYM provides a description of the CFT in the tensor branch

$$S^{\rm bos}_{\rm v+t} = \int \left[\frac{1}{2}d\Phi\wedge\star d\Phi + \frac{1}{2}H\wedge\star H\right] + \sqrt{c}\int\left[-\Phi\,{\rm tr}(F\wedge\star F) + B\wedge{\rm tr}(F\wedge F)\right]$$

VEV

- Instanton strings are the so-called self-dual strings $S \leftarrow \int B\wedge{\rm tr}(F\wedge F)$

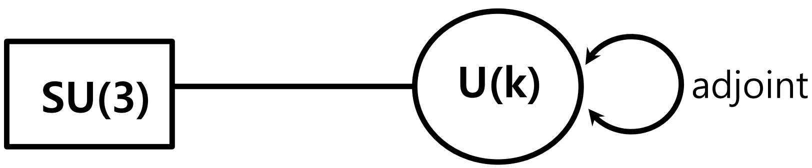
Self-dual strings from instanton strings

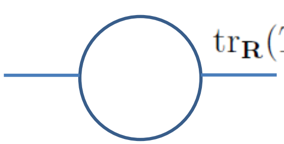
- For various reasons, it has been important to study the self-dual strings' worldsheet QFTs, which are N=(0,4) SCFTs.
- Their partition functions (elliptic genus) are 6d Nekrasov partition functions.
- They can be studied easily if they come from UV gauge theories (GLSM)

- For SU(3), we have Yang-Mills intuitions: self-dual strings = instanton strings

$$F_{\mu\nu} = \star_4 F_{\mu\nu} \qquad k \equiv \frac{1}{8\pi^2}\int \mathrm{tr}\,(F\wedge F) \in \mathbb{Z} \qquad S \leftarrow \int B\wedge \mathrm{tr}(F\wedge F)$$

- SU(3) ADHM has U(k) gauge anomaly, so doesn't provide a good UV description.





$\mathrm{tr}_{\mathbf{R}}(T^aT^b) = D_{\mathbf{R}}\delta^{ab}$

$D_{\mathbf{k}} = 1$

$D_{\mathbf{adj}} = 2k$

fields	$U(k)$	$SU(3)$	$SU(2)_F$	$SU(2)_1$	$SU(2)_2$
$\lambda_{\dot{\alpha}A-}$	**adj**	**1**	**1**	**2**	**2**
$q_{\dot{\alpha}}(\to \psi_{A+})$	**k**	**3**	**1**	**2**	**1**
$a_{\alpha\dot{\beta}}(\to \chi_{\alpha A+})$	**adj**	**1**	**2**	**2**	**1**

$\sim\ 2\cdot 3\cdot 1 + 2\cdot 2k - 2\cdot 2k = 6 \neq 0$

The cure for SU(3)

- Result: can't make N=(0,4) gauge theory. Can have one by sacrificing some SUSY in UV.

- Add the following N=(0,2) superfields to the anomalous SU(3) ADHM :

fields	$U(k)$	$SU(3)$	$SU(2)_F$	$SU(2)_1$	$SU(2)_2$
$\lambda_{\dot{\alpha}A-}$	**adj**	**1**	**1**	**2**	**2**
$q_{\dot{\alpha}}(\to \psi_{A+})$	**k**	$\bar{\mathbf{3}}$	**1**	**2**	**1**
$a_{\alpha\dot{\beta}}(\to \chi_{\alpha A+})$	**adj**	**1**	**2**	**2**	**1**

$$\begin{aligned}
(\phi,\chi) &: \text{ chiral multiplet in } (\bar{\mathbf{k}},\bar{\mathbf{3}}) \\
(b,\xi)+(\tilde{b},\tilde{\xi}) &: \text{ two chiral multiplet in } (\overline{\mathbf{anti}},\mathbf{1}) \\
(\hat{\lambda},\hat{G}) &: \text{ complex Fermi multiplet in } (\mathbf{sym},\mathbf{1}) \\
(\check{\lambda},\check{G}) &: \text{ complex Fermi multiplet in } (\mathbf{sym},\mathbf{1}) \\
(\zeta,G_\zeta) &: \text{ complex Fermi multiplet in } (\bar{\mathbf{k}},\mathbf{1}) \, .
\end{aligned}$$

$$\begin{aligned}
(\tilde{\phi},\tilde{\chi}) &: \text{ chiral multiplet in } (\mathbf{k},\mathbf{1}) \\
(\eta,G_\eta) &: \text{ complex Fermi multiplet in } (\bar{\mathbf{k}},\mathbf{1})
\end{aligned}$$

- anomaly: SU(k) from ADHM ~ $2\cdot 3\cdot 1+2\cdot 2k-2\cdot 2k=6\neq 0$

  from others ~ $+3\cdot 1+2(k-2)-(k+2)-(k+2)-1=-6$

  $D_{\text{sym}}=k+2$

  $D_{\text{anti}}=k-2$

  U(1) $+3\cdot 2\cdot 1^2\cdot k+3\cdot 1^2\cdot k+2\cdot 2^2\cdot\frac{k^2-k}{2}-2^2\cdot\frac{k^2+k}{2}-2^2\cdot\frac{k^2+k}{2}-1^2\cdot k=0$

- Can turn on superpotentials to get the correct SU(3) instanton moduli space: but preserving only N=(0,1) SUSY [H.-C.Kim, SK, J. Park]

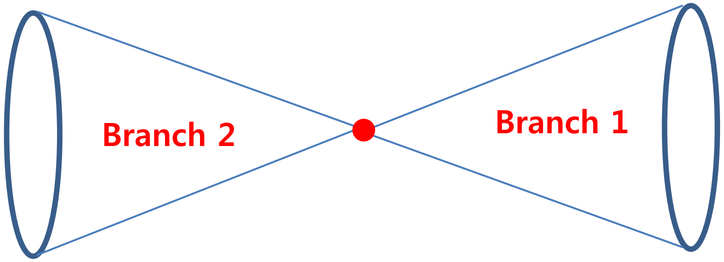
The moduli space & UV completion

- Classical moduli space: vanishing $V(\phi_{\text{ADHM}}, \phi_{\text{others}}) = V_1(\phi_{ADHM}) + V_2(\phi_{\text{others}}, \phi_{\text{ADHM}})$

1) branch 1: extra fields = 0. ADHM fields satisfy

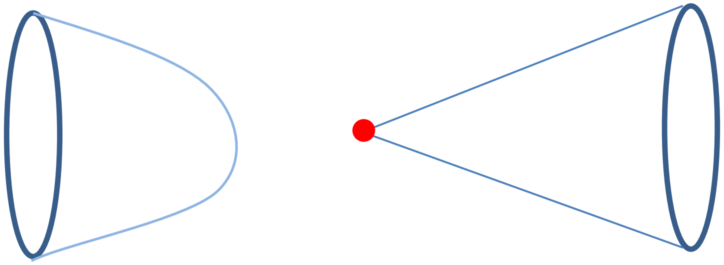
$$D^I \equiv q_{\dot{\alpha}} (\tau^I)^{\dot{\alpha}}{}_{\dot{\beta}} \bar{q}^{\dot{\beta}} + (\tau^I)^{\dot{\alpha}}{}_{\dot{\beta}} [a_{\alpha\dot{\alpha}}, a^{\alpha\dot{\beta}}] = 0$$

- SU(3) instanton moduli space: hyper-Kahler quotient, N=(0,4) SUSY enhancement
- 1-loop correction doesn't spoil this zero potential condition



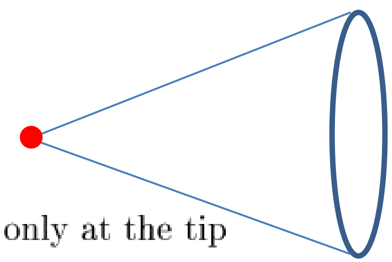
2) branch 2: We find another branch. (k=1)

- Classical: meets 1st branch at small instanton singularity



- Quantum: 1-loop correction only at 2nd branch
  conjecture: detached from the 1st branch (IR decoupling)
  [Melnikov, Quigley, Sethi, Stern] (2012)

- Non-linear sigma model in IR: small instanton singularity.
  Extra light d.o.f. at small instanton singularity. UV completion.



ϕ_{extra} are massless only at the tip

Other observables

- elliptic genus:

$$H_\pm \equiv \frac{H\pm P}{2} \qquad H_- \sim \{Q, \overline{Q}\}$$

$$Z_k(\tau,\epsilon_{1,2},m_a) = \mathrm{Tr}\left[(-1)^F e^{2\pi i\tau H_+} e^{2\pi i\bar{\tau} H_-} e^{2\pi i\epsilon_1(J_1+J_R)} e^{2\pi i\epsilon_2(J_2+J_R)} \cdot \prod_{a\in\text{flavor}} e^{2\pi i m_a F_a}\right]$$

- Easy to compute w/ a UV gauge theory: contour integral

  [Benini,Eager,Hori,Tachikawa] (2013)

- Our U(k) gauge theory: [Flume, Poghossian] [Bruzzo, Fucito, Morales, Tanzini] (2002)

$$Z_k^{SU(3)} = (-1)^{\frac{k^2-k}{2}}\eta^{6k}\sum_{\vec{Y};|\vec{Y}|=k}\prod_{i=1}^{3}\prod_{s\in Y_i}\frac{\theta_1(2u(s))\theta_1(2\epsilon_+-2u(s))\theta_1(\epsilon_++u(s))}{\prod_{j=1}^{3}\theta_1(E_{ij})\theta_1(E_{ij}-2\epsilon_+)\theta_1(\epsilon_+-u(s)-v_j)}$$
$$\times\prod_{i\le j}^{3}\prod_{s_{i,j}\in Y_{i,j};s_i<s_j}\frac{\theta_1(u(s_i)+u(s_j))\theta_1(2\epsilon_+-u(s_i)-u(s_j))}{\theta_1(\epsilon_{1,2}-u(s_i)-u(s_j))}$$

$$E_{ij} = v_i - v_j - \epsilon_1 h_i(s) + \epsilon_2(v_j(s)+1)$$
$$u(s) = v_i - \epsilon_+ - (m-1)\epsilon_1 - (n-1)\epsilon_2$$

- 1d limit, replacing all θ_1 functions to sine functions, agrees with Nekrasov's SU(3) instanton partition function: We found an alternative "ADHM-like" formalism
- Novel results in 2d: For simplicity, let us consider single string k=1

$$Z_1^{SU(3)}(v,\epsilon_{1,2}) = -\frac{\eta^2}{\theta_1(\epsilon_{1,2})}\sum_{i=1}^{3}\frac{\eta^4\theta_1(4\epsilon_+-2v_i)\theta_1(v_i)}{\prod_{j(\neq i)}\theta_1(v_{ij})\theta_1(2\epsilon_+-v_{ij})\theta_1(2\epsilon_++v_j)}$$

Tests

- k=1 (tests also done at k=2,3): computation from topological strings [Haghighat, Klemm, Lockart, Vafa]

$$\log Z(\tau, \epsilon_+, \epsilon_+, \mu) = \sum_{g\geq 0, n\geq 0} (\epsilon_1\epsilon_2)^{g-1}(\epsilon_1+\epsilon_2)^n F_{g,n}(\tau,\mu)$$

$$\begin{aligned} F_{0,0} &= -\left[\frac{\theta_1(2v_1)\theta_1(v_1)}{\theta_1(v_{12})^2\theta_1(v_{13})^2\theta_1(v_2)\theta_1(v_3)} + (1,2,3\to 2,3,1) + (1,2,3\to 3,1,2)\right] \\ &= e^{-\pi i\tau + 2\pi i v_{12} + 2\pi i v_{23}} \sum_{d_0,d_1,d_2=0}^{\infty} N_{d_0,d_1,d_2}\left(\frac{e^{2\pi i\tau}}{e^{2\pi i v_{12}}e^{2\pi i v_{23}}}\right)^{d_0} e^{2\pi d_1 v_{12}} e^{2\pi d_2 v_{23}} \end{aligned}$$

$d_1 \setminus d_2$	0	1	2	3	4	5
0	1	3	5	7	9	11
1	3	4	8	12	16	20
2	5	8	9	15	21	27
3	7	12	15	16	24	32
4	9	16	21	24	25	35
5	11	20	27	32	35	36

Table 1: q^0

$d_1 \setminus d_2$	0	1	2	3	4	5
0	3	4	8	12	16	20
1	4	16	36	60	84	108
2	8	36	56	96	144	192
3	12	60	96	120	180	252
4	16	84	144	180	208	288
5	20	108	192	252	288	320

Table 2: q^1

$d_1 \setminus d_2$	0	1	2	3	4	5
0	5	8	9	15	21	27
1	8	36	56	96	144	192
2	9	56	149	288	465	651
3	15	96	288	456	735	1080
4	21	144	465	735	954	1371
5	27	192	651	1080	1371	**1632**

Table 3: q^2

$d_1 \setminus d_2$	0	1	2	3	4	5
0	7	12	15	16	24	32
1	12	60	96	120	180	252
2	15	96	288	456	735	1080
3	16	120	456	1012	1788	**2796**
4	24	180	735	1788	**2823**	**4356**
5	32	252	1080	**2796**	**4356**	**5760**

Table 4: q^3

black numbers: computed from top. strings

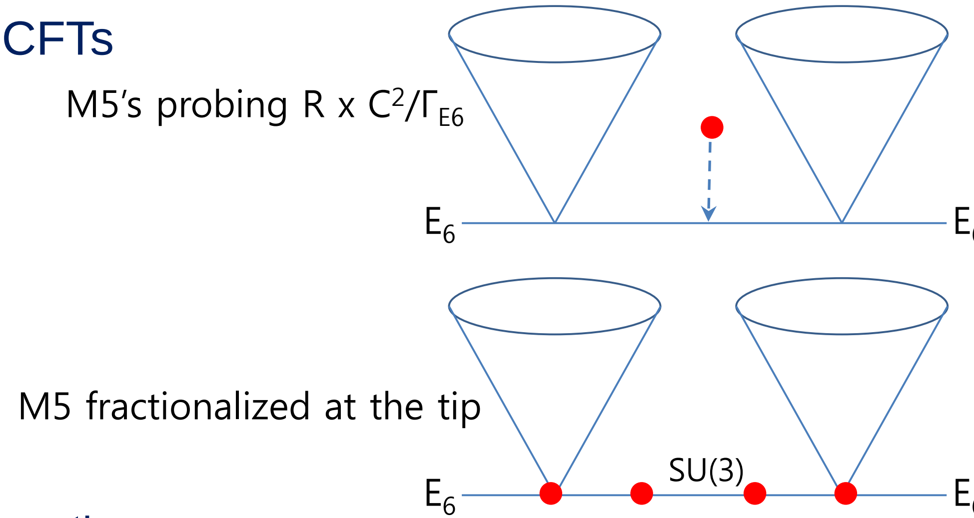
red: our prediction

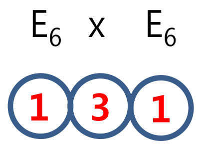
complete agreement

E_6 x E_6 conformal matter

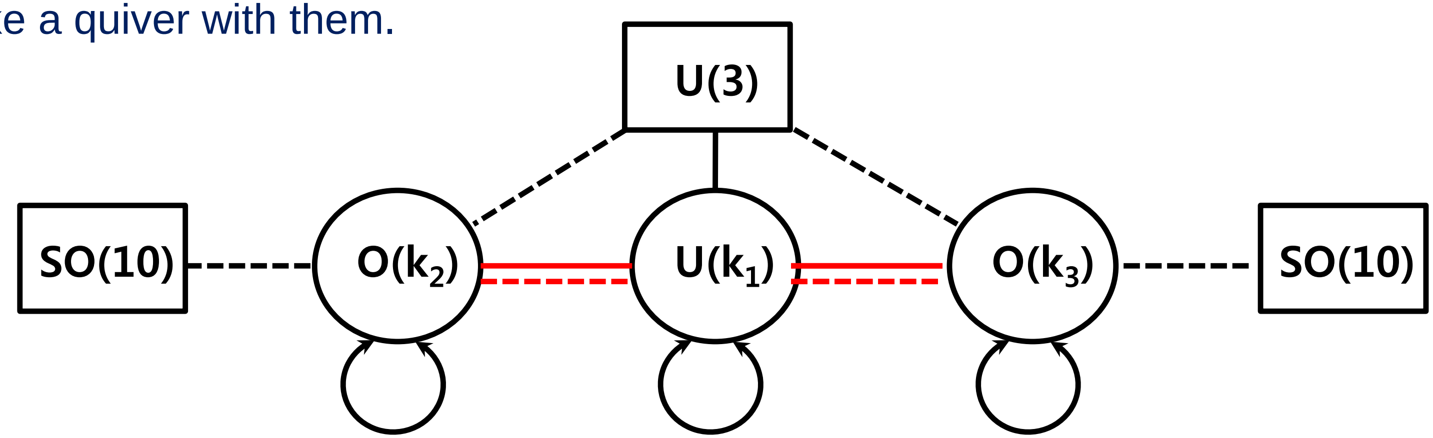
- Strings of more complicated 6d SCFTs
- E_6 x E_6 conformal matter:

  [Del Zotto, Heckman, Tomasiello, Vafa]





- Strings for n=3: our SU(3) strings' gauge theory.
- Strings for n=1 are called E-strings: 2d gauge theory known [J.Kim, SK, K.Lee, J.Park, Vafa]
- Make a quiver with them.



- SO(10) x U(1) x SO(10) enhances to E_6 x E_6: partly checked from elliptic genus

More tests: anomaly inflows from 6d

- 2d anomalies of global symmetries
- Computable from 6d gauge anomaly cancelation w/ 2d defects (anomaly inflow)
- Results from the inflow mechanism [H.-C. Kim, SK, J. Park] (see also [Shimizu, Tachikawa])

$$I_4^{\rm inflow} = I_4^{(1)} + I_4^{(2)} = \Omega^{ij} k_i \left[I_j + \frac{1}{2} k_j \chi(T_4) \right] \qquad I_4^{2d} = -I_4^{\rm inflow}$$

- Agree with the anomalies calculated from our 2d gauge theories

- k instanton strings for G=SU(3): both calculations yield

$$I_4^{2d} = -\frac{3k}{4}{\rm Tr}F_G^2 - 3kc_2(R) - \frac{k}{4}p_1(T_6) - \frac{3k^2}{2}\chi_4(T_4)$$

- E_6 x E_6 conformal matter: both calculations yield

$$\begin{aligned} I_4^{2d} = \quad & k_1(k_2+k_3)\chi_4(T_4) - k_1\left(\frac{3}{4}{\rm Tr}F_{SU(3)}^2 + 3c_2(R) + \frac{p_1(T_6)}{4} + \frac{3}{2}k_1\chi_4(T_4)\right) \\ & + k_2\left(\frac{1}{4}{\rm Tr}F_{E_6^L}^2 + \frac{1}{4}{\rm Tr}F_{SU(3)}^2 - c_2(R) + \frac{p_1(T_6)}{4} - \frac{k_2}{2}\chi_4(T_4)\right) \\ & + k_3\left(\frac{1}{4}{\rm Tr}F_{E_6^R}^2 + \frac{1}{4}{\rm Tr}F_{SU(3)}^2 - c_2(R) + \frac{p_1(T_6)}{4} - \frac{k_2}{2}\chi_4(T_4)\right) \ . \end{aligned}$$

UnHiggsing to exceptional instantons

- 6d Higgsings are reflected in 2d QFT as massive deformations

- Allowed unHiggsing sequences: all exceptional ~ “terminates after finite sequence”

$$(SU(3)) \leftarrow (G_2, n_{\mathbf{7}} = 1) \leftarrow (SO(7), n_{\mathbf{7}} = 0, n_{\mathbf{8}} = 2) \leftarrow (SO(8), n_{\mathbf{8}_v} = n_{\mathbf{8}_s} = n_{\mathbf{8}_c} = 1)$$

$$\leftarrow \begin{cases} (SO(N), n_{\mathbf{N}} = N-7, n_{\mathbf{S}} = \cdots)_{N=9,\cdots,12} \\ (F_4, n_{\mathbf{26}} = 2) \leftarrow (E_6, n_{\mathbf{27}} = 3) \leftarrow (E_7, n_{\frac{1}{2}\mathbf{56}} = 5) \leftarrow (E_8, n_{\text{inst}} = 9) \end{cases}$$

- G_2 & SO(7) instantons w/ 6d matters [Hee-Cheol Kim, Joonho Kim, SK, Jaemo Park] in progress.

- An (anomaly-free) quiver for SO(7) instantons: only SU(4) ⊂ SO(7) is manifest in UV

Standard SU(4) ADHM

$$A_\mu, \lambda_0, \lambda \;:\; \mathcal{N} = (0,4)\ U(k) \text{ vector multiplet}$$
$$q_i, \tilde{q}^i \;:\; (\mathbf{k}, \bar{\mathbf{4}}) + (\bar{\mathbf{k}}, \mathbf{4})$$
$$a, \tilde{a} \;:\; (\mathbf{adj}, \mathbf{1})$$

Extra chiral multiplet to make it a novel “SO(7) ADHM”

$$\phi_i \;:\; (\bar{\mathbf{k}}, \bar{\mathbf{4}})$$
$$b, \tilde{b} \;:\; (\overline{\mathbf{anti}}, \mathbf{1})$$
$$\hat{\lambda} \;:\; (\mathbf{sym}, \mathbf{1})$$
$$\check{\lambda} \;:\; (\mathbf{sym}, \mathbf{1})$$

Extra 2d field induced by 6d hypers in 8

$$\Psi_i \;:\; (\mathbf{k}, \mathbf{1})$$
$$\tilde{\Psi}_i \;:\; (\bar{\mathbf{k}}, \mathbf{1}) \qquad (i = 1, 2)$$

$$\mathbf{8} \to \mathbf{4} + \bar{\mathbf{4}}$$

- Can Higgs SO(7) to G_2 with one 7. Further Higgsing to our alternative SU(3) ADHM.

Exceptional instanton partition functions

- Reduce to 1d. ADHM-like descriptions for exceptional instantons
- Nekrasov partition function in the Coulomb branch: e.g. one G_2 instanton

$$\sum_{i=1}^{3} \frac{2\sinh(2\epsilon_+ - v_i)\cdot 2\sinh\frac{v_i}{2}}{\prod_{j(\neq i)} 2\sinh\frac{v_{ij}}{2}\cdot 2\sinh\frac{2\epsilon_+ - v_{ij}}{2}\cdot 2\sinh\frac{2\epsilon_+ + v_j}{2}}\cdot\frac{1}{2\sinh\frac{v_i}{2}\cdot 2\sinh\frac{2\epsilon_+ - v_i}{2}}$$

One (k=1) G_2 instanton partition function from 1d gauge theory

$$\frac{t^{\frac{3}{2}}(1+t)(1+t\chi_{\mathbf{7}}^{G_2}(v)+t^2)}{\prod_{i<j}(1-te^{-v_{ij}})(1-te^{v_{ij}})} = t^{\frac{3}{2}}\sum_{n=0}^{\infty}\chi_{(0,n)}^{G_2}(v)t^n$$

[Cremonesi, Ferlito, Hanany, Mekareeya] (2014)

- Can add hypermultiplet matters in 7: We can compute for $N_f \leq 3$

- N_f = 1, 2: easy, basically same calculus as above

- N_f = 3: Too many 1d Fermi multiplets. Continuum developed in the 1d Coulomb branch.

(Extensive list of works on pragmatically dividing out "decoupled factors" $Z_{\text{extra}}\, Z_{\text{instanton}}$ from such continua, sometimes with solid arguments, sometimes by accumulation of experiences. [Hayashi, H.-C.Kim, Nishinaka] [Bao, Mitev, Pomoni, Taki, Yagi] [Bergman, Rodriguez-Gomez, Zafrir] [Hwang, J.Kim, SK, Park] …)

- Small puzzle: 5d G_2 SCFTs are predicted to exist till $N_f \leq 4$ [Diaconescu, Entin] (1998)

All by [Hee-Cheol Kim, Joonho Kim, SK, Jaemo Park], work in progress.

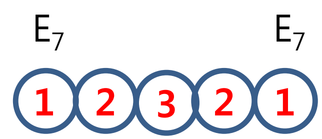
More 6d atoms

- 6d self-dual strings of “exotic atoms”

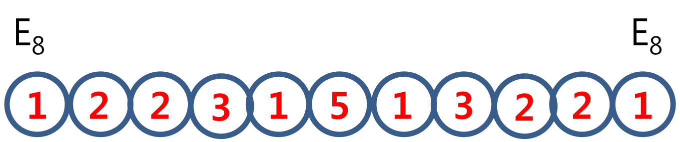
base	3, 2	3, 2, 2	2, 3, 2
gauge symmetry	$G_2 \times SU(2)$	$G_2 \times Sp(1) \times \{0\}$	$SU(2) \times SO(7) \times SU(2)$
matters	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{7}+\mathbf{1},\mathbf{2})$	$\frac{1}{2}(\mathbf{2},\mathbf{8},\mathbf{1})+\frac{1}{2}(\mathbf{1},\mathbf{8},\mathbf{2})$

- They all contain the base ‘3’ : either G_2 with one 7 or SO(7) with two 8’s
- One can form quivers of these gauge theories with those for n=2, which are all known: [Haghighat, Lockhart, Iqbal, Kozcaz, Vafa], or just normal SU(2) or Sp(1) ADHM with matters

- With these atoms, one can study the strings of E_7 x E_7 conformal matter



- We can also understand part of the E_8 x E_8 conformal matter



All by [Hee-Cheol Kim, Joonho Kim, SK, Jaemo Park], work in progress.

Concluding remarks

- We are getting solid clues on 2d gauge theories on self-dual strings:
- related to exceptional instantons' ADHM-like descriptions
- Using tensor branch observables for CFT physics at symmetric phase? (e.g. S^5 x S^1 index)

- Extension to other exceptional instantons…? (allowing reduced UV symmetry)
- For an exceptional group G_r of rank r, we are seeking for ADHM-like UV gauge theories, which exhibit only SU(r+1) subgroup as the UV symmetry.
- $SU(3) \subset G_2$ (already discovered), $SU(8) \subset E_7$, $SU(9) \subset E_8$ (trying hard…)

- Our 2d CFTs = 4d Argyres-Douglas theories on S^2: see also [Del Zotto, Lockhart] (2016)
- More insight on the self-dual strings from AD theories? [Maruyoshi, Song] (2016)