Fluctuations of time-averaged diffusion coefficients in annealed transit time model

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With the aid of single-particle-tracking experiments in living cells, it has been known that the mean square displacement (MSD) shows anomalous diffusion, i.e., $\langle \boldsymbol{r}(t)^2 \rangle \propto t^\alpha$ ($\alpha < 1$). Using single trajectory, one can also have the time-averaged MSD (TAMSD),

$$\overline{\delta^2(\Delta;t)} \equiv \frac{1}{t-\Delta}\int_0^{t-\Delta} dt' \, [\boldsymbol{r}(t'+\Delta) - \boldsymbol{r}(t')]^2. \tag{1}$$

Many experiments show that the TAMSD with fixed Δ seems to be random even when the measurement time t is large. In Brownian motions, $\overline{\delta^2(\Delta;t)}$ converges to a constant, i.e., $\overline{\delta^2(\Delta;t)} \to \langle \boldsymbol{r}(\Delta)^2 \rangle$ as $t \to \infty$. However, single-particle-tracking experiments in living cells do not satisfy such a property (ergodicity). Some stochastic models of anomalous diffusion can explain why such an ergodic property cannot be satisfied. In particular, quenched trap model and continuos-time random walk are typical models that break ergodicity, where distributional behavior of time-averaged observables such as diffusion coefficient is observed intrinsically [1-4].

Here, we consider the annealed transit time model (ATTM) as another prototype of stochastic models that break ergodicity. The ATTM can be described by the Langevin equation with fluctuating diffusivity [5],

$$\frac{d\boldsymbol{r}(t)}{dt} = \sqrt{2D(t)}\boldsymbol{w}(t), \tag{2}$$

where diffusion coefficient $D(t)$ is a stochastic process. Diffusion coefficient does not change (constant) during a random time that we call a sojourn time. Moreover, The diffusion coefficient is coupled with sojourn time of the state. In particular, the diffusion coefficient becomes $\tau^{\gamma-1}$ ($\gamma < 1$) when the sojourn time is τ.

In the previous study [5], it was shown that the MSD shows subdiffusion analytically. However, fluctuations of TAMSDs are not yet studied. In this presentation, we report theoretical results on fluctuations of TAMSDs. In particular, we provide the nth moment of TAMSD. Thus, we obtain the relative standard deviation (RSD) of TAMSDs, defined by

$$\Sigma(t;\Delta) \equiv \frac{\sqrt{\langle [\overline{\delta^2(\Delta;t)} - \langle \overline{\delta^2(\Delta;t)} \rangle]^2 \rangle}}{\langle \overline{\delta^2(\Delta;t)} \rangle}, \tag{3}$$

as a function of the measurement time t for a fixed Δ. Even when the measurement time t goes to infinity, the RSD is still non-zero constant, which implies that fluctuations of TAMSD are intrinsic. Finally, we discuss an experiment related to the ATTM [6].

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