SAMPLE-DEPENDENT FIRST-PASSAGE TIME DISTRIBUTION IN A DISORDERED MEDIUM

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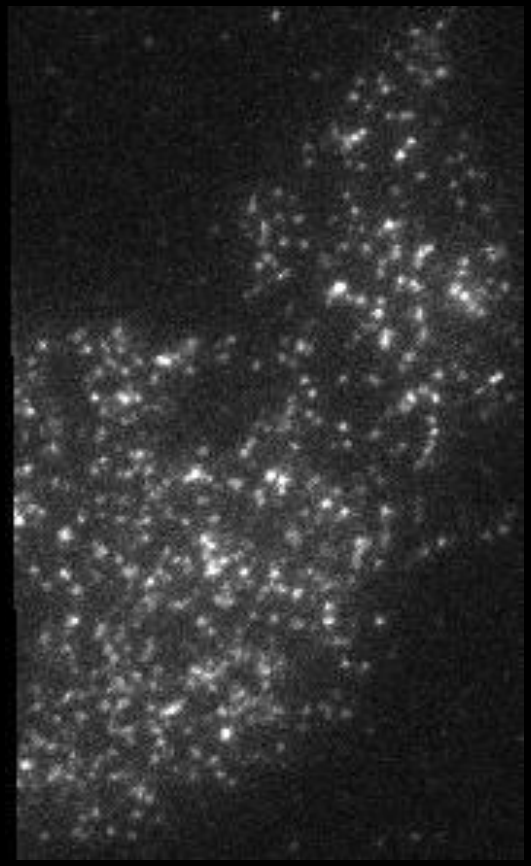
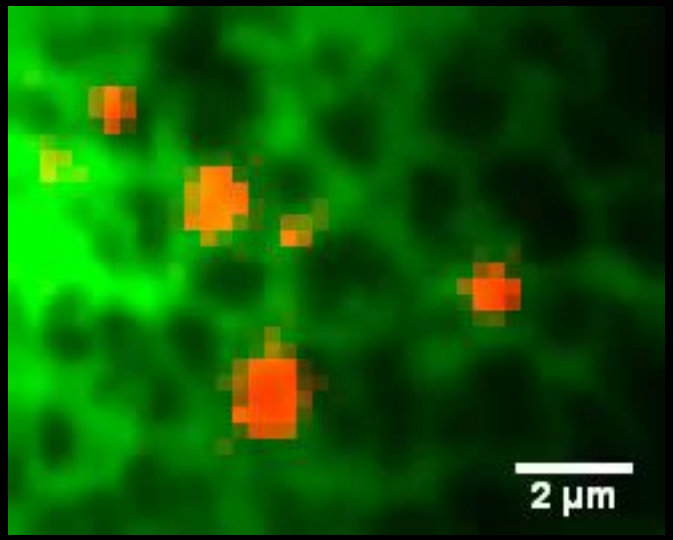
(Sub-)Diffusion in the cell

In vivo imaging experiments offer not only the particle trajectories but also the large structure of the environment.



G protein–coupled receptor on cell membrane

Yongfang Zhao *et al.* private communication

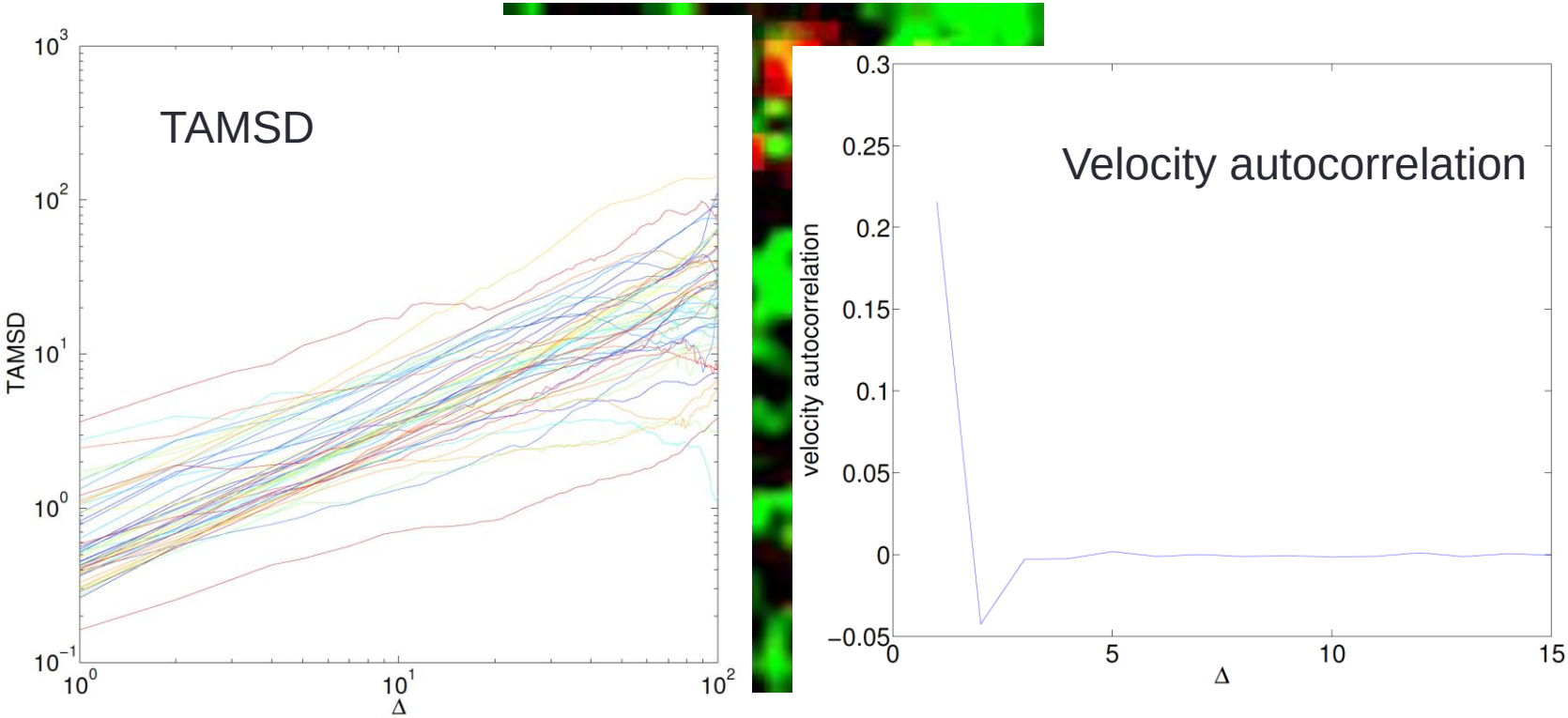


(red) Quantum Dots (QD) in cytoplasm
(green) *endoplasmic reticulum* (ER)

Hui Li *et al.* JACS (2015)

Sub-diffusion observed in experiments

QD in cytoplasm

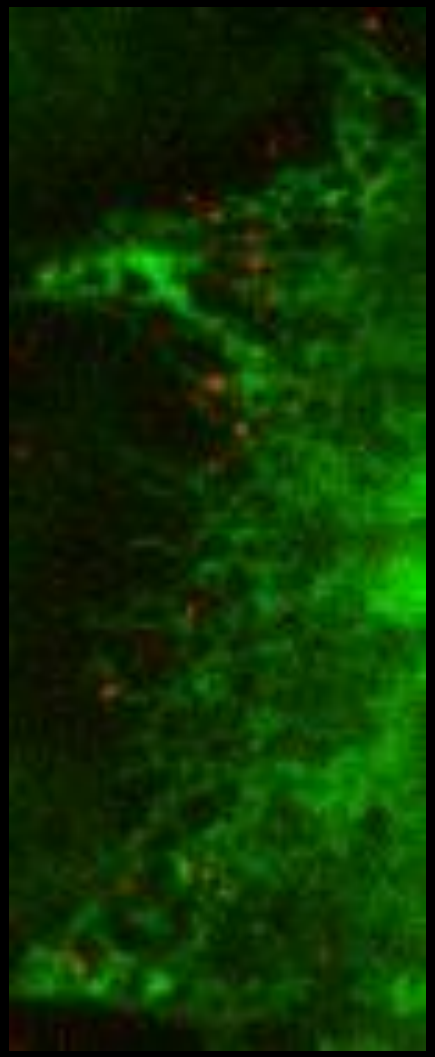


Analysis is still in progress...

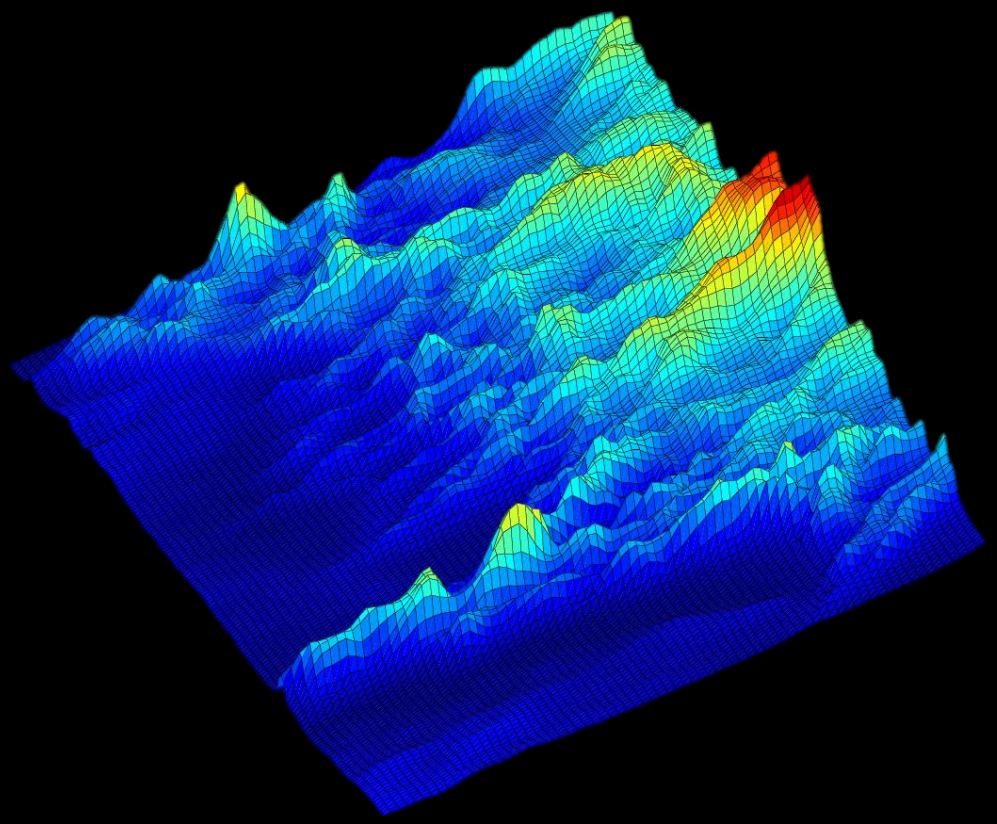
Collaborating with Hui Li

endoplasmic reticulum (ER) structure as a quasi-static landscape

A slice of the ER structure

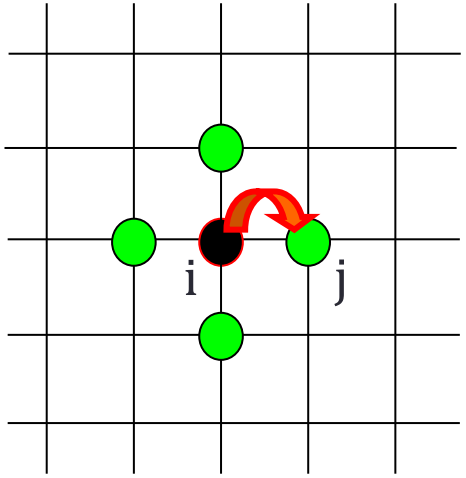


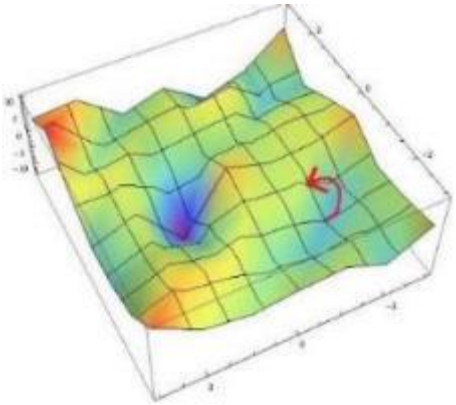
The dynamics on the static disordered samples!



Hui Li, private communication

Quenched trap model (QTM)





Hopping rate:

$$W_{i\to j} = \frac{\omega_0}{2d}\exp(V_i/T)$$

$\{V_i\}$: static random potential

Special case with anomalous diffusion:

$$P(V) = T_g^{-1}\exp\left(V/T_g\right), V < 0$$

The mean waiting time distribution:

$$P(\tau) = \mu\omega_0^{-\mu}\tau^{-(1+\mu)}, \mu \equiv T/T_g$$

Continuous Time Random Walk (CTRW):

NO static potential.

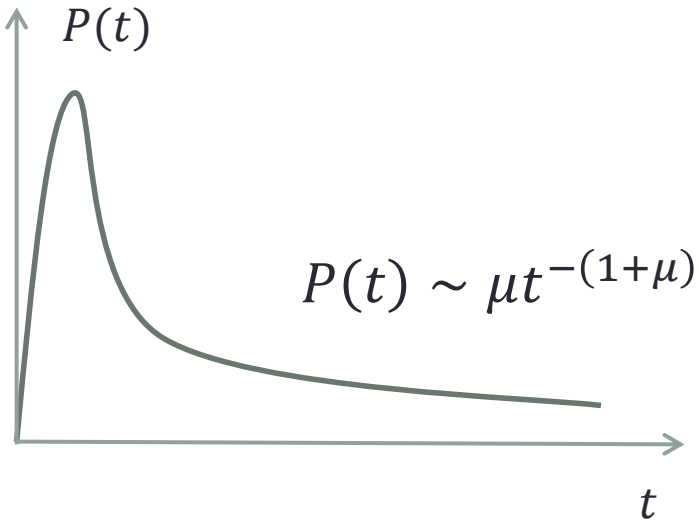
The waiting time is independently generated for each hopping.

QTM $\Longleftrightarrow$ CTRW, $d > 2$

RG in real-space, Machta, J. Phys. A, (1985)

Arguments, Bouchaud and George, Phys. Rep. (1990)

The CTRW time scale



The total walking time of a N-step walk

$$t_N = \sum_{i=1}^{N} t_i$$

A fat tail case: $P(t_i = t) \sim \mu t^{-(1+\mu)}$, $\mu < 1$

$$\langle t_N \rangle / N = \int_1^{\infty} dt\, t P(t) \to \infty$$

$P(t_N/N)$: one-side Levy stable distribution

See e.g. Bouchaud and George 1990.

$$t_{typ} \equiv \exp(\langle \ln t_N \rangle) \sim N^{1/\mu}$$

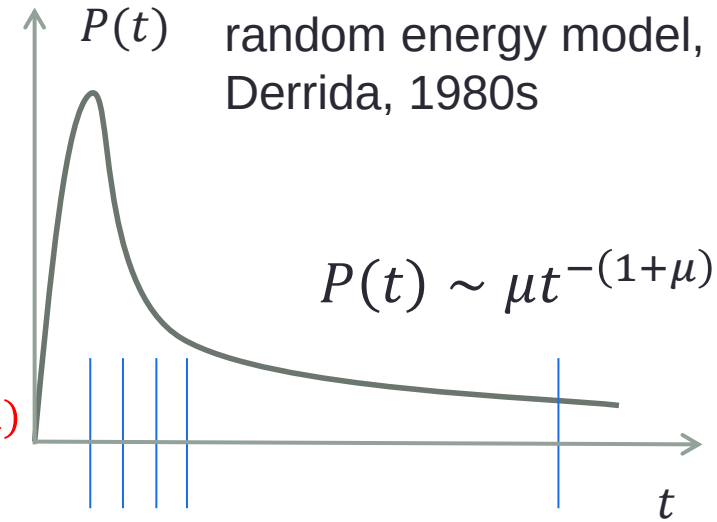
The first-passage time of CTRW

Another approach (Extreme value)

the maximum value dominates the sum

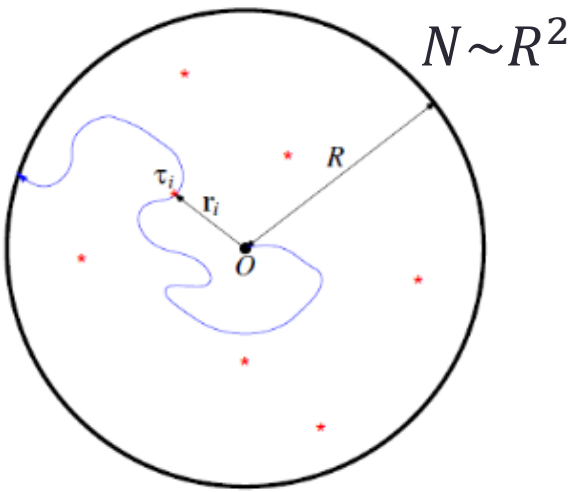
$$t_N = \sum_{i=1}^{N} t_i$$

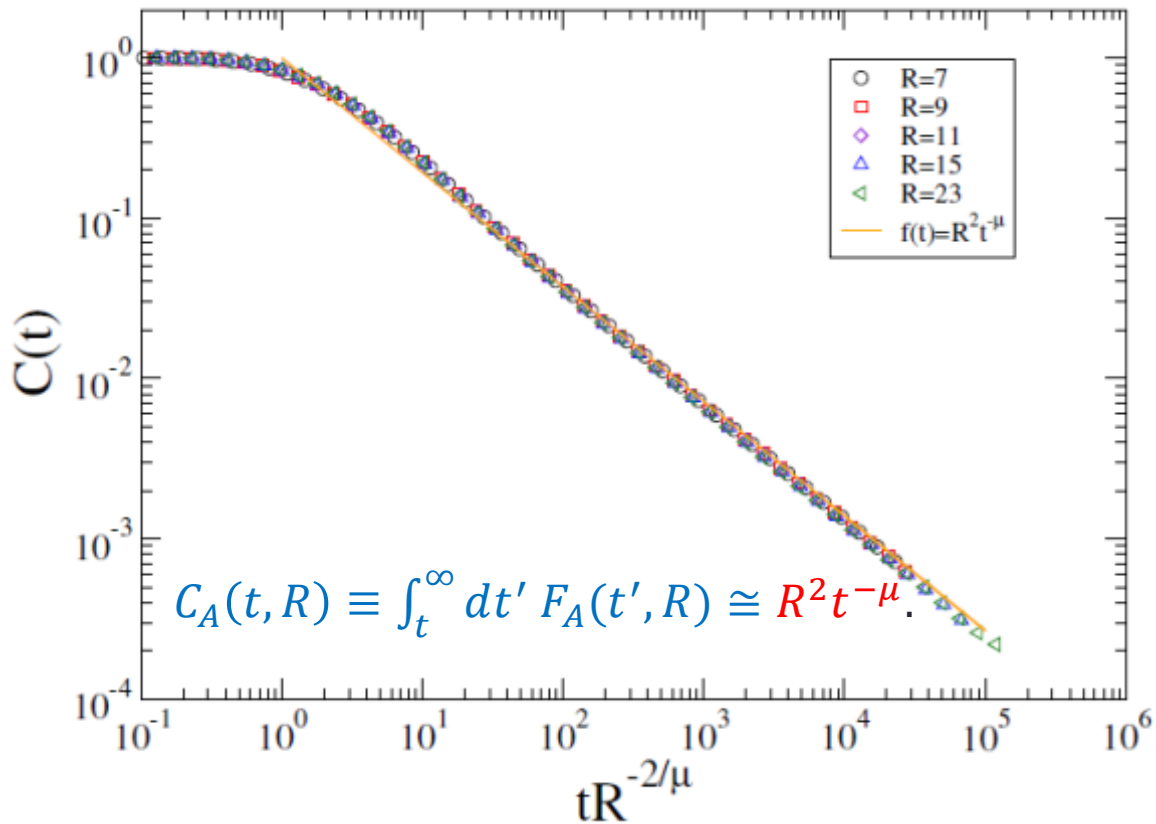
$$P(t_N = t) \approx P_N(t_{max} = t)$$
$$= N\mu(1 - t^{-\mu})^{N-1} t^{-(1+\mu)}$$



random energy model, Derrida, 1980s

$$P(t) \sim \mu t^{-(1+\mu)}$$





$$C_A(t,R) \equiv \int_t^\infty dt'\, F_A(t',R) \cong R^2 t^{-\mu}.$$

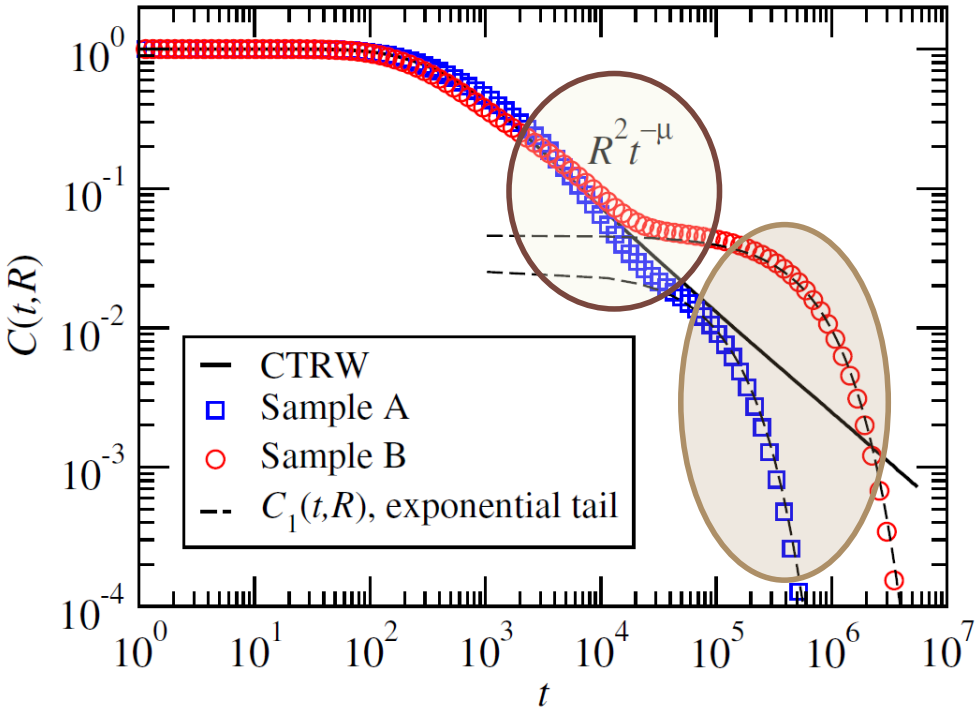
The time scales in quenched potential

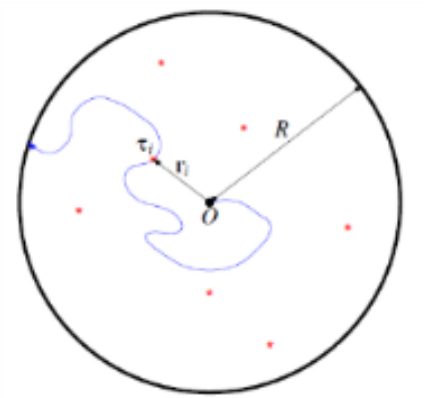
The CTRW time scale
(irrelevant to the disorder sample)

$$\tau_{CTRW} \sim R^{2/\mu}$$

The mean waiting time of a jump out of the deepest trap of the given sample

$$\tau_{QTM} \sim R^{d/\mu}$$

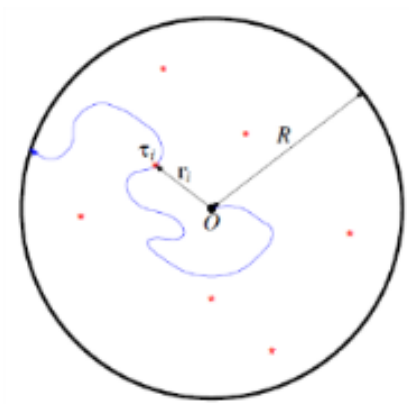




Cumulative distribution of waiting time for a jump out of the deepest trap:

$$C_1(t, R|\tau_Q, r) \simeq w(r, R)\exp(-e_d t/\tau_Q)$$

Counting strong traps

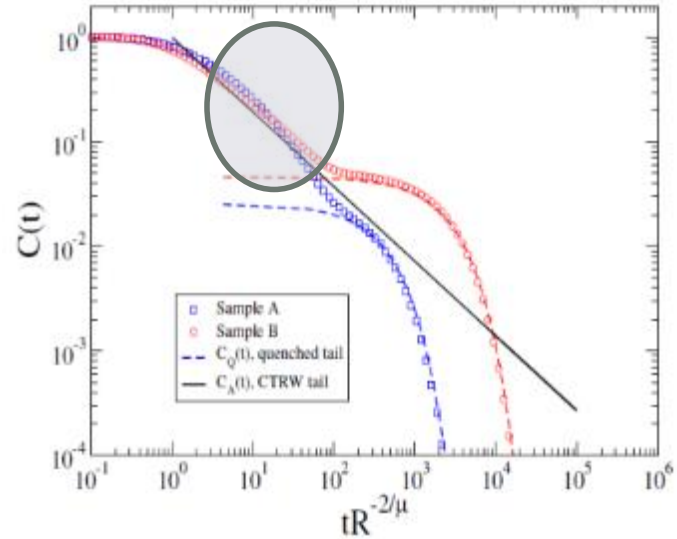


A filtered landscape only with deep traps $\tau_i > \tau_c$.

$$C(t,R) = \sum_{\Gamma} W_\Gamma C_\Gamma(t)$$

$$= \sum_{\text{trap } i} \sum_{\Gamma \in \Lambda_i} W_\Gamma \, C_\Gamma(t)$$

$$\approx \sum_{trap\ i} C_1(t, R|\tau_i, r_i)$$



$$C_1(t, R|\tau, r) \approx w(r, R) \exp(-e_d t/\tau)$$

Γ: path

W_Γ: the probability for path Γ, $\sum_\Gamma W_\Gamma = 1$

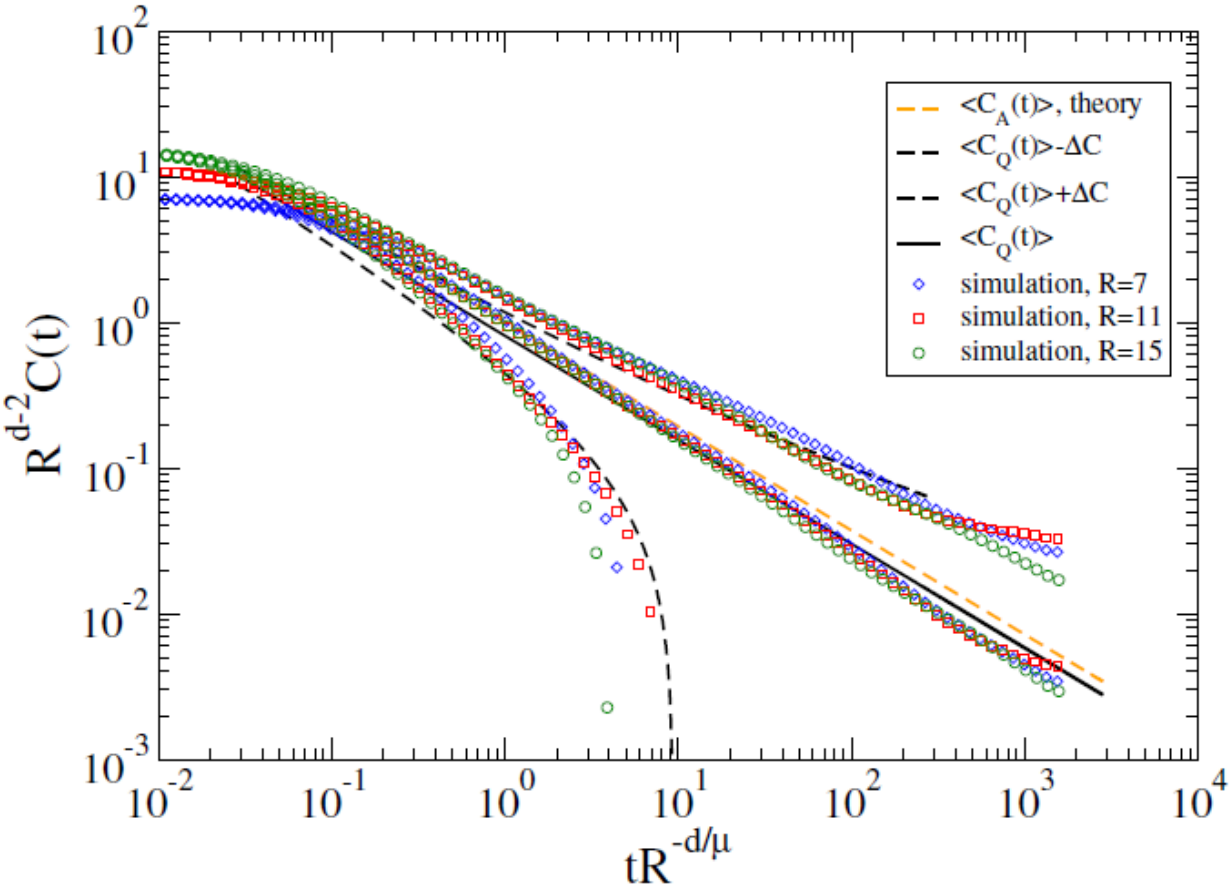
Λ_i: the set of all the paths visit trap i

Statics over disorder ensemble

$$C(t,R|\{\tau_i,r_i\}) \approx \sum_{trap\ i} C_1(t,R|\tau_i,r_i)$$

$\langle C(t)\rangle$

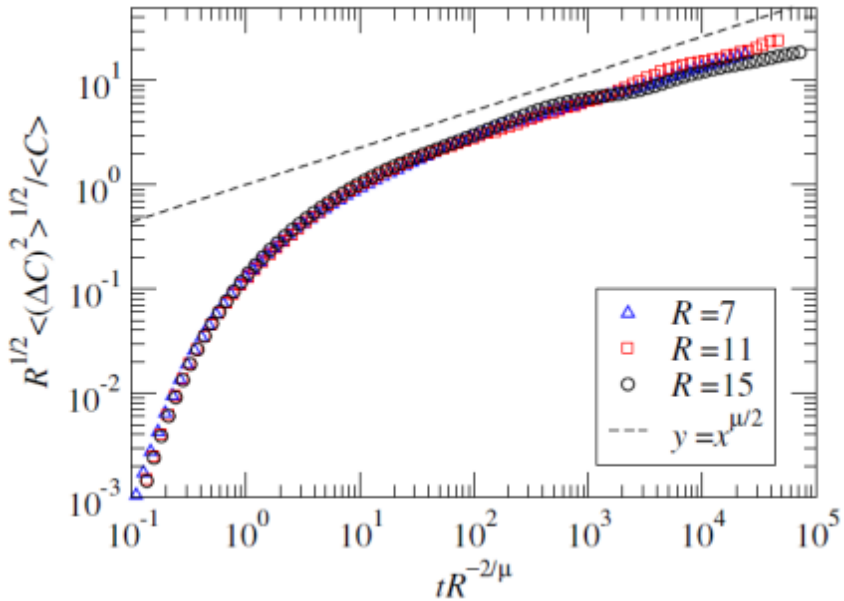
$[\Delta C(t)]$



$d < 4$

$$\langle C(t,R)\rangle \propto R^2 t^{-\mu},$$
$$\Delta C(t,R) \propto R^{2-d/2} t^{-\mu/2}.$$

Sample-dependent FPT distribution



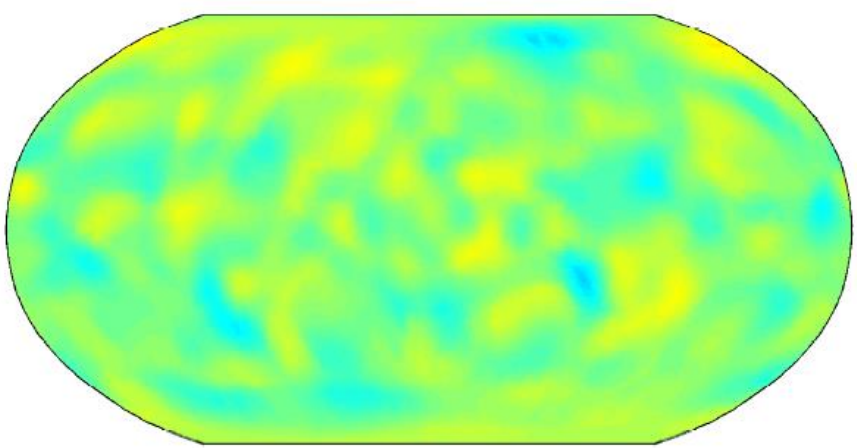
Sample-to-sample fluctuation overwhelms the mean distribution !

$$\frac{\langle[\Delta C(t,R)]^2\rangle^{1/2}}{\langle C(t,R)\rangle} = R^{(\alpha_d-2)/2} f(tR^{-2/\mu}),$$

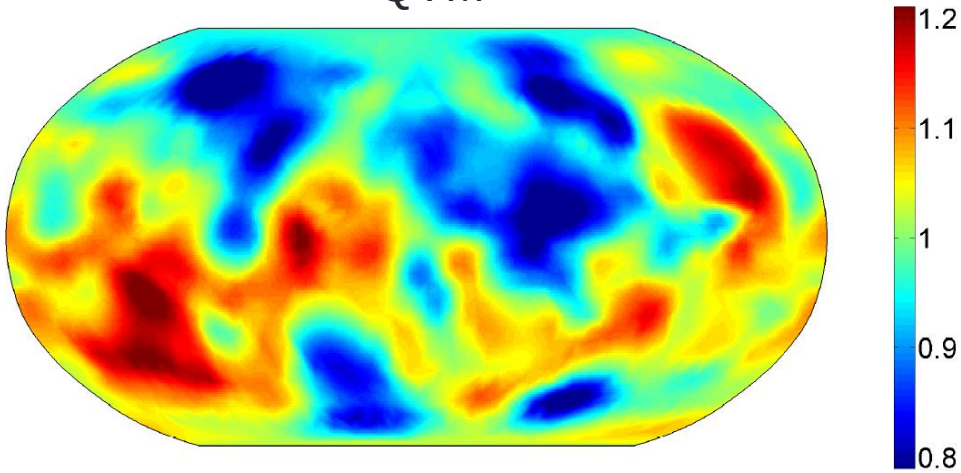
$$f(z) \sim z^{\mu/2} \text{ for } z \gg 1.$$

Earth plot of the hit map

CTRW



QTM



The hit map of sites on the absorbing surface

Summary

More details in arXiv:1507.07409

- Quasi-static structures in living cell largely determine the passive transport of macro-molecules.
- We propose an analytic approach to estimate the FPT distribution in given quenched samples.
- The FPT distribution in a sub-diffusive quenched trap model is sample-dependent for finite-size systems. (even for $d > 2$!)
- The earth map is proposed as a novel technique to detect the inner structures of the cell.

Thank you!