Quantum Entanglement and Decoherence

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KIAS

I think of my lifetime in physics as divided into three periods.

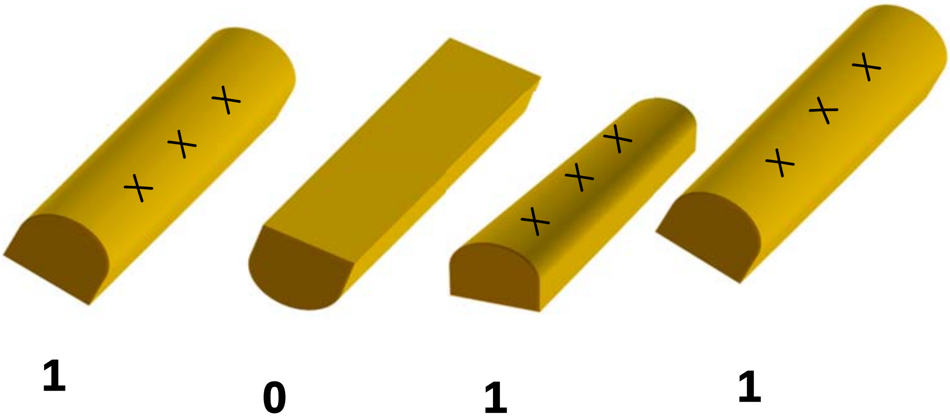
In the first period … I was in the grip of the idea that Everything is Particles, …

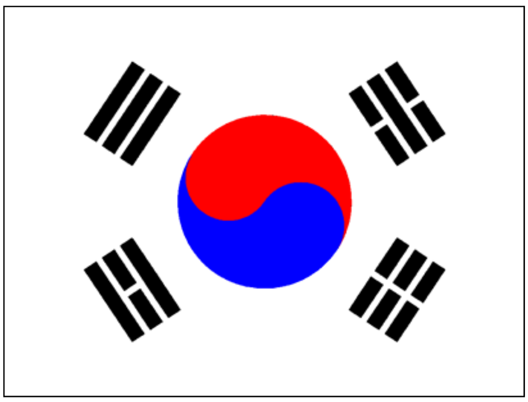
I call my second period Everything is Fields.
…

Now I am in the grip of a new vision, that Everything is Information.

– John Archibald Wheeler, *Geon, Black Holes, and Quantum Foam* –

Yoot = *Ancient* Korean Binary Dice

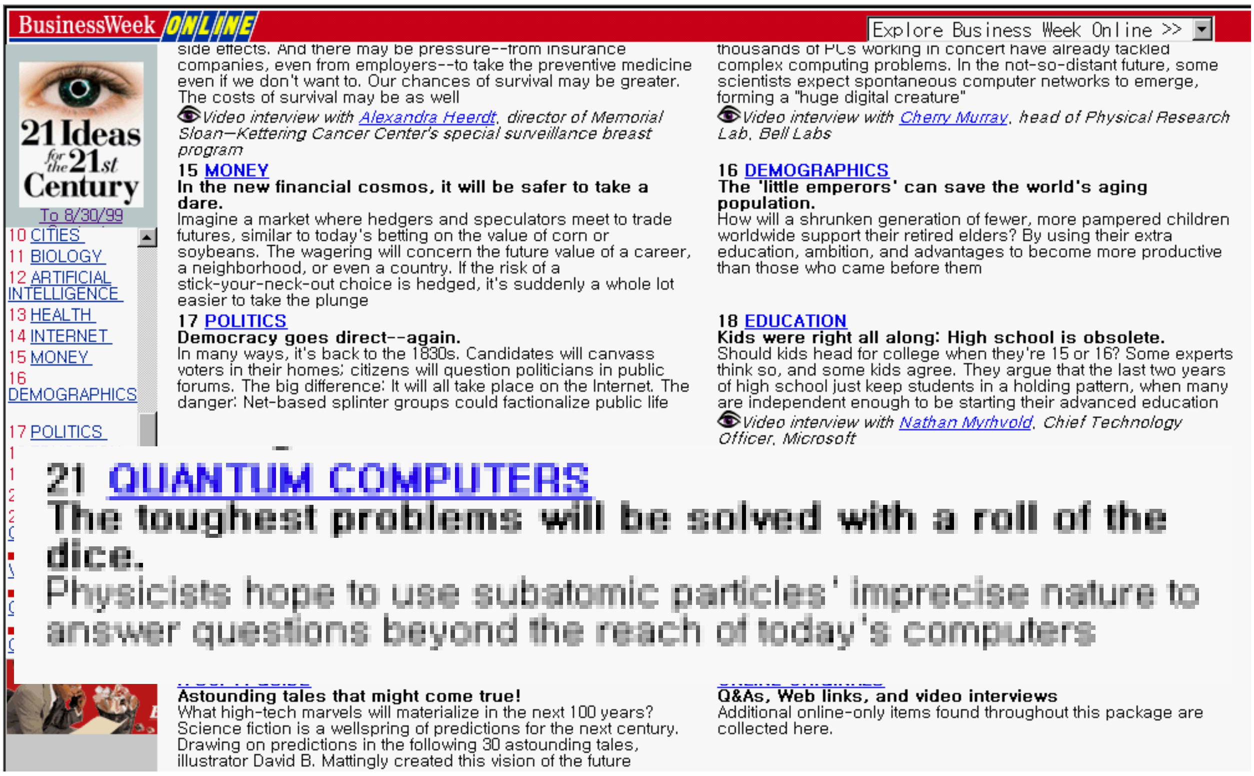


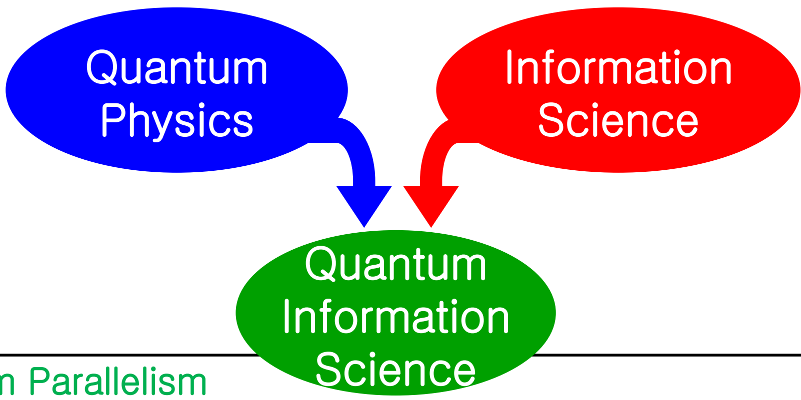


$$|\varphi\rangle = a|0\rangle + b|1\rangle$$

Qubit = quantum binary digit

Businessweek 21 Ideas for the 21st Century





Quantum Parallelism

→ Quantum computing is exponentially larger and faster than digital computing.

Quantum Fourier Transform, Quantum Database Search, Quantum Many-Body Simulation (Nanotechnology)

No Clonability of Quantum Information, Irrevesability of Quantum Measurement

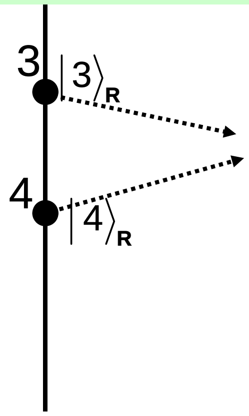
→ Quantum Cryptography (Absolutely secure digital communication)

Quantum Correlation by Quantum Entanglement

→ Quantum Teleportation, Quantum Superdense Coding, Quantum Cryptography, Quantum Imaging

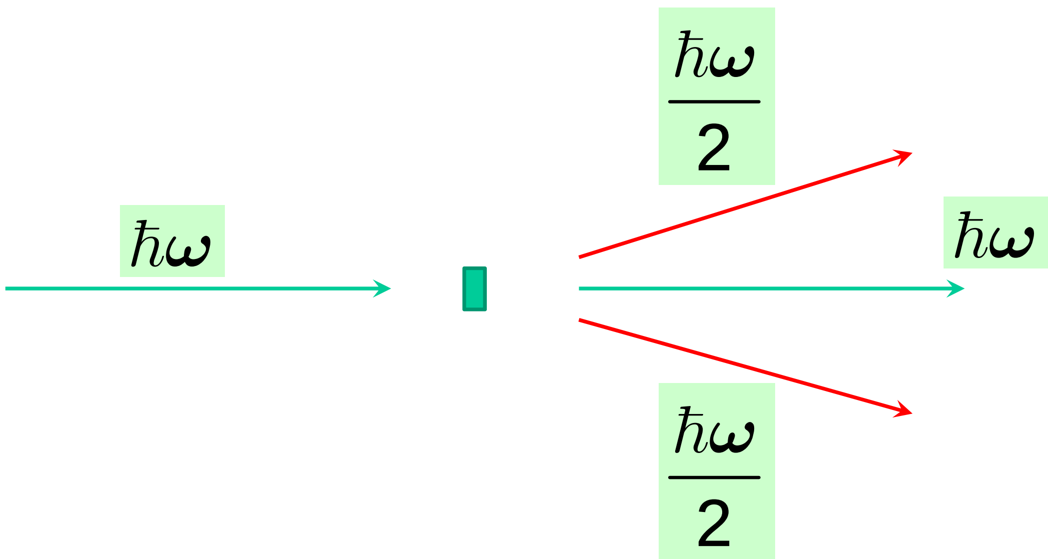
$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_R + |4\rangle_R\right)$$

$$\rho_{LR} = |\varphi\rangle\langle\varphi| = \begin{array}{c|cc} & \langle 3| & \langle 4| \\ \hline |3\rangle & \frac{1}{2} & \frac{1}{2} \\ |4\rangle & \frac{1}{2} & \frac{1}{2} \end{array}$$



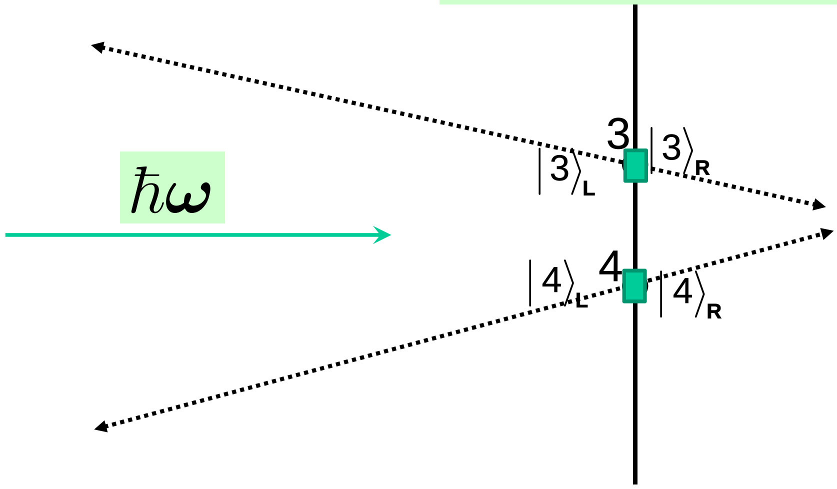
Parametric Down Conversion

: Splitting a photon into two



$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_L\,|3\rangle_R + |4\rangle_L\,|4\rangle_R\right)$$

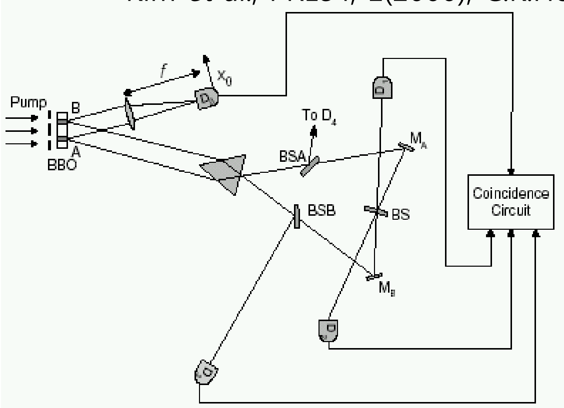
$$\rho_{LR} = |\varphi\rangle\langle\varphi|$$

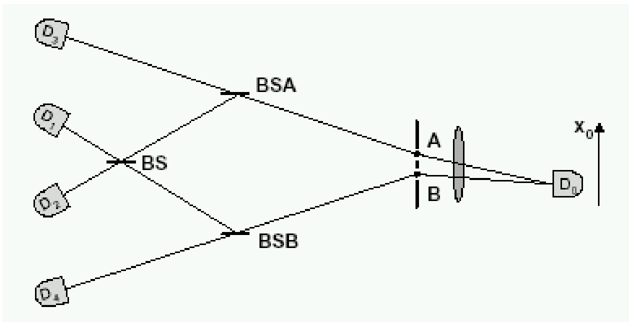


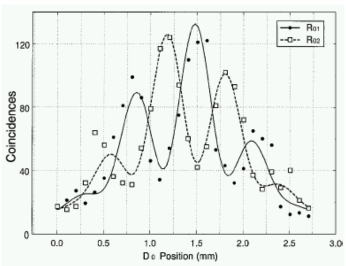
C K Hong and T Noh (1998)
Y-H Kim and Y Shi (2000)

Delayed Choice Quantum Erasure
REVISITED

Kim *et al.*, PRL84, 1(2000); C.K.Hong and T.G.Noh, JOSA B15, 1192(1998)







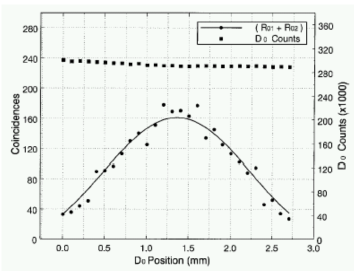


FIG. 4. $R_{01} + R_{02}$ is shown. The solid line is a fit to the sinc function given in Eq. (6). The single counting rate of D_0 is constant over the scanning range.

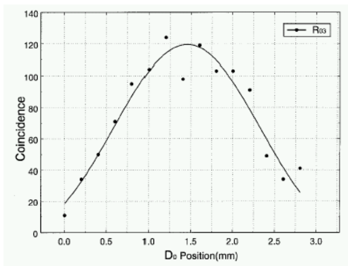
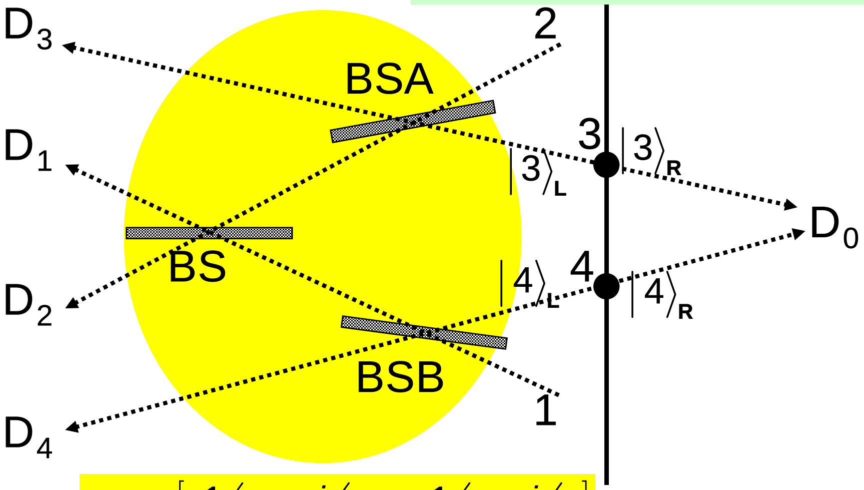


FIG. 5. R_{03} is shown. Absence of interference is clearly demonstrated. The solid line is a fit to the sinc function given in Eq. (6).

$$\rho'_{LR} = U_L|\varphi\rangle\langle\varphi|U_L^+ = \frac{1}{2}\left\{\left[\left(-\frac{|1\rangle_L}{2} + i\frac{|2\rangle_L}{2} + \frac{|3\rangle_L}{\sqrt{2}}\right)|3\rangle_R + \left(i\frac{|1\rangle_L}{2} - \frac{|2\rangle_L}{2} + \frac{|4\rangle_L}{\sqrt{2}}\right)|4\rangle_R\right]\right\}\{h.c.\}$$

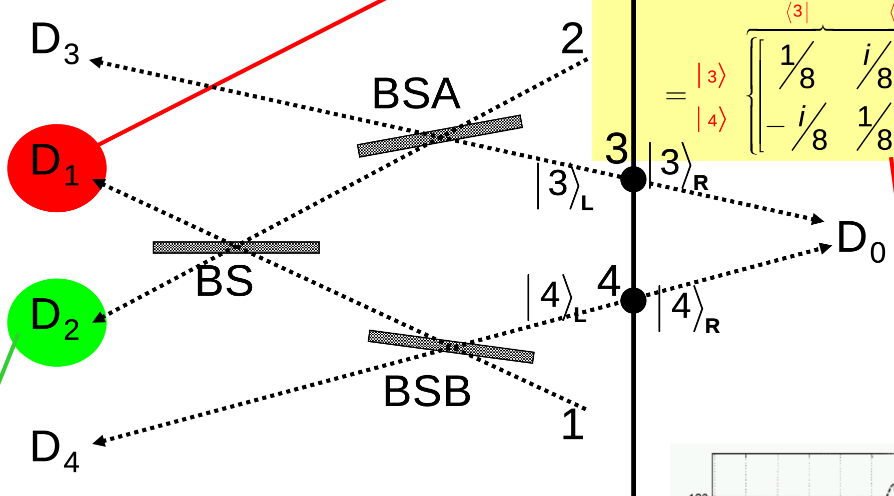


$$U_L = \begin{bmatrix} 1/2 & i/2 & -1/2 & i/2 \\ i/2 & 1/2 & i/2 & -1/2 \\ 0 & i/\sqrt{2} & 1/\sqrt{2} & 0 \\ i/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$\rho'_{R|D_1} = {}_L\langle 1|U_L|\varphi\rangle\langle\varphi|U_L^+|1\rangle_L$$

$$= \frac{1}{8}\{-|3\rangle_R + i|4\rangle_R\}\{-{}_R\langle 3| - i\,{}_R\langle 4|\}$$

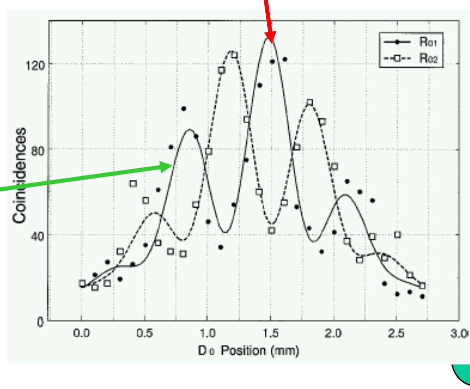
$$= \begin{array}{c c} & \begin{array}{cc} \langle 3| & \langle 4| \end{array} \\ \begin{array}{c} |3\rangle \\ |4\rangle \end{array} & \begin{bmatrix} 1/8 & i/8 \\ -i/8 & 1/8 \end{bmatrix} \end{array}$$

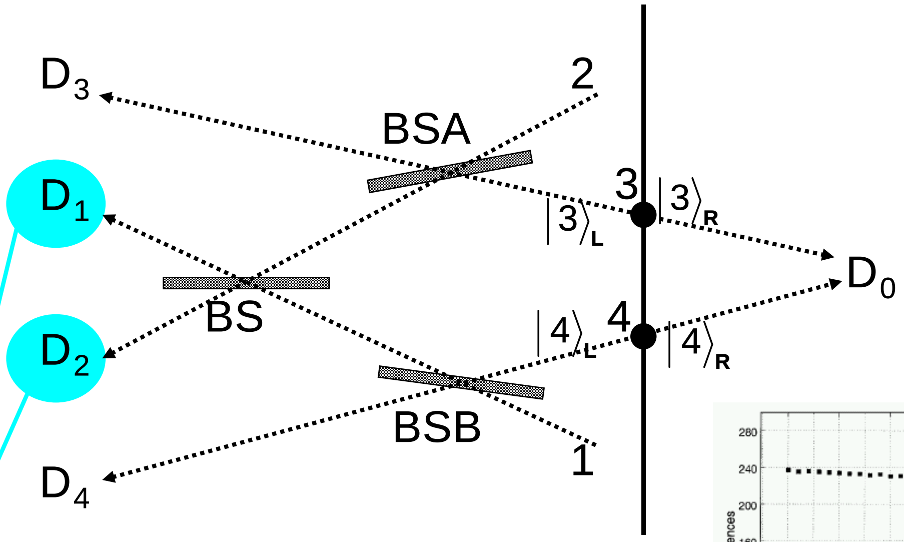


$$\rho'_{R|D_2} = {}_L\langle 2|U_L|\varphi\rangle\langle\varphi|U_L^+|2\rangle_L$$

$$= \frac{1}{8}\{i|3\rangle_R - |4\rangle_R\}\{-i\,{}_R\langle 3| - {}_R\langle 4|\}$$

$$= \begin{array}{c c} & \begin{array}{cc} \langle 3| & \langle 4| \end{array} \\ \begin{array}{c} |3\rangle \\ |4\rangle \end{array} & \begin{bmatrix} 1/8 & -i/8 \\ i/8 & 1/8 \end{bmatrix} \end{array}$$





$$\rho'_{\mathbf{R}|(\mathbf{D1+D2})} = \begin{bmatrix} 1/8 & i/8 \\ -i/8 & 1/8 \end{bmatrix} + \begin{bmatrix} 1/8 & -i/8 \\ i/8 & 1/8 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 \\ 0 & 1/4 \end{bmatrix}$$

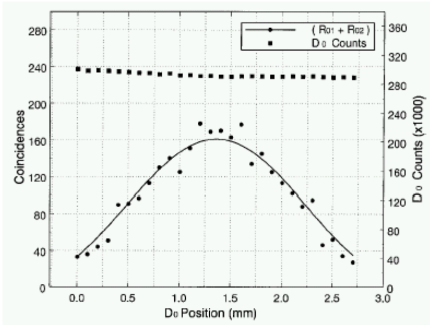
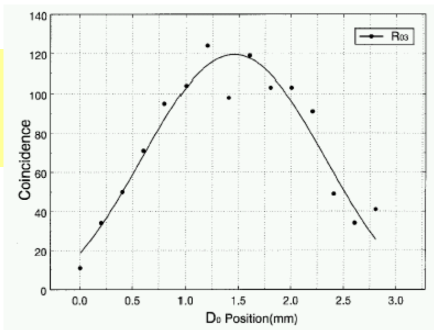


FIG. 4. $R_{01} + R_{02}$ is shown. The solid line is a fit to the sinc function given in Eq. (6). The single counting rate of D_0 is constant over the scanning range.

$$\rho'_{\mathbf{R}|\mathbf{D3}} = {}_{\mathbf{L}}\langle 3|U_{\mathbf{L}}|\varphi\rangle\langle\varphi|U_{\mathbf{L}}^{+}|3\rangle_{\mathbf{L}} = \frac{1}{4}|3\rangle_{\mathbf{R}}\,{}_{\mathbf{R}}\langle 3| = \begin{bmatrix} 1/4 & 0 \\ 0 & 0 \end{bmatrix}$$



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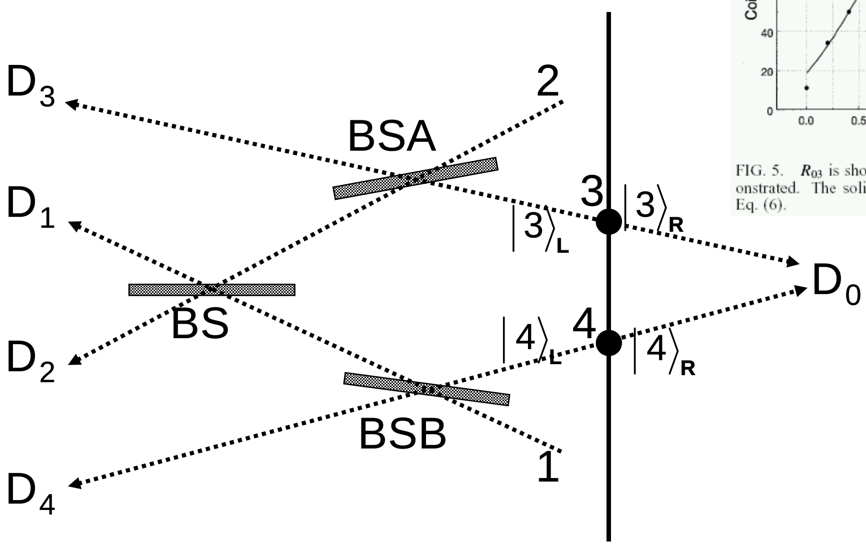


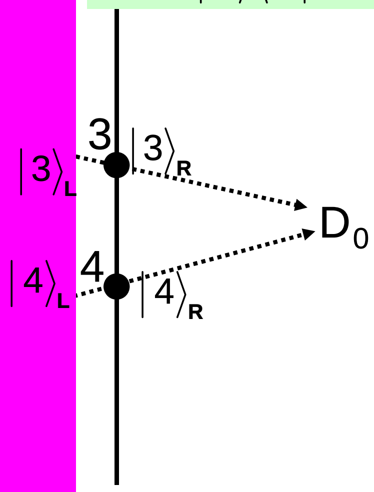
$$\rho'_{\mathbf{R}|\mathbf{D4}} = {}_{\mathbf{L}}\langle 4|U_{\mathbf{L}}|\varphi\rangle\langle\varphi|U_{\mathbf{L}}^{+}|4\rangle_{\mathbf{L}} = \frac{1}{4}|4\rangle_{\mathbf{R}}\,{}_{\mathbf{R}}\langle 4| = \begin{bmatrix} 0 & 0 \\ 0 & 1/4 \end{bmatrix}$$

Environment

Decoherence

$$|\varphi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle_L |3\rangle_R + |4\rangle_L |4\rangle_R\right)$$

$$\rho_{LR} = |\varphi\rangle\langle\varphi|$$

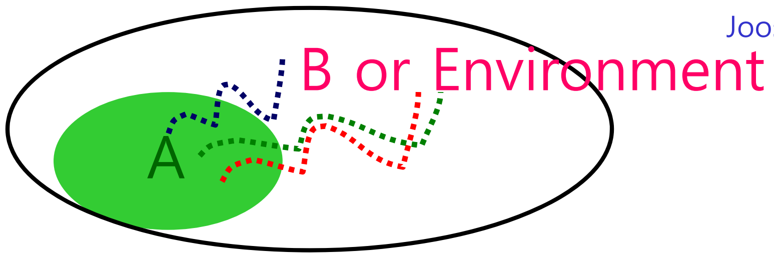


$$\rho_R = \mathrm{tr}_L \rho_{LR} = \frac{1}{2}\left\{|3\rangle_{R\,R}\langle 3| + |4\rangle_{R\,R}\langle 4|\right\} = \frac{1}{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\rho'_{R|(D_1+D_2+D_3+D_4)} = \begin{bmatrix}1/4 & 0\\ 0 & 1/4\end{bmatrix} + \begin{bmatrix}1/4 & 0\\ 0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 1/4\end{bmatrix} = \begin{bmatrix}1/2 & 0\\ 0 & 1/2\end{bmatrix}$$

Entanglement and Decoherence

Joos, Zeh, Zurek



When system A is entangled with environment, state of A cannot be described by a state vector, but by a density matrix.

$$|Y\rangle_{AB} = a\,|0\rangle_A|0\rangle_B + b\,|1\rangle_A|1\rangle_B \neq |\psi\rangle_A|\phi\rangle_B$$

$$\rho_A = \mathrm{Tr}_B\,\rho_{AB} = \mathrm{Tr}_B\,|Y\rangle_{AB\,AB}\langle Y| = \boxed{\begin{matrix}|a|^2 & 0\\ 0 & |b|^2\end{matrix}}$$

ab*

ba*

$$\neq |\psi\rangle_{A\,A}\langle\psi|$$

When a quantum system is entangled with its environment through some interaction, there is no pure state description available for the system even though pure state description of the system plus environment as a whole might be possible.

In this case, the system loses its coherence, in other words, is decohered or the decoherence occurred on the system. When the whole is entangled, the part is decohered.

The system entangled with environment is not a pure state any more and is called a mixed state, which cannot be described by a ket vector or a state function, but by a density matrix.

While the conventional idea for the emergence of classical physics from quantum physics is based on the smallness of planck constant and has nothing to do with the existence of external system or environment, the decoherence idea requires the interaction between the system and its environment.

Zurek, QP/0306072

Old Quantum Theory

$$\oint I\,d\theta = nh$$

Einstein's forgotten question:

How would you quantize non-integrable systems?

New quantization

$$x \rightarrow \hat{x}$$

$$p \rightarrow \hat{p}$$

$$\hat{x}\hat{p} \neq \hat{p}\hat{x}$$

$$\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar\hat{1}$$

Mysterious Connection Between Linearity of QM & Special Relativity

- Weinberg: Can QM be nonlinear?
- Experiments: Not so positive result.
- Polchinski, Gisin:
  If QM is nonlinear,
  communication faster than light is possible.

No Cloning Theorem

An Unknown Quantum State Cannot Be Cloned.

Zurek, Wootters
Diks

<Proof>

$$U\left(|\alpha\rangle|0\rangle\right)=|\alpha\rangle|\alpha\rangle$$

$$U\left(|\beta\rangle|0\rangle\right)=|\beta\rangle|\beta\rangle \qquad |\alpha\rangle\neq|\beta\rangle$$

$$\text{Let } |\gamma\rangle=\frac{1}{\sqrt{2}}\left(|\alpha\rangle+|\beta\rangle\right).$$

$$\text{Then } \quad U\left(|\gamma\rangle|0\rangle\right)=\frac{1}{\sqrt{2}}\left(|\alpha\rangle|\alpha\rangle+|\beta\rangle|\beta\rangle\right)\neq|\gamma\rangle|\gamma\rangle$$

If an unknow quantum state Can be cloned …

- Quantum States can be measured as accurately as possible ???

  $|\psi\rangle \Rightarrow |\psi\rangle, |\psi\rangle, |\psi\rangle, |\psi\rangle, |\psi\rangle, |\psi\rangle, \ldots$

  measure, measure, measure, measure, measure, measure, …

- Commnunication Faster Than Light?

  $|\psi\rangle = |0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B$ for "0"

  $= |+\rangle_A |-\rangle_B - |-\rangle_A |+\rangle_B$ for "1"

Communication Faster Than Light

when unknown quantum states can be copied.

- Alice wants to send Bob "1."
  Alice measures her qubit in $|+\rangle, |-\rangle$.
  → Alice's state will become $|+\rangle$ *or* $|-\rangle$.
- Bob's state will become $|-\rangle$ *or* $|+\rangle$.
  → Let's assume it is $|+\rangle$.
- Bob makes many copies of this.
  → For half of them, he measures them in $\{|+\rangle, |-\rangle\}$, and gets 100% $|+\rangle$.
  → For the others, he measures them in $\{|0\rangle, |1\rangle\}$, and gets 50% $|0\rangle$ and 50% $|1\rangle$.
  → Thus Bob can conclude that Alice measured her qubit in $\{|+\rangle, |-\rangle\}$.

F O R U M O N P H Y S I C S & S O C I E T Y
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July 2006
Vol. 35, No. 3

ARTICLES (1)

Intelligent Design in the Physics Classroom?

Travis Norsen

A dangerous enemy has infiltrated our science classrooms and is infecting our students' minds. The enemy is a profoundly unscientific theory masquerading as legitimate science. Its presence in the science classroom blurs the distinction between real science and arbitrary dogma and "makes students stupid" by leaving them less able to distinguish reasonable ideas from unreasonable ones – a skill that is surely one of the main goals of teaching science in the first place.

You probably suspect the enemy I'm talking about is Intelligent Design (ID). Yes, ID has infiltrated some science classrooms. Yes, ID is specifically designed to blur the distinction between real science and religious dogma. And yes, the phrase "makes students stupid" is straight out of Pennsylvania Judge John Jones' recent finding that "ID is not science" and shouldn't be taught in the biology classroom.[1] But, in part because of Jones' excellent analysis, I don't think ID is a terribly significant danger. It is too transparently unscientific, too widely recognized for what it really is: a thinly-veiled attempt to inject religious creationism into the science classroom.

The enemy I'm worried about is something else – something just as unscientific as ID, but more dangerous because it is not widely recognized as such: the Copenhagen interpretation of quantum mechanics.

The Copenhagen interpretation, so named because of the Danish roots of its main author Niels Bohr, grew out of the paradoxical nature of sub-atomic particles revealed by experiments in the 1920s: electrons sometimes acted like particles but sometimes like waves. This is a paradox because particles are, by definition, localized entities that follow definite trajectories while waves are not confined to any particular path or region of space. How could the same thing be both confined and not confined, both a particle and a wave? Paradox indeed!

Luckily, the two conflicting aspects never appear simultaneously. The experimental situations in which the particle and wave properties manifest themselves are, in a sense, mutually exclusive. The famous Heisenberg Uncertainty Principle codifies this separation: any experiment which reveals the precise particle character of an electron will necessarily obscure the wave character completely, and vice versa.

If one wants to achieve a coherent physical understanding of the nature of the electron, however, this is not very satisfying. Bohr's approach was not so much to resolve the paradox as to embrace it. Naming his philosophy "complementarity," he posited that the electron's wave and particle natures were mutually incompatible – yet still jointly exhaustive – perspectives. A complete theoretical description of the electron would have to include both wave and particle aspects; yet, like the experimental situations in which they are revealed, the very concepts of "wave" and "particle" could not be applied simultaneously. According to the Copenhagen view, physicists can never really understand the surprising experimental results or the real nature of the electron. We must simply embrace the paradox and quit looking for a coherent physical picture.

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Objectivity, retrocausation, and the experiment of Englert, Scully, and Walther

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Sri Aurobindo Ashram, Pondicherry 605002, India

(Received 5 August 1998; accepted 17 September 1998)

In a recent contribution to this journal [Am. J. Phys. **64**, 1468–1475 (1996)] I wrongly asserted that retrocausation in the Englert, Scully, and Walther (ESW) experiment (a double-slit interference experiment with atoms) can occur only until the atom arrives at the screen. In their response, Englert, Scully, and Walther [preceding paper] point out my fallacy but give an incomplete analysis of its origin. In this paper I trace this fallacy to a deep-seated preconception about time and reality. I show that among the two possible realistic interpretations of standard quantum mechanics, the reality-of-states view and the reality-of-phenomena view, only the latter is viable. It follows that retrocausation is a necessary feature of any realistic account of the ESW experiment based on standard quantum mechanics. Finally I elucidate the sense in which the spatial properties of quantum systems are objective, and show that they are extrinsic rather than intrinsic.

I. INTRODUCTION

In a recent article[1] I analyzed the thought experiment of Englert, Scully, and Walther[2,3] (ESW) from two ''metaphysical'' perspectives, the reality-of-states view and the reality-of-phenomena view. In that article I arrived at a wrong conclusion, for which I wish to express my sincere apologies to the readers of this journal. I compounded my mistake by attributing my views to Englert, Scully, and Walther. My apologies also to these authors! It ought to be mentioned, however, that I was argued into misrepresenting their views by the anonymous referee of my article. He/she not only agreed with my erroneous conclusion but also thought that ESW would likewise agree with it. For this the referee cannot be blamed, however, for it is only in their

slits. The cavities are designed to force each atom to emit a microwave photon. They are separated from each other by a pair of shutters between which a photosensor is placed (see Fig. 1). Each atom leaves a mark where it hits the screen. If one simply looks at the distribution of marks created by a large number of atoms, no interference pattern is seen. But quantum mechanics predicts that if the experimenters open the shutters (this can be done well after the corresponding atom has reached the screen) and consider separately the cases in which the sensor responds and the cases in which it does not respond, they will be able to discern two complementary interference patterns. Alternatively, they can leave the shutters closed and ascertain the cavity that contains the photon. They thus appear to have a choice between either

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Quantum Measurement

Is the moon there when nobody looks?
-Einstein

Decoherence

Is the moon classical only when somebody looks?

Entanglement

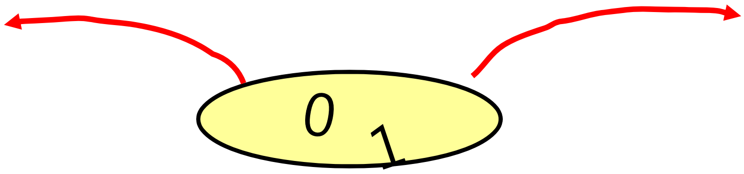
Quantum correlation

Alice

x

Bob

y



Classical physics: x and y are decided when picked up. $0_A 1_B$ or $1_A 0_B$

Quantum physics: x and y are decided when measured.

{0,1} basis → 0 or 1

{+,-} basis → + or -

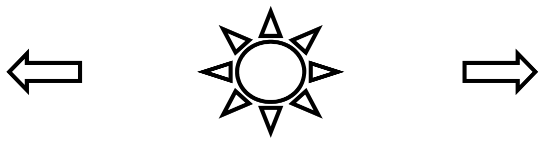
$$
\begin{aligned}
|\Psi\rangle &= \frac{1}{\sqrt{2}}\left(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\right) \\
&= \frac{1}{\sqrt{2}}\left(|+\rangle_A|-\rangle_B - |-\rangle_A|+\rangle_B\right) \\
&= \frac{1}{\sqrt{2}}\left(|0'\rangle_A|1'\rangle_B - |1'\rangle_A|0'\rangle_B\right)
\end{aligned}
$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right),\ |-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

Entanglement

EPR & Nonlocality

Alice



Bob

$$\frac{1}{\sqrt{2}}\left(|1\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} - |0\rangle_{\mathrm{A}}|1\rangle_{\mathrm{B}}\right) \neq |\psi\rangle_{\mathrm{A}} \otimes |\phi\rangle_{\mathrm{B}}$$

Local Hidden Variable → Bell's Inequality

Aspect's Experiment → Quantum Mechanics is nonlocal!

GHZ State: $\frac{1}{\sqrt{2}}\left(|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}}|0\rangle_{\mathrm{C}} + |1\rangle_{\mathrm{A}}|1\rangle_{\mathrm{B}}|1\rangle_{\mathrm{C}}\right)$

Charles Bennett's Hippie analogy

I don't know what I think.

You don't know what you think.

But we know what we think.

---

qubit $\{|0\rangle, |1\rangle\}$

A

$$|\psi\rangle_A = a|0\rangle + b|1\rangle = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\{|00\rangle \equiv |0\rangle_A|0\rangle_B,\ |01\rangle,\ |10\rangle,\ |11\rangle\}$$

A B

$$|\Psi\rangle_{AB} = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$= \begin{bmatrix} |0\rangle & |1\rangle \end{bmatrix}_A \underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{= UDV} \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}_B$$

$= UDV$: singular value decomposition

$$= \begin{bmatrix} |0'\rangle & |1'\rangle \end{bmatrix}_A \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \begin{bmatrix} |0''\rangle \\ |1''\rangle \end{bmatrix}_B$$

$$= d_1|0'0''\rangle + d_2|1'1''\rangle$$; Schmidt Decomposition

If $d_2 = 0$, or $d_1 = 0$, $|\Psi\rangle_{AB} = |0'\rangle_A|0''\rangle_B$; Separable or Product State.

OTHERWISE, Entangled State!

$$|\Psi\rangle_{AB} \neq |\psi\rangle_A |\phi\rangle_B$$

$$\langle O \rangle = \langle \psi | O | \psi \rangle = \mathrm{Tr}\{ O |\psi\rangle\langle\psi| \} = \mathrm{Tr}\{ O \rho \}$$

$$\langle O_A \rangle_A = \mathrm{Tr}_{AB}\{ O_A \otimes I_B \, \rho_{AB} \} = \sum_{j \in A} \sum_{k \in B} {}_A\langle j| \, {}_B\langle k| O_A \otimes I_B \, \rho_{AB} |k\rangle_B |j\rangle_A$$

$$= \sum_{j \in A} {}_A\langle j| O_A \underbrace{\left\{ \sum_{k \in B} {}_B\langle k| \rho_{AB} |k\rangle_B \right\}}_{\equiv \rho_A} |j\rangle_A$$

$$= \mathrm{Tr}_A \{ O_A \rho_A \}$$

$$\rho_A = \mathrm{Tr}_B \, \rho_{AB}$$

$$|\Psi\rangle = |0\rangle_A \underbrace{\{ a|0\rangle + b|1\rangle \}_B}_{\equiv \vec{\alpha}} + |1\rangle_A \underbrace{\{ c|0\rangle + d|1\rangle \}_B}_{\equiv \vec{\beta}}$$

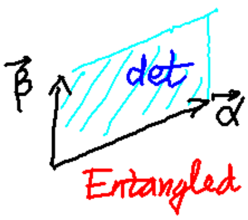
$$|\Psi\rangle = \vec{\alpha} |0\rangle_A + \vec{\beta} |1\rangle_A \qquad \text{cf. } |\psi\rangle = a|0\rangle + b|1\rangle$$

Not Scalar   complex scalar

If $\vec{\alpha} \,//\, \vec{\beta}$ or $\det \begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$,

$\vec{\alpha} = |\vec{\alpha}| \, |\xi\rangle_B$ and $\vec{\beta} = |\vec{\beta}| e^{i\phi} |\xi\rangle_B$ normalized

$|\Psi\rangle = \{ |\vec{\alpha}| \, |0\rangle + |\vec{\beta}| e^{i\phi} |1\rangle \}_A |\xi\rangle_B$; separated



$$0 \leq \left|\det\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right| = |ad-bc| \leq \frac{1}{2}$$

with $|a|^2+|b|^2+|c|^2+|d|^2=1$

$C = 2|ad-bc|$; concurrence

measure of Entanglement

Entangled $\Longrightarrow$ $\nexists$ $|\psi\rangle_A$ or $|\phi\rangle_B$

$\Longrightarrow$ still $\exists$ ρ_A and ρ_B

$$\rho_A = \mathrm{Tr}_B\, \rho_{AB}$$

$$\rho_B = \mathrm{Tr}_A\, \rho_{AB}$$

$$\rho_A = \mathrm{Tr}_B\, \rho_{AB} = \sum_{j\in B} \langle j|\Psi\rangle\langle\Psi|j\rangle = \sum_j \langle j|\left(d_1|0'\rangle|0''\rangle + d_2|1'\rangle|1''\rangle\right)(h.c.)|j\rangle$$

$$= |d_1|^2\, |0'\rangle\langle 0'| + |d_2|^2\, |1'\rangle\langle 1'| \Rightarrow \begin{bmatrix} |d_1|^2 & 0 \\ 0 & |d_2|^2 \end{bmatrix}$$

$$= \sum_j \langle j|\left(\vec{\alpha}\,|0\rangle_A + \vec{\beta}\,|1\rangle_A\right)\left(\vec{\alpha}^\dagger\,{}_A\langle 0| + \vec{\beta}^\dagger\,{}_A\langle 1|\right)|j\rangle$$

$$= \alpha^\dagger\alpha\, |0\rangle\langle 0| + \beta^\dagger\alpha\, |0\rangle\langle 1| + \alpha^\dagger\beta\, |1\rangle\langle 0| + \beta^\dagger\beta\, |1\rangle\langle 1| = \begin{bmatrix} \alpha^\dagger\alpha & \beta^\dagger\alpha \\ \alpha^\dagger\beta & \beta^\dagger\beta \end{bmatrix}$$

cf. $\begin{bmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{bmatrix}$

$$= \begin{bmatrix} |a|^2+|b|^2 & ac^*+bd^* \\ a^*c+b^*d & |c|^2+|d|^2 \end{bmatrix} \xrightarrow{\text{diag}} \lambda^2 - (\lambda_1+\lambda_2)\lambda + \lambda_1\lambda_2 = 0$$

or $\lambda^2 - \underbrace{\mathrm{Tr}\,\rho_A}_{=1}\cdot\lambda + \det\rho_A = 0$

$$\lambda = \frac{1 \pm \sqrt{1-4\det\rho_A}}{2} = |d_1|^2, |d_2|^2$$

$$\det\rho_A = \left(|a|^2+|b|^2\right)\left(|c|^2+|d|^2\right) - \left(ac^*+bd^*\right)\left(a^*c+b^*d\right) = |ad-bc|^2 = \frac{1}{4}C^2 \,!$$

$\uparrow$ concurrence

$$\lambda_{1,2} = \frac{1 \pm \sqrt{1-C^2}}{2}$$

$$\rho_A \Rightarrow \begin{bmatrix} |d_1|^2 & 0 \\ 0 & |d_2|^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

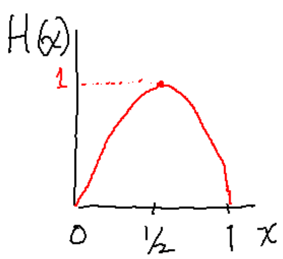
$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$: $|0'\rangle$ pure state

$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$: $|1'\rangle$ pure state

$\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$: $|0'\rangle$ 50%, $|1'\rangle$ 50% maximally mixed state

$$H(\lambda_1) = H(\lambda_2) = S(\rho_A) = -\mathrm{Tr}\{\rho_A \log_2 \rho_A\}$$

$$= -\lambda_1 \log_2 \lambda_1 - \lambda_2 \log_2 \lambda_2$$



- Purification
  - ✓ Church of Larger Hilbert space

- Measure of Entanglement
  - ✓ bipartite: C, S
  - ✓ tripartite: GHZ, W
  - ✓ multipartite: should not be a scalar

Bipartite Qubit Entanglement
Hardy's Nonlocality

"Not about the expectation values"

- ➢ Bell's inequality, CHSH
  - ✓ bipartite qubit entanglement
  - ✓ inequality for the expectation values

- ➢ GHZ
  - ✓ tripartite qubit entanglement
  - ✓ not about the expectation values

- ➢ Hardy's nonlocality: The SIMPLEST
  - ✓ bipartite qubit entanglement
  - ✓ not about the expectation values

Bell's inequality, CHSH

Alice
$Q = \pm 1$
$R = \pm 1$

1 particle ⟷ 1 particle

Bob
$S = \pm 1$
$T = \pm 1$

$$QS + RS + RT - QT = (Q+R)S + (R-Q)T$$

$$QS + RS + RT - QT = \pm 2$$

$$\begin{aligned}\mathbf{E}(QS+RS+RT-QT) &= \sum_{qrst} p(q,r,s,t)(qs+rs+rt-qt)\\ &\le \sum_{qrst} p(q,r,s,t)\times 2\\ &= 2.\end{aligned}$$

$$\begin{aligned}\mathbf{E}(QS+RS+RT-QT) &= \sum_{qrst} p(q,r,s,t)qs + \sum_{qrst} p(q,r,s,t)rs\\ &\quad + \sum_{qrst} p(q,r,s,t)rt - \sum_{qrst} p(q,r,s,t)qt\\ &= \mathbf{E}(QS)+\mathbf{E}(RS)+\mathbf{E}(RT)-\mathbf{E}(QT).\end{aligned}$$

$$\mathbf{E}(QS)+\mathbf{E}(RS)+\mathbf{E}(RT)-\mathbf{E}(QT) \le 2.$$

Bell's inequality, CHSH inequality (Classical)

$$|\psi\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$Q = Z_1 \qquad S = \frac{-Z_2 - X_2}{\sqrt{2}}$$

$$R = X_1 \qquad T = \frac{Z_2 - X_2}{\sqrt{2}}.$$

$$\langle QS\rangle = \frac{1}{\sqrt{2}};\ \langle RS\rangle = \frac{1}{\sqrt{2}};\ \langle RT\rangle = \frac{1}{\sqrt{2}};\ \langle QT\rangle = -\frac{1}{\sqrt{2}}.$$

$$\langle QS\rangle + \langle RS\rangle + \langle RT\rangle - \langle QT\rangle = 2\sqrt{2}.$$

Violation of inequality (Quantum)

from Nielsen and Chuang's book

GHZ: Tripartite Qubit Entanglement

Not expectation values

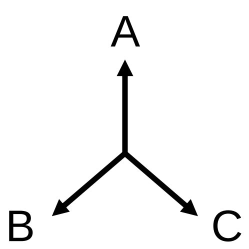
Z-basis: $|0\rangle$ with -1, $|1\rangle$ with $+1$

X-basis: $|a\rangle \approx |0\rangle + |1\rangle$ with -1, $|b\rangle \approx |0\rangle - |1\rangle$ with $+1$

$|0\rangle \approx |a\rangle + |b\rangle$, $|1\rangle \approx |a\rangle - |b\rangle$

Y-basis: $|c\rangle \approx |0\rangle + i|1\rangle$ with -1, $|d\rangle \approx |0\rangle - i|1\rangle$ with $+1$

$|0\rangle \approx |c\rangle + |d\rangle$, $|1\rangle \approx -i(|c\rangle - |b\rangle)$



$$\begin{aligned}|GHZ\rangle &= \frac{1}{\sqrt{2}}\{|000\rangle + |111\rangle\} \quad : \text{Z-basis}\\ &\approx (a+b)(a+b)(a+b) + (a-b)(a-b)(a-b)\\ &\approx aaa + abb + bab + bba \quad : xxx = -1\\ &\approx (a+b)(c+d)(c+d) - (a-b)(c-d)(c-d)\\ &\approx acd + adc + bcc + bdd \quad : xyy = +1\end{aligned}$$

$xxx = -1$
$xyy = +1$
$yxy = +1$
$yyx = +1$

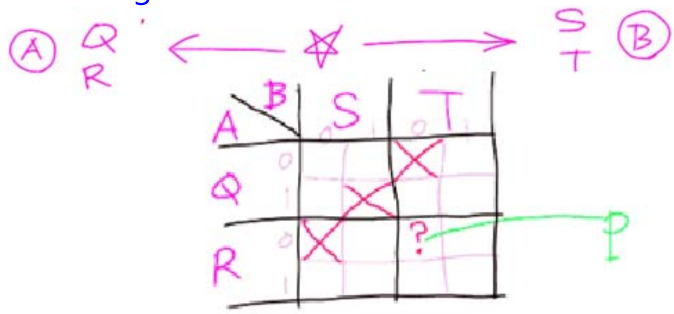
$$(x^2y^2)_A (x^2y^2)_B (x^2y^2)_C = 1 \quad ?$$

Hardy's nonlocality

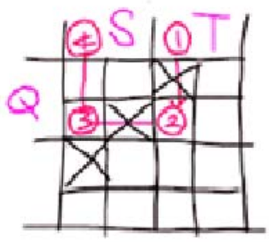
Bipartite Qubit Entanglement

Not expectation values

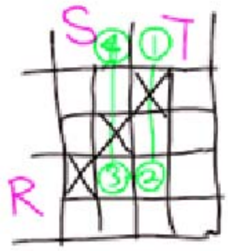
counterfactual argument



Local Determinism



S=0



S=1

	Q	R	S	T	
1	0	0	0	0	Q=0, T=0
2	0	0	0	1	R=0, S=0
3	0	0	1	0	Q=0, T=0
4	0	0	1	1	
5	0	1	0	0	Q=0, T=0
6	0	1	0	1	
7	0	1	1	0	Q=0, T=0
8	0	1	1	1	
9	1	0	0	0	R=0, S=0
10	1	0	0	1	R=0, S=0
11	1	0	1	0	Q=1, S=1
12	1	0	1	1	Q=1, S=1
13	1	1	0	0	
14	1	1	0	1	
15	1	1	1	0	Q=1, S=1
16	1	1	1	1	Q=1, S=1

Q=0, T=0 !

Q=1, S=1 !

R=0, S=0 !

R=0, T=0 ?

$$\left|\Psi\right\rangle = a\left|00\right\rangle + b\left|01\right\rangle + c\left|10\right\rangle + \underbrace{d\left|11\right\rangle}_{=0} \quad \text{with } |a|^2+|b|^2+|c|^2=1$$

$$\left\langle 00''\middle|\Psi\right\rangle = 0 = a\,{}_B\left\langle 0''\middle|0\right\rangle_B + b\,{}_B\left\langle 0''\middle|1\right\rangle_B$$

$$\left\langle 0'0\middle|\Psi\right\rangle = 0 = a\,{}_A\left\langle 0'\middle|0\right\rangle_A + c\,{}_A\left\langle 0'\middle|1\right\rangle_A$$

$$p = \left|\left\langle 0'0''\middle|\Psi\right\rangle\right|^2 = |a|^2 \underbrace{\left|{}_A\left\langle 0'\middle|0\right\rangle_A\right|^2}_{=p_A} \underbrace{\left|{}_B\left\langle 0''\middle|0\right\rangle_B\right|^2}_{=p_B} = \frac{p_A p_B (1-p_A)(1-p_B)}{1-p_A p_B}$$

Maximize p !

$$\frac{\partial p}{\partial p_A} = \frac{\partial p}{\partial p_B} = 0 \;\Rightarrow 1-2p_A + p_A{}^2 p_B = 1-2p_A + p_A{}^2 p_B = 0$$

$$p_A = p_B = 1, \frac{-1\pm\sqrt{5}}{2} \;\;\Rightarrow\;\; p_A = p_B = \frac{\sqrt{5}-1}{2} = g: \text{ golden mean}$$

$$p = g^5 \approx 9\ \%$$

$$\begin{aligned}
\left|\Psi\right\rangle &= a\left|00\right\rangle + b\left|01\right\rangle + c\left|10\right\rangle + \cancel{d}\left|11\right\rangle \quad \text{with } d=0\\
&= \cancel{a'}\left|0'0\right\rangle + b'\left|0'1\right\rangle + c'\left|1'0\right\rangle + d'\left|1'1\right\rangle \quad \text{with } a'=0\\
&= \cancel{a''}\left|00''\right\rangle + b''\left|01''\right\rangle + c''\left|10''\right\rangle + d''\left|11''\right\rangle \quad \text{with } a''=0\\
&= \alpha\left|0'0''\right\rangle + \beta\left|0'1''\right\rangle + \gamma\left|1'0''\right\rangle + \delta\left|1'1''\right\rangle \quad \text{with } \alpha \neq 0\,?
\end{aligned}$$

with $|a|^2+|b|^2+|c|^2=1$ for normalization.

Since $a'=0$,

$$\left\langle 0'0\middle|\Psi\right\rangle = a\left\langle 0'\middle|0\right\rangle + c\left\langle 0'\middle|1\right\rangle = 0.$$

Since $a''=0$,

$$\left\langle 00''\middle|\Psi\right\rangle = a\left\langle 0''\middle|0\right\rangle + b\left\langle 0''\middle|1\right\rangle = 0.$$

$$\begin{aligned}
\alpha &= \left\langle 0'0''\middle|\Psi\right\rangle\\
&= \left\langle 0'0''\right|\left\{\left|00\right\rangle\left\langle 00\right| + \left|01\right\rangle\left\langle 01\right| + \left|10\right\rangle\left\langle 10\right| + \left|11\right\rangle\left\langle 11\right|\right\}\left|\Psi\right\rangle\\
&= \left\langle 0'\middle|0\right\rangle\left\langle 0''\middle|0\right\rangle a + \left\langle 0'\middle|0\right\rangle\left\langle 0''\middle|1\right\rangle b + \left\langle 0'\middle|1\right\rangle\left\langle 0''\middle|0\right\rangle c\\
&= a\left\langle 0'\middle|0\right\rangle\left\langle 0''\middle|0\right\rangle - a\left\langle 0'\middle|0\right\rangle\left\langle 0''\middle|0\right\rangle - a\left\langle 0'\middle|0\right\rangle\left\langle 0''\middle|0\right\rangle\\
&= -a\left\langle 0'\middle|0\right\rangle\left\langle 0''\middle|0\right\rangle
\end{aligned}$$

Let us call the probability of R=+1 and T=+1 p.

Let $s = |\langle 0'|0\rangle|^2$ and $t = |\langle 0''|0\rangle|^2$.

Since $|a|^2 + |b|^2 + |c|^2 = 1$, $a\langle 0'|0\rangle + c\langle 0'|1\rangle = 0$, and $a\langle 0''|0\rangle + b\langle 0''|1\rangle = 0$,

$$p = |\alpha|^2 = \frac{s(1-s)t(1-t)}{1-st}.$$

To find the s and t maximizing p,

$$\frac{\partial p}{\partial s} = \frac{t(1-t)}{(1-st)^2}\left(1-2s+s^2 t\right) = 0$$

and

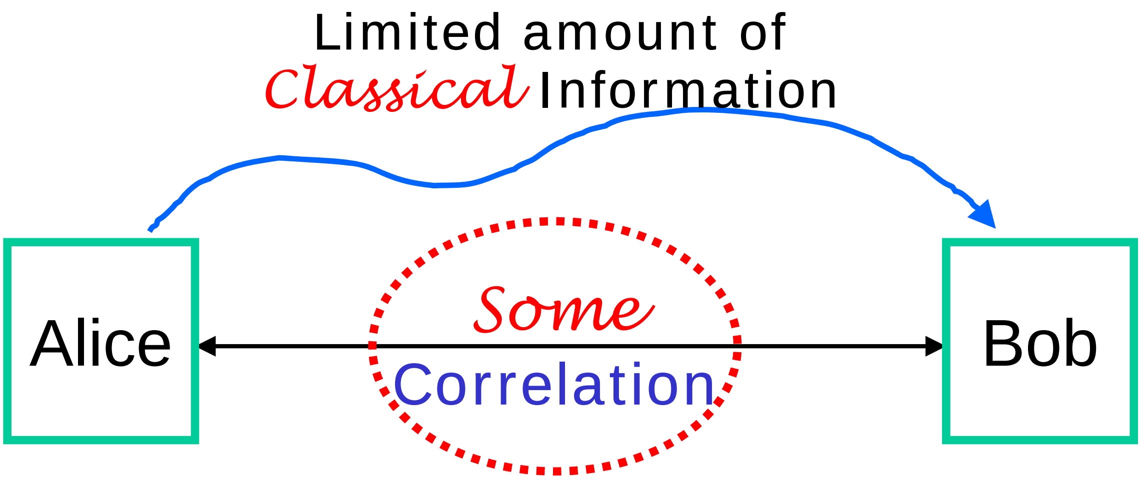
$$\frac{\partial p}{\partial t} = \frac{s(1-s)}{(1-st)^2}\left(1-2t+t^2 s\right) = 0.$$

Thus p is maximized when

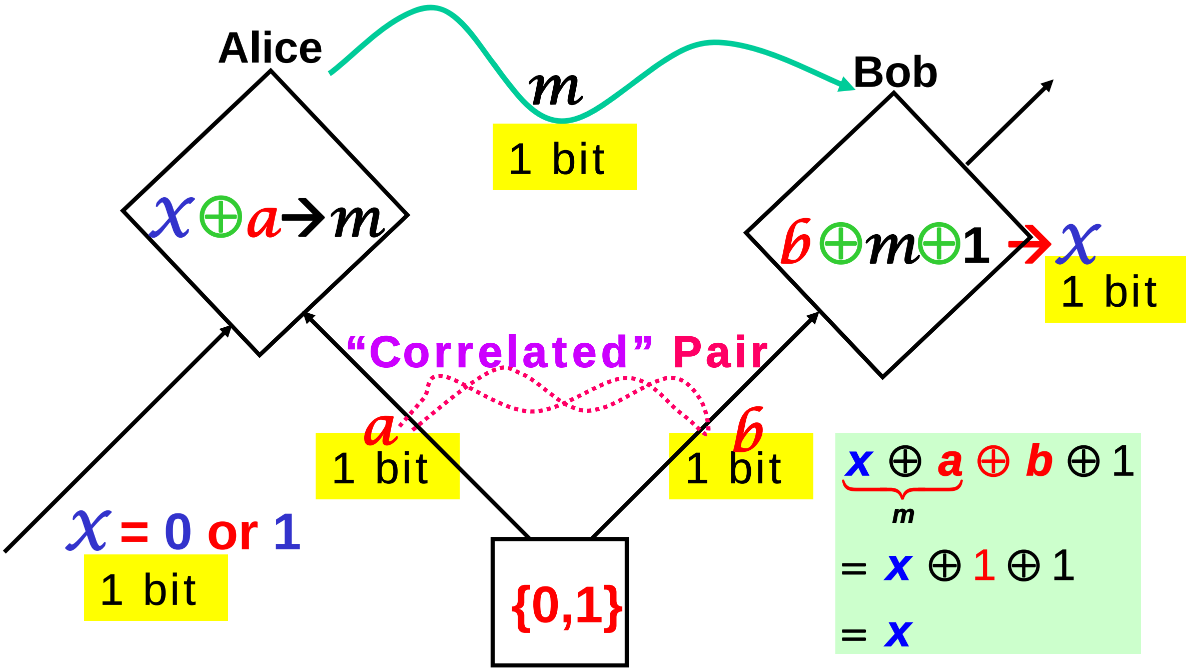
$$s = t = \frac{-1+\sqrt{5}}{2} = g \text{ (golden mean)}$$

$$p = g^5 \approx 9\%$$

Contrary to the classical case with reality assumed,
quantum mechanical case could give nonzero probability of
R=T=+1 case. This is the first showup of golden mean in quantum mechanics.

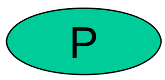


Classical Teleportation?

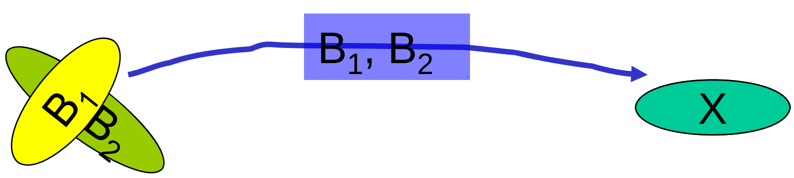


Quantum Teleportation

- Transportation
  - Continuous movement through space



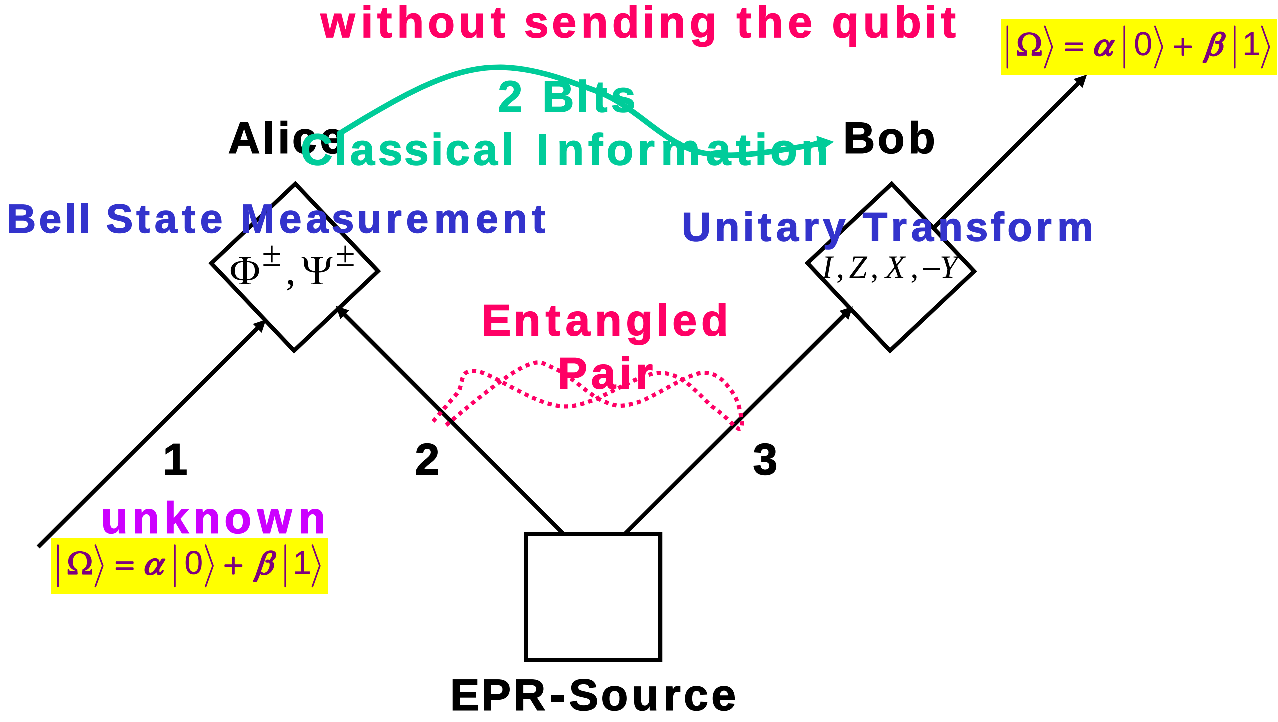
- Quantum Teleportation



Bennett

Quantum Teleportation

Transmit an unknown qubit without sending the qubit



Nonlocality implied in Q. Teleportation

Alice — Bob

$|\psi\rangle = a|0\rangle + b|1\rangle$ $\quad \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$

$ax + by \qquad xx + yy$

$(a\ \ b)\begin{pmatrix} x \\ y \end{pmatrix} \quad (x\ \ y)\begin{pmatrix} x \\ y \end{pmatrix}$

Bell meas. at Alice
+ cl. Comm [2 Bit]
+ U_B (Bob)

not enough to transfer the whole values of a & b!

Nonlocal!

$$| \ \rangle_Q | \ \rangle_A | \ \rangle_B$$

$$\left(a|0\rangle_Q + b|1\rangle_Q\right)\left\{\frac{|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B}{\sqrt{2}}\right\}$$

$$\equiv (ax+by)\left\{\frac{xx+yy}{\sqrt{2}}\right\}$$

$$= \frac{a}{\sqrt{2}}xxx + \frac{a}{\sqrt{2}}xyy + \frac{b}{\sqrt{2}}yxx + \frac{b}{\sqrt{2}}yyy \equiv |\Xi\rangle$$

$$= \frac{1}{2}\underbrace{\left(\frac{xx+yy}{\sqrt{2}}\right)}_{|\Phi^+\rangle}(ax+by) + \frac{1}{2}\underbrace{\left(\frac{xx-yy}{\sqrt{2}}\right)}_{|\Phi^-\rangle}(ax-by)$$

$$+ \frac{1}{2}\underbrace{\left(\frac{xy+yx}{\sqrt{2}}\right)}_{|\Psi^+\rangle}(ay+bx) + \frac{1}{2}\underbrace{\left(\frac{xy-yx}{\sqrt{2}}\right)}_{|\Psi^-\rangle}(ay-bx)$$

$$\left(a|0\rangle_Q + b|1\rangle_Q\right)\left\{\frac{|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B}{\sqrt{2}}\right\} \equiv (ax+by)\left\{\frac{xx+yy}{\sqrt{2}}\right\}$$

$$= \begin{pmatrix} a & b \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} / \sqrt{2} = \begin{pmatrix} a & b \end{pmatrix}\begin{pmatrix} xx & xy \\ yx & yy \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} / \sqrt{2}$$

$$= \begin{pmatrix} a & b \end{pmatrix}\begin{pmatrix} xx & 0 \\ 0 & yy \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} / \sqrt{2} + \begin{pmatrix} a & b \end{pmatrix}\begin{pmatrix} 0 & xy \\ yx & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} / \sqrt{2}$$

$$= \begin{pmatrix} xx & yy \end{pmatrix}\begin{pmatrix} ax \\ by \end{pmatrix} / \sqrt{2} + \begin{pmatrix} xy & yx \end{pmatrix}\begin{pmatrix} ay \\ bx \end{pmatrix} / \sqrt{2}$$

$$= \begin{pmatrix} xx & yy \end{pmatrix} HH \begin{pmatrix} ax \\ by \end{pmatrix} / \sqrt{2} + \begin{pmatrix} xy & yx \end{pmatrix} HH \begin{pmatrix} ay \\ bx \end{pmatrix} / \sqrt{2}$$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{xx+yy}{\sqrt{2}} & \frac{xx-yy}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{ax+by}{\sqrt{2}} \\ \frac{ax-by}{\sqrt{2}} \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{xy+yx}{\sqrt{2}} & \frac{xy-yx}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{ay+bx}{\sqrt{2}} \\ \frac{ay-bx}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{1}{2}\left(\frac{xx+yy}{\sqrt{2}}\right)(ax+by) + \frac{1}{2}\left(\frac{xx-yy}{\sqrt{2}}\right)(ax-by)$$

$$+ \frac{1}{2}\left(\frac{xy+yx}{\sqrt{2}}\right)(ay+bx) + \frac{1}{2}\left(\frac{xy-yx}{\sqrt{2}}\right)(ay-bx)$$

Bell Measurement

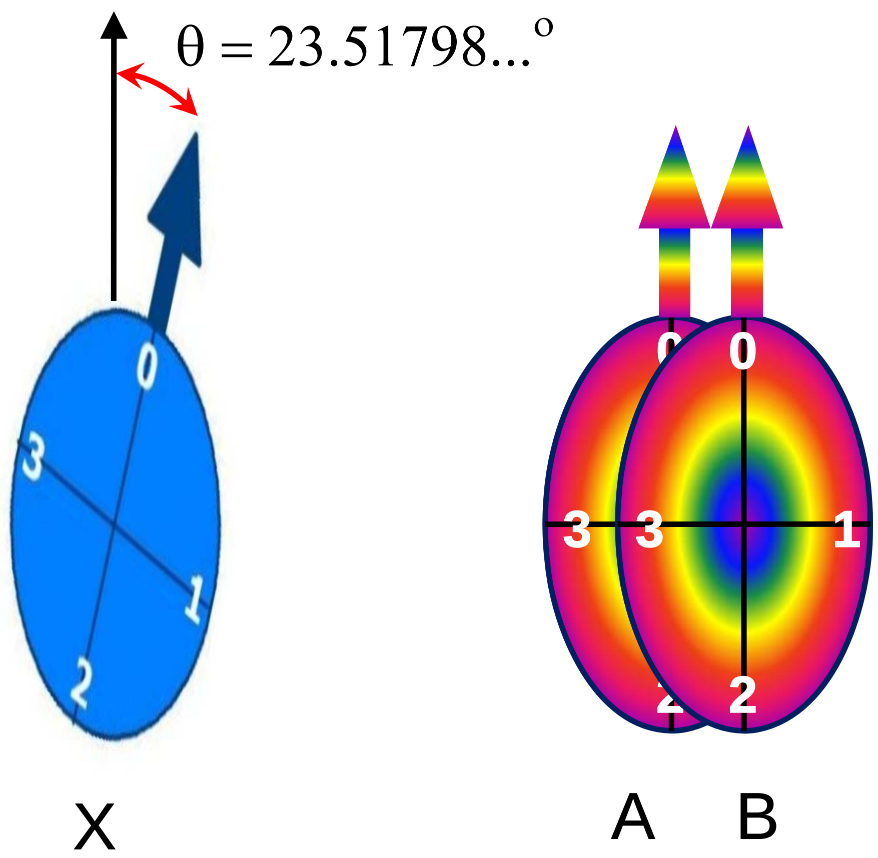
$$X_{A|Q} \equiv |0\rangle_Q\,{}_Q\langle 0| \otimes I_A + |1\rangle_Q\,{}_Q\langle 1| \otimes X_A$$
$$X_A|0\rangle_A = |1\rangle_A \quad \text{and} \quad X_A|1\rangle_A = |0\rangle_A$$
$$H_Q|0\rangle_Q = \frac{1}{\sqrt{2}}\left(|0\rangle_Q + |1\rangle_Q\right)$$
$$\text{and} \quad H_Q|1\rangle_Q = \frac{1}{\sqrt{2}}\left(|0\rangle_Q - |1\rangle_Q\right)$$

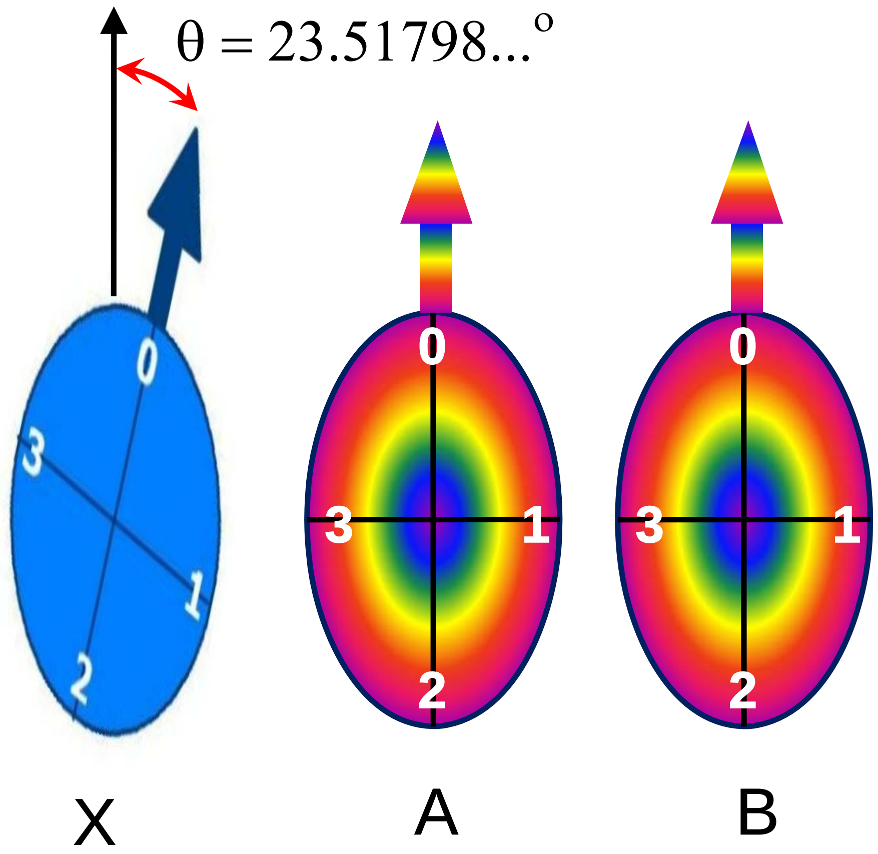
$$H_Q X_{A|Q}\left\{\begin{aligned}&\frac{1}{2}\left(\frac{xx+yy}{\sqrt{2}}\right)(ax+by) + \frac{1}{2}\left(\frac{xx-yy}{\sqrt{2}}\right)(ax-by)\\ &\quad + \frac{1}{2}\left(\frac{xy+yx}{\sqrt{2}}\right)(ay+bx) + \frac{1}{2}\left(\frac{xy-yx}{\sqrt{2}}\right)(ay-bx)\end{aligned}\right\}$$

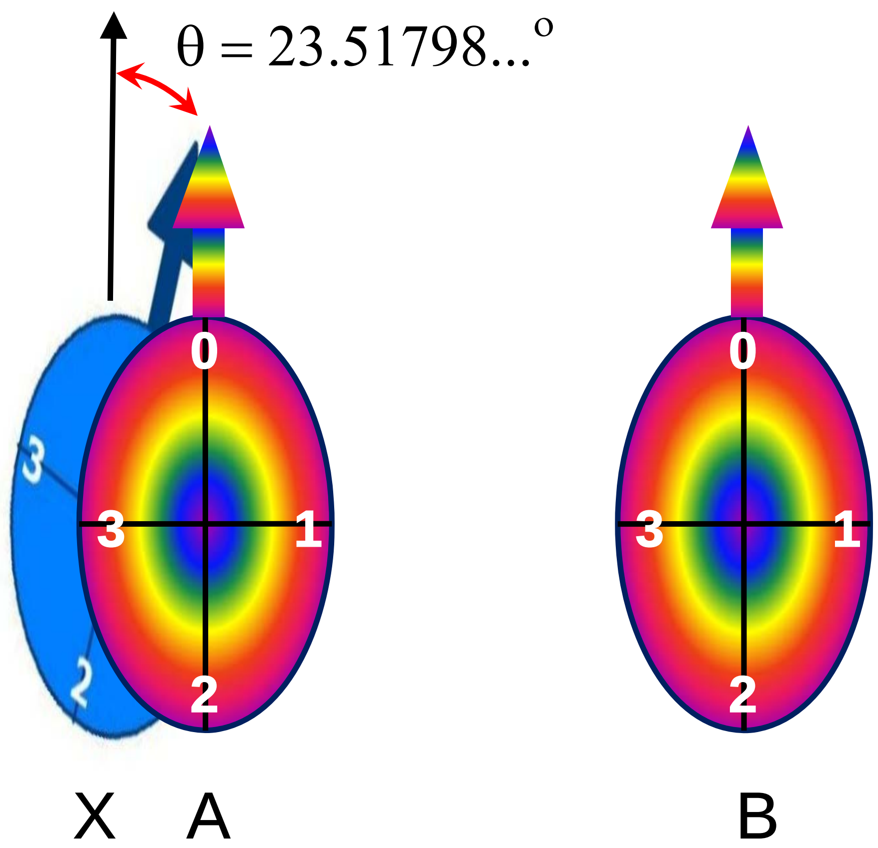
$$= H_Q\left\{\begin{aligned}&\frac{1}{2}\left(\frac{xx+yx}{\sqrt{2}}\right)(ax+by) + \frac{1}{2}\left(\frac{xx-yx}{\sqrt{2}}\right)(ax-by)\\ &\quad + \frac{1}{2}\left(\frac{xy+yy}{\sqrt{2}}\right)(ay+bx) + \frac{1}{2}\left(\frac{xy-yy}{\sqrt{2}}\right)(ay-bx)\end{aligned}\right\}$$

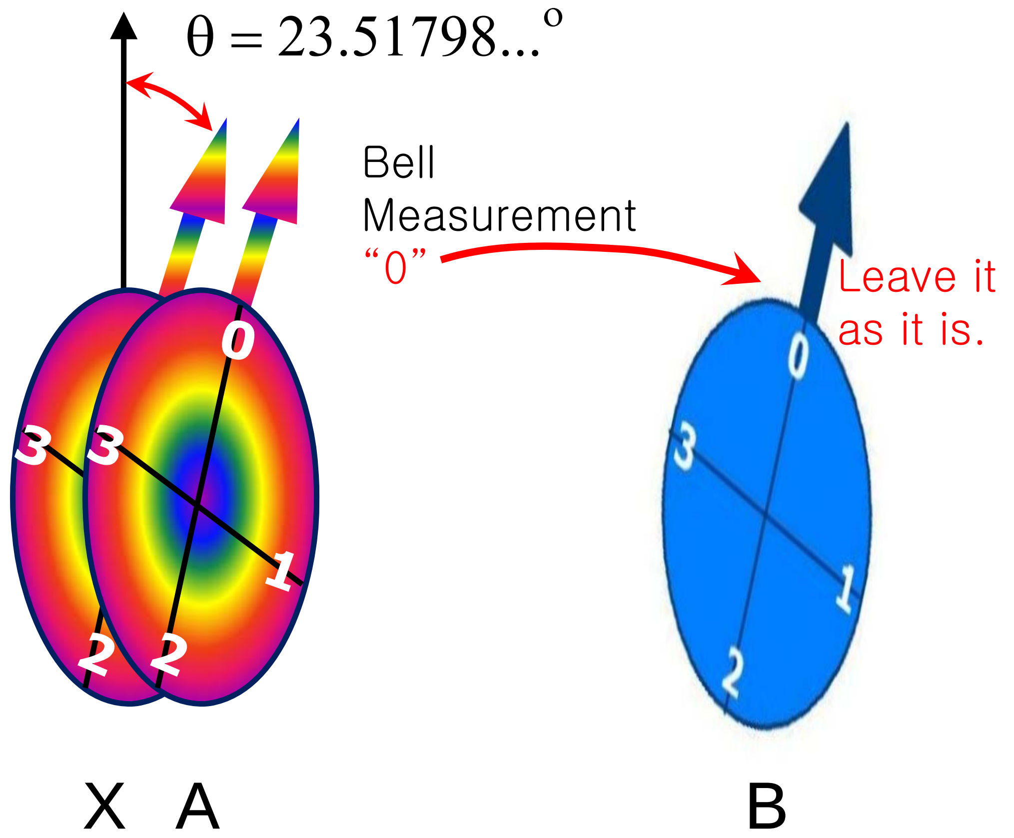
$$= H_Q\left\{\begin{aligned}&\frac{1}{2}\left(\frac{x+y}{\sqrt{2}}\right)x(ax+by) + \frac{1}{2}\left(\frac{x-y}{\sqrt{2}}\right)x(ax-by)\\ &\quad + \frac{1}{2}\left(\frac{x+y}{\sqrt{2}}\right)y(ay+bx) + \frac{1}{2}\left(\frac{x-y}{\sqrt{2}}\right)y(ay-bx)\end{aligned}\right\}$$

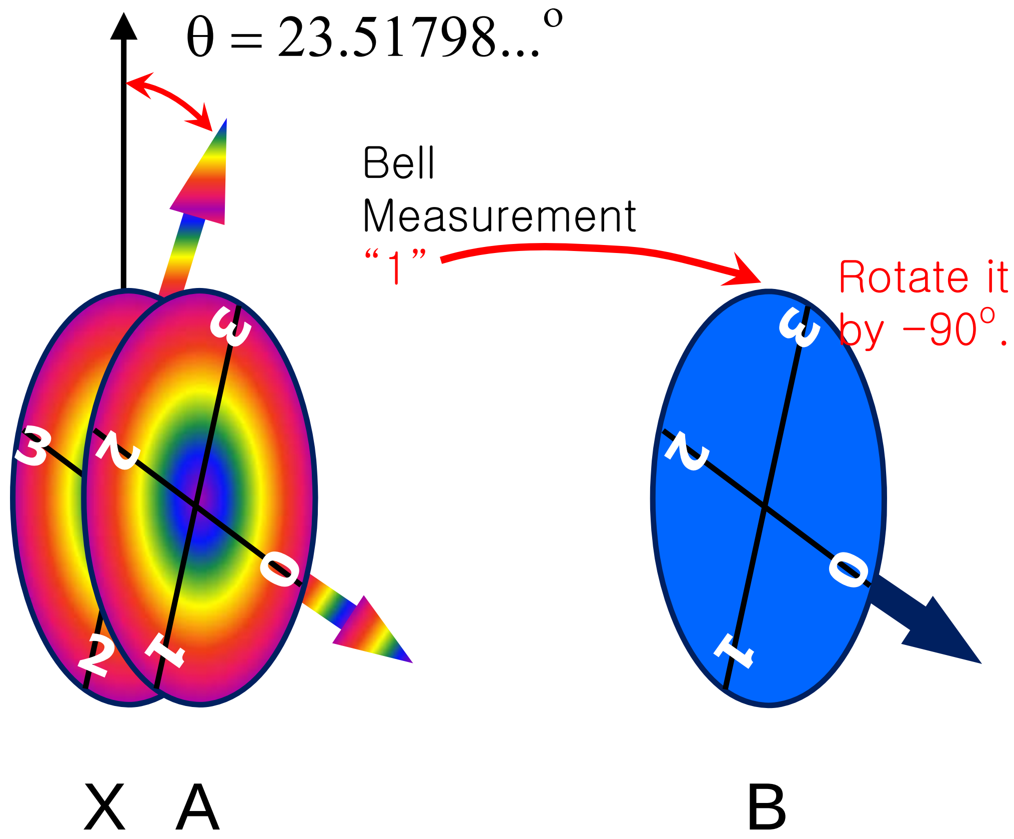
$$= \frac{1}{2}xx(ax+by) + \frac{1}{2}yx(ax-by) + \frac{1}{2}xy(ay+bx) + \frac{1}{2}yy(ay-bx)$$

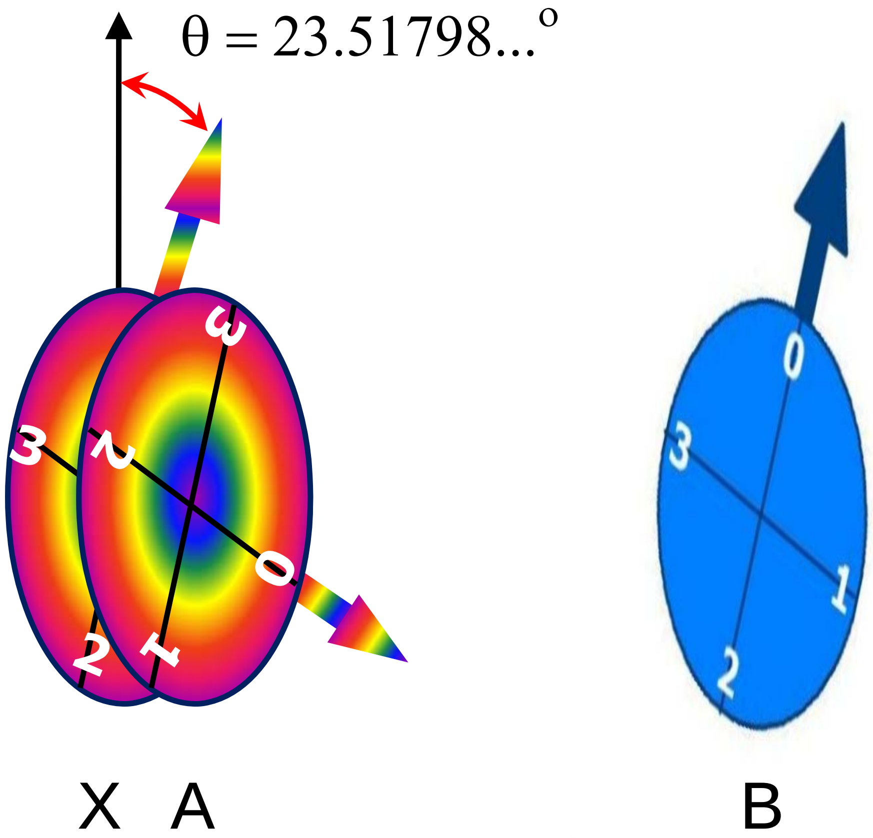



θ = 23.51798...°
0
3
1
2
0
3
1
2
0
3
1
2
X
A
B

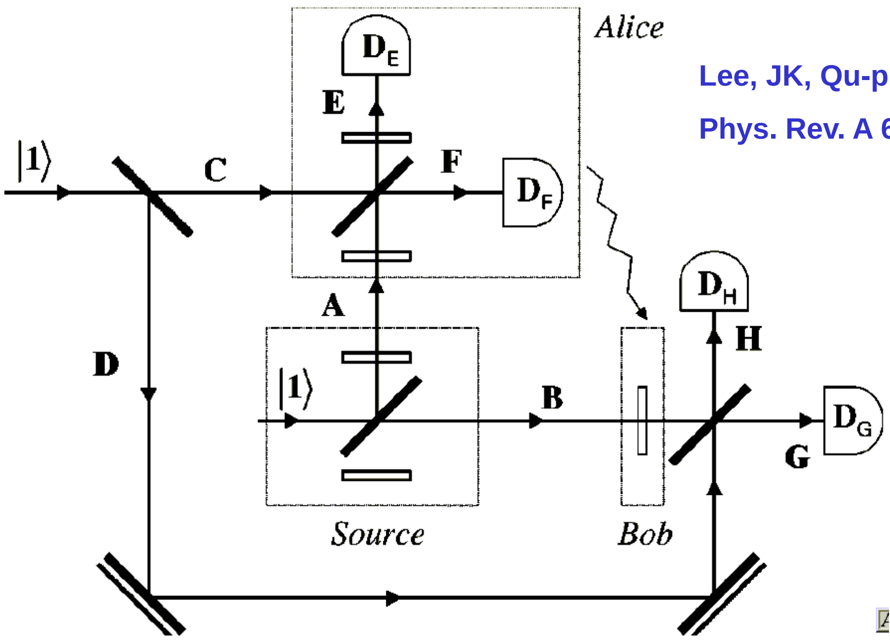

θ = 23.51798...°
3
2
0
3
1
2
0
3
1
2
X
A
B


θ = 23.51798...°
Bell
Measurement
"0"
Leave it
as it is.
0
3
1
2
X
A
B


θ = 23.51798...°
Bell
Measurement
"1"
Rotate it
by −90°.
3
2
0
1
X
A
B



Quantum Teleportation
using Single Particle Entanglement



Lee, JK, Qu-ph/0007106;

Phys. Rev. A 63, 012305 (2001)

PRL88,070402 (2002) or QP/0204158

"Active" Teleportation of a Quantum Bit

S. Giacomini, F. Sciarrino, E. Lombardi and F. De Martini

: 88, NUMBER 7 PHYSICAL REVIEW LETTERS 18 FEBRUARY 2002

Teleportation of a Vacuum–One-Photon Qubit

Egilberto Lombardi,[1] Fabio Sciarrino,[1] Sandu Popescu,[2,3] and Francesco De Martini[1]
Istituto Nazionale di Fisica della Materia, Dipartimento di Fisica, Università "La Sapienza," Roma, 00185 Italy
[2]*H. H. Wills Physics Laboratory, University of Bristol, United Kingdom*
[3]*Hewlett-Packard Laboratories, Stoke Gifford, Bristol BS12-6QZ, United Kingdom*
(Received 24 September 2001; published 30 January 2002)

We report the experimental realization of teleporting a one-particle entangled qubit. The qubit is physically implemented by a two-dimensional subspace of states of a mode of the electromagnetic field, specifically, the space spanned by the vacuum and the one-photon state. Our experiment follows the line suggested by Lee and Kim [Phys. Rev. A 63, 012305 (2000)] and Knill, Laflamme, and Milburn [Nature (London) 409, 46 (2001)]. An unprecedented large value of the teleportation "fidelity" has been attained: $F = (95.3 \pm 0.6)\%$.

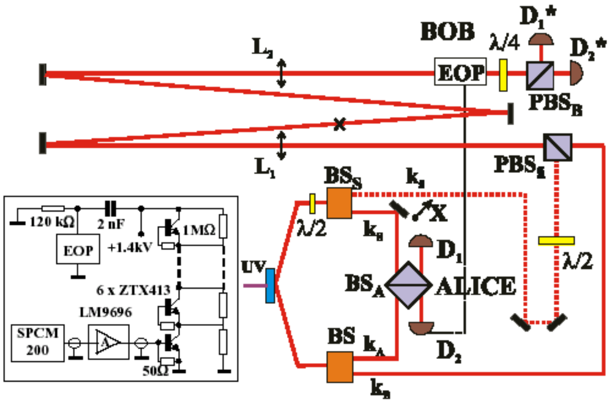
articles

A scheme for efficient quantum computation with linear optics

E. Knill*, R. Laflamme* & G. J. Milburn†
* Los Alamos National Laboratory, MS B265, Los Alamos, New Mexico 87545, USA
† Centre for Quantum Computer Technology, University of Queensland, St. Lucia, Australia

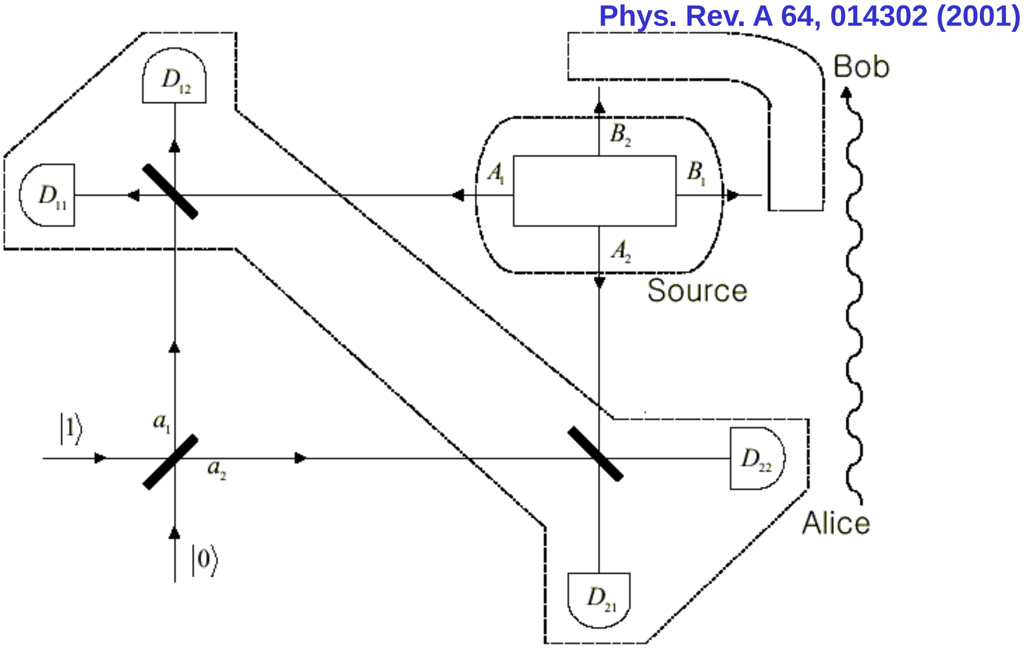
46 NATURE | VOL 409 | 4 JANUARY 2001 | www.nature.com

26 Apr 2002



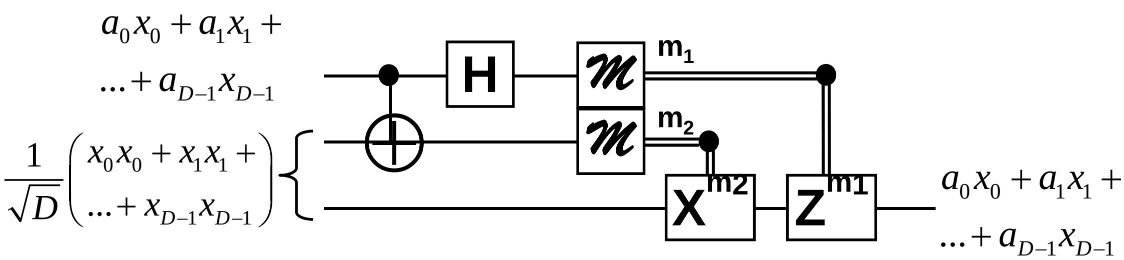
Quantum Teleportation of Single Particle Entanglement

Lee, Qu-ph/0104097;
Phys. Rev. A 64, 014302 (2001)



Generalized QTP: qudit

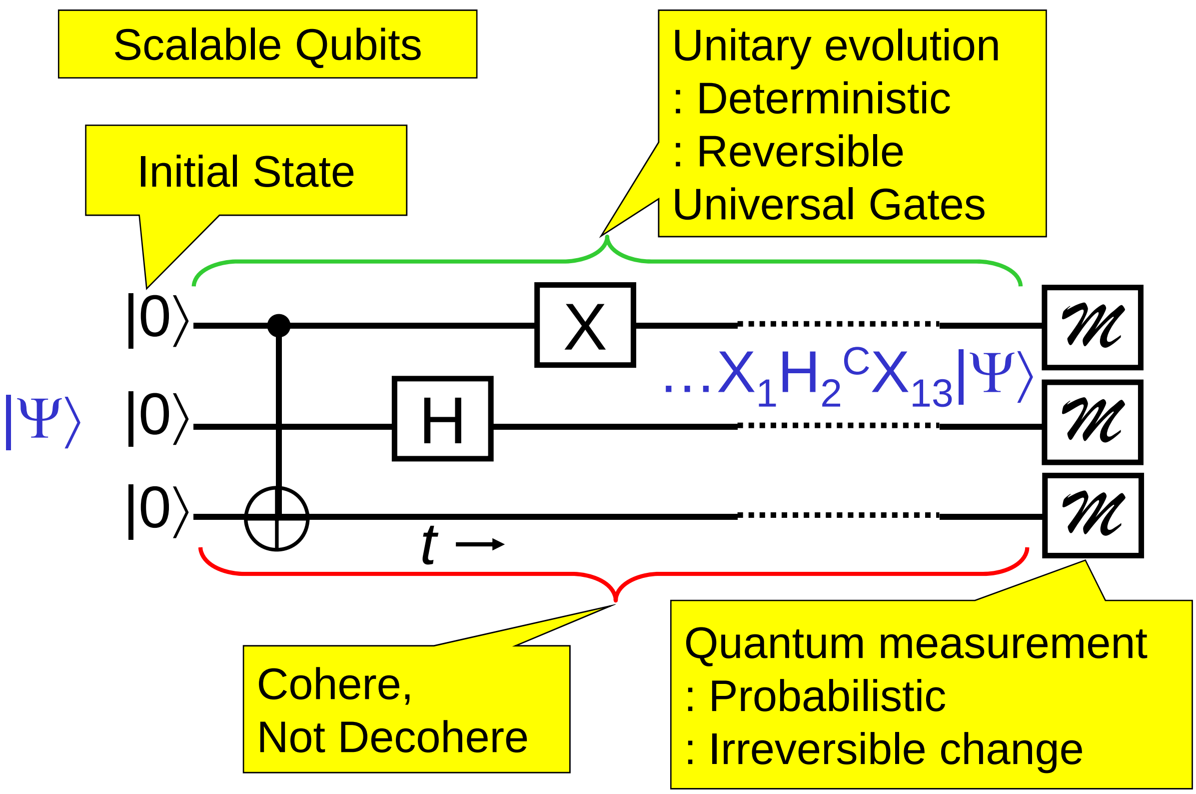
$$x_1 \equiv |0\rangle, x_2 \equiv |1\rangle, \ldots x_{D-1} \equiv |D-1\rangle$$



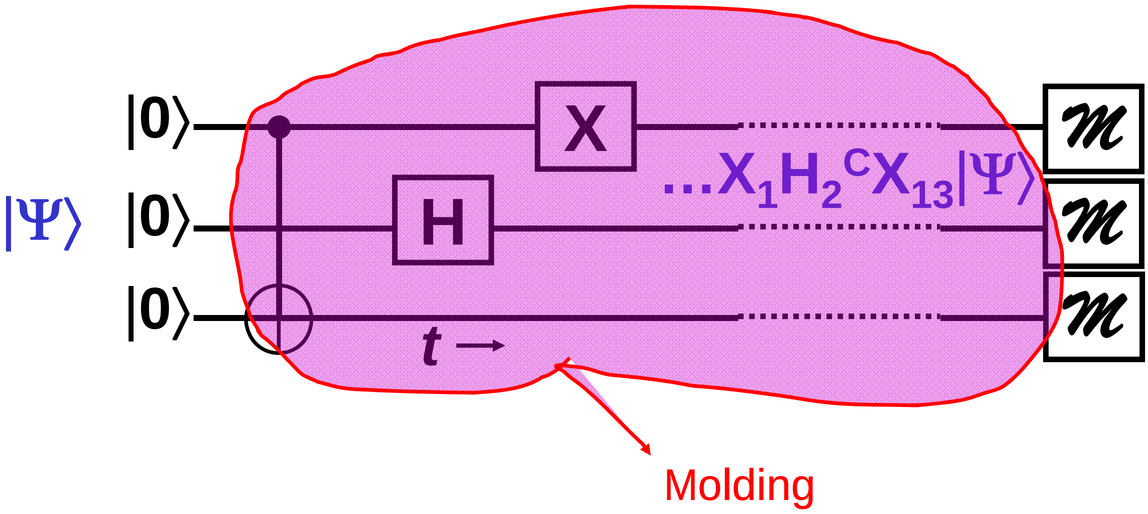
- **Generalized Bell Measurements**
- **Modulo-D Summation**
- **Generalized Hadamard Transformation**

Quantum Network

DiVincenzo, Qu-Ph/0002077

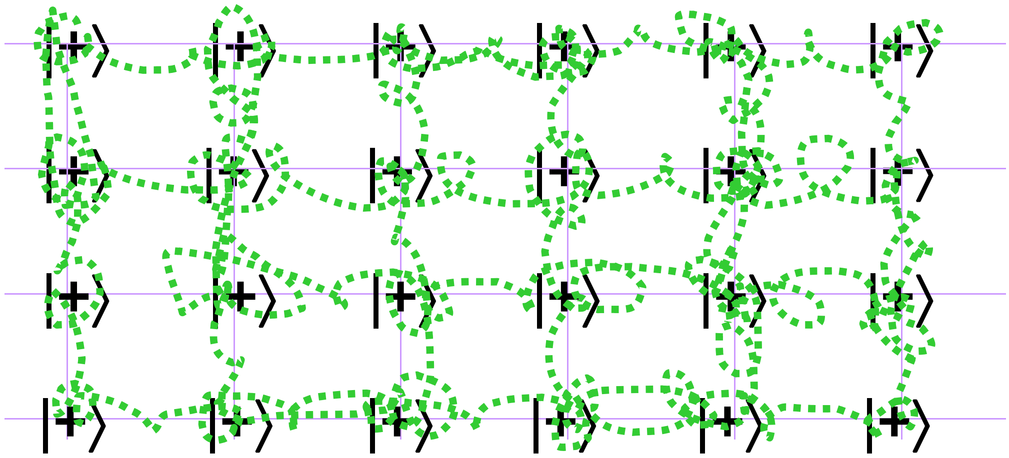


Molding a Quantum State



Sculpturing a Quantum State

- Cluster State Quantum Computing -



1. Initialize each qubit in |+⟩ state.
2. Contolled-Phase between the neighboring qubits.
3. Single qubit manipulations and single qubit measurements only [Sculpturing].
   No two qubit operations!

Cont-Z and $|+\rangle$

- Commuting with each other
- Symmetric w.r.t. control and target

Even superposition of computational basis states

Opt Comm 337, 79-82 (2015)
Jaewan Kim, Juhui Lee, Se-Wan Ji, Hyunchul Nha, Petr M. Anisimov, Jonathan P. Dowling
(Cross Kerr Nonlinear Optical Interaction)

$$Z_{12}|+\rangle_1|+\rangle_2 = Z_{12}\left(\frac{|0\rangle_1+|1\rangle_1}{\sqrt{2}}\right)\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right)$$

$$= Z_{12}\frac{|0\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) + Z_{12}\frac{|1\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right)$$

$$= \frac{|0\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2+|1\rangle_2}{\sqrt{2}}\right) + \frac{|1\rangle_1}{\sqrt{2}}\left(\frac{|0\rangle_2-|1\rangle_2}{\sqrt{2}}\right)$$

$$= \frac{1}{\sqrt{2}}|0\rangle_1|+\rangle_2 + \frac{1}{\sqrt{2}}|1\rangle_1|-\rangle_2 \quad \text{Bell State}$$

$$Z_{12}Z_{23}|+\rangle_1|+\rangle_2|+\rangle_3 = Z_{21}Z_{23}|+\rangle_1\frac{|0\rangle_2}{\sqrt{2}}|+\rangle_3 + Z_{21}Z_{23}|+\rangle_1\frac{|1\rangle_2}{\sqrt{2}}|+\rangle_3$$

$$= \frac{1}{\sqrt{2}}|+\rangle_1|0\rangle_2|+\rangle_3 + \frac{1}{\sqrt{2}}|-\rangle_1|1\rangle_2|-\rangle_3 \quad \text{GHZ state}$$

B. C. Sanders and G. J. Milburn, Phys. Rev. A 45, 1919 (1992).

M. Paternostra et al., Phys. Rev. A 67, 023811 (2003).

Wang W.-F. et al., Chin. Phys. Lett. 25, 839 (2008)

Nguyen B. A. and J. Kim, Phys. Rev. A 80, 042316 (2009).

An Interesting Observation of Exponential Function

Opt Comm 337, 79-82 (2015)
Jaewan Kim, Juhui Lee, Se-Wan Ji, Hyunchul Nha, Petr M. Anisimov, Jonathan P. Dowling
(Cross Kerr Nonlinear Optical Interaction)

$$e^x = \underbrace{1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^{d-1}}{(d-1)!}}_{d \text{ terms}}$$

$$+ \frac{x^d}{d!} + \frac{x^{d+1}}{(d+1)!} + \frac{x^{d+2}}{(d+2)!} + \cdots + \frac{x^{2d-1}}{(2d-1)!}$$

$$+ \frac{x^{2d}}{(2d)!} + \frac{x^{2d+1}}{(2d+1)!} + \frac{x^{2d+1}}{(2d+1)!} + \cdots + \frac{x^{3d-1}}{(3d-1)!}$$

$$+ \cdots$$

$$= f_0(x) + f_1(x) + f_2(x) + \cdots + f_{d-1}(x)$$

$$= \sum_{k=0}^{d-1} f_k(x)$$

$$f_k(x) = \sum_{m=0}^{\infty} \frac{x^{k+md}}{(k+md)!} \quad \text{for } k = 0,1,\cdots,d-1$$

$$f_0(x) = 1 + \frac{x^d}{d!} + \frac{x^{2d}}{(2d)!} + \cdots$$

$$f_1(x) = \frac{x}{1!} + \frac{x^{d+1}}{(d+1)!} + \frac{x^{2d+1}}{(2d+1)!} + \cdots$$

$$\vdots$$

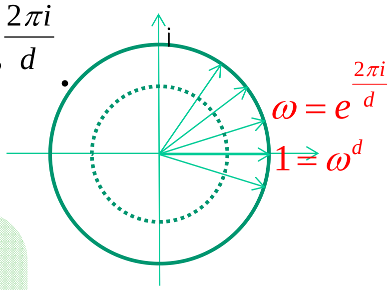
$$f_{d-1}(x) = \frac{x^{d-1}}{(d-1)!} + \frac{x^{2d-1}}{(2d-1)!} + \frac{x^{3d-1}}{(3d-1)!} + \cdots$$

$$f'_{d-1}(x) = f_{d-2}(x)\,,\ f'_{d-2}(x) = f_{d-3}(x)\,,\ \cdots\,,\ f'_1(x) = f_0(x)\,,\ f'_0(x) = f_{d-1}(x)$$

$$\text{As } x \to \infty,\quad \frac{f_l(x)}{e^x} \to \frac{1}{d}\,. \qquad O\left(e^{-\frac{2\pi^2}{d^2}x}\right)$$

Conjugate Relations

$$e_s(x) \equiv e^{\omega^s x} \quad \text{with} \quad \omega = e^{\frac{2\pi i}{d}}.$$



$$\begin{cases} f_k(x) = \dfrac{1}{d}\displaystyle\sum_{s=0}^{d-1} \omega^{-ks} e_s(x) \\ e_s(x) = \displaystyle\sum_{k=0}^{d-1} \omega^{ks} f_k(x) \end{cases}$$

An Interesting Observation of a Coherent State

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

$$\begin{aligned}
&= \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^0}{\sqrt{0!}}\right)|0\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^1}{\sqrt{1!}}\right)|1\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^2}{\sqrt{2!}}\right)|2\rangle + \cdots + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{d-1}}{\sqrt{(d-1)!}}\right)|(d-1)\rangle \\
&+ \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^d}{\sqrt{d!}}\right)|d\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{d+1}}{\sqrt{(d+1)!}}\right)|d+1\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{d+2}}{\sqrt{(d+2)!}}\right)|d+2\rangle + \cdots + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d-1}}{\sqrt{(2d-1)!}}\right)|(2d-1)\rangle \\
&+ \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d}}{\sqrt{(2d)!}}\right)|2d\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d+1}}{\sqrt{(2d+1)!}}\right)|2d+1\rangle + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{2d+2}}{\sqrt{(2d+2)!}}\right)|2d+2\rangle + \cdots + \left(e^{-\frac{|\alpha|^2}{2}}\frac{\alpha^{3d-1}}{\sqrt{(3d-1)!}}\right)|(3d-1)\rangle \\
&+ \cdots \\
&= \frac{|0_d\rangle}{\sqrt{d}} + \frac{|1_d\rangle}{\sqrt{d}} + \frac{|1_d\rangle}{\sqrt{d}} + \cdots + \frac{|1_d\rangle}{\sqrt{d}} \\
&= \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle
\end{aligned}$$

with $|k_d\rangle = \sqrt{d}\cdot e^{-\frac{|\alpha|^2}{2}}\sum_{m=0}^{\infty}\frac{\alpha^{k+md}}{\sqrt{(k+md)!}}|k+md\rangle$

$$\frac{\langle k_d|k_d\rangle}{d} = e^{-|\alpha|^2}\sum_{m=0}^{\infty}\frac{\left(|\alpha|^2\right)^{k+md}}{(k+md)!} \xrightarrow[\text{as } |\alpha|^2\to\infty]{} \frac{1}{d}.$$

$\therefore$ As $|\alpha|^2 \to \infty, \langle k_d|k_d\rangle = 1$ and $\langle k_d|l_d\rangle = \delta_{kl}$.

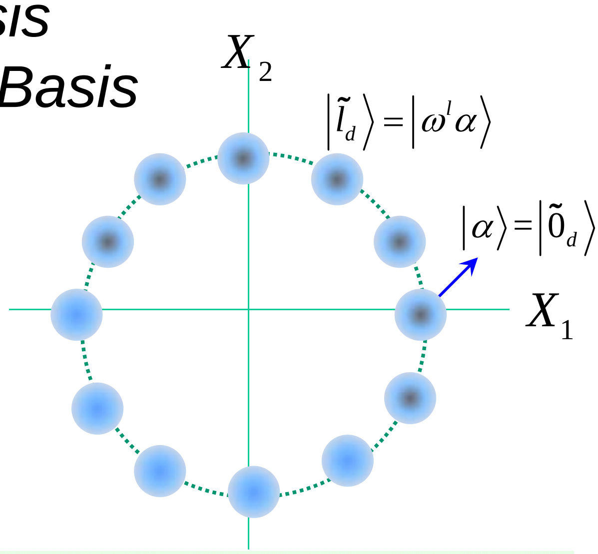
$$O(e^{-\frac{2\pi^2}{d^2}|\alpha|^2})$$

practically $|\alpha| \geq d$

Pseudo-Number Basis and *Pseudo-Phase* Basis

Define $|\tilde{l}_d\rangle = |\omega^l\alpha\rangle$.

$$\begin{cases} |k_d\rangle = \dfrac{1}{\sqrt{d}}\displaystyle\sum_{l=0}^{d-1}\omega^{-kl}|\tilde{l}_d\rangle \\ |\tilde{l}_d\rangle = \dfrac{1}{\sqrt{d}}\displaystyle\sum_{l=0}^{d-1}\omega^{lk}|k_d\rangle \end{cases}$$



$|0_d\rangle$ is the 0th Pseudo-number state (Computational basis) and an Even Superposition of $\underbrace{\text{Pseudo-Phase States}}_{\text{(Coherent States; Conjugate Basis States)}}$.

$|\alpha\rangle = |\tilde{0}_d\rangle$ is the 0th Pseudo-phase state (Conjugate basis) and an Even Superposition of $\underbrace{\text{Pseudo-Number States}}_{\text{(Computational Basis States)}}$.

Qubits and Qu*d*its

Computational Basis

$$\{|0\rangle, |1\rangle\}$$

Conjugate Basis

$$\{|+\rangle, |-\rangle\}$$

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Computational Basis

: pseudo-number basis

$$\{|0_d\rangle, |1_d\rangle, \cdots, |(d-1)_d\rangle\}$$

Conjugate Basis

: pseudo-phase basis

$$\{|\tilde{0}_d\rangle = |\alpha\rangle, |\tilde{1}_d\rangle = |\omega\alpha\rangle, \cdots, |\widetilde{(d-1)}_d\rangle\}$$

$$\omega = e^{\frac{2\pi i}{d}}$$

$$\begin{cases} |k_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{s=0}^{d-1} \omega^{-kl} |\tilde{l}_d\rangle \\ |\tilde{l}_d\rangle = \dfrac{1}{\sqrt{d}} \displaystyle\sum_{k=0}^{d-1} \omega^{lk} |k_d\rangle \end{cases}$$

Qubit Operators and Qu*d*it Operators

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{CNOT} = X_{12} = |0\rangle_1 {}_1\langle 0| \otimes I_2 + |1\rangle_1 {}_1\langle 1| \otimes X_2$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & & \\ 0 & 1 & \multicolumn{2}{c}{\smash{\raisebox{.5\normalbaselineskip}{0}}} \\ \hline & & 0 & 1 \\ \multicolumn{2}{c|}{\smash{\raisebox{.5\normalbaselineskip}{0}}} & 1 & 0 \end{array}\right]$$

$$\text{C-Z} = Z_{12} = |0\rangle_1 {}_1\langle 0| \otimes I_2 + |1\rangle_1 {}_1\langle 1| \otimes Z_2$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & & \\ 0 & 1 & \multicolumn{2}{c}{\smash{\raisebox{.5\normalbaselineskip}{0}}} \\ \hline & & 1 & 0 \\ \multicolumn{2}{c|}{\smash{\raisebox{.5\normalbaselineskip}{0}}} & 0 & -1 \end{array}\right]$$

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = |+\rangle\langle 0| + |-\rangle\langle 1|$$

$$X = \begin{bmatrix} 0 & 1 & 0 & & & \\ & 0 & 1 & \ddots & & \\ & & 0 & \ddots & \ddots & \\ & & & \ddots & 1 & 0 \\ & & & & 0 & 1 \\ 1 & & & & & 0 \end{bmatrix}$$

$$\omega = e^{\frac{2\pi i}{d}}$$

$$Z = \begin{bmatrix} 1 & 0 & 0 & \cdots & & 0 \\ & \omega & 0 & \ddots & & \\ & & \omega^2 & \ddots & \ddots & \vdots \\ & & & \ddots & 0 & 0 \\ & & & & \omega^{d-2} & 0 \\ 0 & & & & & \omega^{d-1} \end{bmatrix} = \sum_{k=0}^{d-1} \omega^k |k_d\rangle\langle k_d| = \omega^{\hat{n}}$$

$$\text{C-Z} = Z_{12} = |0_d\rangle_1 {}_1\langle 0_d| \otimes I_2 + |1_d\rangle_1 {}_1\langle 1_d| \otimes Z_2 + |2_d\rangle_1 {}_1\langle 2_d| \otimes Z^2{}_2 + \cdots$$

$$= \sum_{k=0}^{d-1} \sum_{l=0}^{d-1} |k_d\rangle_1 {}_1\langle k_d| \omega^{kl} |l_d\rangle_2 {}_2\langle l_d| = \omega^{\hat{n}_1 \hat{n}_2}$$

$$H = \frac{1}{\sqrt{d}} \begin{bmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & \omega & \omega^2 & \omega^3 & \cdots & \\ 1 & \omega^2 & \omega^4 & \omega^6 & & \\ \vdots & \omega^3 & \omega^6 & \ddots & & \\ \vdots & \vdots & & & \ddots & \\ 1 & & & & & \omega^{(d-1)^2} \end{bmatrix} = \sum_{k=0}^{d-1} |\tilde{k}_d\rangle\langle k_d|$$

Cf. D. L. Zhou et al., Phys. Rev. A 68, 062303 (2003)

Generalized Controlled-Z Operator

$$H = -\chi \hat{n}_1 \hat{n}_2$$

Cross Kerr Interaction

C. F. Roos et al., Phys. Rev. A 77, 040302(R) (2008).
Michael Siomau, Ali A. Kamli, Sergey A. Moiseev, and Barry C. Sanders, Phys. Rev. A 85, 050303(R) (2012)

$$|\chi L| = \frac{2\pi}{d},\ d \lesssim |\alpha|$$

($d \cong$ 10~1000 ?!)

$$U = e^{i\chi L \hat{n}_1 \hat{n}_2} = e^{\frac{2\pi i}{d}\hat{n}_1\hat{n}_2} = \omega^{\hat{n}_1\hat{n}_2} = Z_{12}$$

Opt Comm 337, 79-82 (2015)
Jaewan Kim, Juhui Lee, Se-Wan Ji, Hyunchul Nha, Petr M. Anisimov, Jonathan P. Dowling
(Cross Kerr Nonlinear Optical Interaction)

$$Z_{12}|\alpha\rangle_1|\alpha\rangle_2 = \omega^{\hat{n}_1\hat{n}_2}\frac{1}{d}\sum_{k=0}^{d-1}|k_d\rangle\sum_{l=0}^{d-1}|l_d\rangle$$

$$= \frac{1}{d}\sum_{k=0}^{d-1}\sum_{l=0}^{d-1}\omega^{kl}|k_d\rangle|l_d\rangle$$

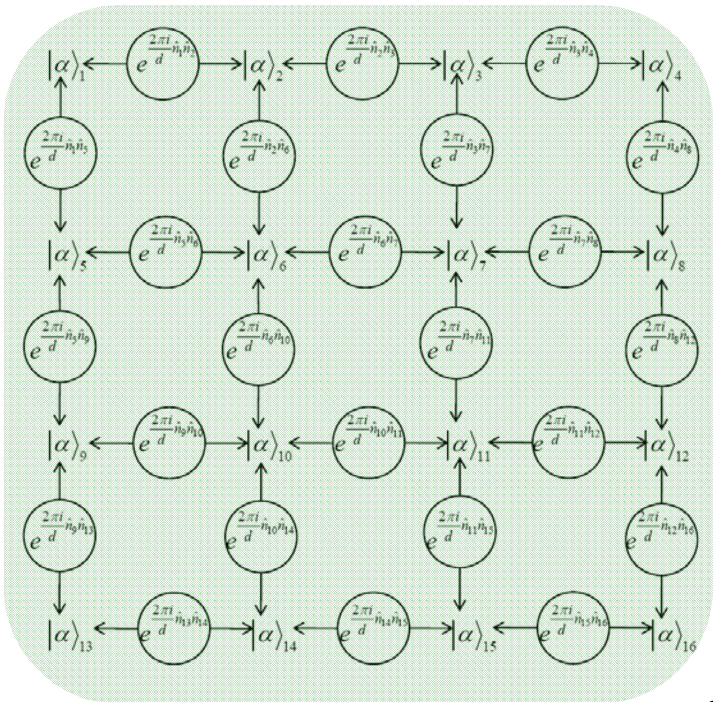
$$= \frac{1}{\sqrt{d}}\sum_{l=0}^{d-1}|\tilde{l}_d\rangle|l_d\rangle = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}|k_d\rangle|\tilde{k}_d\rangle$$

Maximal Entanglement of
Pseudo-Number State and Pseudo-Phase State

Opt Comm 337, 79-82 (2015)
Jaewan Kim, Juhui Lee, Se-Wan Ji, Hyunchul Nha, Petr M. Anisimov, Jonathan P. Dowling
(Cross Kerr Nonlinear Optical Interaction)

"Deterministic" Generation of a Qu*d*it Cluster State

$$\sum_{\langle p,q\rangle} e^{\frac{2\pi i}{d}\hat{n}_p\hat{n}_q}\prod_{r\in \mathrm{lattice}}|\alpha\rangle_r$$



Tayloring

Scissors:
measurements in pseudo-number basis (Z)

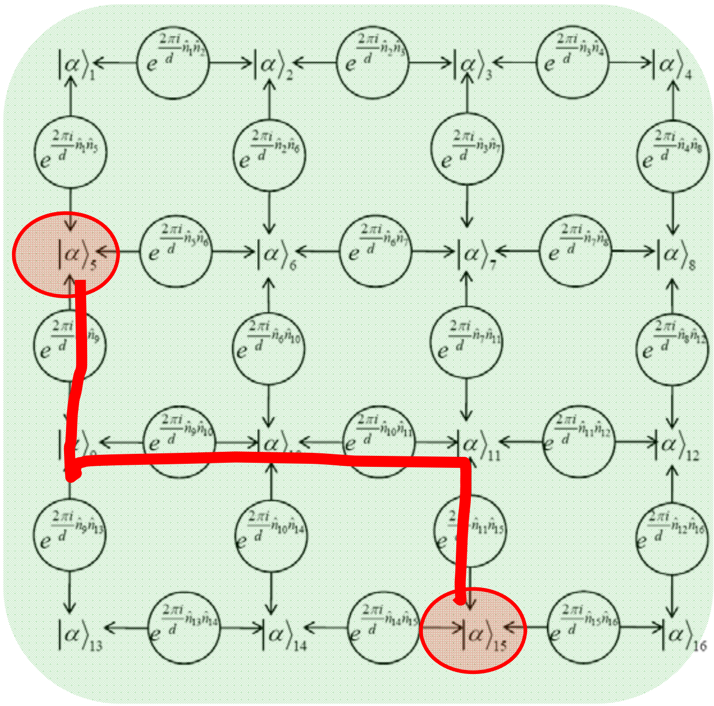
Stitches:
measurements in pseudo-phase basis (X)

Bell State

Tayloring

Scissors:
measurements in pseudo-number basis (Z)

Stitches:
measurements in pseudo-phase basis (X)

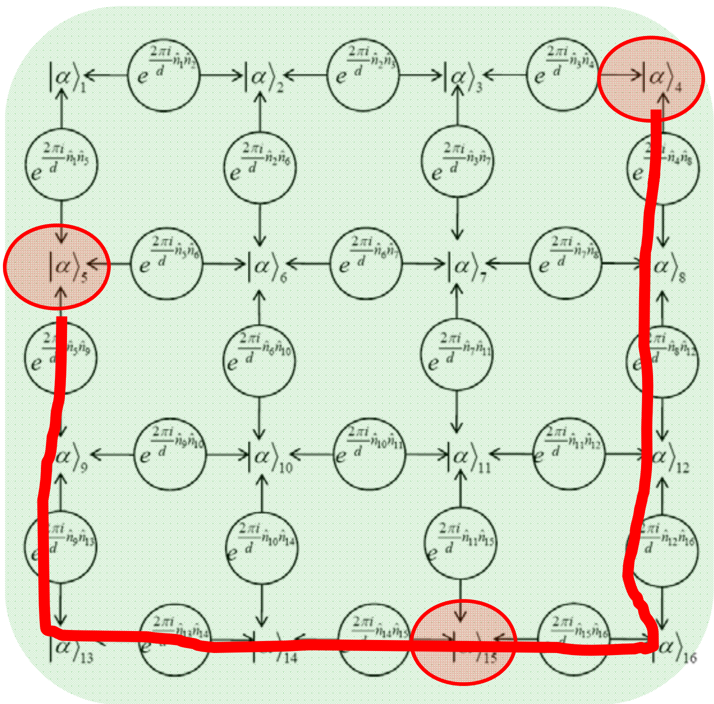


GHZ State

Tayloring

Scissors:
measurements in pseudo-number basis (Z)

Stitches:
measurements in pseudo-phase basis (X)



Entanglement of pure states

$$|\Psi\rangle_{AB} = a_0 |0\rangle_A |0\rangle_B + a_1 |1\rangle_A |1\rangle_B$$

$$\rho_A = \mathrm{Tr}_B(\rho_{AB}) = \mathrm{Tr}_B\left(|\Psi\rangle_{AB}\,{}_{AB}\langle\Psi|\right) = \boxed{\begin{matrix} |a_0|^2 & 0 \\ 0 & |a_1|^2 \end{matrix}} = \rho_B$$

$$S(\rho_{AB}) = 0 \leq S(\rho_A) = S(\rho_B)$$

$$C = 2\,\mathrm{Det} = 2 a_0 a_1, \quad p_1 = |a_1|^2 = 1 - |a_0|^2 = 1 - p_0$$

$$C^2 = 4 a_0^2 a_1^2 = 4 a_0^2 (1 - a_0^2)$$

$$\Rightarrow a_0^2 = \frac{1 + \sqrt{1 - C^2}}{2}$$

Wootters, PRL80, 2245(1998)

$$\mathcal{E} = -p_0 \log_2 p_0 - p_1 \log_2 p_1$$
$$= -a_0^2 \log_2 a_0^2 - (1 - a_0^2) \log_2 (1 - a_0)^2$$

Entanglement of mixed states

Wootters, PRL80, 2245(1998)

SEPARABLE

$$\rho_{AB} = \sum_i p_i \rho_{A_i} \otimes \rho_{B_i}$$

OTHERWISE ENTANGLED

$$R = \sqrt{\sqrt{\rho_{AB}}\,(\sigma_y \otimes \sigma_y)\,\rho_{AB}^*\,(\sigma_y \otimes \sigma_y)\sqrt{\rho_{AB}}}$$

eigenvalues: $\{\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4\}$

$$C(\rho_{AB}) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$$\Rightarrow \mathcal{E}(\rho_{AB}) = h\left(\frac{1 + \sqrt{1 - C^2}}{2}\right)$$

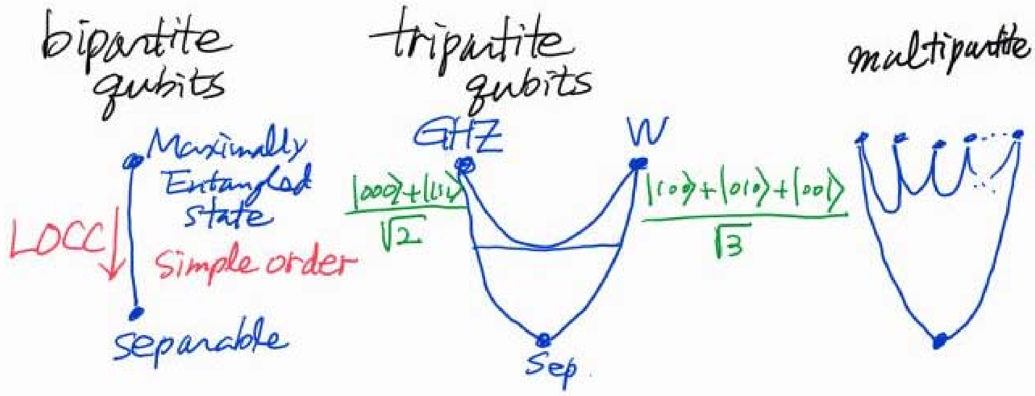
with $h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$

Entanglement Measure

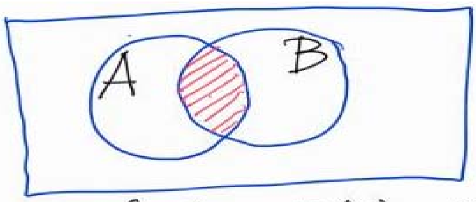
mixed state $\rho_A \longrightarrow |\Psi\rangle_{AB}$; purification

ensemble $\{\rho_{AB}\} \longrightarrow |\Phi\rangle_{AB}$; distillation

low entanglements — maximal entanglement



Discord



Olivier & Zurek (2001)
Henderson & Vedral (2001)

$$\begin{cases} I(B;A) = H(A) + H(B) - H(AB) \\ J_{cl}(B|A) = H(B) - H(B|A) \end{cases}$$

$$I(B;A) - J_{cl}(B|A) = H(A) - H(AB) + H(B|A)$$

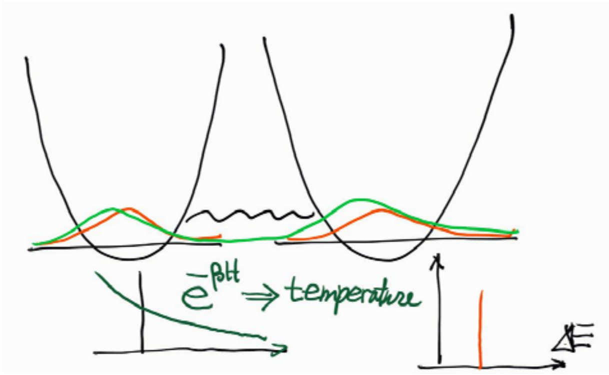
$$\Rightarrow D(B|A) = I(B;A) - J(B|A)$$

$$= \min_{\{E_a\}} \sum_a p_a S(\rho_{B|a}) + S(A) - S(AB)$$

$\hookrightarrow = 0$ for maximally entangled states!!!

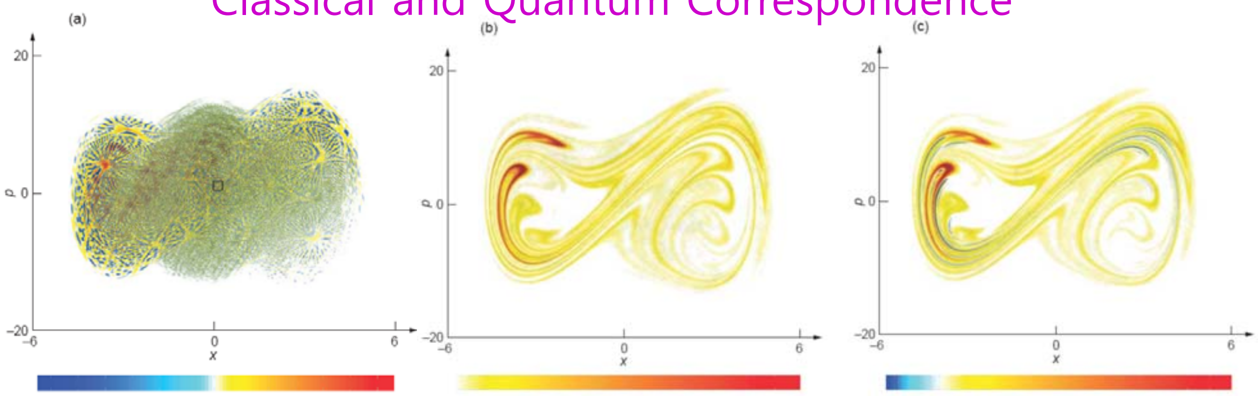
Decoherence → Thermalization?

Srednicki, PRL71, 666(1993)



Decoherence in a Chaotic Driven Double-Well System

Classical and Quantum Correspondence



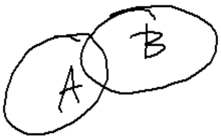
This numerical study (Habib et al. 1998) of a chaotic driven double-well system described by the Hamiltonian $H = p^2/2m - Ax^2 + Bx^4 + Fx\cos(\omega t)$ with $m = 1$, $A = 10$, $B = 0.5$, $F = 10$, and $\omega = 6.07$ illustrates the effectiveness of decoherence in the transition from quantum to classical. These parameters result in a chaotic classical system with a Lyapunov exponent $\lambda \cong 0.5$. The three snapshots taken after 8 periods of the driving force illustrate phase space distributions in (a) the quantum case, (b) the classical case, and (c) the quantum case but with decoherence ($D = 0.025$). The initial condition was always the same Gaussian, and in

Quantum to Classical
Habib, Shizume, and Zurek
'Decoherence, Chaos, and the Correspondence Principle'
Phys. Rev. Lett. 80, 4361 (1998)

Zurek, QP/0306072

Dynamics

$i\hbar|\dot{\psi}\rangle = H|\psi\rangle$ or $\dot{\rho} = -\frac{i}{\hbar}[H,\rho]$ for closed system



$$\rho'_A = \mathrm{Tr}_B\left(U_{AB}\,\rho_A \otimes |0\rangle_B\,{}_B\langle 0|\,U_{AB}^\dagger\right) = \sum_{\mu\in B} \underbrace{{}_B\langle\mu|U_{AB}|0\rangle_B}_{=M_\mu}\,\rho_A\,\underbrace{{}_B\langle 0|U_{AB}^\dagger|\mu\rangle_B}_{=M_\mu^\dagger}$$

$$\rho'_A = \$(\rho_A) = \sum_\mu M_\mu \rho_A M_\mu^\dagger$$

$$\sum_\mu M_\mu^\dagger M_\mu = \sum_\mu \langle 0|U_{AB}^\dagger|\mu\rangle\langle\mu|U_{AB}|0\rangle = 1$$

Preskill's
Lecture Note on QI&Comp

$$\rho(dt) = \$_t(\rho(0)) = \sum_\mu M_\mu(dt)\rho(0)M_\mu^\dagger(dt) = \rho(0) + O(dt)$$

$$\$_{t=0} = 1$$

$$\begin{cases} M_0 = 1 + O(dt) \\ M_{\mu\neq 0} \sim O(\sqrt{dt}) \end{cases}$$; quantum jumps

$$\begin{cases} M_0 = 1 + \left(-\frac{i}{\hbar}H + K\right)dt \\ M_{\mu>0} = \sqrt{dt}\,L_\mu \end{cases}$$

Hermitian

$$1 = \sum_\mu M_\mu^\dagger M_\mu = 1 + dt\left(2K + \sum_{\mu\neq 0} L_\mu^\dagger L_\mu\right)$$

$$\therefore K = -\frac{1}{2}\sum_{\mu>0} L_\mu^\dagger L_\mu$$

$$\dot{\rho} = \mathcal{L}[\rho] = -\frac{i}{\hbar}[H,\rho] + \sum_{\mu\neq 0}\left(L_\mu \rho L_\mu^\dagger - \frac{1}{2}L_\mu^\dagger L_\mu \rho - \frac{1}{2}\rho L_\mu^\dagger L_\mu\right)$$

Localization

$$|x\rangle|\chi\rangle \xrightarrow{t} |x\rangle|\chi_x\rangle = |x\rangle S_x|\chi\rangle$$

Joos et al.(2003)

$$\int d^3x\ \varphi(x)|x\rangle|\chi\rangle \xrightarrow{t} \int d^3x\ \varphi(x)|x\rangle S_x|\chi\rangle$$

$$\rho(x,x') = \varphi(x)\varphi^*(x')\left\langle \chi|S^\dagger_{x'}S_x|\chi\right\rangle$$

$$\rho(x,x',t) = \rho(x,x',0)\exp\left\{-\Lambda t(x-x')^2\right\}$$

$$\Lambda = \frac{k^2 N v \sigma_{eff}}{V}$$

Localization rate

Localization rate Λ in $cm^{-2}s^{-1}$ for three sizes of "dust particles" and various types of scattering processes (from [10]). This quantity measures how fast interference between different positions disappears as a function of distance in the course of time.

	$a = 10^{-3}$cm dust particle	$a = 10^{-5}$cm dust particle	$a = 10^{-6}$cm large molecule
Cosmic background radiation	10^{6}	10^{-6}	10^{-12}
300 K photons	10^{19}	10^{12}	10^{6}
Sunlight (on earth)	10^{21}	10^{17}	10^{13}
Air molecules	10^{36}	10^{32}	10^{30}
Laboratory vacuum (10^3 particles/cm^3)	10^{23}	10^{19}	10^{17}

$$i\frac{\partial\rho}{\partial t} = [H_{internal},\rho] + i\left.\frac{\partial\rho}{\partial t}\right|_{scatt.}$$

$$i\frac{\partial\rho(x,x',t)}{\partial t} = \frac{1}{2m}\left(\frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial x^2}\right)\rho - i\Lambda(x-x')^2\rho$$

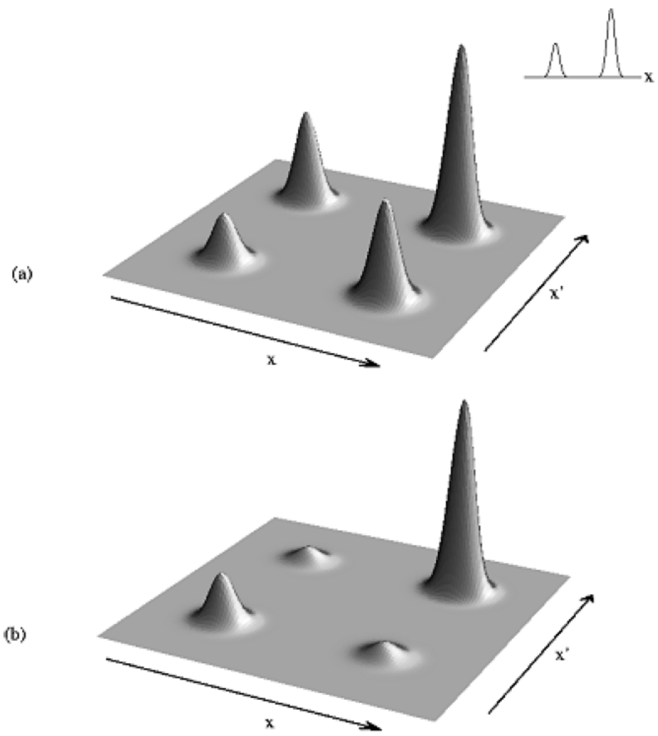
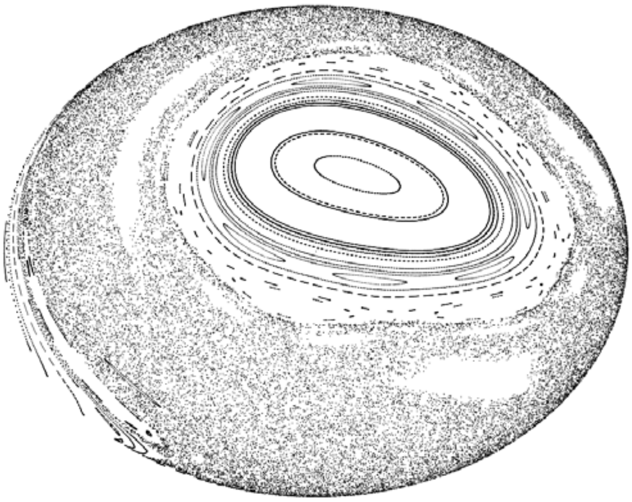
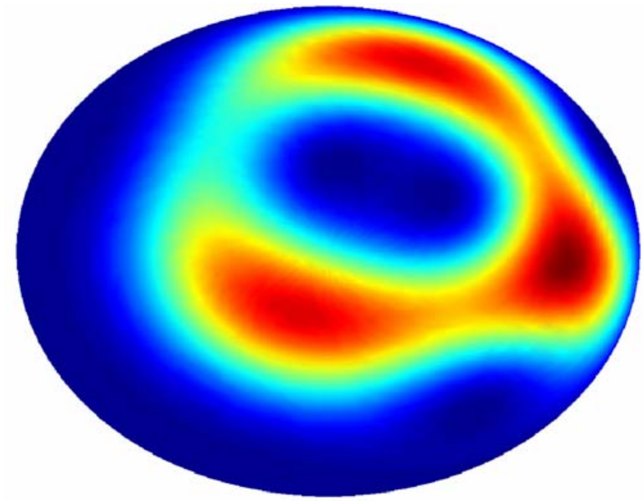


Fig. 1. (a) Density matrix of a superposition of two Gaussian wave packets. The wave function is shown in the inset. Coherence between the two parts of the wave function is represented by the two off-diagonal peaks. (b) The density matrix after interference is partially destroyed by decoherence. The position distribution, along the diagonal, is not changed appreciably.

Joos et al.(2003)

Chaotic Dynamics of a Rotator
Classical vs. Quantum



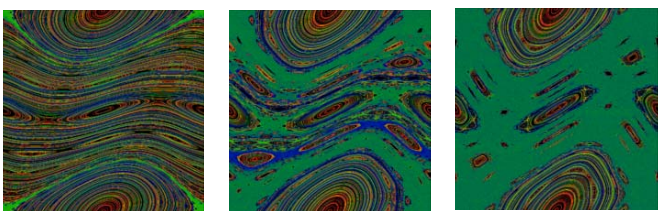


Simplest, faithful, nonlinear

- Standard map (Chirikov)

$$I_{n+1} = I_n - K \sin \phi_n$$

$$\phi_{n+1} = \phi_n + I_{n+1}$$

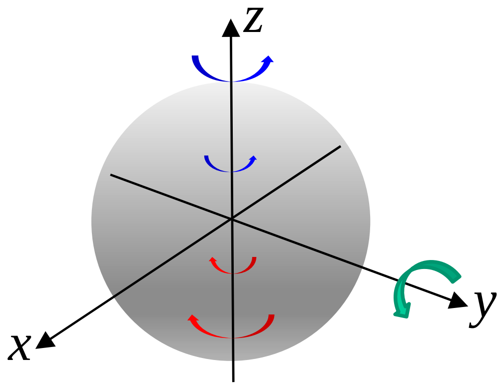


$K = 0.6$ 0.971635   1.2

from wikipedia

Simplest, faithful, nonlinear

- Rotor
  - Rotation around y-axis (parameter a)
  - Twist around z-axis (parameter b)



Classical dynamics

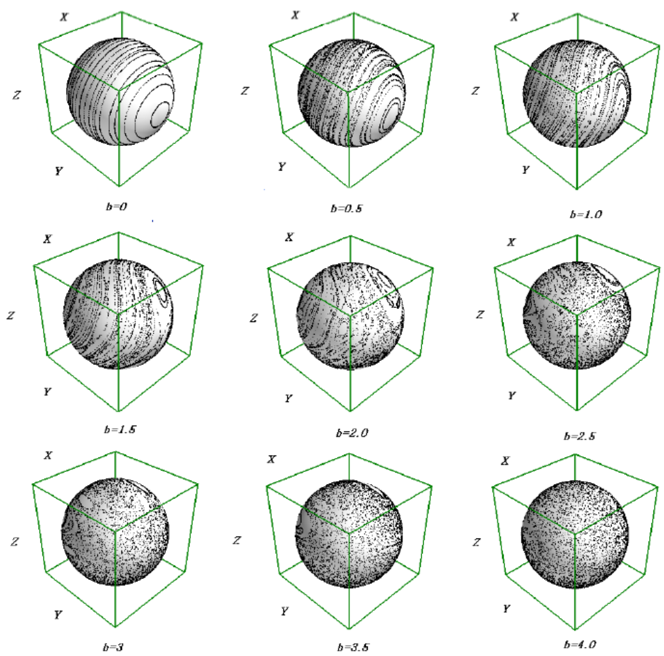


FIG. 2. Trajectories of Classical Kicked Top.Trajectories are regular with small nonlinearity. Chaotic trajectories emerges when nonlinearity becomes large.

Unitary dynamics

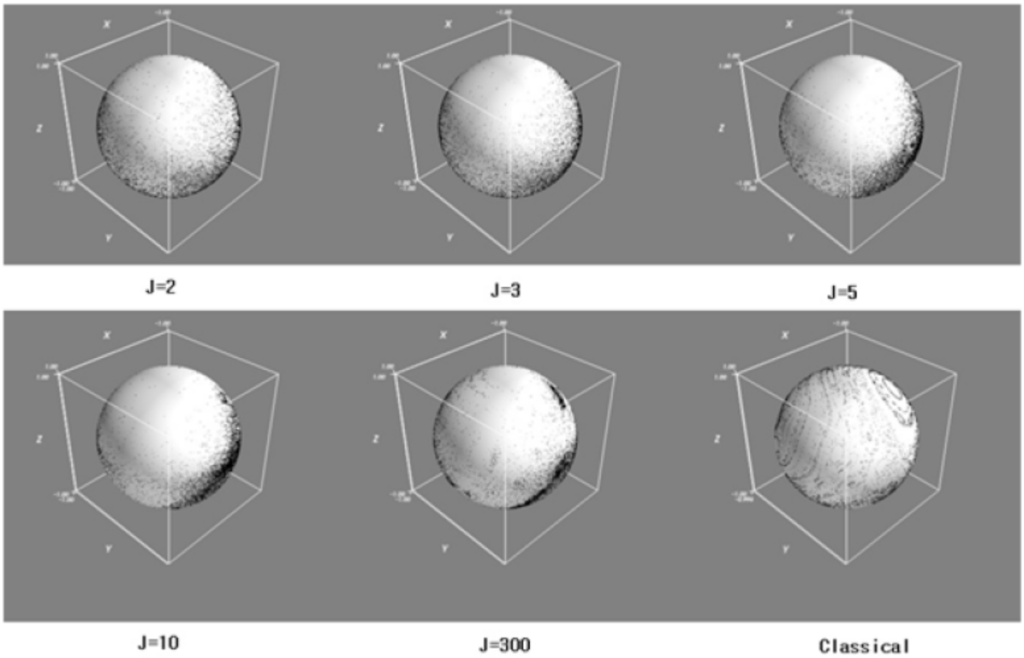
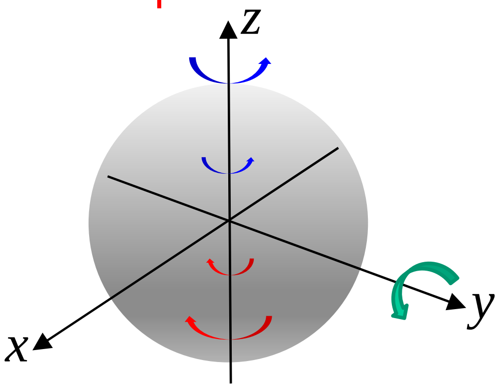


FIG. 8. Unitary evolution with fixed b=2. Trajectories with various quantum number J. Trajectory does not become Close to classical one even when J is very large

Simplest, faithful, nonlinear

- Rotor
  - Rotation around y-axis (parameter a)
  - Twist around z-axis (parameter b)
  - Collapse to spin coherent state (decoherence)



Unitary + decoherence

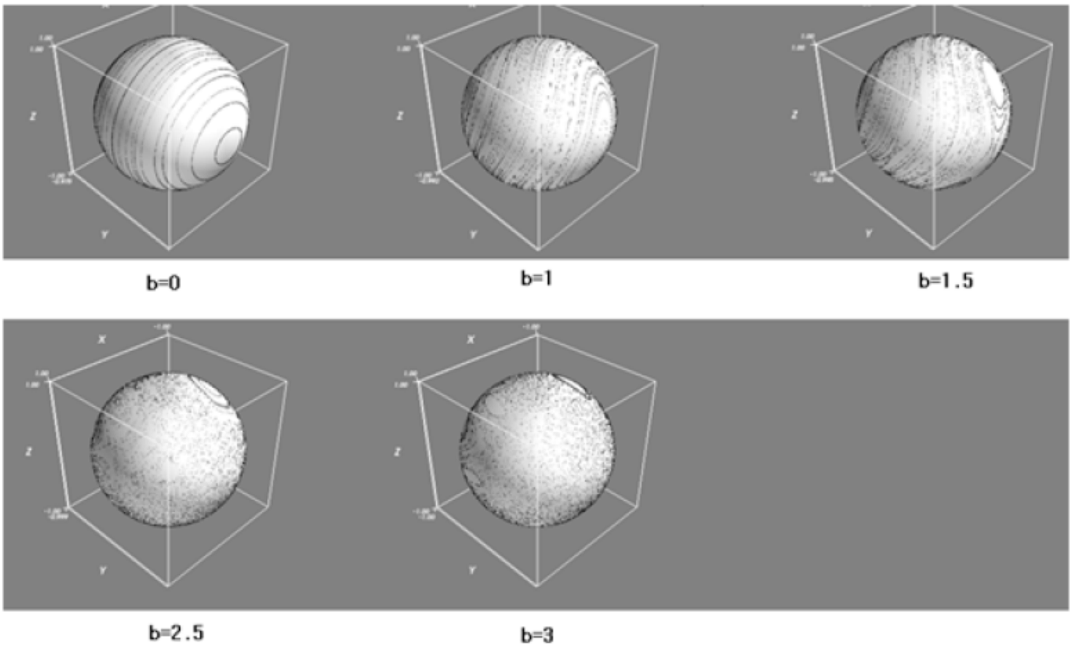


FIG. 13. Quantum Kicked Top with collapse to spin coherent state.Trajectories are almost identical to classical trajectory.J=500

Unitary + decoherence

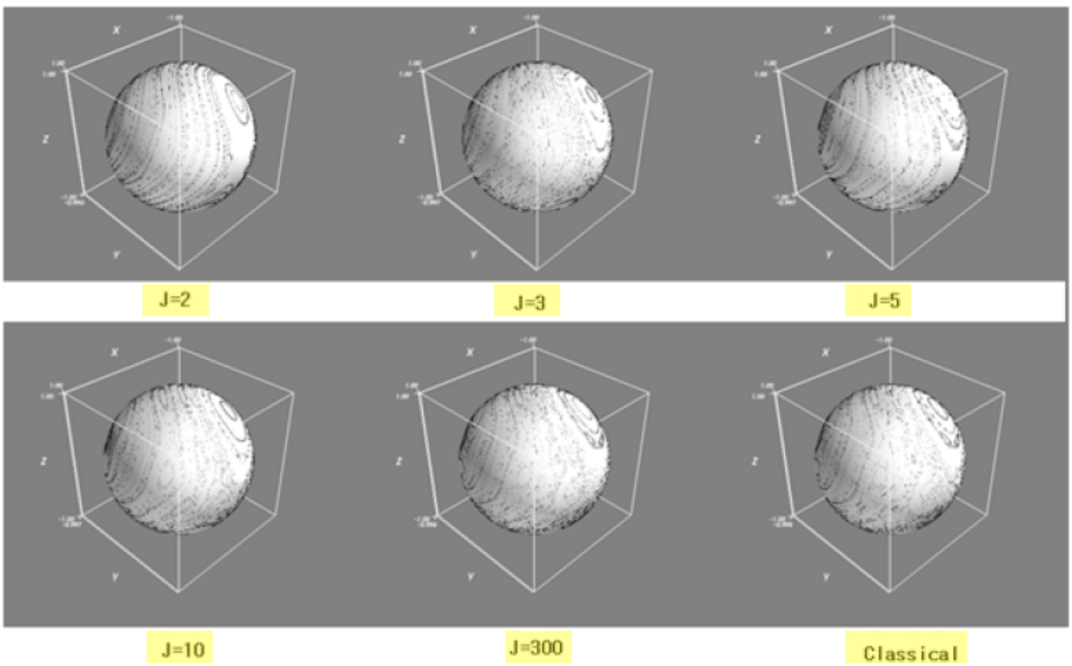


FIG. 17. Evolution of quantum top with collapse. Trajectories with various J and b is fixed. b=2. Trajectories becomes more similar to classical one as J increases

- Entanglement of System and Environment with preferred basis in the interaction

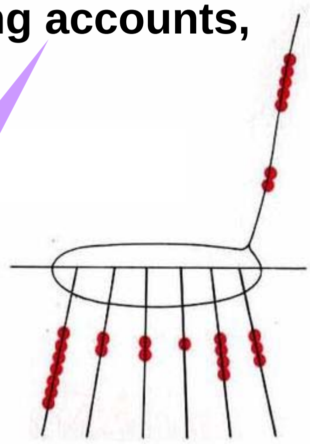
➔Decoherence of the System

➔Emergence of classical physics

QUIPU [kí:pu]

1. Quantum Information Processing Unit

2. A device consisting of a cord with knotted strings of various colors attached, used by the ancient Peruvians for recording events, keeping accounts, etc.



* Peruvian Information Processing Unit for Communication and Computation