2015/11/27 @ KIAS Workshop

Information Balance in Quantum Measurement determines Optimal Quantum Teleportation

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In general,

Information Gain	Information Sharing
	
by measurement	by communication

(Key Idea of this work will be…)

Measurement plays an important role in Quantum Communications

Information

"The amount of uncertainty before we learn (measure)"

quantifying the resource needed to store information

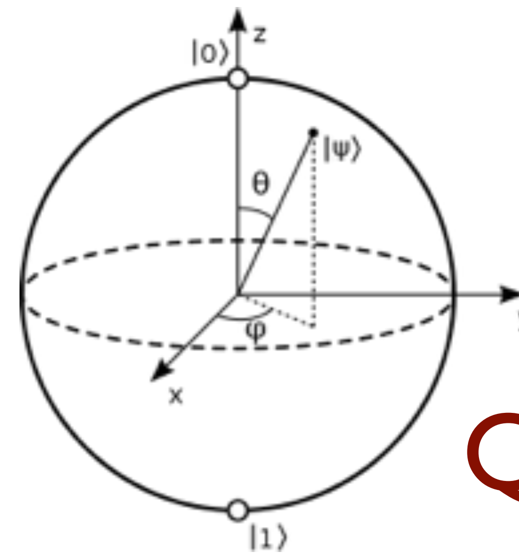
Bit

random variable X

with probability distribution $p_1, \ldots, p_n$

Shannon Entropy

$$H(X) \equiv H(p_1, \ldots, p_n) \equiv -\sum_x p_x \log p_x$$



Qubit

quantum state ρ

von Neumann Entropy

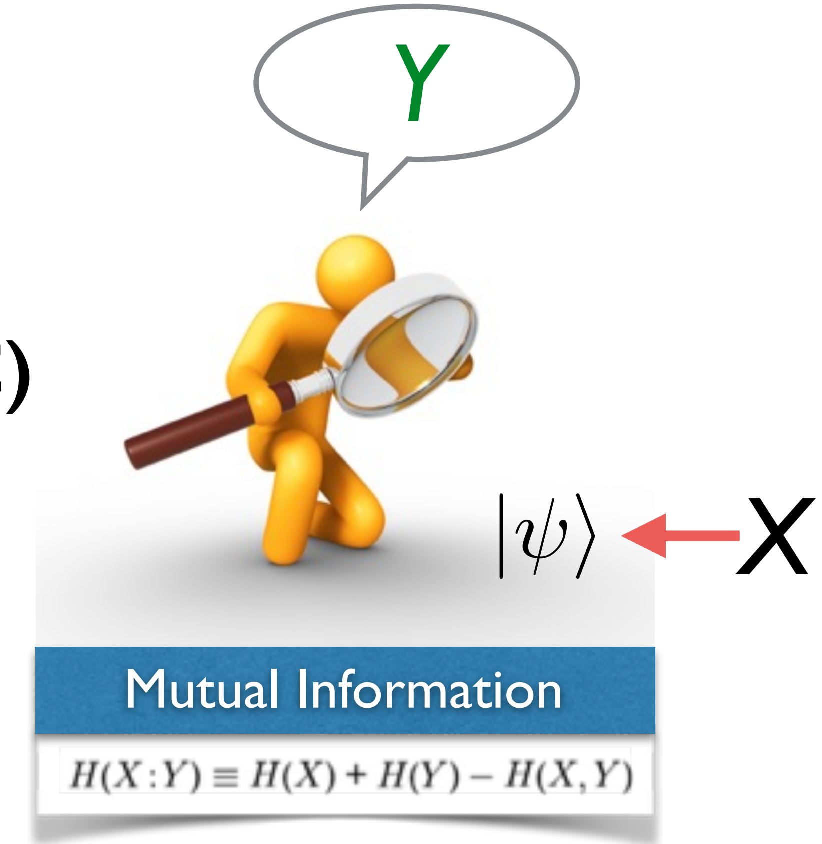
$$S(\rho) \equiv -\mathrm{tr}(\rho \log \rho)$$

Information gain by Measurement

"How much information has gained by measurement ?"

(C-C)

(C-Q-C)



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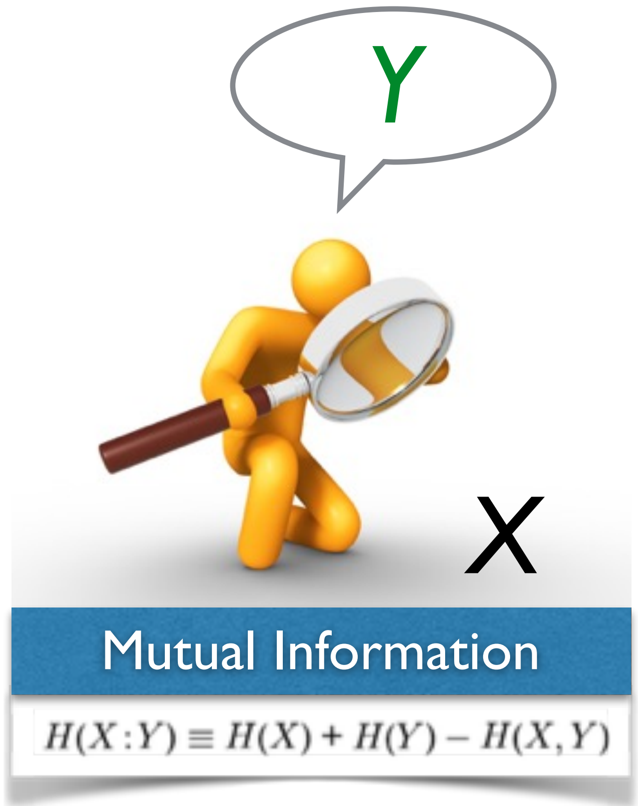
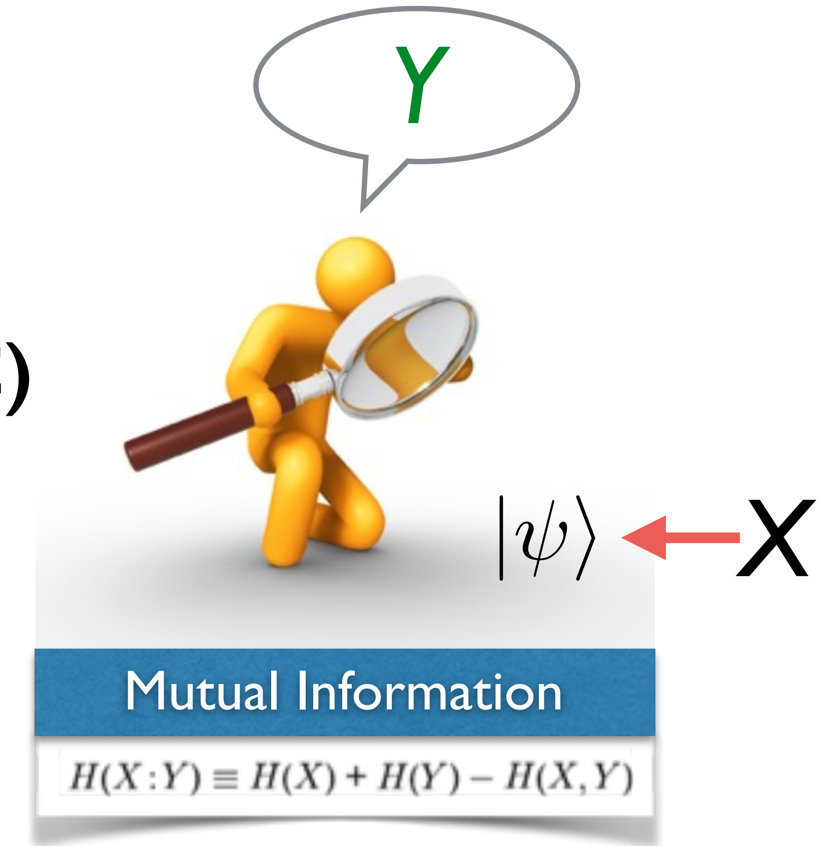


(C-Q-C)



(Q-C)

F. Buscemi et al. PRL (2008);T. Sagawa et al. PRL (2008)

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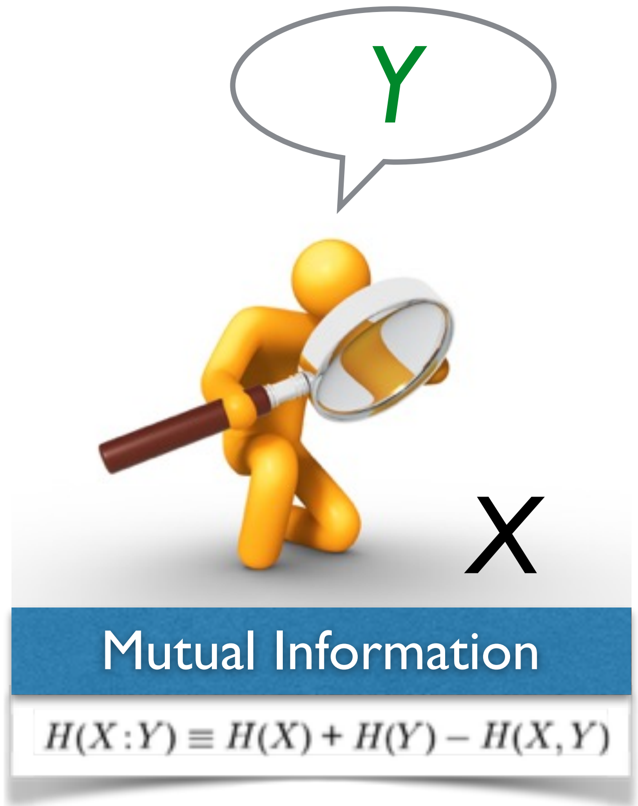

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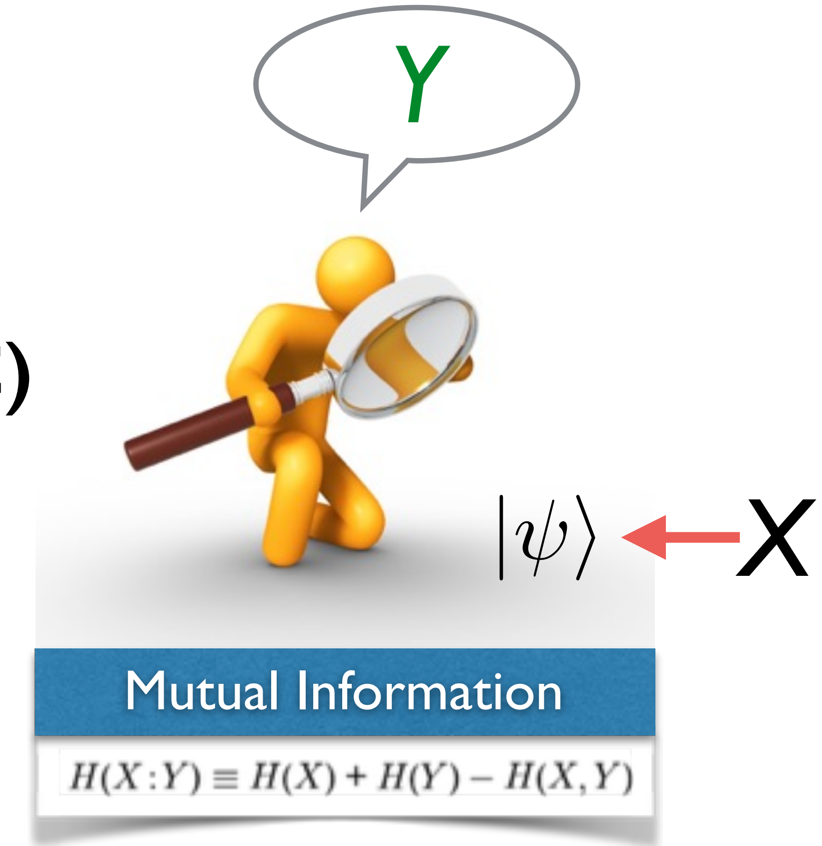
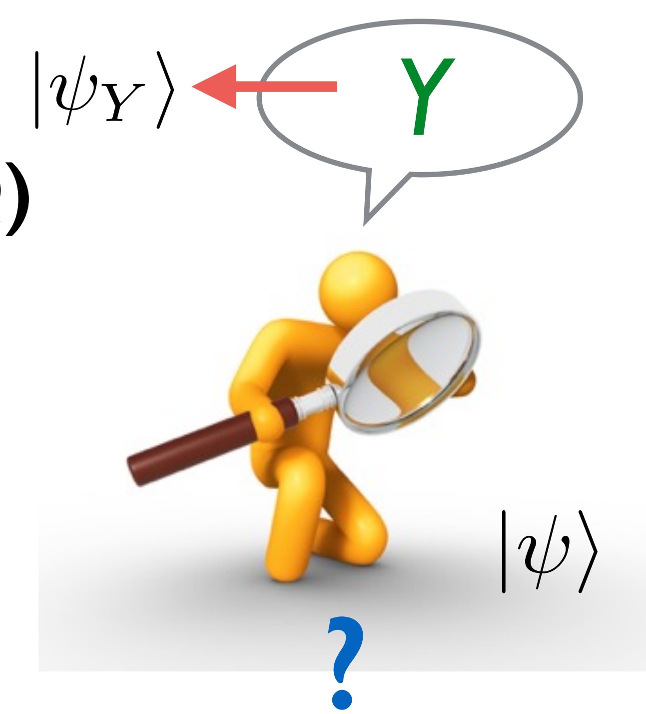


(Q-C)

(Q-C-Q)



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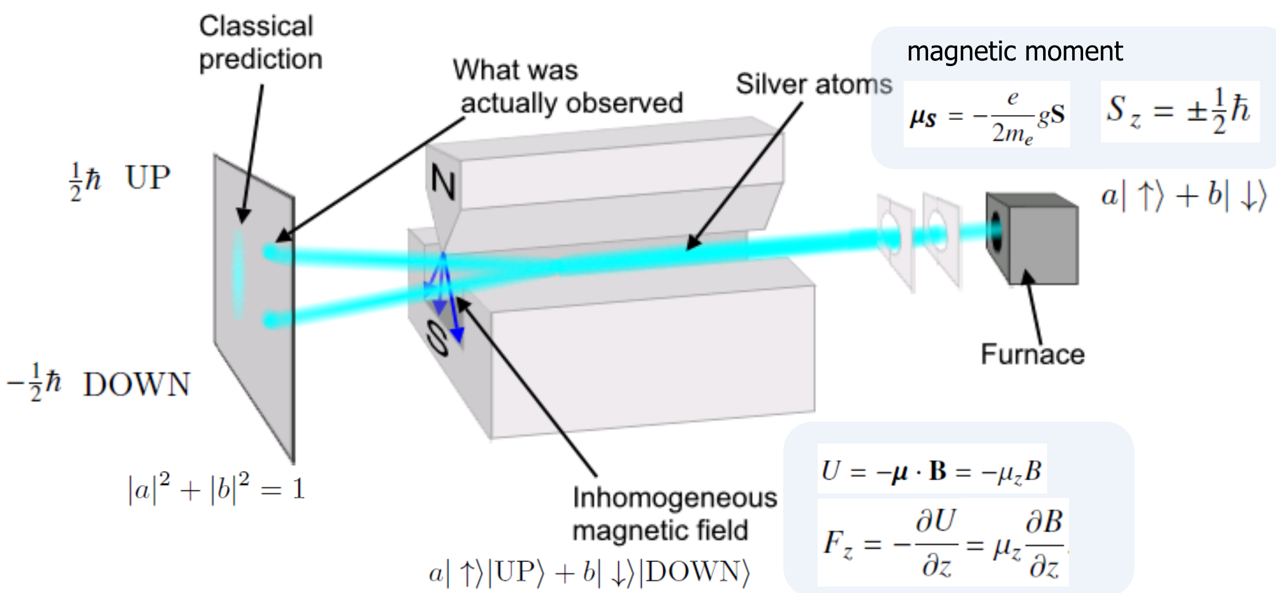
Quantum Measurement

"Quantum measurement lies at the heart of fundamental quantum physics."



Quantum measurement process =

quantum system to be measured +
measurement apparatus (probe)

General quantum measurement can be described by a set of operators

$$\{\hat{A}_r | r = 1, \ldots, N\}$$

satisfying the completeness relation (probability sum = 1)

$$\sum_{r=1}^{N} \hat{A}_r^\dagger \hat{A}_r = \mathbb{1}$$

The probability that the outcome is r $p(r, |\psi\rangle) = \langle\psi| \hat{A}_r^\dagger \hat{A}_r |\psi\rangle$

The post measurement state $|\psi_r\rangle = \dfrac{\hat{A}_r |\psi\rangle}{\sqrt{p(r, |\psi\rangle)}}$

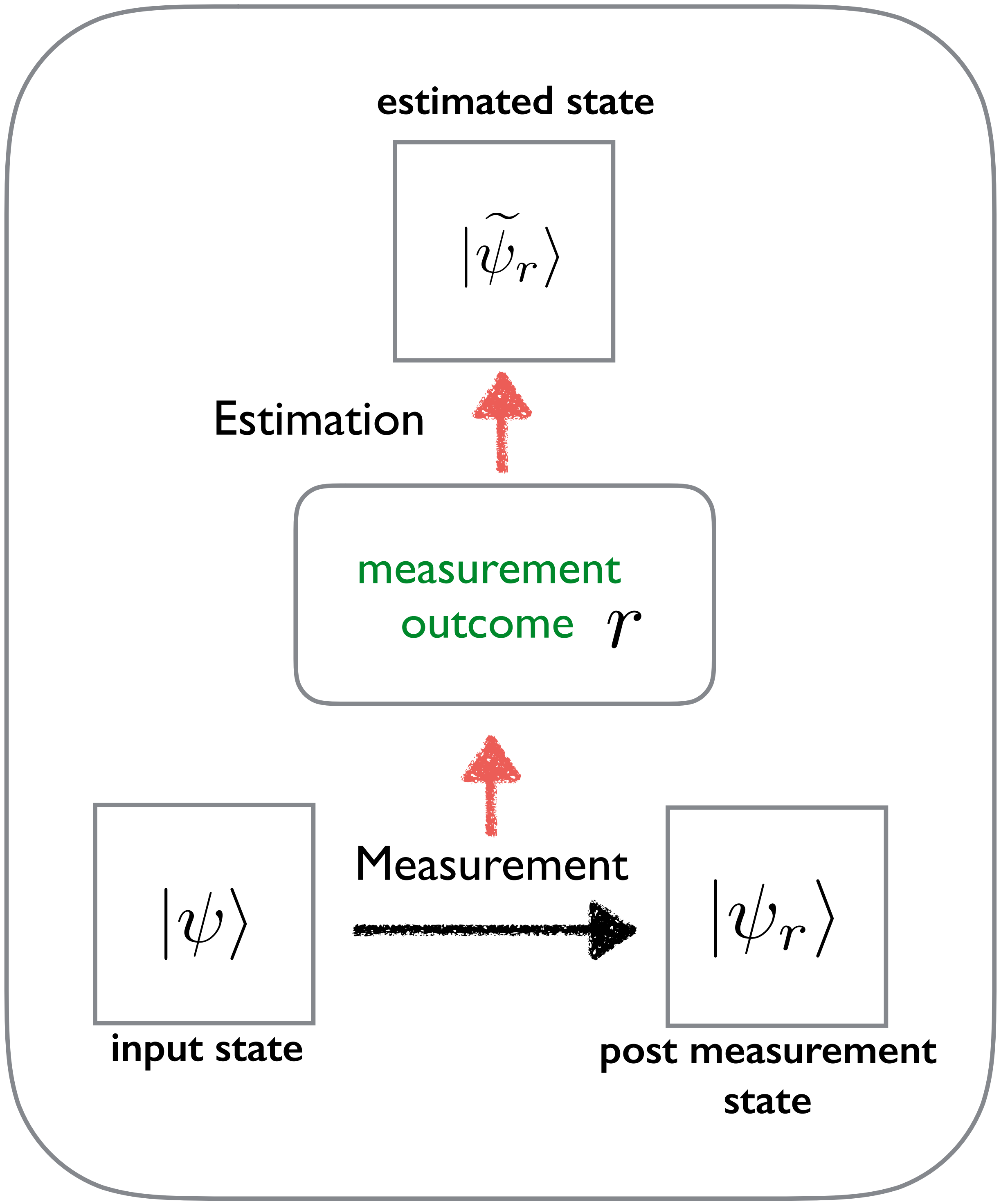
Each operator can be written by singular-value decomposition $\hat{A}_r = \hat{V}_r \hat{D}_r \hat{U}_r$

$\hat{U}_r$ $\hat{V}_r$ unitary operator $\hat{D}_r = \sum_{i=0}^{d-1} \lambda_i^r |v_i^r\rangle \langle v_i^r|$ is a diagonal matrix,

Singular values $\lambda_0^r \geq \lambda_1^r \geq \ldots \geq \lambda_{d-1}^r \geq 0$

Information Gain and Disturbance

"The relation between information gain and disturbance by measurement ?"



The amount of information gain and disturbance ?

- closeness of the states

by fidelity (or distance)

$$F(\rho,\sigma) = \mathrm{Tr}\left[\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right]$$

for pure state

$$F(\rho,\sigma) = \sqrt{\langle\phi|\psi\rangle\langle\psi|\phi\rangle} = |\langle\phi|\psi\rangle|$$

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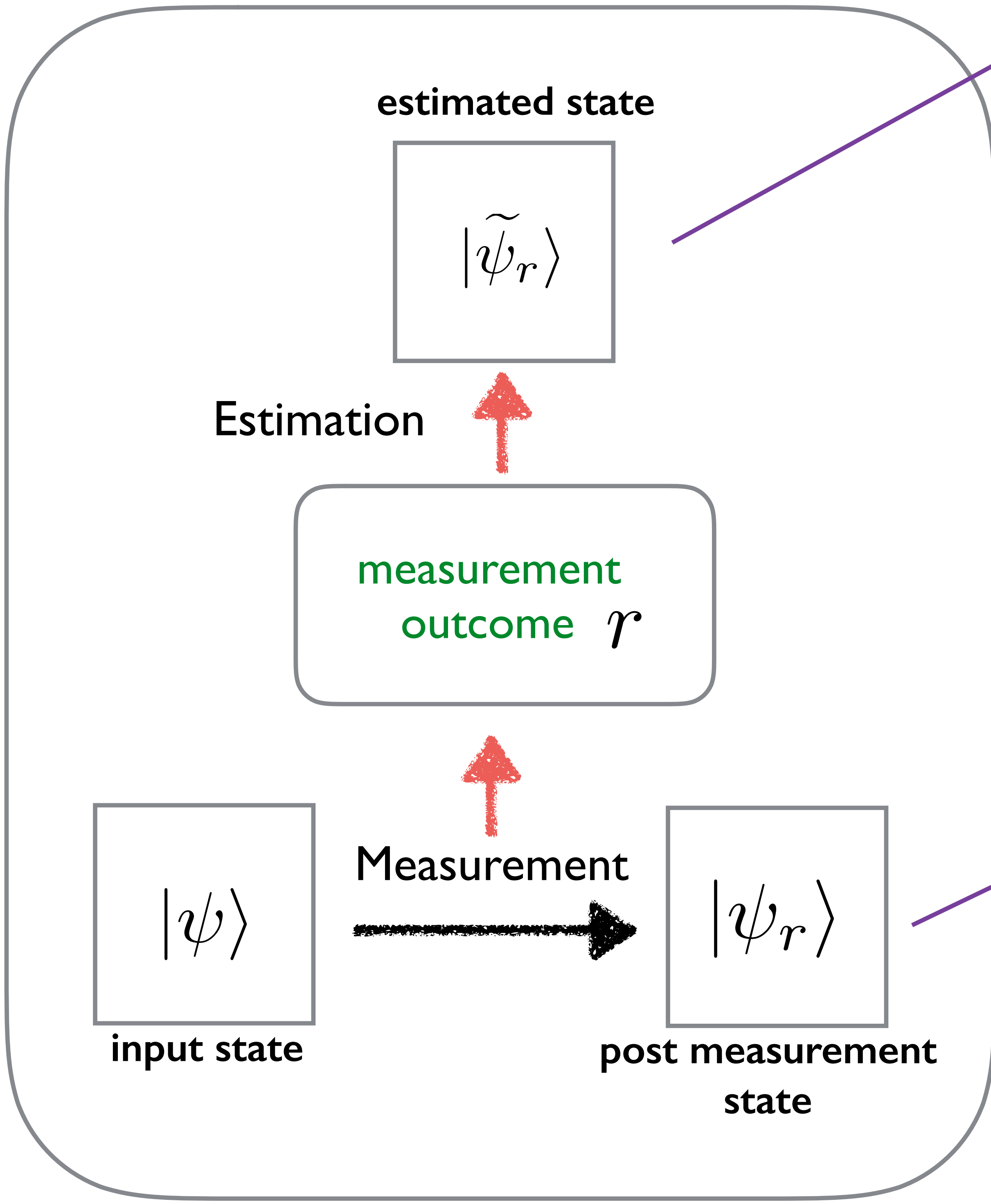


Information Gain

by averaging the estimation fidelity $|\langle\psi|\widetilde{\psi_r}\rangle|^2$

by using the optimal estimation strategy:

we guess that the state is the singular basis of measurement operators with maximal value

$$G_{max} = \frac{1}{d(d+1)}(d + \sum_{r=1}^{N} (\lambda_0^r)^2)$$

$$1/d \leq G_{max} \leq 2/(d+1)$$

Disturbance

by averaging the operation fidelity $|\langle\psi|\psi_r\rangle|^2$

$$F = \frac{1}{d(d+1)}\left[d + \sum_{r=1}^{N}\left(\sum_{i=0}^{d-1}\lambda_i^r\right)^2\right]$$

Information Gain and Disturbance

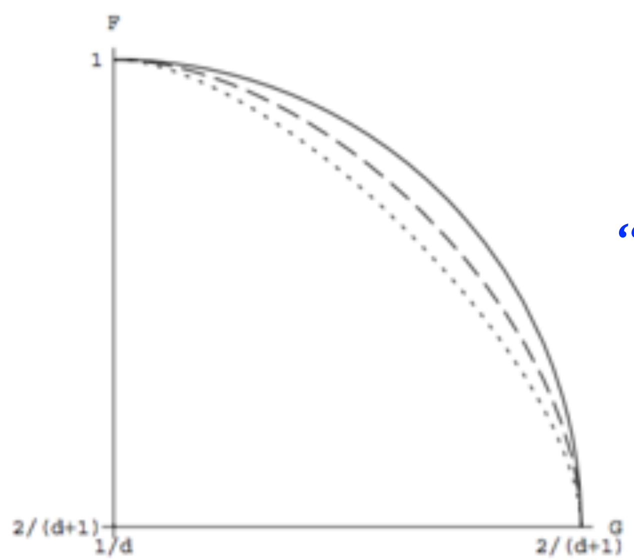
"The relation between information gain and disturbance by measurement ?"

Information Gain

estimated state

Trade-off between info-gain and disturbance

K. Banaszek, PRL 86, 1366 (2001)



"The more information is obtained from measurement, the more its state is disturbed."

"We gain information by measurement"

+

"Measurement disturbs a quantum state"

$$\sqrt{F - \frac{1}{d+1}} \leq \sqrt{G - \frac{1}{d+1}} + \sqrt{(d-1)\left(\frac{2}{d+1} - G\right)}$$

basis of

al value

$\lambda_0^r)^2)$

$/(d+1)$

$|\widetilde{\psi_r}\rangle|^2$

$|\psi_r\rangle|^2$

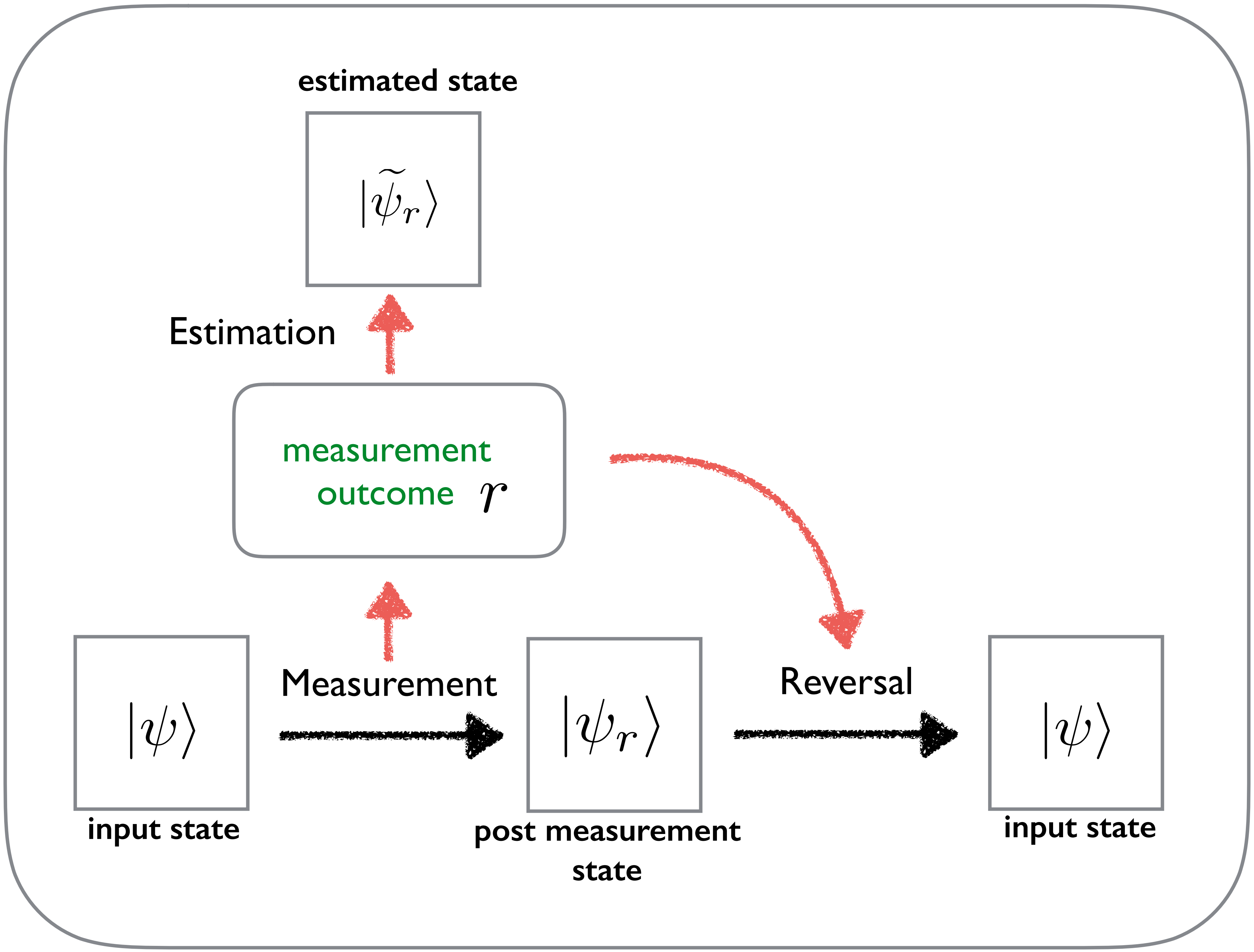
input state

post measurement state

$$F = \frac{1}{d(d+1)}\left[d + \sum_{r=1}^{N}\left(\sum_{i=0}^{d-1}\lambda_i^r\right)^2\right]$$

Reversing Quantum Measurement

"Can we reverse (undo) the Quantum Measurement ?"



Reversing Quantum Measurement

"Can we reverse (undo) the Quantum Measurement ?"

Common belief

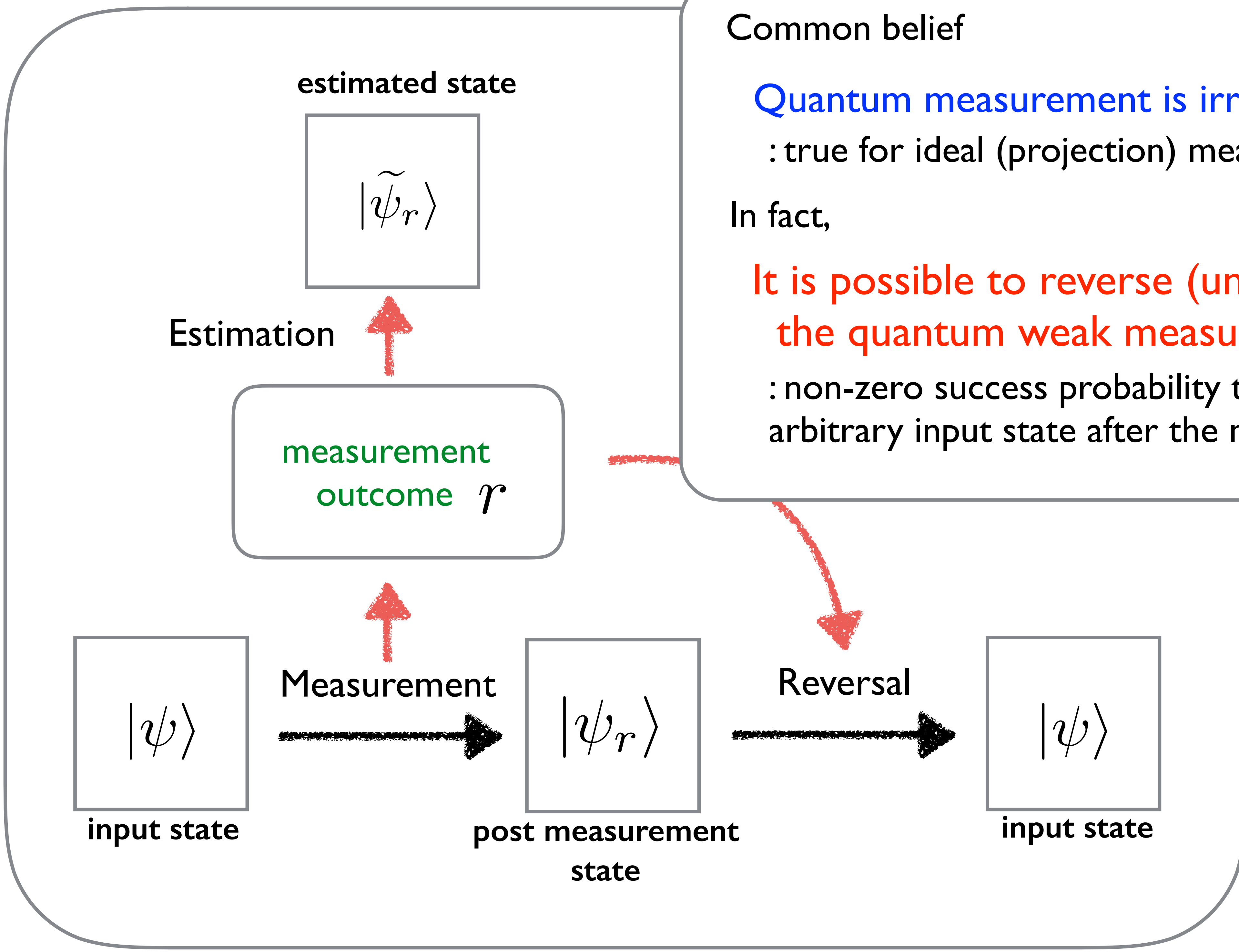
Quantum measurement is irreversible ?

: true for ideal (projection) measurements

In fact,

It is possible to reverse (undo) the quantum weak measurement !!

: non-zero success probability to retrieve the arbitrary input state after the measurement



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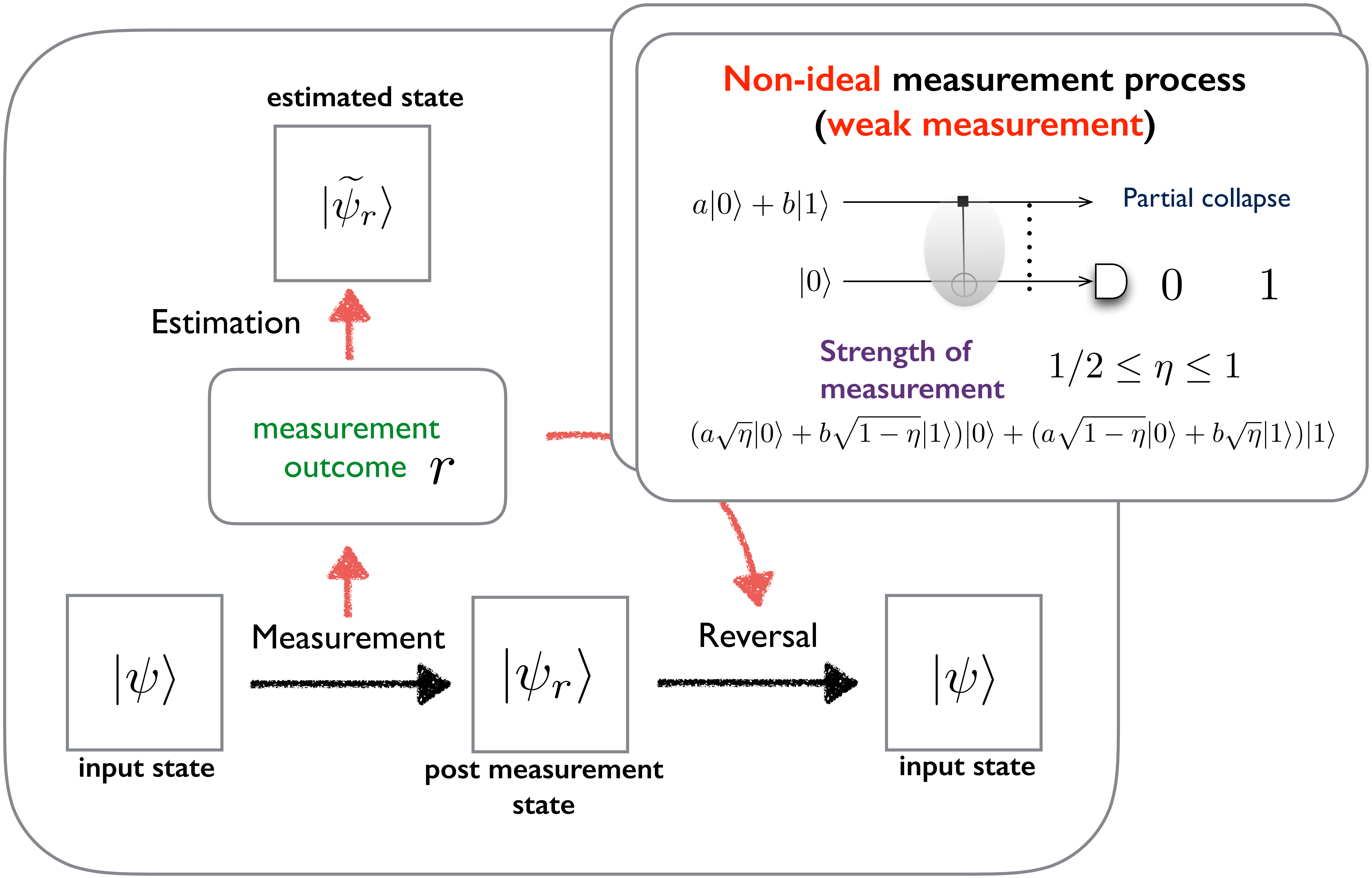

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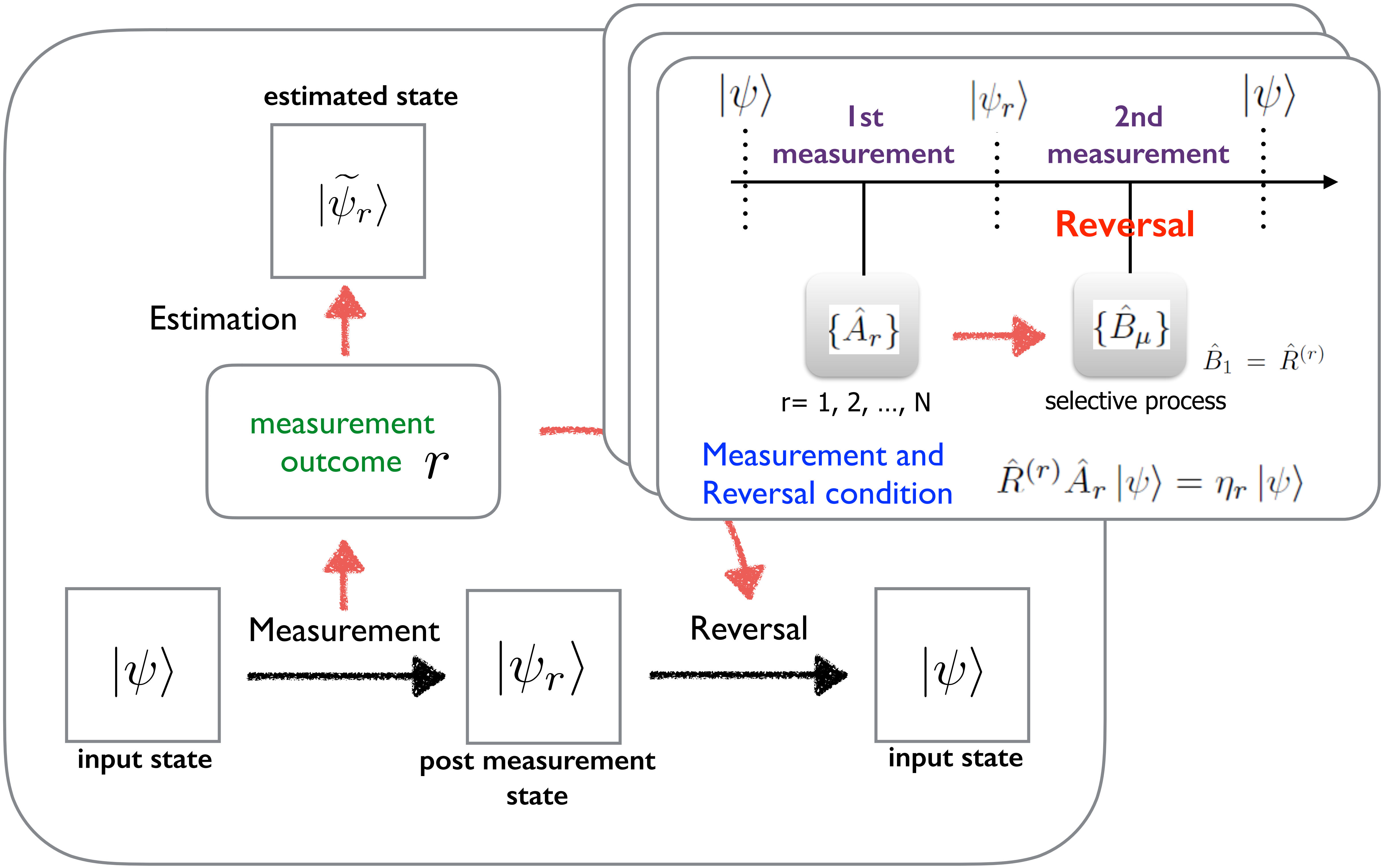

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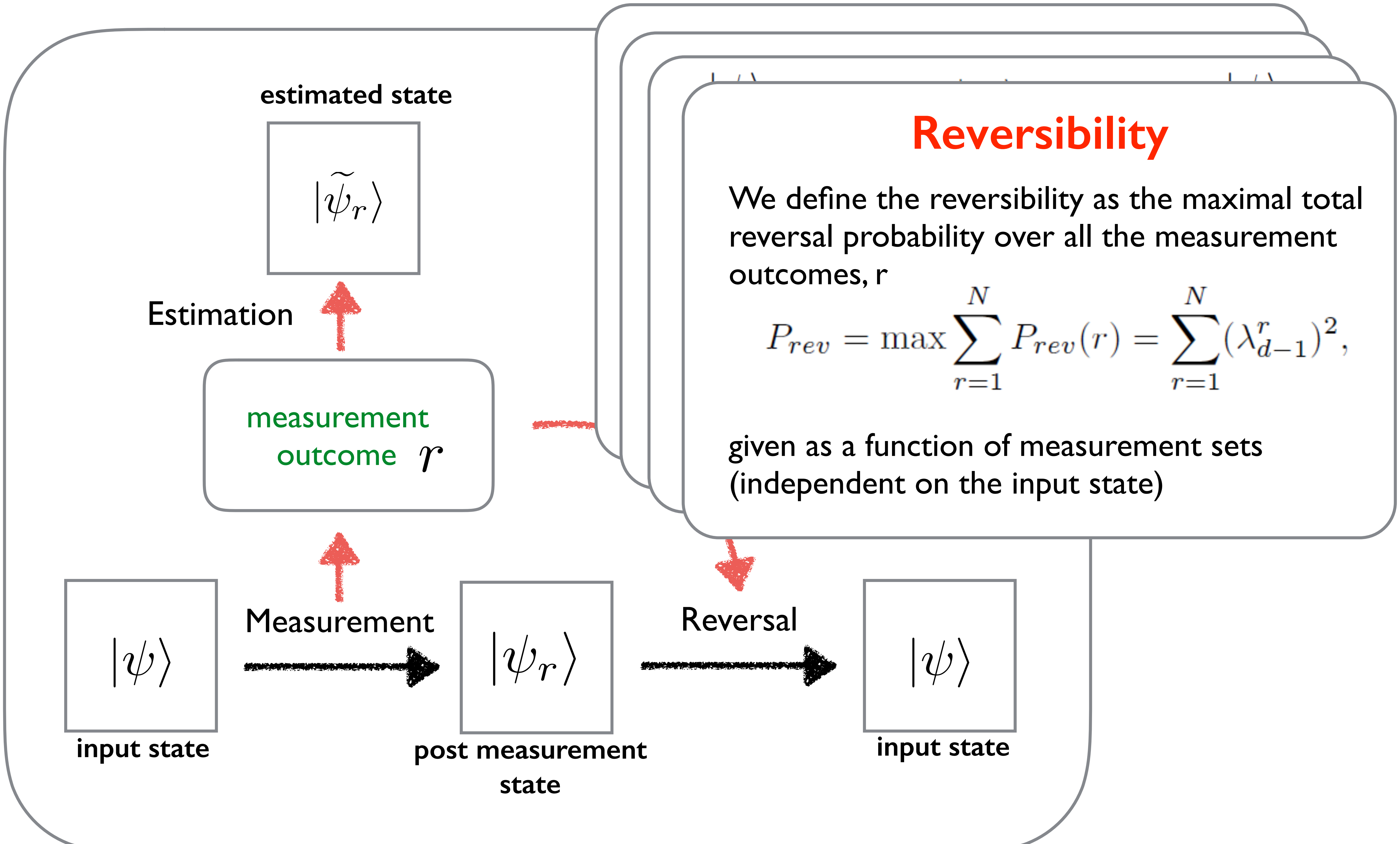


Reversibility

We define the reversibility as the maximal total reversal probability over all the measurement outcomes, r

$$P_{rev} = \max \sum_{r=1}^{N} P_{rev}(r) = \sum_{r=1}^{N} (\lambda^r_{d-1})^2,$$

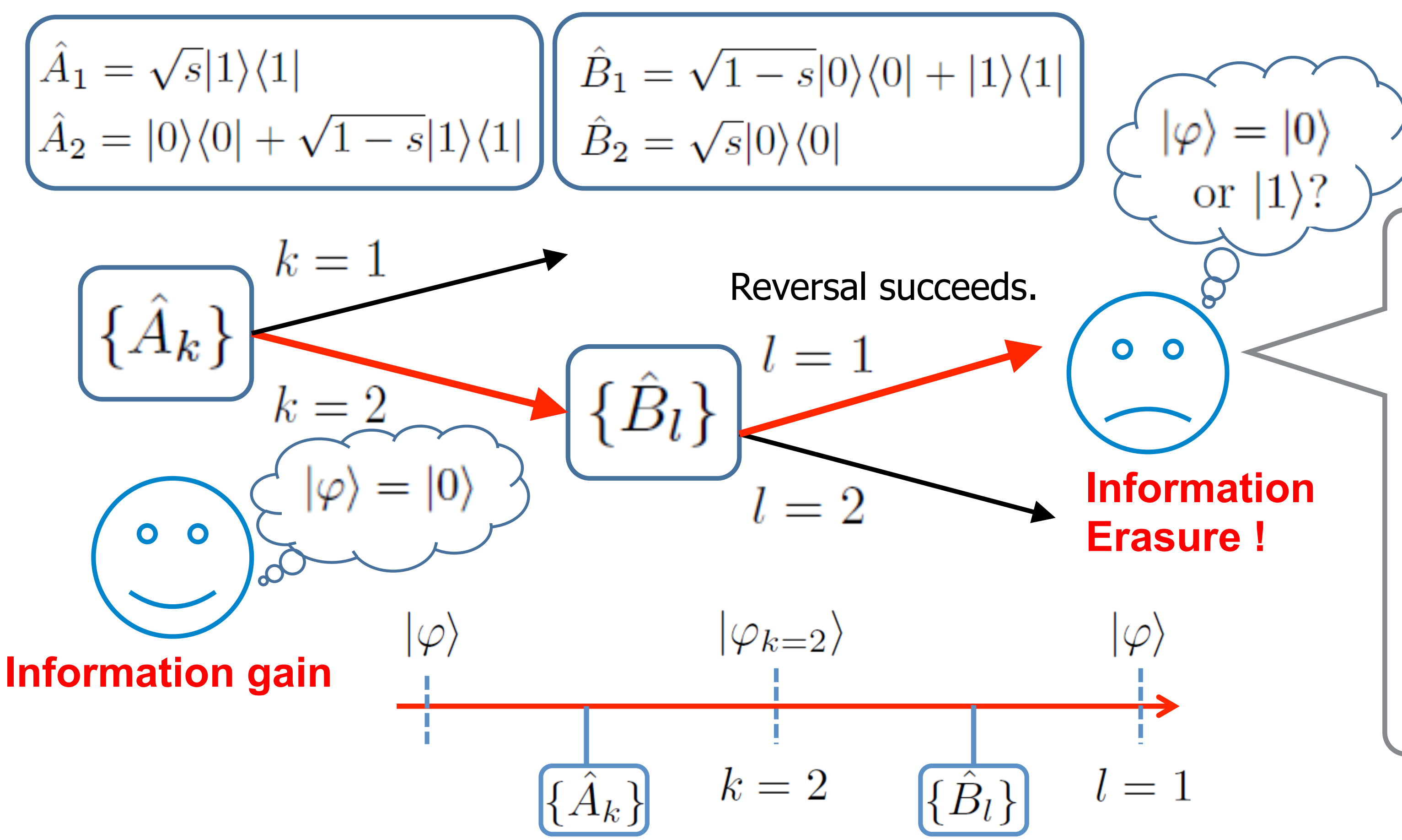
given as a function of measurement sets (independent on the input state)

Result 1

Trade-off relation between info gain and reversibility

[1] Y. W. Cheong and S.-W. Lee*, Phys. Rev. Lett. 109, 150402 (2012).

After reversing quantum measurement, where is the already obtained information ?



The same amount of information is erased by measurement reversal !!

Quantitative bound of information gain and reversibility of quantum measurment

$$\boxed{d(d+1)G_{max} + (d-1)P_{rev} \leq 2d}$$

For qubit

$$6G_{max} + P_{rev} = 4$$

Result 1

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$\hat{A}_1 = \sqrt{s}|1\rangle\langle 1|$

$\hat{B}_1 = \sqrt{1-s}|0\rangle\langle 0| + |1\rangle\langle 1|$

Trade-off between info-gain and reversibility

"The more information is obtained from quantum measurement, the less possible it is to undo the measurement."

↓

Information balance (no-cloning of quantum information)

of

ed

$\{\hat{A}_k\}$ $k = 2$ $\{\hat{B}_l\}$ $l = 1$

Quantitative bound of information gain and reversibility of quantum measurment

$$\boxed{d(d+1)G_{max} + (d-1)P_{rev} \leq 2d}$$

For qubit

$$6G_{max} + P_{rev} = 4$$

Result II

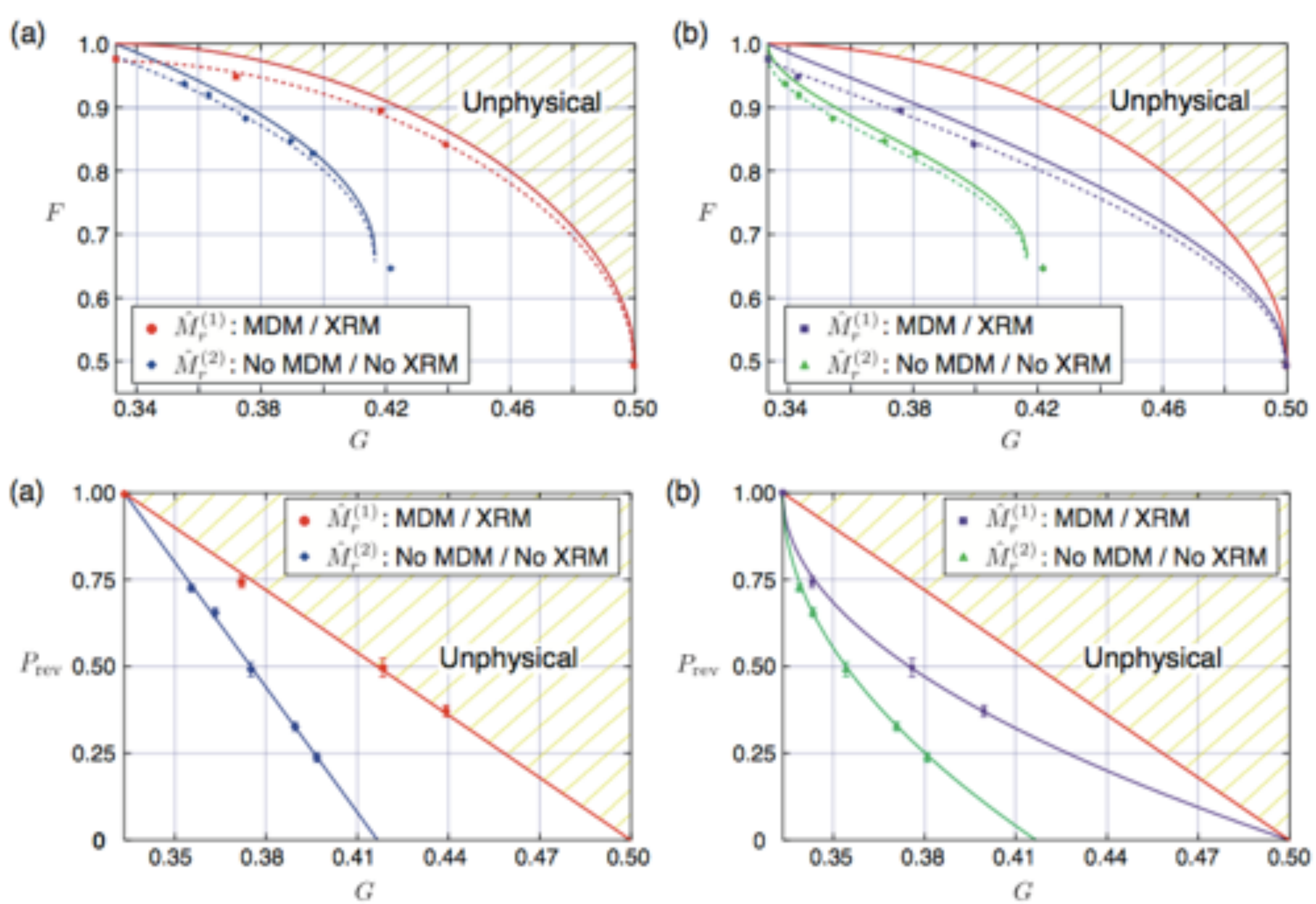
Experimental proof of fundamental bounds in quantum measurements

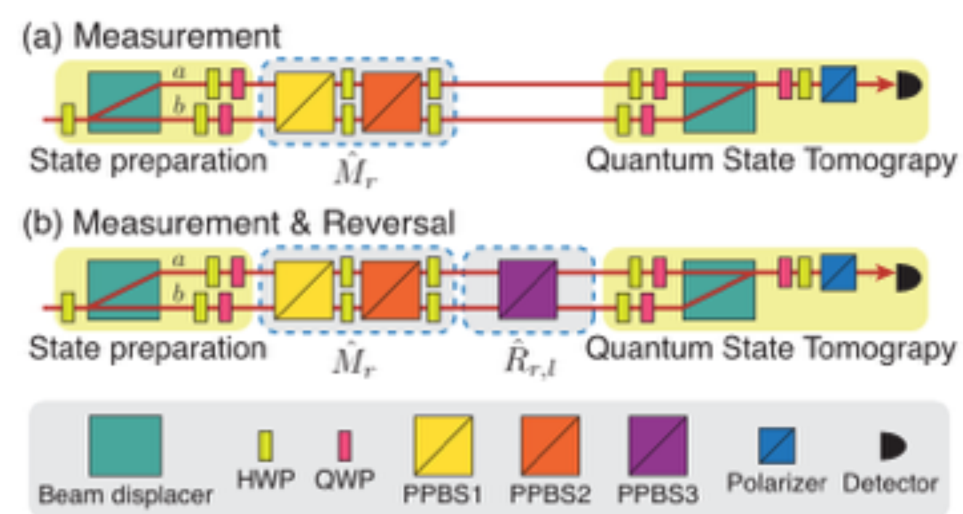
[2] H.-T. Lim*, Y.-S. Ra, K.-H, Hong, S.-W. Lee*, and Y.-H. Kim*, Phys. Rev. Lett. 113, 020504 (2014)

There exist fundamental bounds in quantum measurements !!

MRM (maximal reversible measurement), MDM (minimal disturbance measurement)

Experimental proof of for qutrit systems
collaboration with Prof. Y.-H. Kim's group in POSTECH



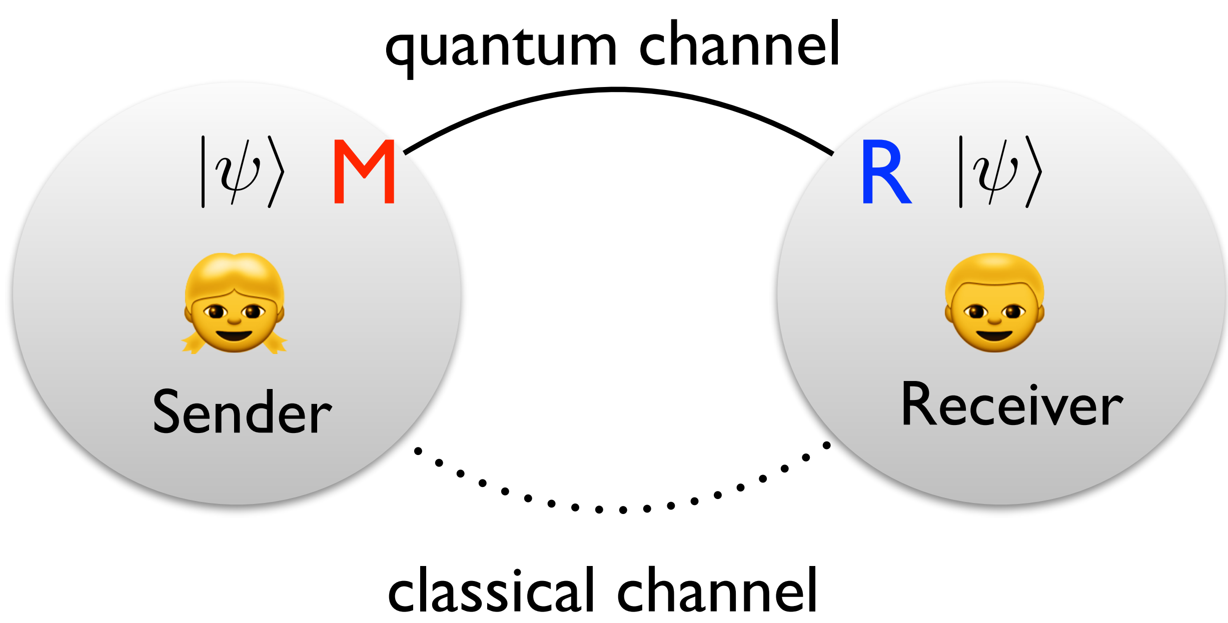


We can find the optimal measurement sets

with Maximal Reversibility and Minimal Disturbance

Quantum Teleportation

"A quantum task to transfer an arbitrary quantum state to remote place"



Teleportation Protocol

1. **Entanglement** is distributed between the sender and receiver
2. **Measurement** is performed on the input and a part of the quantum channel
3. **Measurement result is shared** through the classical channel
4. **Reversing operation** is performed on the out part of the quantum channel

mode a	mode b	mode c
unknown state $|\psi\rangle$	entangled quantum channel $|\Psi\rangle$	
joint measurements M $\hat{A}_r = |W_r\rangle\langle W_r|$ (projection)		
		R Reversing operation $\hat{R}^{(r)}$

Quantum Teleportation

- After sender's joint measurement (for an outcome r)

$$\mathrm{Tr}_{a,b}\left[\hat{A}_{r,ab}|\psi\rangle_a\langle\psi|\otimes|\Psi\rangle_{bc}\langle\Psi|\hat{A}^\dagger_{r,ab}\right]$$
$$={}_{ab}\langle W_r| \cdot |\psi\rangle_a\langle\psi|\otimes|\Psi\rangle_{bc}\langle\Psi| \cdot |W_r\rangle_{ab}$$
$$=\hat{M}_{r,a\to c}\ |\psi\rangle_a\langle\psi|\ \hat{M}^\dagger_{r,a\to c},$$

where

$$\boxed{\hat{M}_{r,a\to c} \equiv {}_{ab}\langle W_r|\Psi\rangle_{bc}}$$

effective overall measurement

- Reversing operation $\hat{R}^{(r)}_c\hat{M}_{r,a\to c}|\psi\rangle_a = \eta_r|\psi\rangle_c$
- Overall teleportation process

$$\mathrm{Tr}_{a,b}\left[\hat{R}^{(r)}_c\hat{A}_{r,ab}|\psi\rangle_a\langle\psi|\otimes|\Psi\rangle_{bc}\langle\Psi|\hat{A}^\dagger_{r,ab}\hat{R}^{(r)\dagger}_c\right]$$
$$=\hat{R}^{(r)}_c\hat{M}_{r,a\to c}\ |\psi\rangle_a\langle\psi|\ \hat{M}^\dagger_{r,a\to c}\hat{R}^{(r)\dagger}_c = |\eta_r|^2|\psi\rangle_c\langle\psi|.$$

"Quantum teleportation can be regarded as a quantum measurement and reversal process"

In general, **effective measurement operator** can be defined as

$$\hat{M}_r \equiv {}_{\mathrm{senders}}\langle W_r||\Psi\rangle_{\mathrm{channel}}$$

$$\sum_r \hat{M}^\dagger_r\hat{M}_r = \mathbb{1}_{\mathrm{senders}}$$

$|W_r\rangle$: local joint measurement basis

$|\Psi\rangle$: entangled quantum channel

with its optimal reversing operation $\hat{R}^{(r)}$

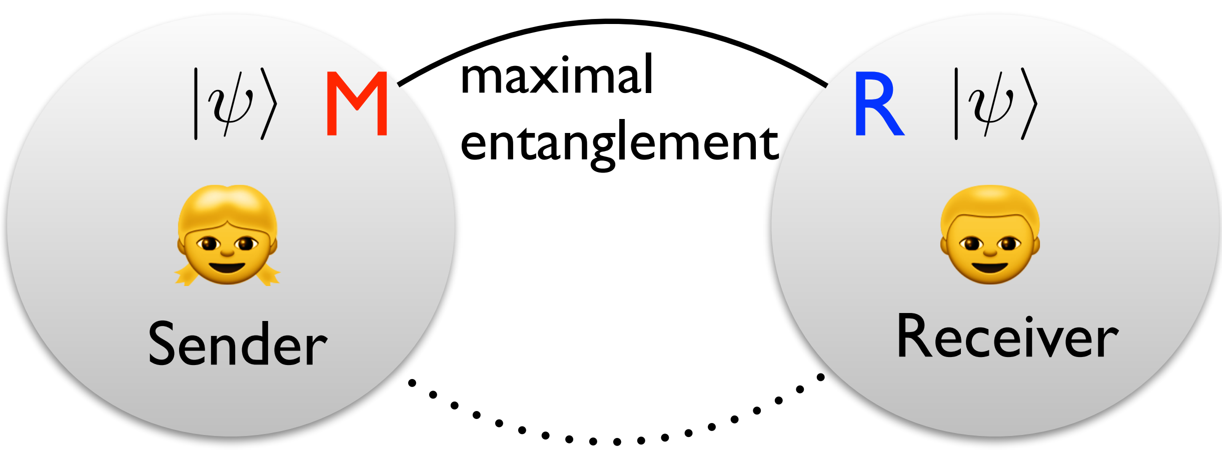
Result III

Information balance determines optimal quantum teleportation

[3] S.-W. Lee, in preperation.

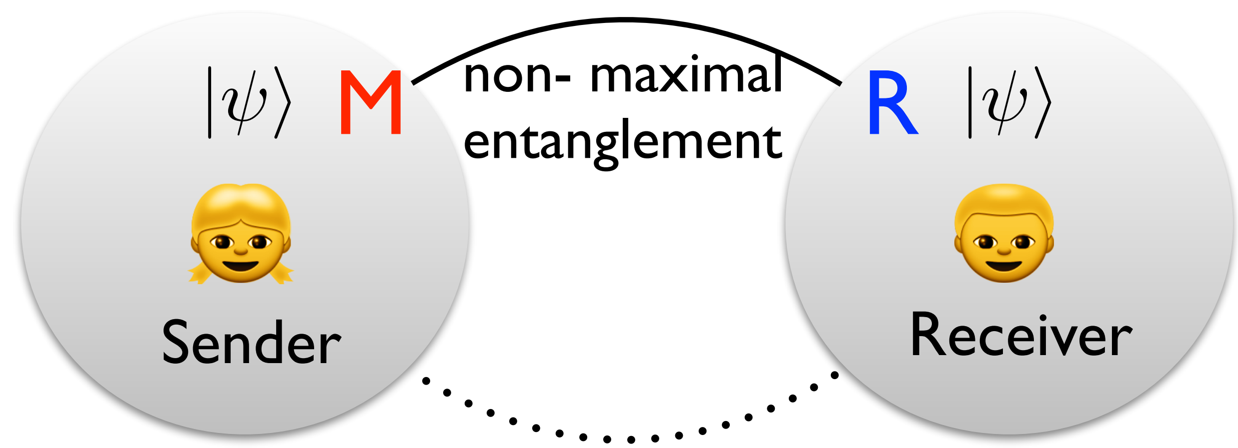
Non-ideal quantum channel or measurement → Information gain by the sender → Maximal success prob. of teleportation = Reversibility

No info gain — All info transferred



Success prob. = 1

Info gain — Partial info transferred



Success prob. < 1

Conditions for optimal quantum communications

- ✓ minimize the information gain by the (effective nonlocal) measurement
- ✓ maximize the reversibility of the measurement

(Example)

- quantum channel $|\Psi\rangle_{bc} = \cos\frac{\theta}{2}|0\rangle_b|0\rangle_c + \sin\frac{\theta}{2}|1\rangle_b|1\rangle_c$ where $0 \le \theta \le \frac{\pi}{2}$

  $\theta = 0$: product state $\theta = \frac{\pi}{2}$: maximal entanglement

- joint measurement

$$
\begin{aligned}
|W_1\rangle &= \cos\frac{\phi}{2}|0\rangle_a|0\rangle_b + \sin\frac{\phi}{2}|1\rangle_a|1\rangle_b \\
|W_2\rangle &= \sin\frac{\phi}{2}|0\rangle_a|0\rangle_b - \cos\frac{\phi}{2}|1\rangle_a|1\rangle_b \\
|W_3\rangle &= \cos\frac{\phi}{2}|0\rangle_a|1\rangle_b + \sin\frac{\phi}{2}|1\rangle_a|0\rangle_b \\
|W_4\rangle &= \sin\frac{\phi}{2}|0\rangle_a|1\rangle_b - \cos\frac{\phi}{2}|1\rangle_a|0\rangle_b,
\end{aligned}
$$

  $\phi = \frac{\pi}{2}$: Bell basis

- effective measurement

$$\hat{M}_r \equiv {}_{\text{senders}}\langle W_r||\Psi\rangle_{\text{channel}}$$

$$\hat{M}_1{}^\dagger\hat{M}_1 + \hat{M}_2{}^\dagger\hat{M}_2 + \hat{M}_3{}^\dagger\hat{M}_3 + \hat{M}_4{}^\dagger\hat{M}_4 = \mathbb{1}$$

$$
\begin{aligned}
\hat{M}_1 &= {}_{ab}\langle W_1||\Psi\rangle_{bc} = \cos\frac{\theta}{2}\cos\frac{\phi}{2}|0\rangle_c\ {}_a\langle 0| + \sin\frac{\theta}{2}\sin\frac{\phi}{2}|1\rangle_c\ {}_a\langle 1| \\
\hat{M}_2 &= {}_{ab}\langle W_2||\Psi\rangle_{bc} = \cos\frac{\theta}{2}\sin\frac{\phi}{2}|0\rangle_c\ {}_a\langle 0| - \sin\frac{\theta}{2}\cos\frac{\phi}{2}|1\rangle_c\ {}_a\langle 1| \\
\hat{M}_3 &= {}_{ab}\langle W_3||\Psi\rangle_{bc} = \cos\frac{\theta}{2}\cos\frac{\phi}{2}|0\rangle_c\ {}_a\langle 1| + \sin\frac{\theta}{2}\sin\frac{\phi}{2}|1\rangle_c\ {}_a\langle 0| \\
\hat{M}_4 &= {}_{ab}\langle W_4||\Psi\rangle_{bc} = \cos\frac{\theta}{2}\sin\frac{\phi}{2}|0\rangle_c\ {}_a\langle 1| - \sin\frac{\theta}{2}\cos\frac{\phi}{2}|1\rangle_c\ {}_a\langle 0|
\end{aligned}
$$

- optimal reversing operators

$$
\begin{aligned}
\hat{R}^{(1)} &= \tan\frac{\theta}{2}\tan\frac{\phi}{2}|0\rangle\langle 0| + |1\rangle\langle 1|, \\
\hat{R}^{(2)} &= \begin{cases} \hat{\sigma}_x\left(\cot\frac{\theta}{2}\tan\frac{\phi}{2}|0\rangle\langle 0| + |1\rangle\langle 1|\right)i\hat{\sigma}_y, & \text{if } \theta \ge \phi, \\ \hat{\sigma}_x\left(|0\rangle\langle 0| + \tan\frac{\theta}{2}\cot\frac{\phi}{2}|1\rangle\langle 1|\right)i\hat{\sigma}_y, & \text{if } \theta < \phi \end{cases} \\
\hat{R}^{(3)} &= \hat{\sigma}_x\left(\tan\frac{\theta}{2}\tan\frac{\phi}{2}|0\rangle\langle 0| + |1\rangle\langle 1|\right) \\
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  $\phi = \frac{\pi}{2}$: Bell basis

- effe

  M

  $\hat{M}$

Reversibility = the highest success probability of the teleportation

$$P_{\mathrm{rev}} = 2\sin^2\left(\frac{\min[\theta,\phi]}{2}\right) = 1 - \cos(\min[\theta,\phi]).$$

(standard teleportation)

$P_{\mathrm{rev}} = 1$ when $\theta = \phi = \pi/2$

higher than the ones by previously known protocols

P. Agrawal and A. K. Pati, Phys. Lett. A **305**, 12 (2002).

- optimal reversing operators

$$\hat{R}^{(1)} = \tan\frac{\theta}{2}\tan\frac{\phi}{2}|0\rangle\langle 0| + |1\rangle\langle 1|,$$

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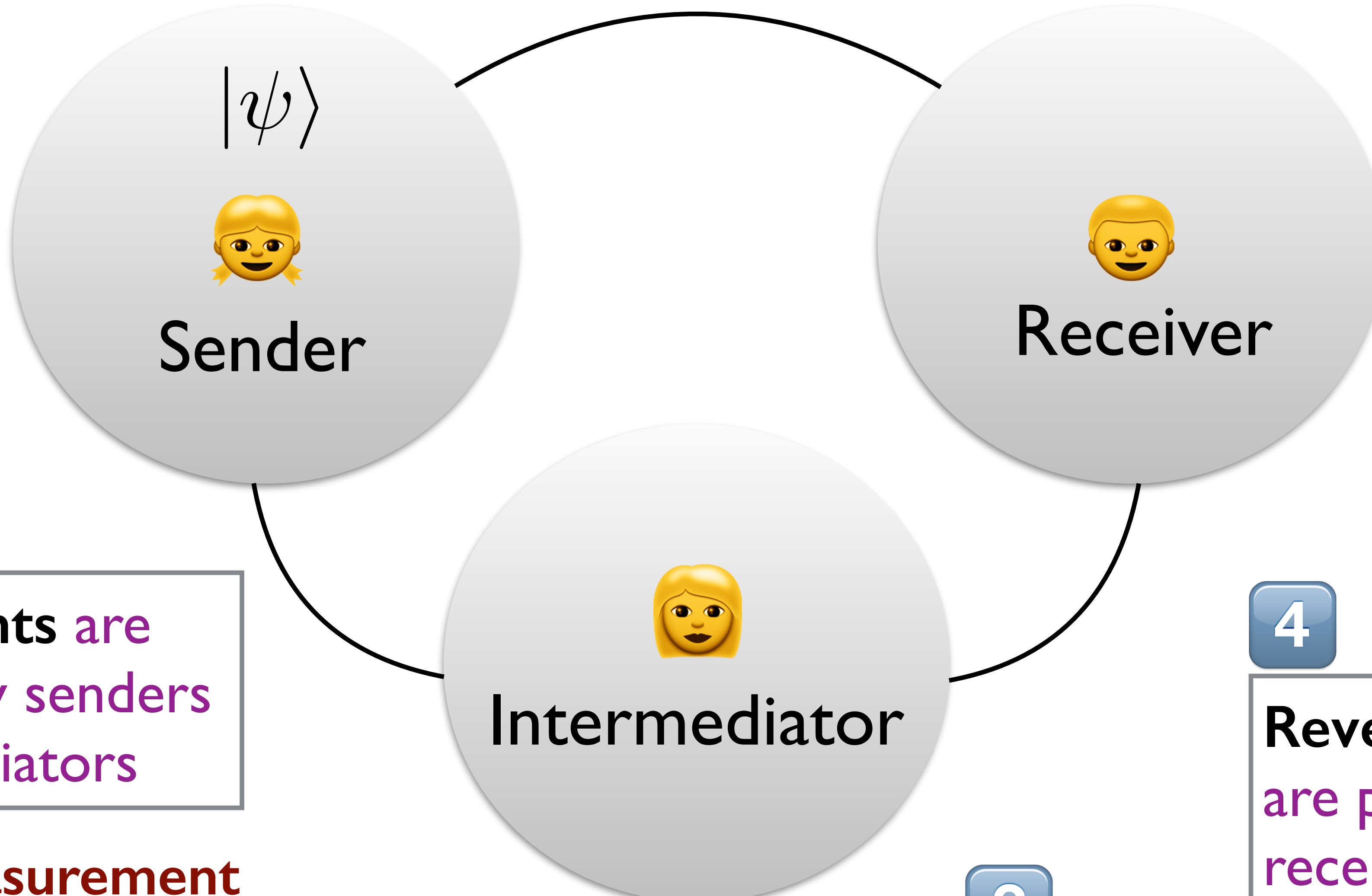
Multiparty Quantum Teleportation

1

Entanglement is shared between senders, intermediators and receivers



2

Measurements are performed by senders and intermediators

effective measurement

$$\hat{M}_{r'} = {}_{\text{inter}}\langle Y_l|_{\text{send}}\langle W_r|\Psi\rangle_{\text{ch}}$$

$$\sum_{r'} \hat{M}^{\dagger}_{r'}\hat{M}_{r'} = \mathbb{1}_{\text{send+inter}}$$

$|Y_l\rangle$: measurement basis by intermediator

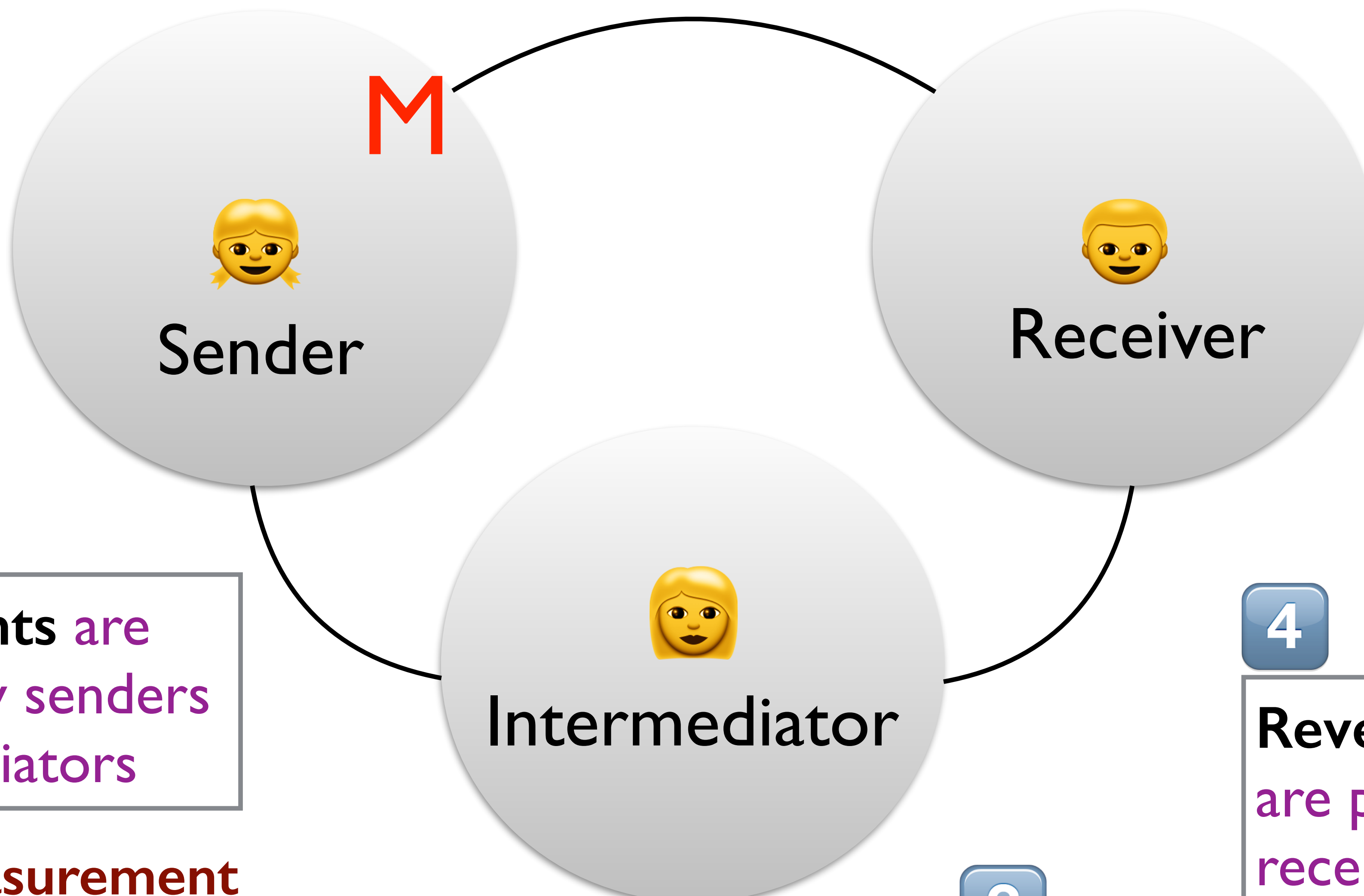
3

Measurement results are sent from senders and intermediators to receivers through classical channels

4

Reversing operations are performed on the receivers' parties

Multiparty Quantum Teleportation



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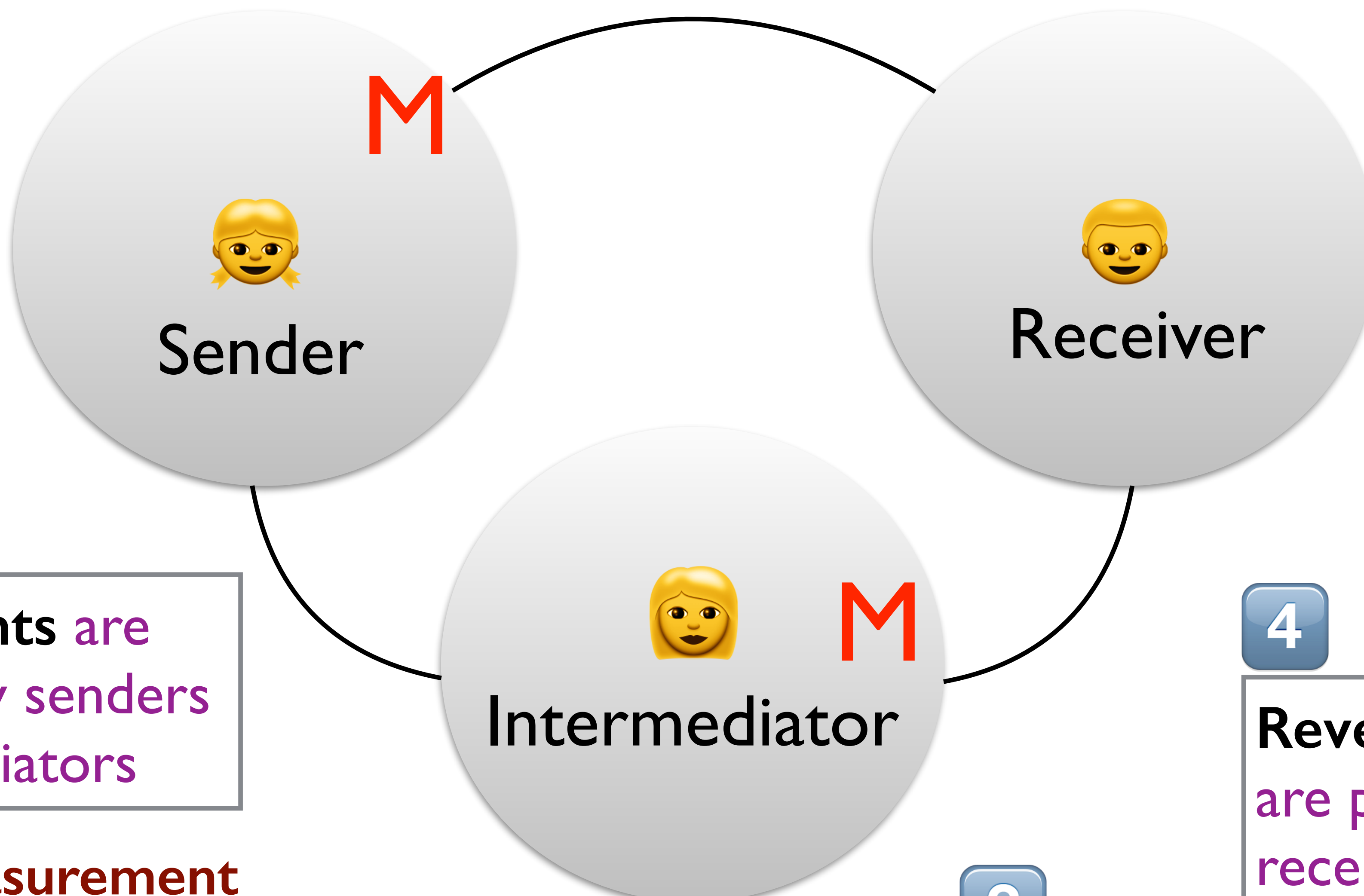

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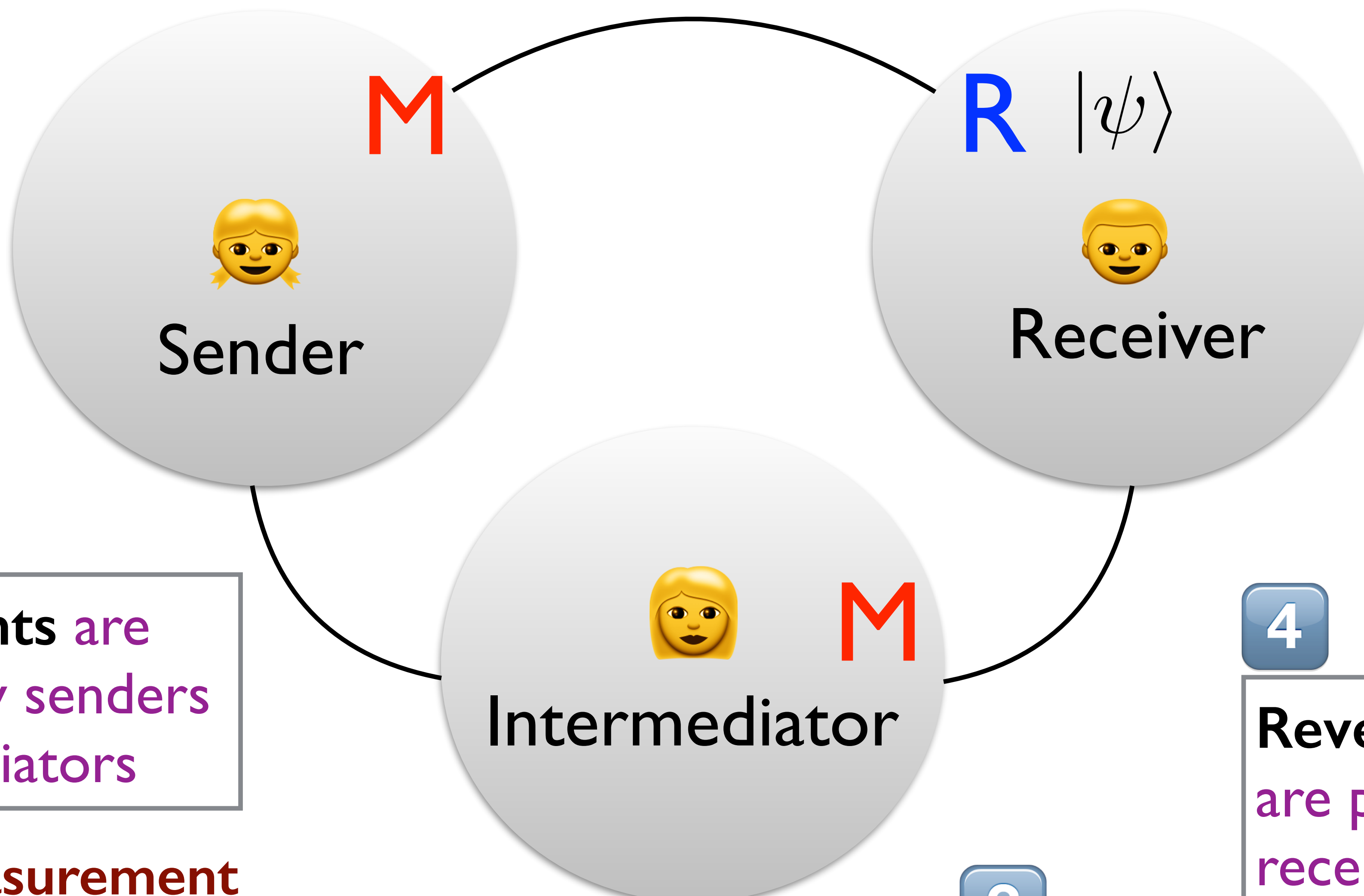

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Measurements are performed by senders and intermediators

effective measurement

$$\hat{M}_{r'} = {}_{\mathrm{inter}}\langle Y_l|_{\mathrm{send}}\langle W_r|\Psi\rangle_{\mathrm{ch}}$$

$$\sum_{r'} \hat{M}^{\dagger}_{r'}\hat{M}_{r'} = \mathbb{1}_{\mathrm{send+inter}}$$

$|Y_l\rangle$: measurement basis by intermediator

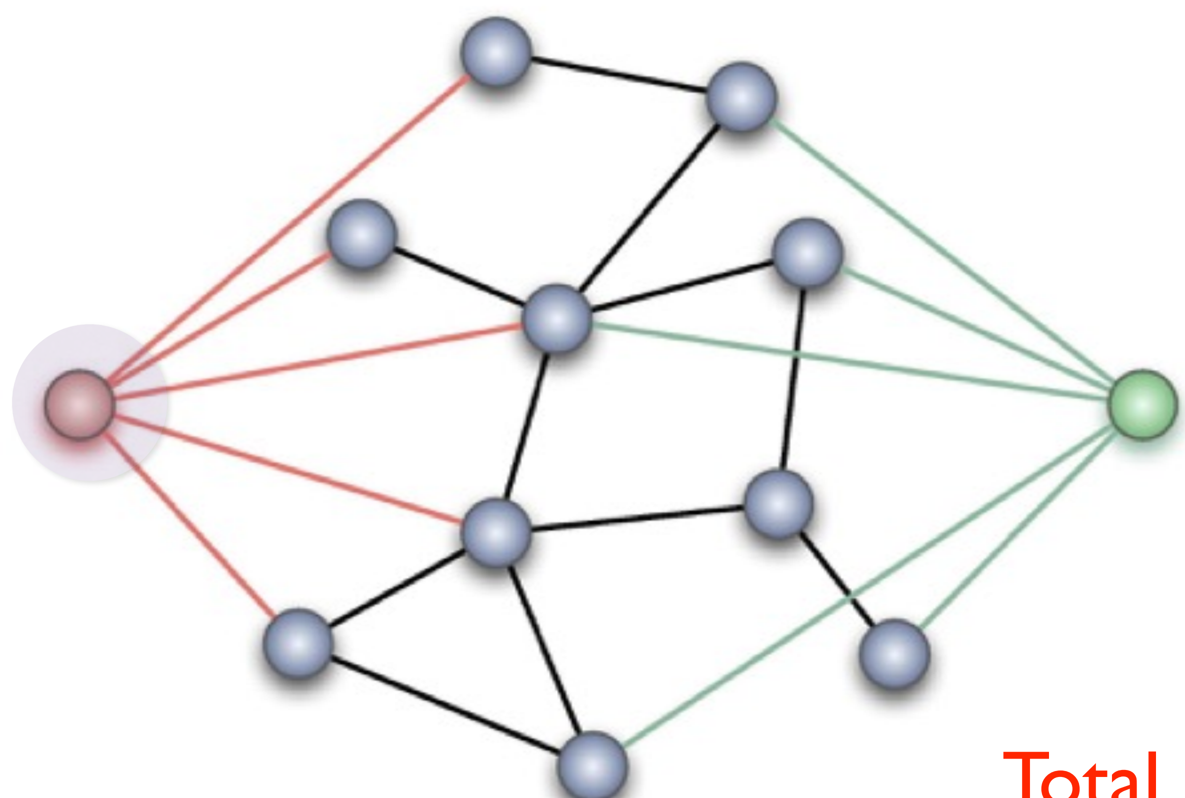
3

Measurement results are sent from senders and intermediators to receivers through classical channels

4

Reversing operations are performed on the receivers' parties

Quantum communications
in arbitrary quantum network



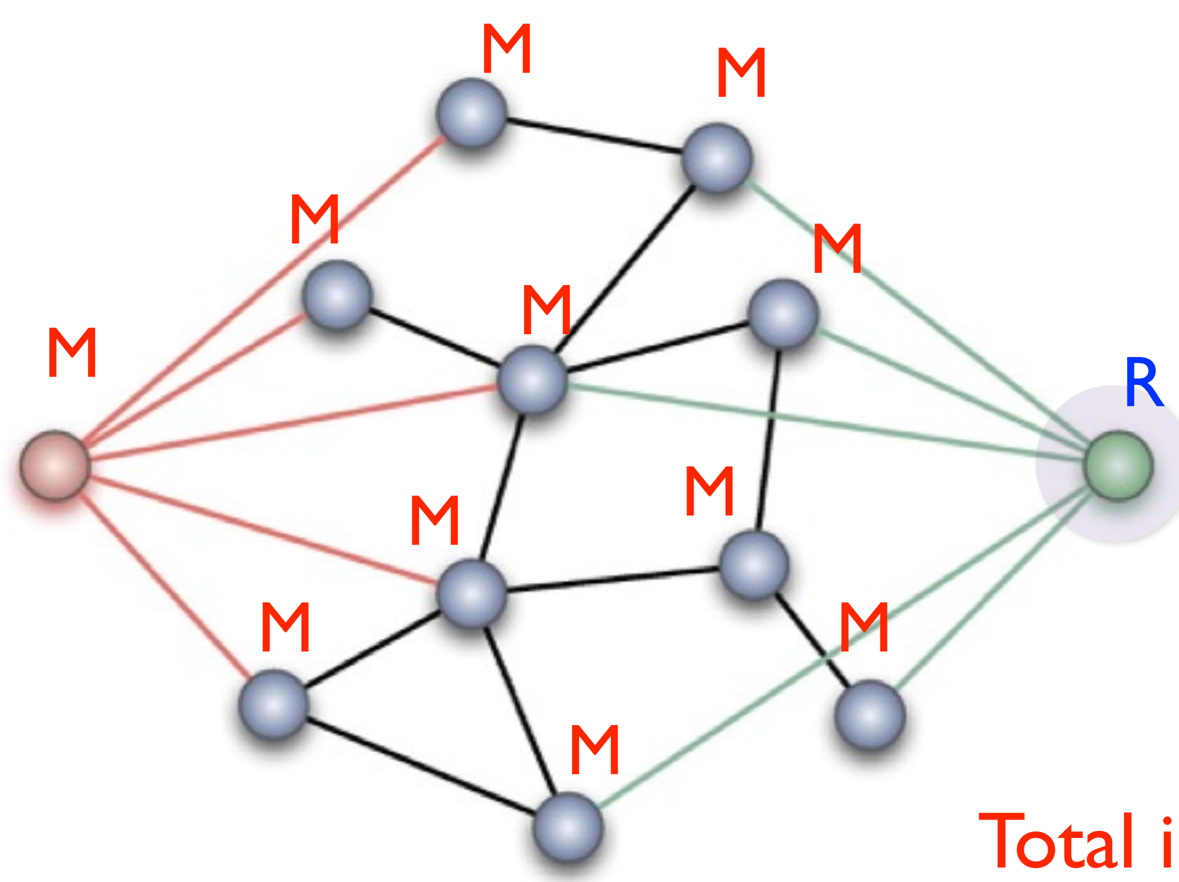
Total information balance !

Conclusions

[1] S.-W. Lee, in preperation.

Information balance determines
optimal protocols of any quantum communications !

Quantum communications
in arbitrary quantum network



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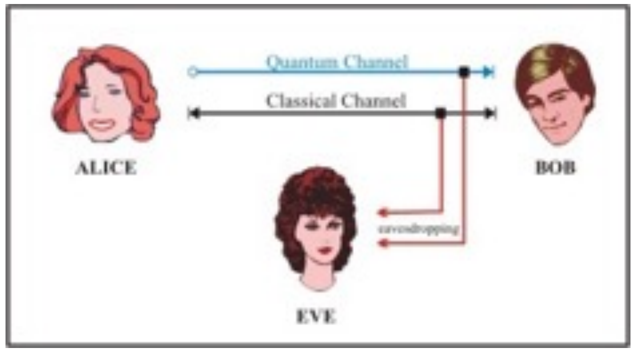
Applications and possible further studies

Impossibility of weak measurement and reversal attack to QKD



QKD (Quantum Key Distribution)

Intercept - Weak measurement - Reversal attack ?

Quantitative and fundamental proof of its impossibility based on the ***Information balance*** in quantum measurement

A global information balance in quantum measurements

Quantitative Bound for
Information gain + disturbance + reversibility

Decoherence Suppression

by Weak measurement and Reversal

A.N.Korotkov et al., PRA 81 040103(R) (2010)   Y.-S. Kim et al., Nature Phy 8 117 (2012)

by Zeno effects with weak measurements

G. A. Paz-Silva et al., PRL 108 080501 (2012)

The laws of thermodynamics for quantum state transfers ?

and so on…

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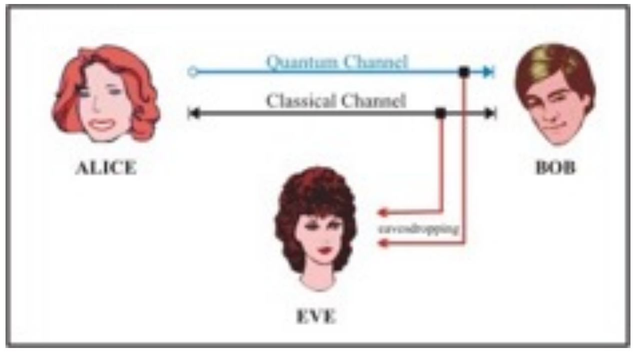

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Quantitat

Informati

Thank you !

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