QUANTUM MECHANICS

(Non.relativistic Theory) 3rd Ed.

Landau and Lifshitz

This irreversibility is of fundamental significance. We shall see later (at the end of §18) that the basic equations of quantum mechanics are in themselves symmetrical with respect to a change in the sign of the time; here quantum mechanics does not differ from classical mechanics. The irreversibility of the process of measurement, however, causes the two directions of time to be physically non-equivalent, i.e. creates a difference between the future and the past.

Irreversibility of Quantum Projective Measurement

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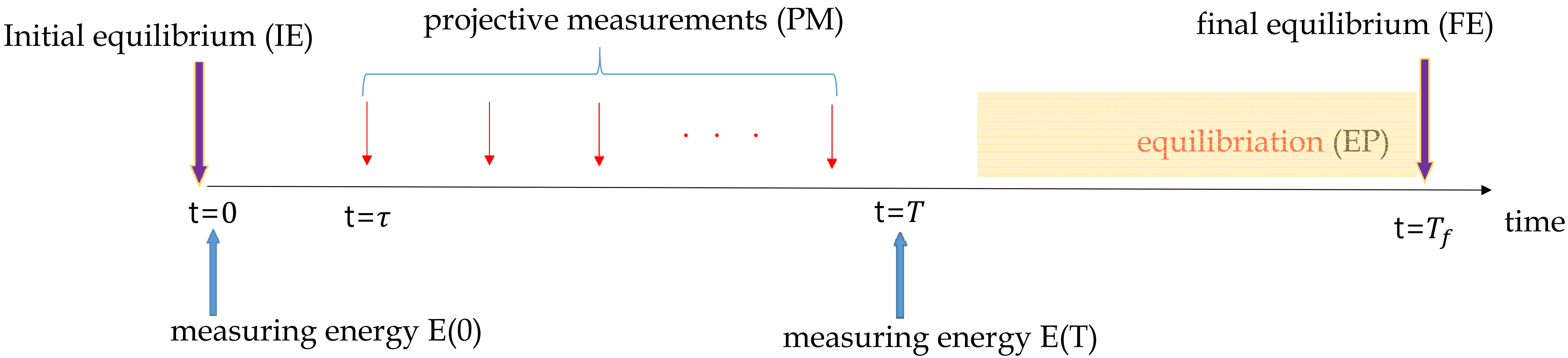
setting



System Hamiltonian remains constant
The final equilibrium is thermodynamically identical to the initial equilibrium

thermodynamic entropy production

Recording the energy changes $W = E(T) - E(0)$

Statistics of W gives the probability distribution $P(W)$

One can prove a relation,

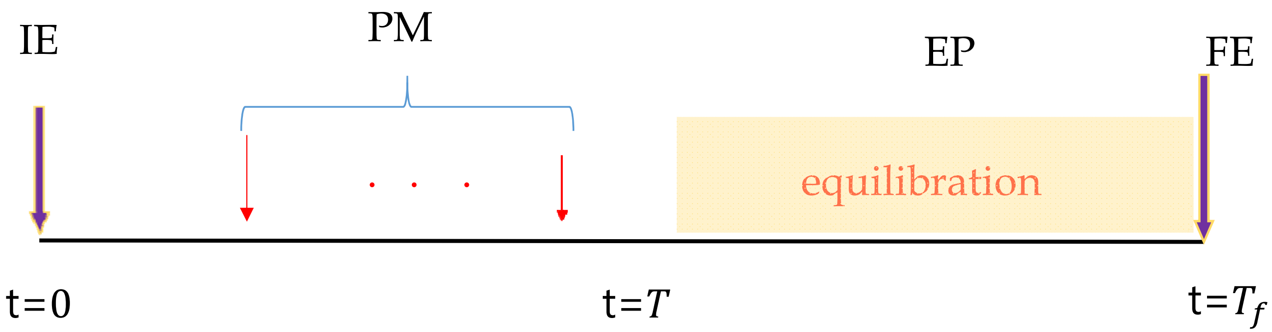
$$P(W) = e^{\beta W} \; P(-W)$$ (Crooks type)

$$\int_{-\infty}^{\infty} dW \, P(W) e^{-\beta W} = \langle e^{-\beta W} \rangle = 1$$ (Jarzynski type)

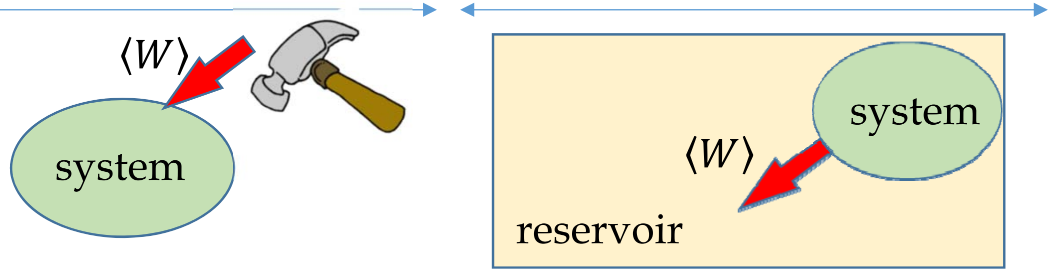
$\langle e^{-\beta W} \rangle \geq e^{-\beta <W>}$ (Jensen inequliaty)

$$\langle W \rangle \geq 0$$

Energy of a measured system increases!



System energy change $\langle E(T) - E(0)\rangle = \langle W\rangle \geq 0$ $\quad \langle E(T_f) - E(T)\rangle$=-$\langle W\rangle$ for $\langle E(T_f)\rangle = \langle E(0)\rangle$



The reservoir during EP absorbs the energy in the form of heat : $\langle Q_r\rangle$=$\langle W\rangle = T\Delta S_r$

Total entropy change between FE and IE increases:

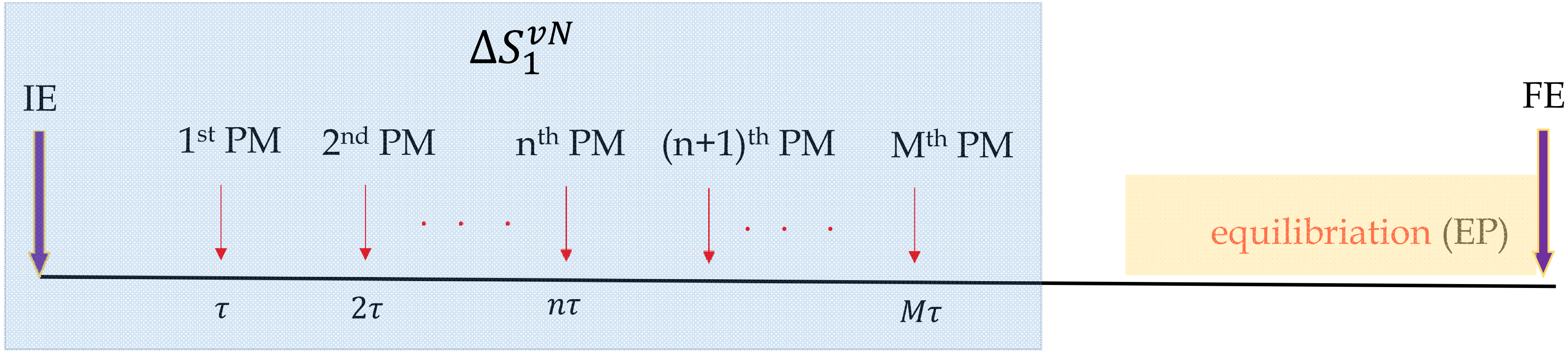
$$S_r(T_f)\text{-}S_r(0) + S_s(T_f)\text{-}S_s(0) = \Delta S_r = \langle W\rangle/T \geq 0$$

Information entropy production

The von Neumann entropy

$$S^{vN} = -Tr\,\rho \ln \rho$$

ρ : density matrix



Density matrix after the (n+1)th PM : ρ_{n+1}

$$\rho_{n+1} = \sum_{\alpha} P_\alpha U(\tau)\rho_n U^+(\tau) P_\alpha$$

P_α : projection operator for a measurement outcome α

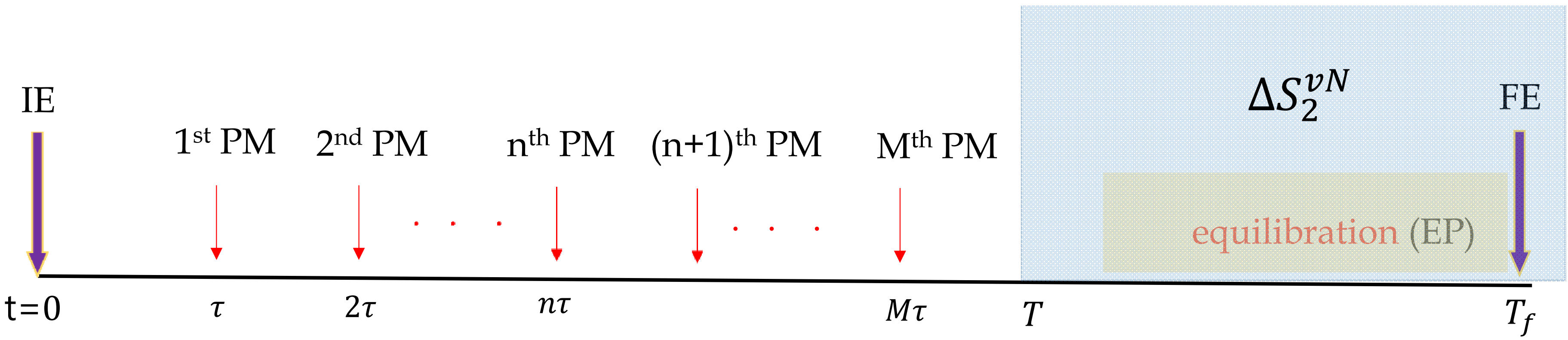
$U(\tau)$: unitary time evolution operator

Change in the von Neumann entropy between the (n+1)th and the nth PM

$$\Delta S^{vN}(n) = -Tr\, \rho_{n+1} \ln \rho_{n+1} + Tr\, \rho_n \ln \rho_n \geq 0$$

The von Neumann entropy production due to PM is positive

$$\Delta S_1^{vN} = \sum_{n=0}^{M-1} \Delta S^{vN}(n) \geq 0$$



Entropy production during EP

$$\Delta S_2^{vN} = -Tr\,\rho_{T_f}\ln\rho_{T_f} + Tr\,\rho_T\ln\rho_T + \Delta S_r \geq 0$$

$$= -\Delta S_1^{vN} + \Delta S_r \geq 0$$

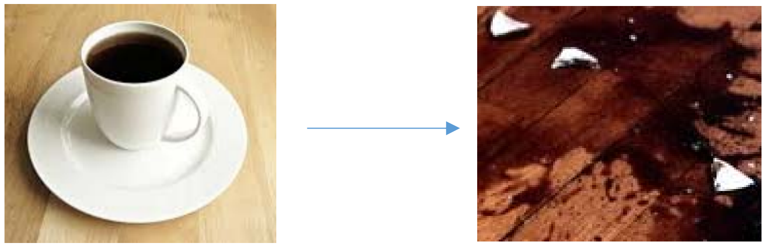
Using $\rho_{T_f} = \rho_0$ and no entropy production between the time T and the last measurement

$$\Delta S_r \geq \Delta S_1^{vN}$$

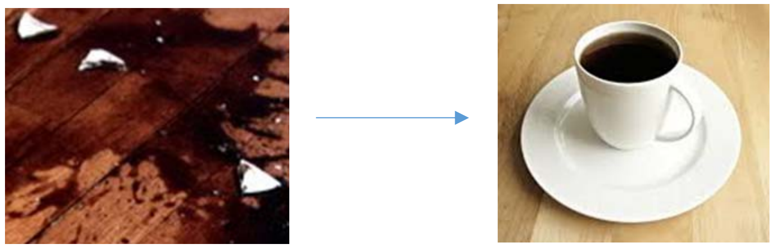
Landauer principle: the erase of a single bit of information requires at least an entropy change of $\ln 2$

time's arrow

(a) entropy *increasing* event

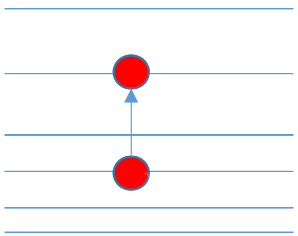


(b) entropy *decreasing* event



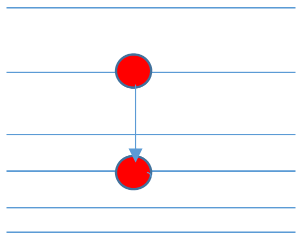
If you watch a video clip of the event (b), you will recognize that
it is in fact the video clip recording the event (a) but is ran event.
Probability to observe the event (a) is much higher than the event (b)

entropy *increasing* event



$W > 0$

entropy *decreasing* event



$W < 0$

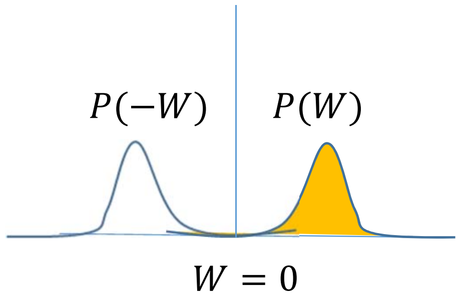
The arrow of time associated with projective measurements depends on the asymmetry between $P(W)$ and $P(-W)$

Time asymmetry (Jensen Shannon divergence)

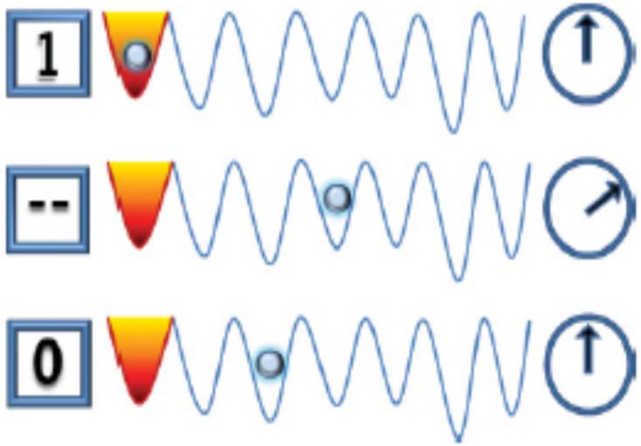
$$A = \int_{-\infty}^{\infty} dW\, P(W) \ln \left[\frac{2P(W)}{P(W) + P(-W)} \right] \qquad (0 \leq A \leq \ln 2\,)$$

$A = 0$ if $P(W) = P(-W)$

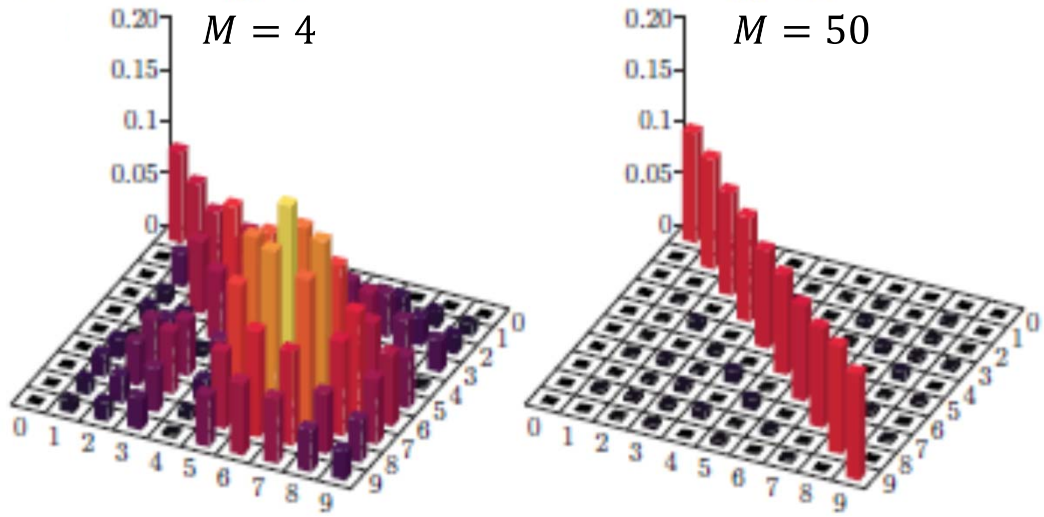
$A = ln2$ if $P(W)$ has no overlap with $P(-W)$



Model system



Tomography of a density matrix



System approaches infinite temperature
upon repeated measurements

At high temperatures

$$A \approx C\beta^2$$

At low temperatures

$$A \approx (1-\Gamma)\ln 2$$

survival probability of the ground state