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Consistent theory for generalized causal non-locality

Generalization of Born's rule

KIAS meeting, Seaclaud, Pusan, 2015

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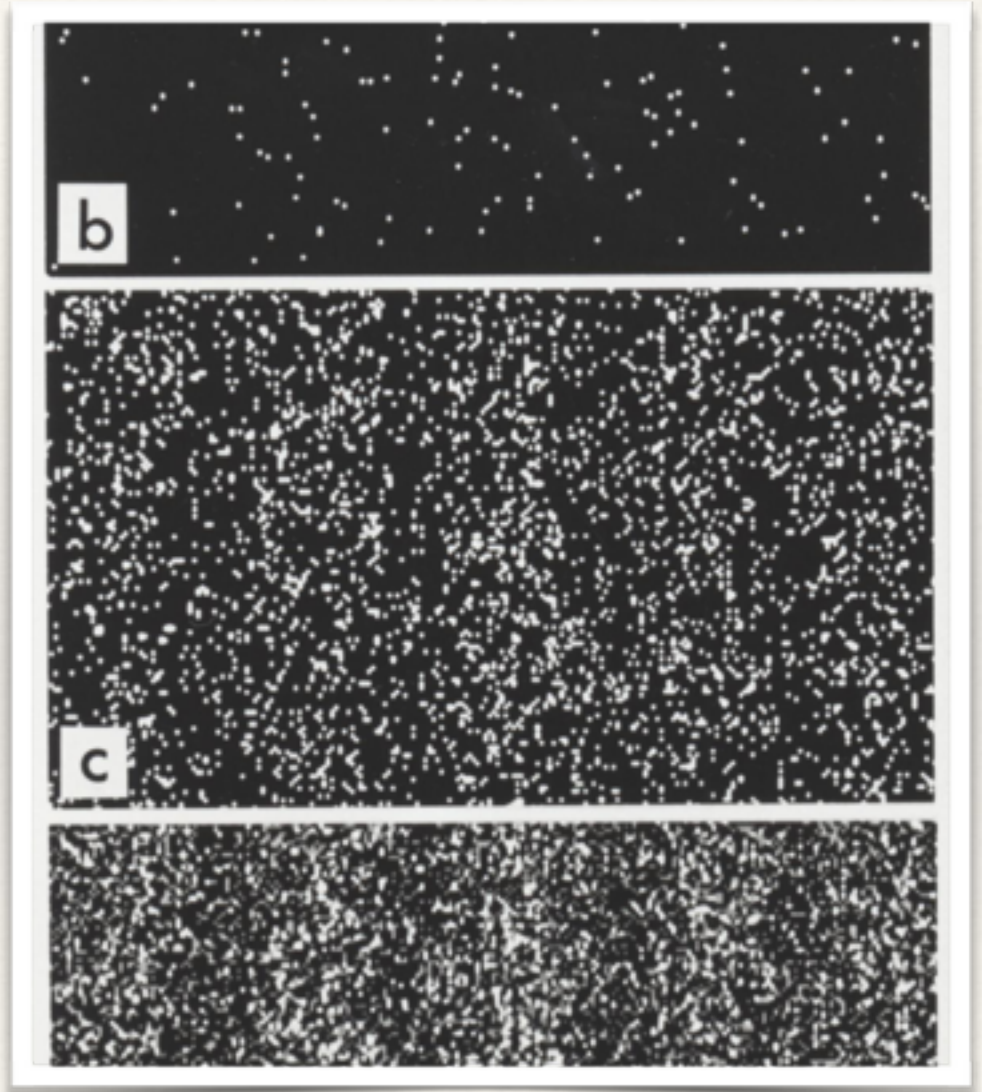
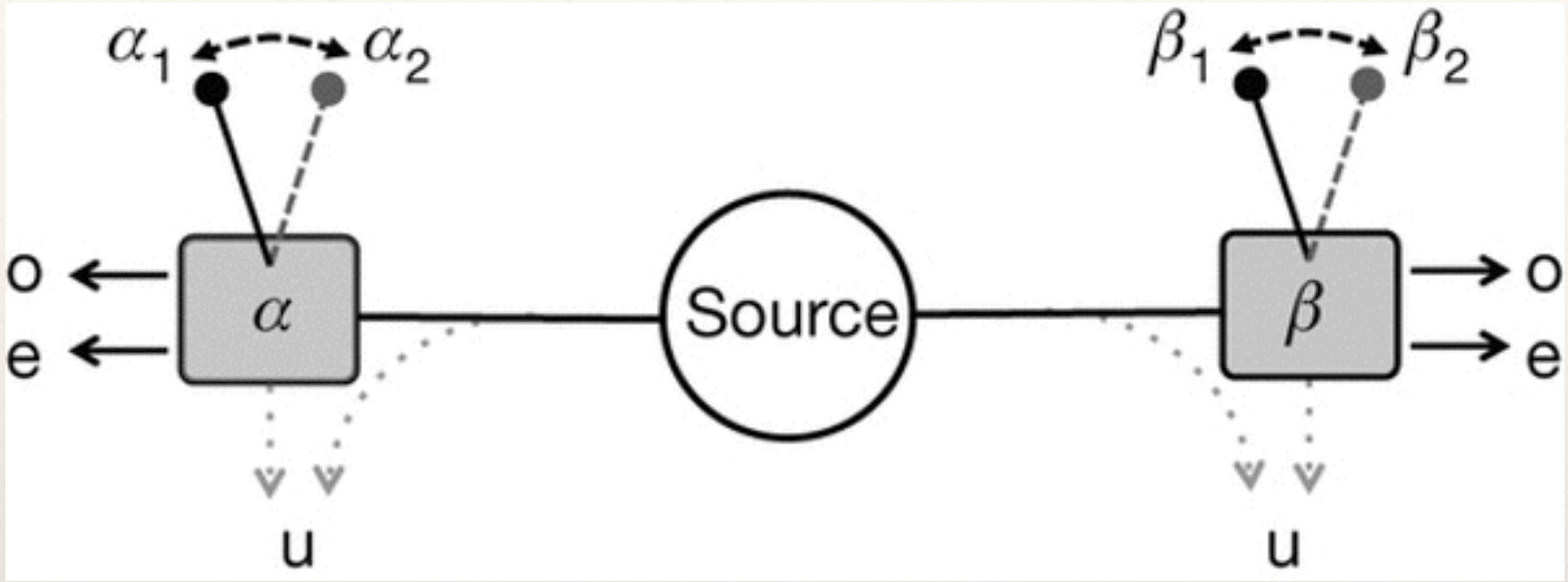


Challenging history in QM

- Completeness of quantum mechanics has been argued from the vary beginning of history of quantum mechanics (EPR-Bohr, 1935).
- Mathematical criteria to determine the completeness of quantum mechanics had been proposed (Bell's inequality, 1964).
- Experimental verification of Bell's theorem had been made through *quantum optical setup* (parametric down conversion, 1981).
- Recent claim of loophole free Bell test. (2015?)

Basic setup for Bell test



- Standard Bell-CHSH (two measurements- two outcomes)

$$\begin{aligned} CHSH =& \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \\ =& \langle \hat{A}_1 \otimes (\hat{B}_1 + \hat{B}_2) + \hat{A}_2 \otimes (\hat{B}_1 - \hat{B}_2) \rangle \end{aligned}$$

$$E(AB) + E(A'B) + E(AB') - E(A'B') < 2$$

Quantum violation and beyond

$$E(AB) + E(A'B) + E(AB') - E(A'B') < 2\sqrt{2}$$

- Generalization of Bell theorem to an arbitrary quantum system is a main challenge in ***modern QM***.
- Question about the foundational origin of the Cirelson's bound is to be asked by a group of people as like Aharanov, Shimony, Popescu, Rohrlich.
- Information theoretic importance of a quantum state can be characterized by the criteria. (van Dam, PhDthesis and so on)
    - Key words: entanglement, quantum steering, communication complexity, nonlocal box, information causality…

Axioms in quantum mechanics

- Axioms in QM
  - State-Hilbert space (complex vector space)
  - Measurement- Observable (Hermitian operator)
  - Dynamics- Schrodinger equation
  - Symmetry- Identical particles
  - Probability- Born's rule

BORN RULE, (Not principle)

- Similarly to the axioms in special relativity (physics is conserved w.r.t. RoF; speed of light is constant in RoF), QM can be derived from
  - Non-locality?
  - No signaling?

Contradictable

Conjecture leading to PR box (1991)

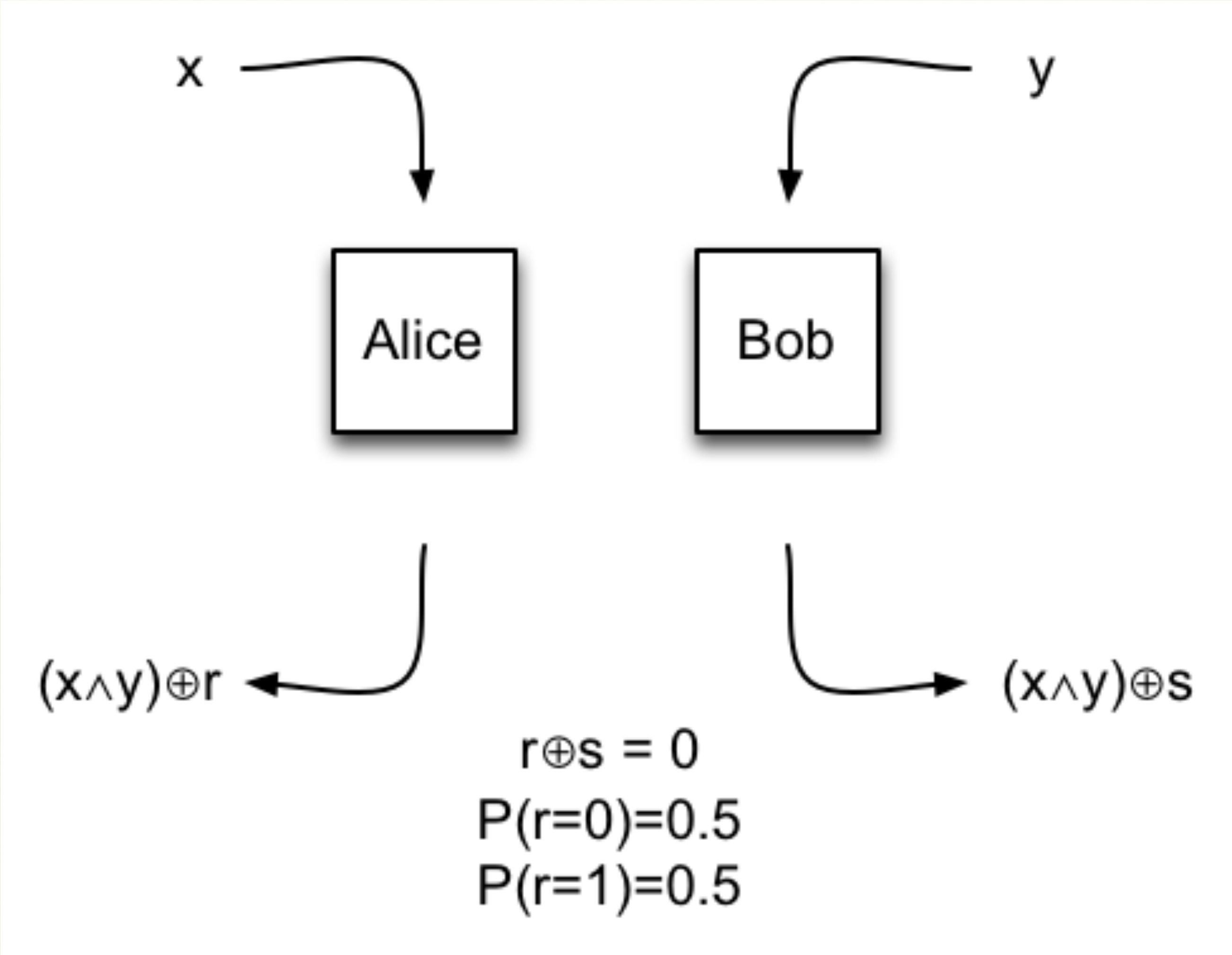
Non-locality+No-signaling

Cirelson Bound

$$2\sqrt{2}$$

The conjecture was not correct

pf) Non-local box leading to the CHSH correlation 4 still satisfies the no-signalling theorem.



The first question(s)!

- What kind of the theory allows us to achieve the correlation for the nonlocal box,
(or Classical correlation, Quantum correlation)?

- Is it possible to obtain a dynamical equation for such kind of super-correlated (super-quantum) systems?

Arguable derivation of Born's Rule

$$P(x) = |\psi(x)|^2 = |\langle x|\psi\rangle|^2$$

- Gleason's theorem (Probability of composite events)
- Wallace's statement
- Deutsch's Decision theory based upon the maximum likelihood approach. (*Proc. R. Soc. Lond.* **A,** 1999)
- Zureck's Environment Induced (envariant) approach (Principle of indifference, Baysian) Phys. Rev. A 71, 052105 (2005);idem, Phys. Rev. Lett. 106, 250402 (2011)
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Theory for the correlation with arbitrary bound of CHSH function (Recent research results)

$$P(x) = |\psi(x)|^p, \text{where } 1 \leq p$$

- Theory without Born's rule produces new correlation function leading to non-local box.
- Such a generalization gives arbitrary violation of CHSH while it satisfies No-signaling postulate.
- Generalized trigonometric function is given as the solution of the p-norm probabilities whose dynamics can be described by non-linear Schrodinger type equation.
- Recently the result was test by experimental group in Canada

Generalized Sine Function

The 3rd assumption : $\left(\frac{d \sin_p \theta}{d\theta}\right)^2 + \left(\frac{d \cos_p \theta}{d\theta}\right)^2 = C.$
From $\sin_p^p \theta + \cos_p^p \theta = 1$, we have
$\frac{d \cos_p \theta}{d\theta} = -\sin_p^{p-1} \theta \left(1 - \sin_p^p \theta\right)^{\frac{1}{p}-1} \frac{d \sin_p \theta}{d\theta}.$
Then, the assumption becomes

$$\left(\frac{d \sin_p \theta}{d\theta}\right)^2 + \left[-\sin_p^{p-1} \theta \left(1 - \sin_p^p \theta\right)^{\frac{1}{p}-1} \frac{d \sin_p \theta}{d\theta}\right]^2 = C.$$

Thus, $\left(\frac{d \sin_p \theta}{d\theta}\right) [1 + \sin_p^{2(p-1)} \theta \left(1 - \sin_p^p \theta\right)^{2\left(\frac{1}{p}-1\right)}]^{\frac{1}{2}} = C^{\frac{1}{2}}$
or by substituting $x = \sin_p^p \theta$,

$$\frac{1}{p}\frac{dx}{d\theta}\left[x^{2\left(\frac{1}{p}-1\right)} + (1-x)^{2\left(\frac{1}{p}-1\right)}\right]^{\frac{1}{2}} = C^{\frac{1}{2}}.$$

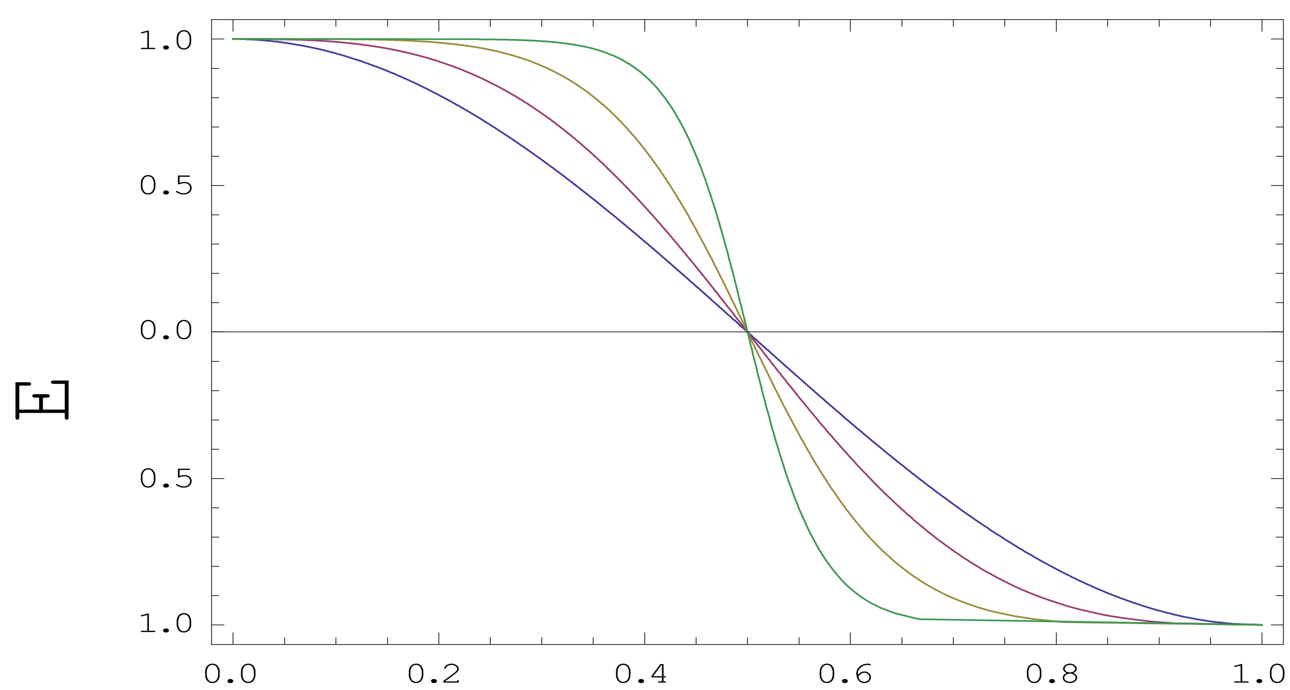
Therefore, $\theta = \sin_p^{-1} x^{1/p} = \frac{C^{\frac{1}{2}}}{p}\int_0^x dx \left[x^{2\left(\frac{1}{p}-1\right)} + (1-x)^{2\left(\frac{1}{p}-1\right)}\right]^{\frac{1}{2}}.$
It can be rewritten as

$$\sin_p^{-1} x = C^{\frac{1}{2}} \int_0^x dx \left[1 + \left(x(1-x^p)^{-\frac{1}{p}}\right)^{2(p-1)}\right]^{\frac{1}{2}}$$

Correlation from the generalized trigonometric function
W. Son, ArXiv:1401.1012

Generalized Sine Function

From this sine function with normalization by a period, $E(\theta) = 1 - 2\sin_p^p \theta$ is shown as



Remarks

- A theory to make the non-local box can be generated if one allows to release the constraints made by Born's rule
- The wave function for the super-quantum correlation have been derived.
- Non-linear dynamic equation can be derived from the solution.
- The factor can be measured in an experimentally testable way.

- Thanks for your attention!