KIAS

Protecting entanglement from decoherence via weak quantum measurement

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POSTECH

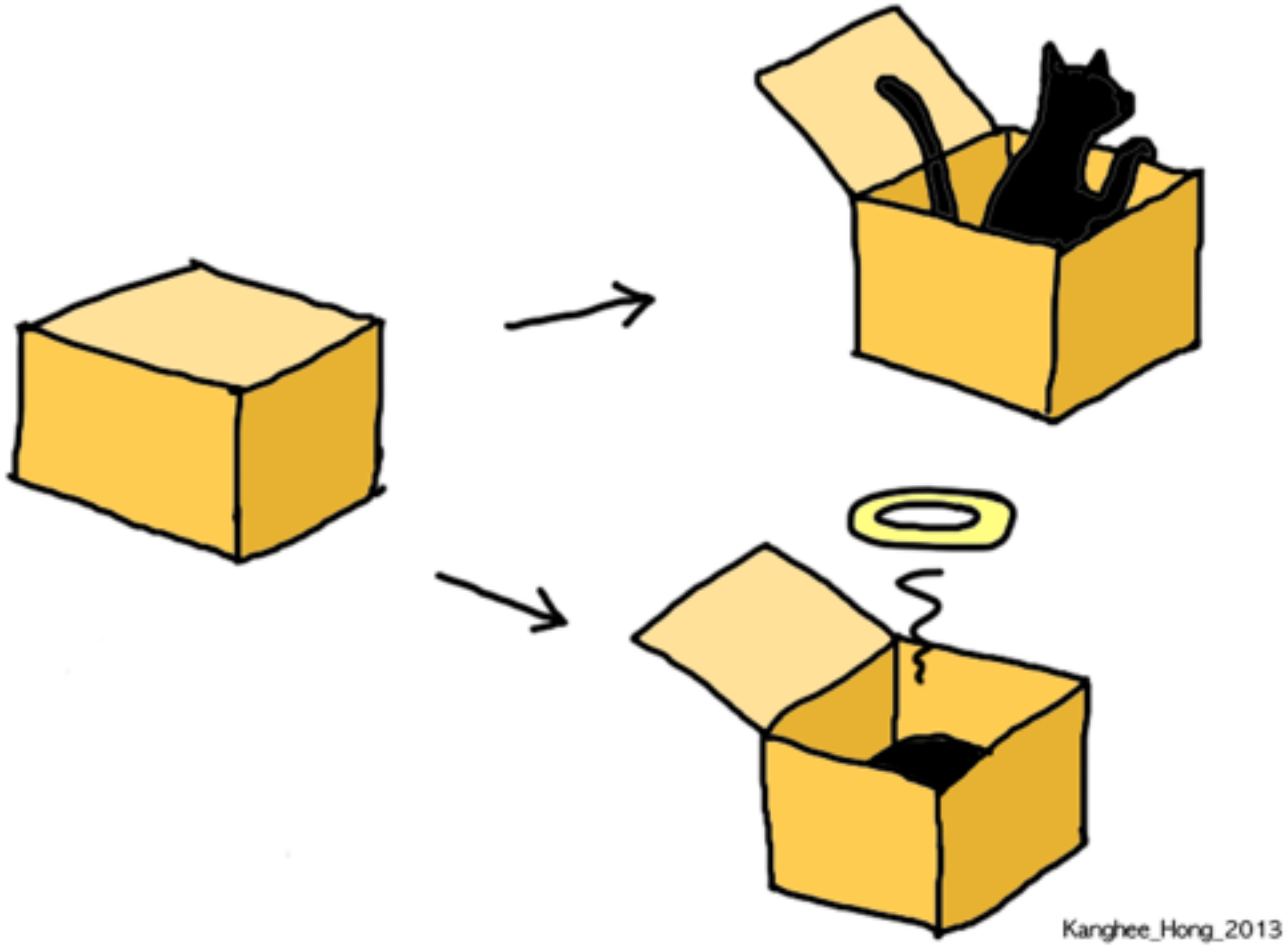
Nov 28, 2015

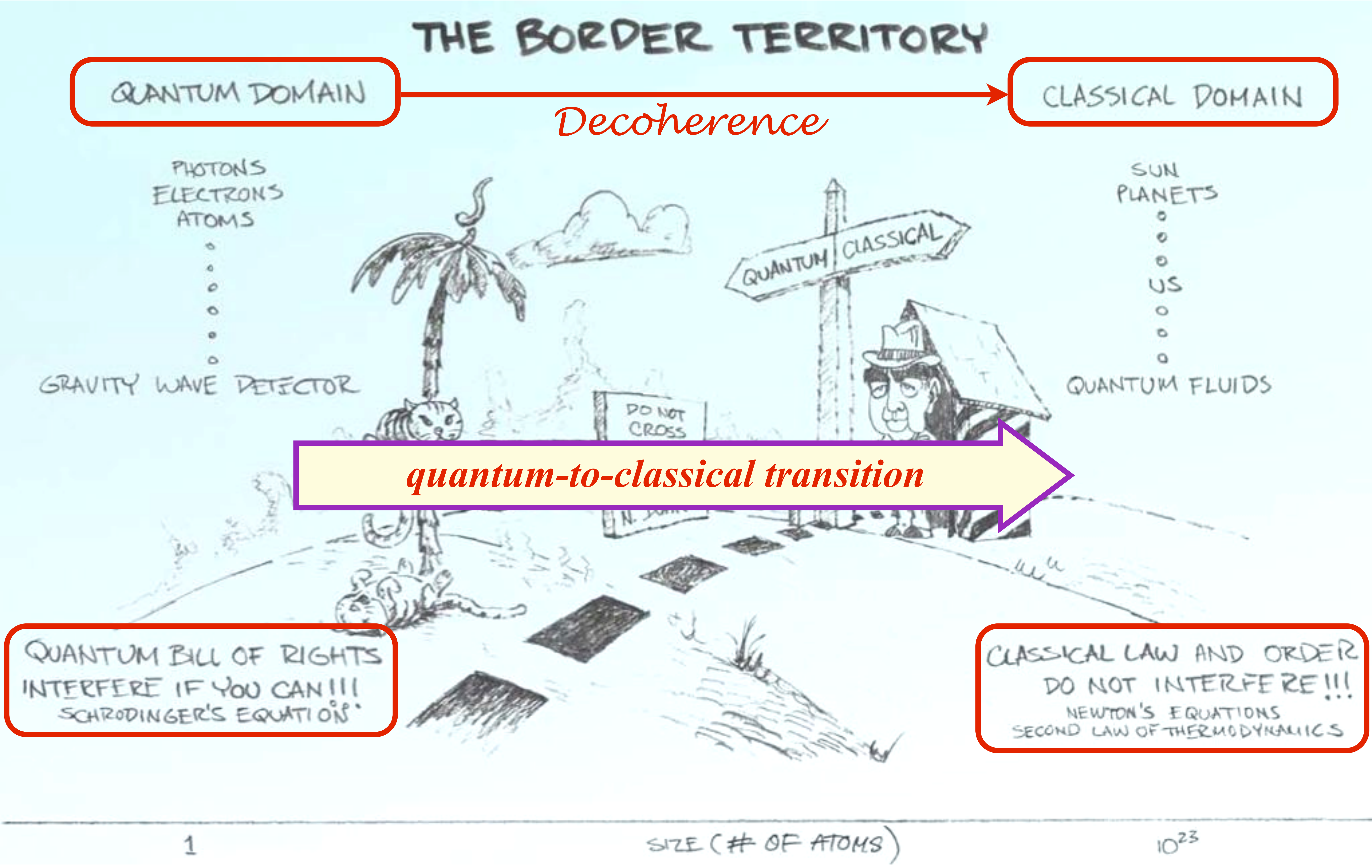
The Schrödinger's cat

Kanghee_Hong_2013

$$|\Psi\rangle = |\text{alive}\rangle + |\text{dead}\rangle$$

Observing the Schrödinger’s cat causes abrupt quantum-to-classical transition



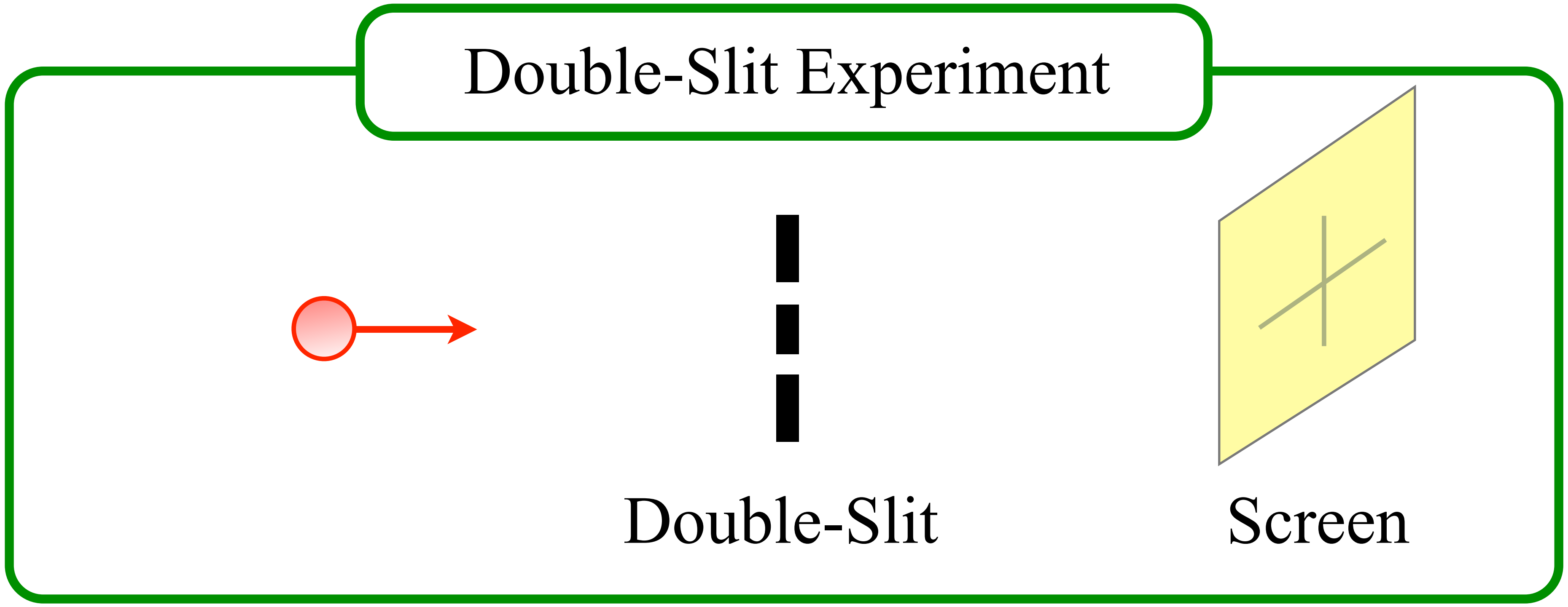


Decoherence

quantum-to-classical transition

Zurek, Los Alamos Science (2002)

Quantum domain: particles exhibit interference due to quantum superposition

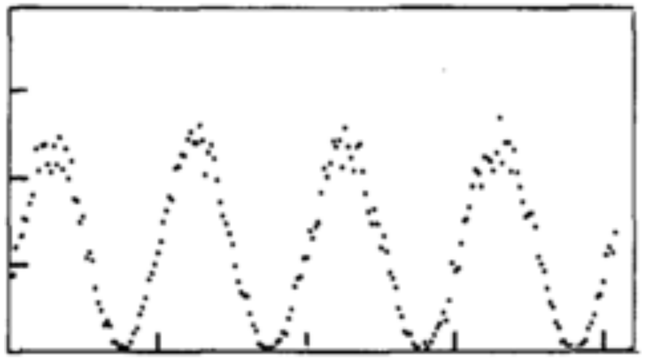


Double-Slit Experiment

Double-Slit

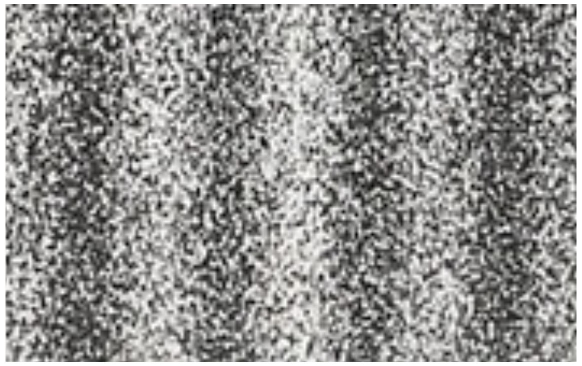
Screen

Photon



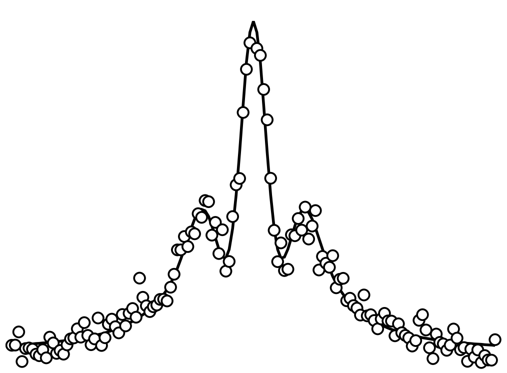
Europhys. Lett. 1, 173 (1986)

Electron



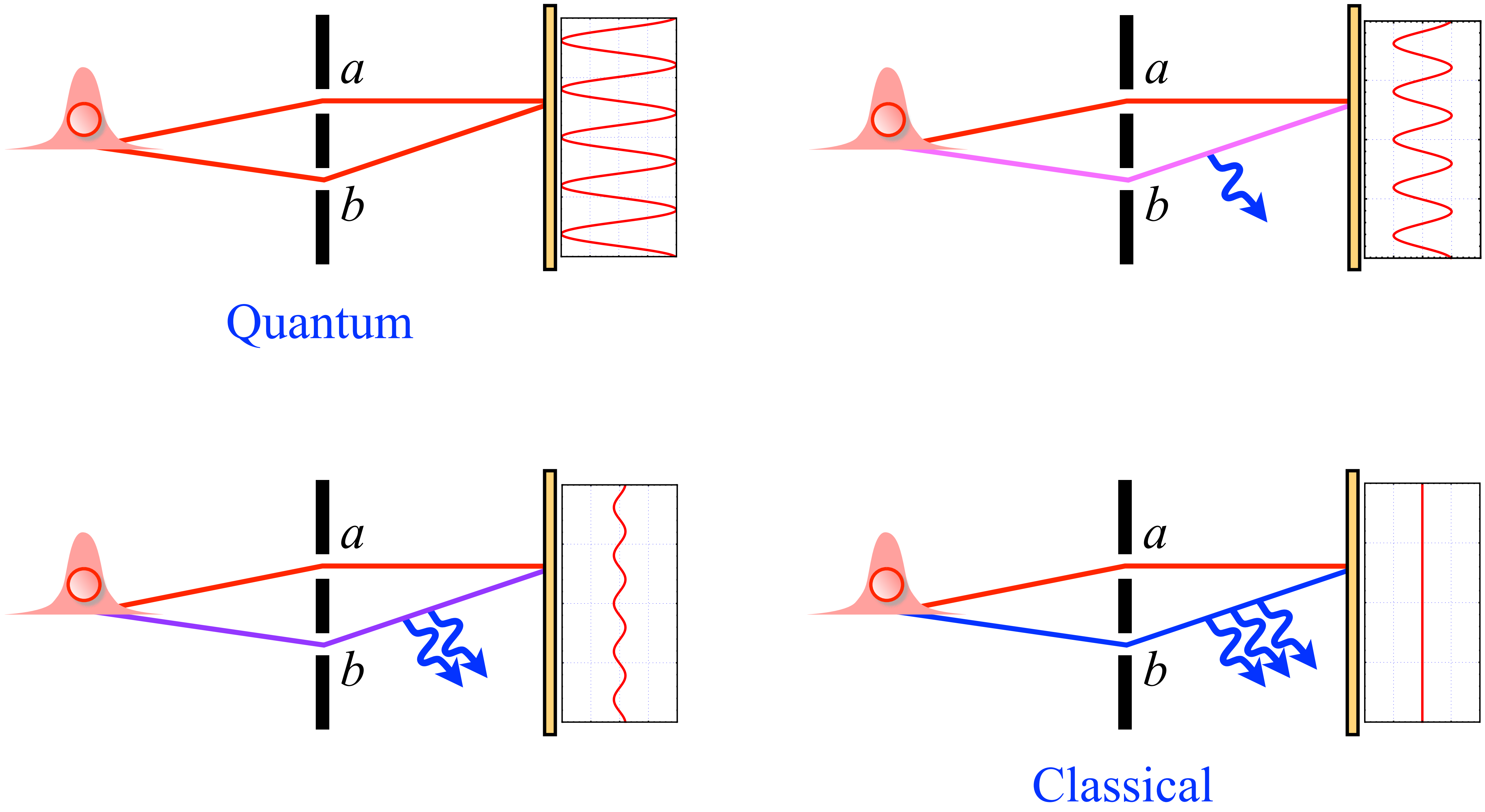
Am. J. Phys. 41, 639 (1973)

Fullerene (C_{60})



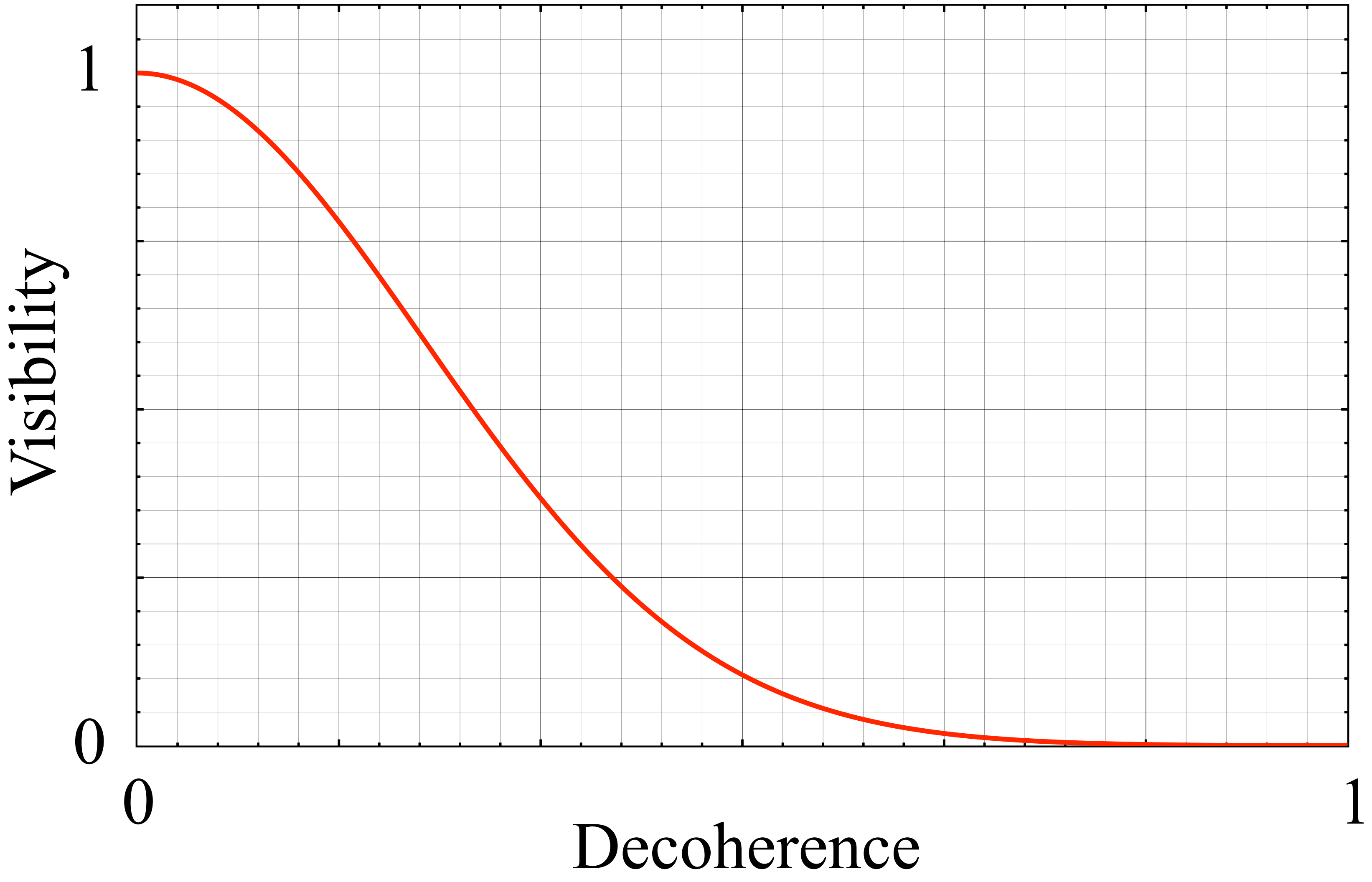
Nature 401, 680 (1999)

Decoherence causes gradual quantum-to-classical transition



Ra *et al.*, PNAS **110**, 1227 (2013)

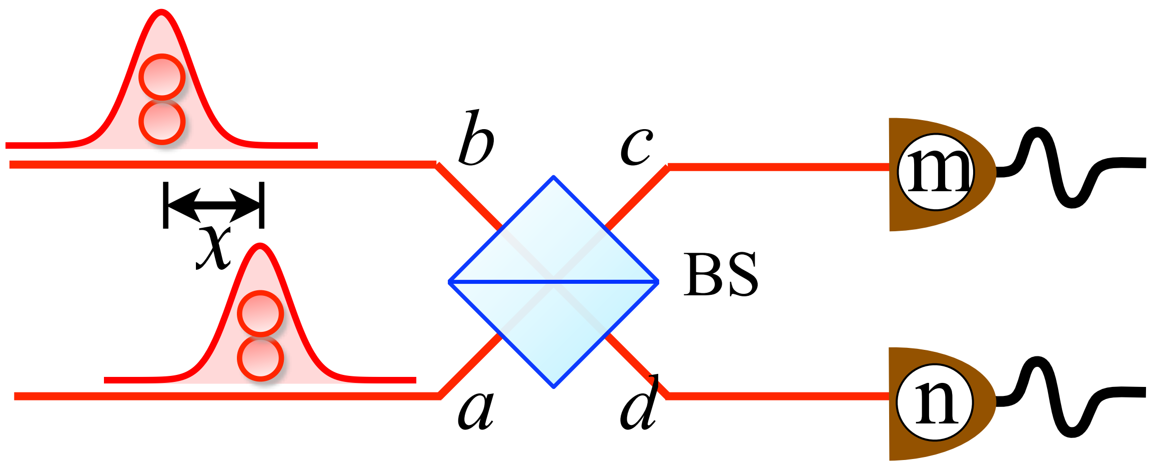
Quantum-to-classical transition always monotonic?

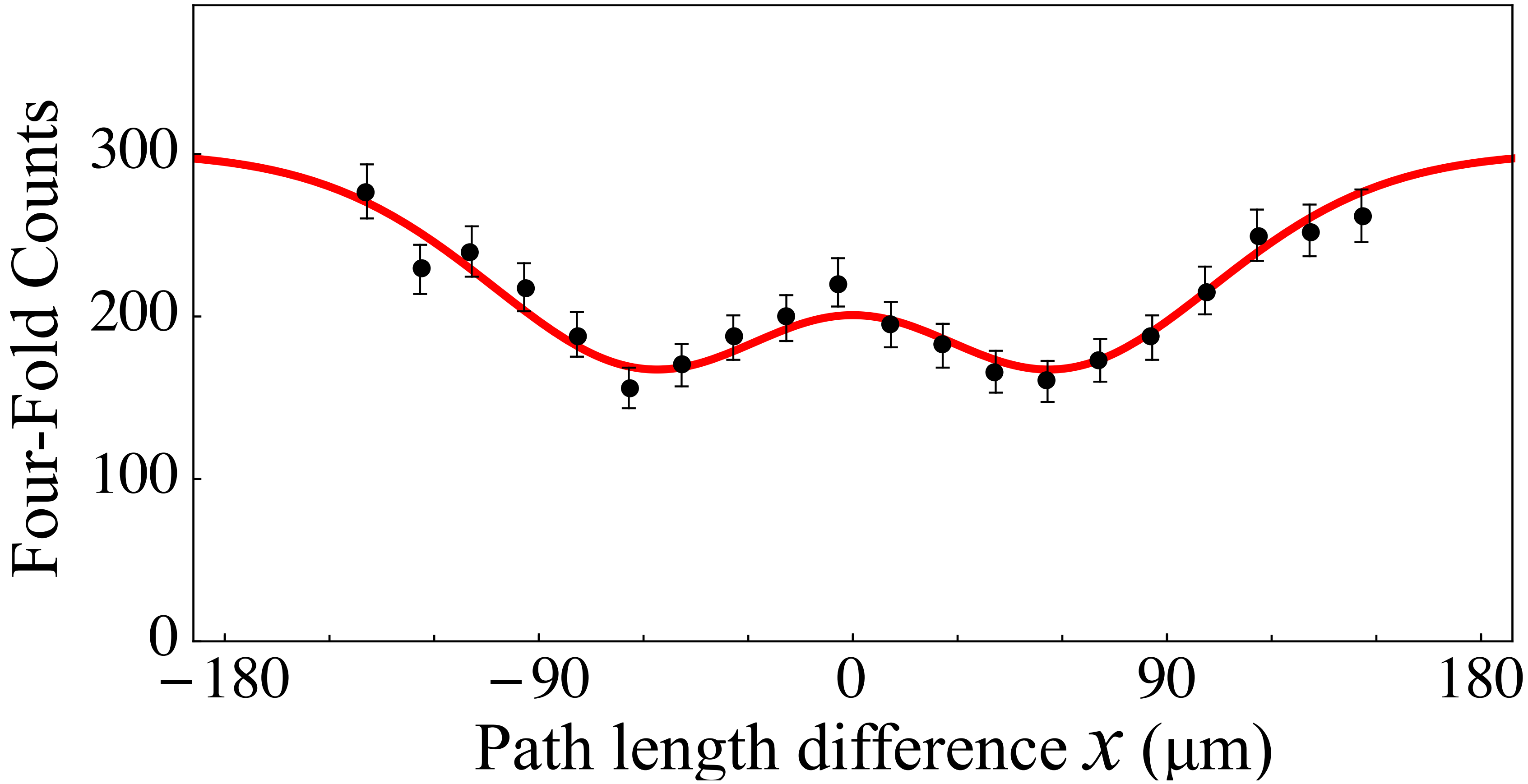


Ra *et al.*, PNAS **110**, 1227 (2013)

Digression

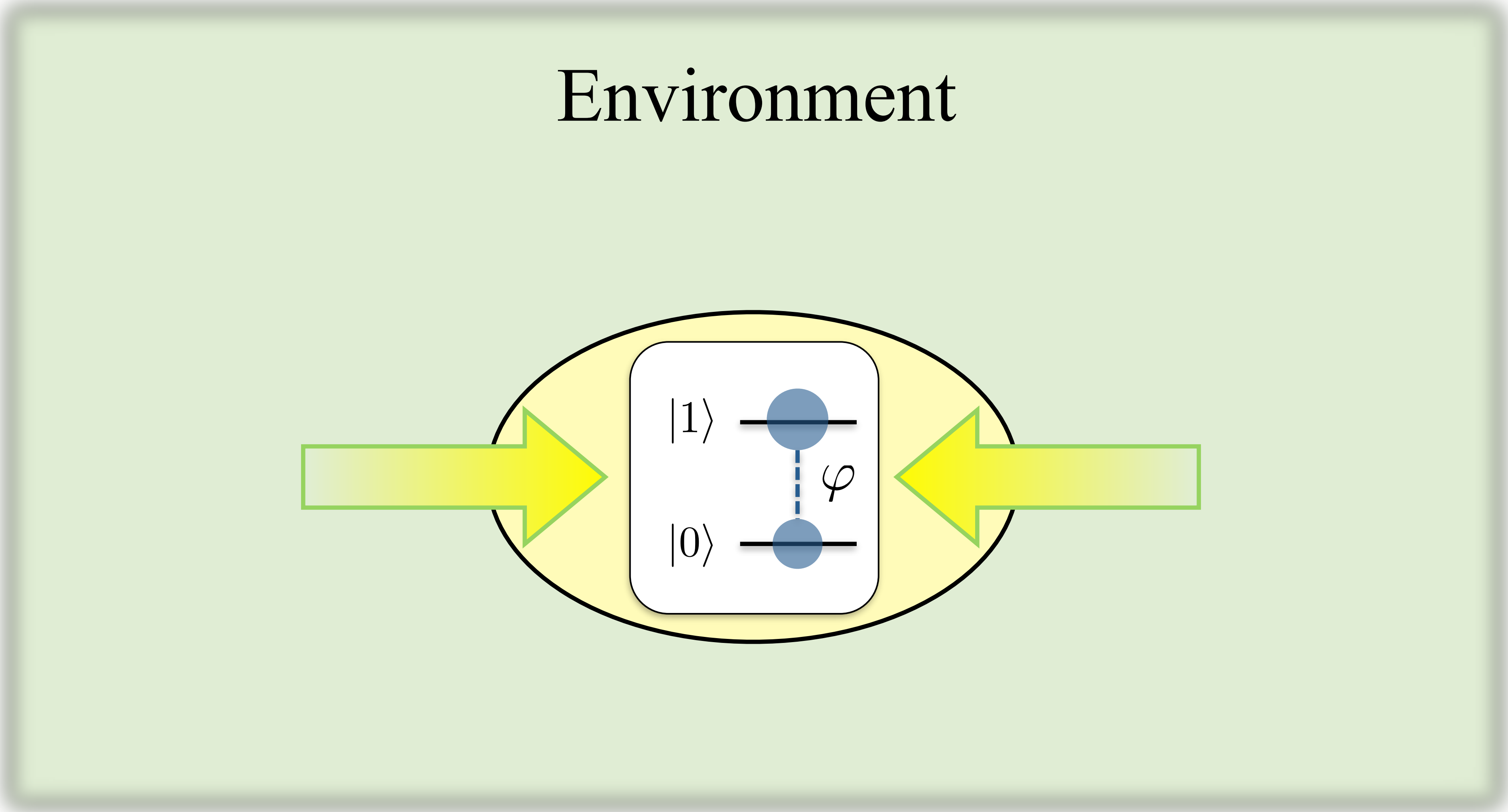
Non-monotonic quantum-to-classical transition in multi-particle interference





Ra *et al.*, PNAS **110**, 1227 (2013)

Decoherence is due to unwanted interaction between the system and environment



$$\rho_S \otimes \rho_E \longrightarrow \rho_{SE}$$

Photonic qubit

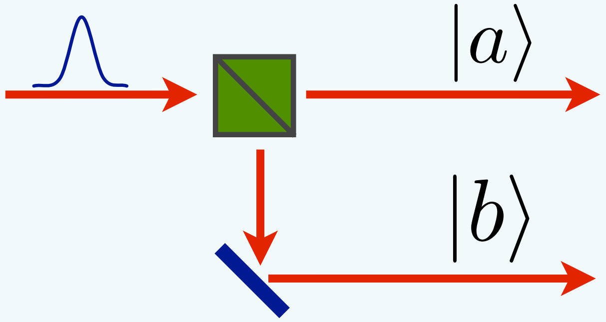
Single-photon state
Choose a degree of freedom

$$\left.\begin{array}{l}\text{Single-photon state}\\ \text{Choose a degree of freedom}\end{array}\right\} \text{Photonic Qubit} \quad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Path ("Dual-Rail" Qubit)

$|0\rangle \rightarrow |a\rangle$

$|1\rangle \rightarrow |b\rangle$

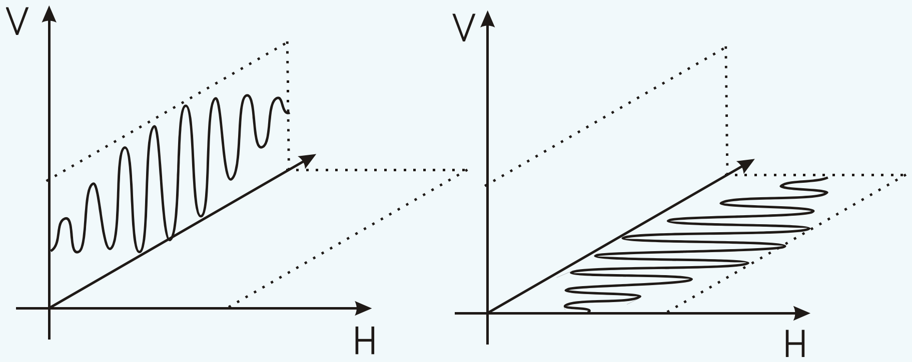


$$|\psi\rangle = \alpha|a\rangle + \beta|b\rangle$$

Polarization

$|0\rangle \rightarrow |H\rangle$

$|1\rangle \rightarrow |V\rangle$

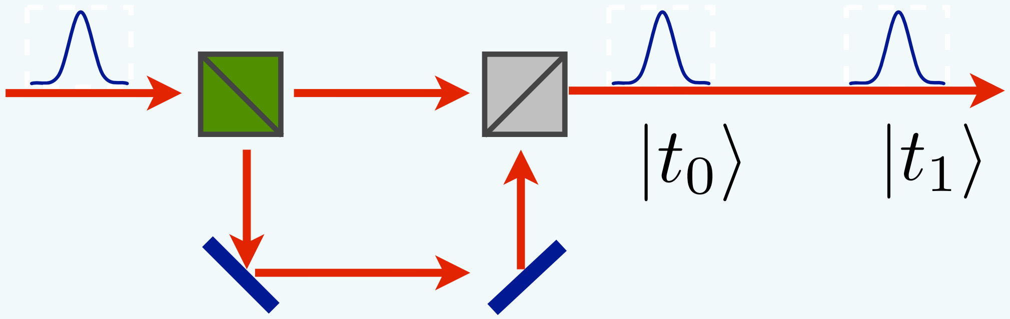


$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$$

Time-bin

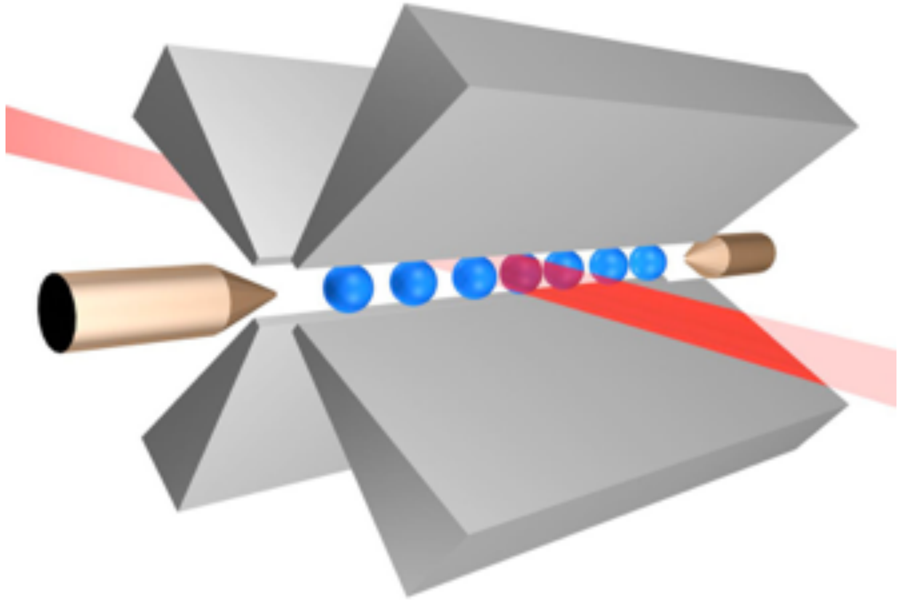
$|0\rangle \rightarrow |t_0\rangle$

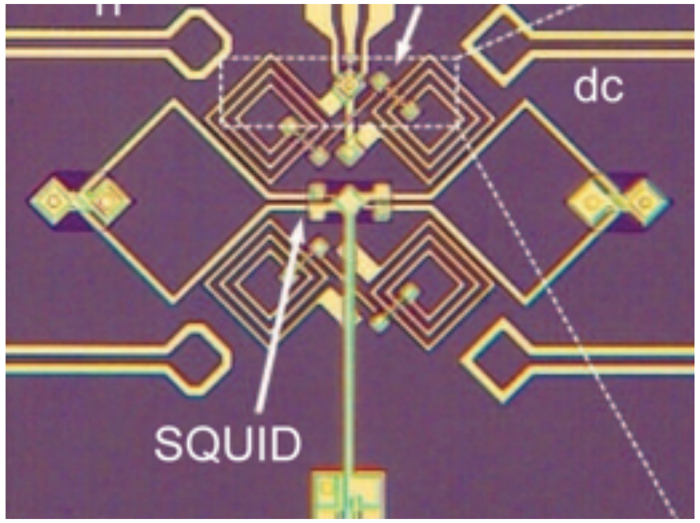
$|1\rangle \rightarrow |t_1\rangle$

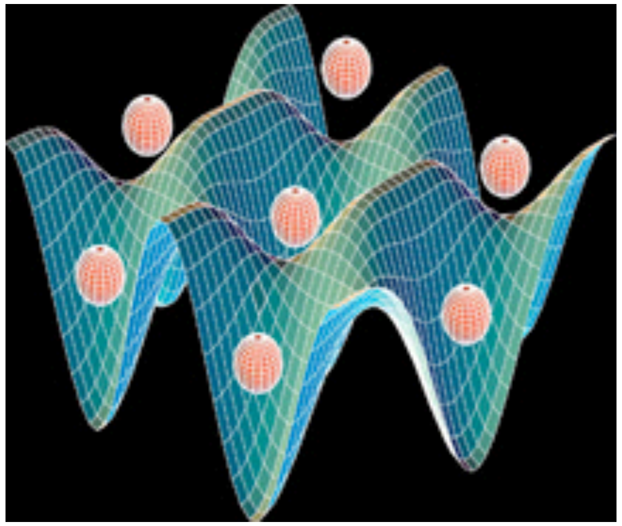


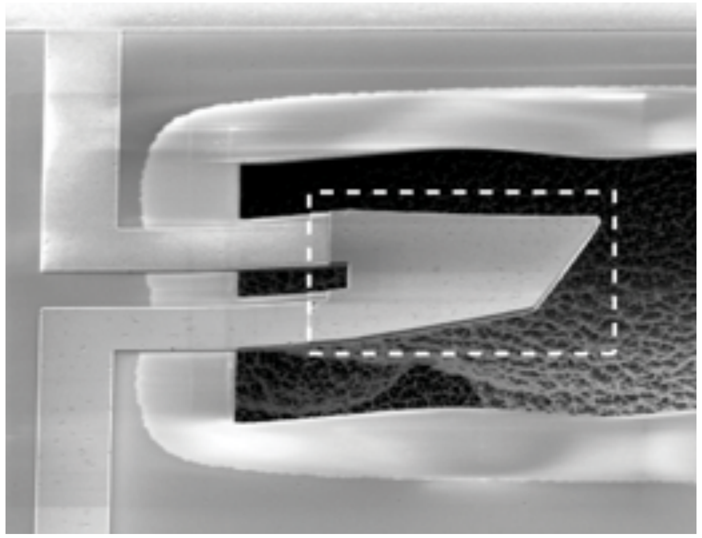
$$|\psi\rangle = \alpha|t_0\rangle + |t_1\rangle$$

Other qubits

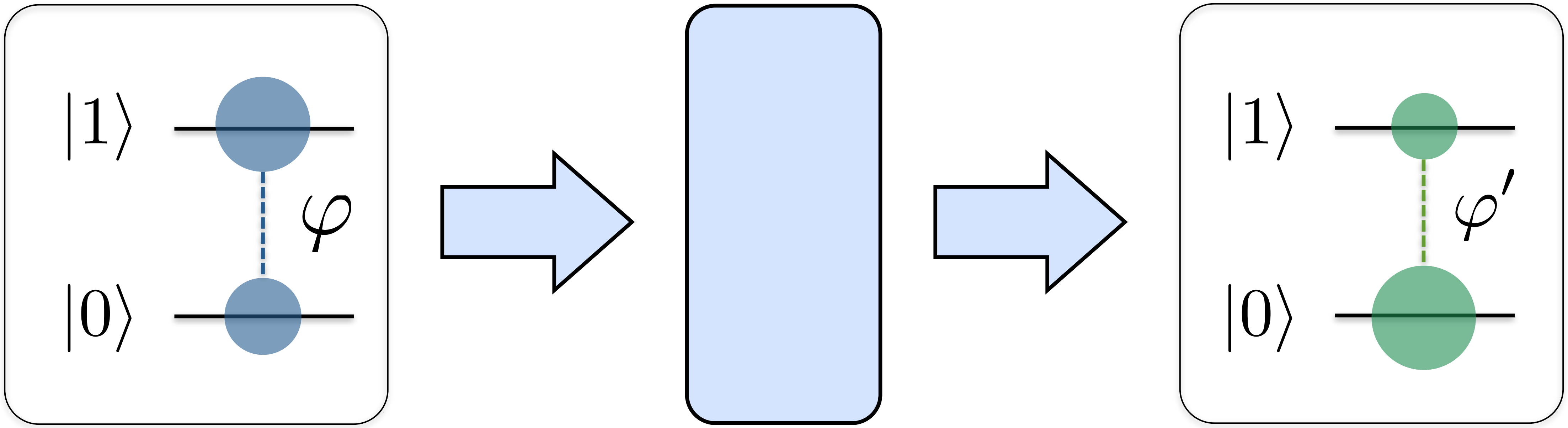






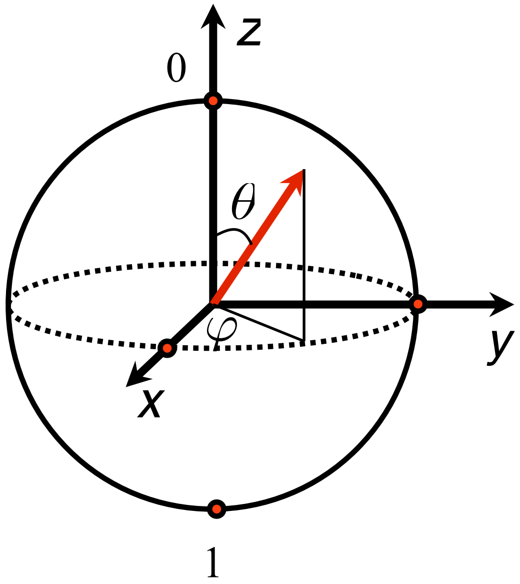


Single-qubit under unitary operation

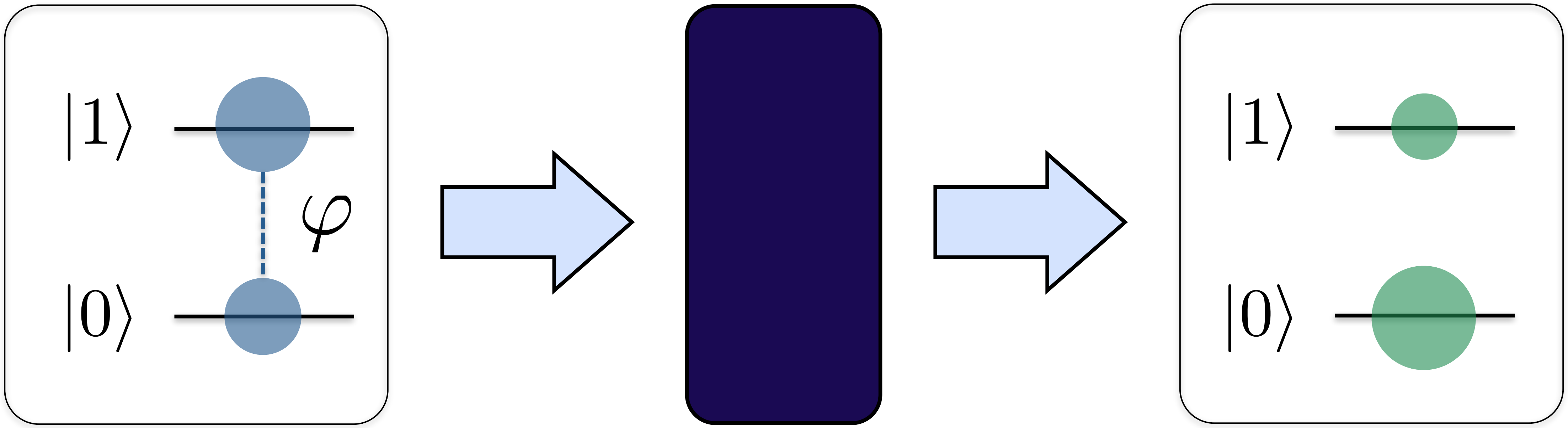


“Dual-Rail” Qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longleftrightarrow |\psi'\rangle = \alpha'|0\rangle + \beta'|1\rangle$$

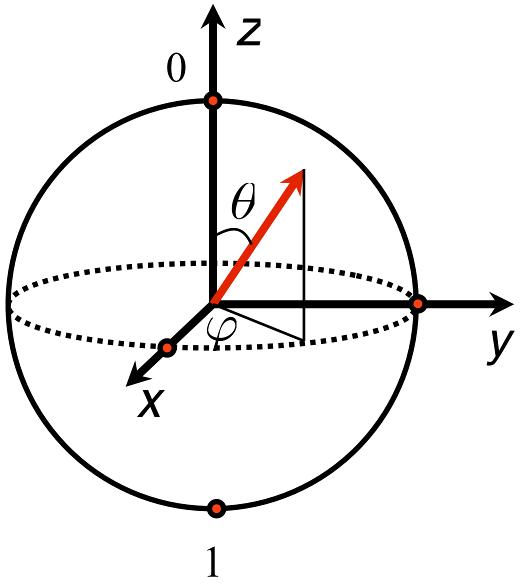


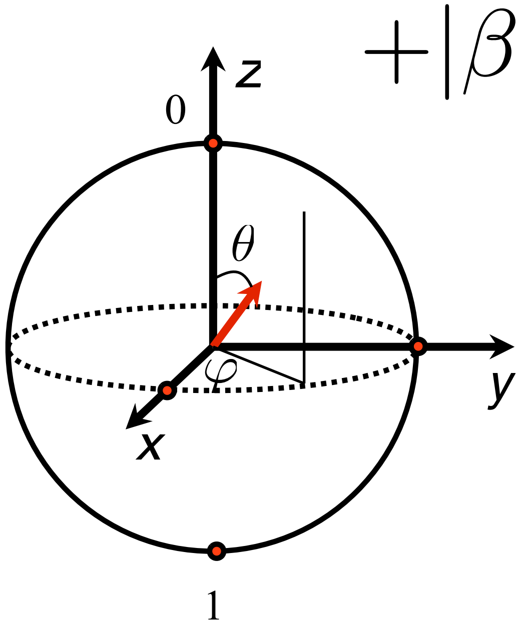
Single-qubit under decoherence



"Dual-Rail" Qubit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow \rho' = |\alpha'|^2|0\rangle\langle 0| + |\beta'|^2|1\rangle\langle 1|$$





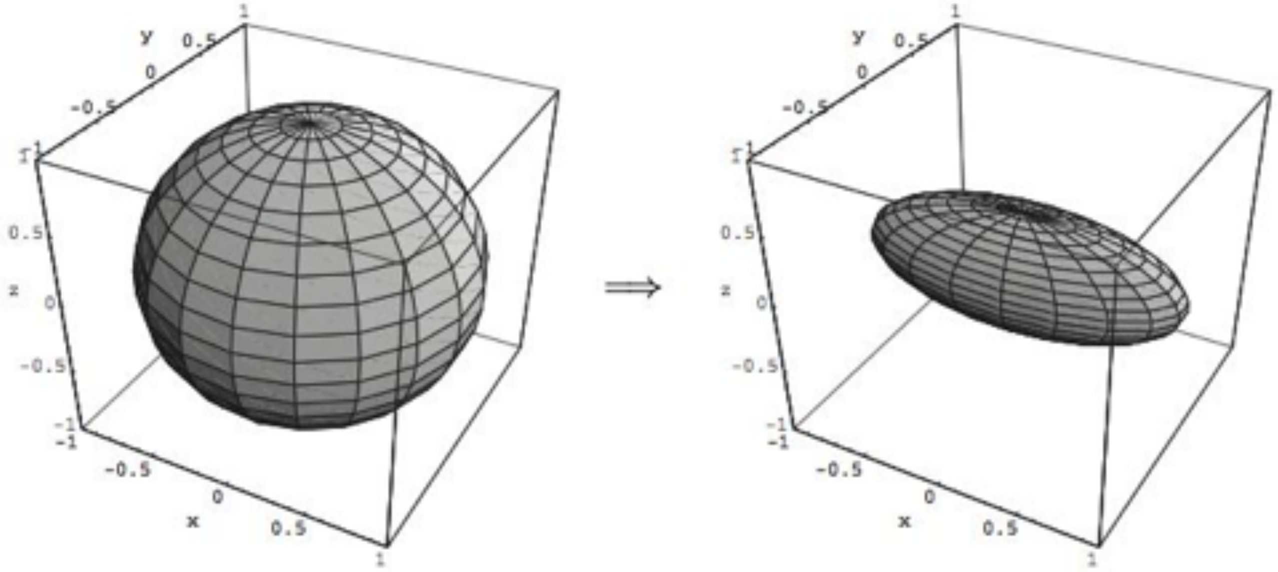
Modeling decoherence as quantum errors

$$\mathcal{E}(\rho) = \sum_i E_i \rho E_i^\dagger$$

Bit-flip error

$$E_0 = \sqrt{p} I = \sqrt{p} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

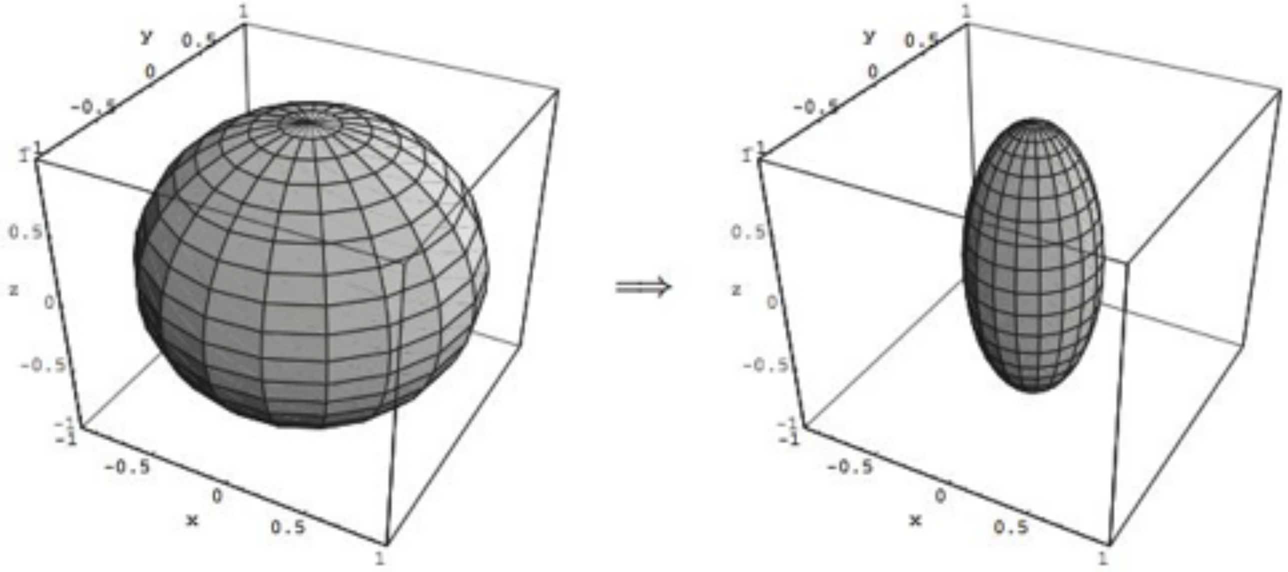
$$E_1 = \sqrt{1-p} X = \sqrt{1-p} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



Phase-flip error

$$E_0 = \sqrt{p} I = \sqrt{p} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_1 = \sqrt{1-p} Z = \sqrt{1-p} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$



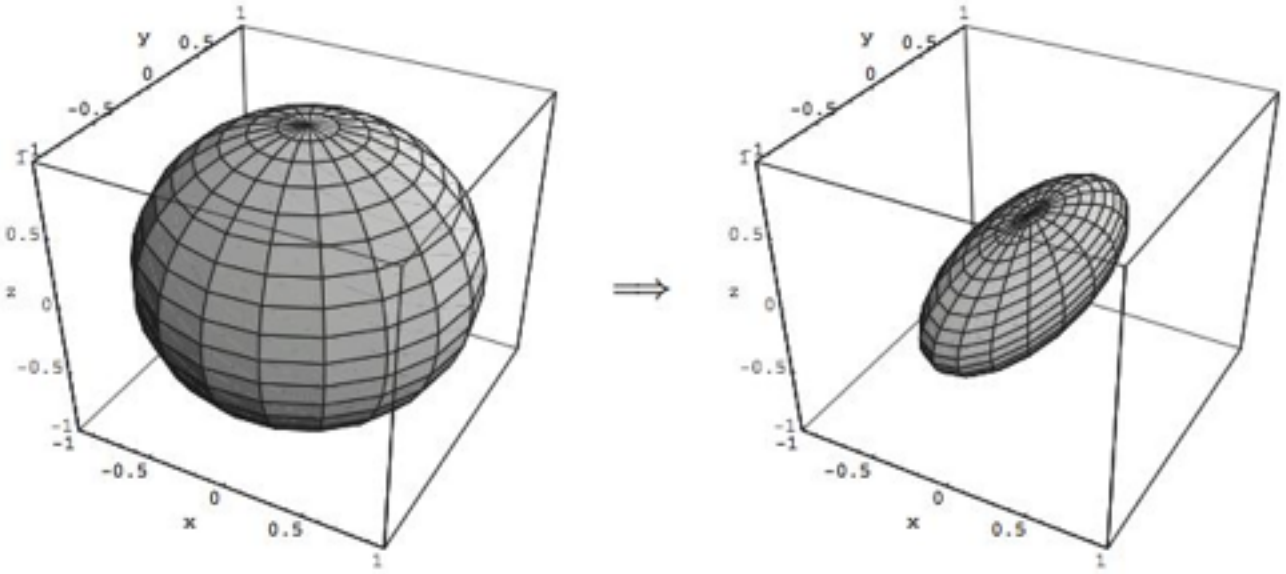
Nielsen & Chuang, QCQI (2000)

Modeling decoherence as quantum errors

Bit-Phase flip

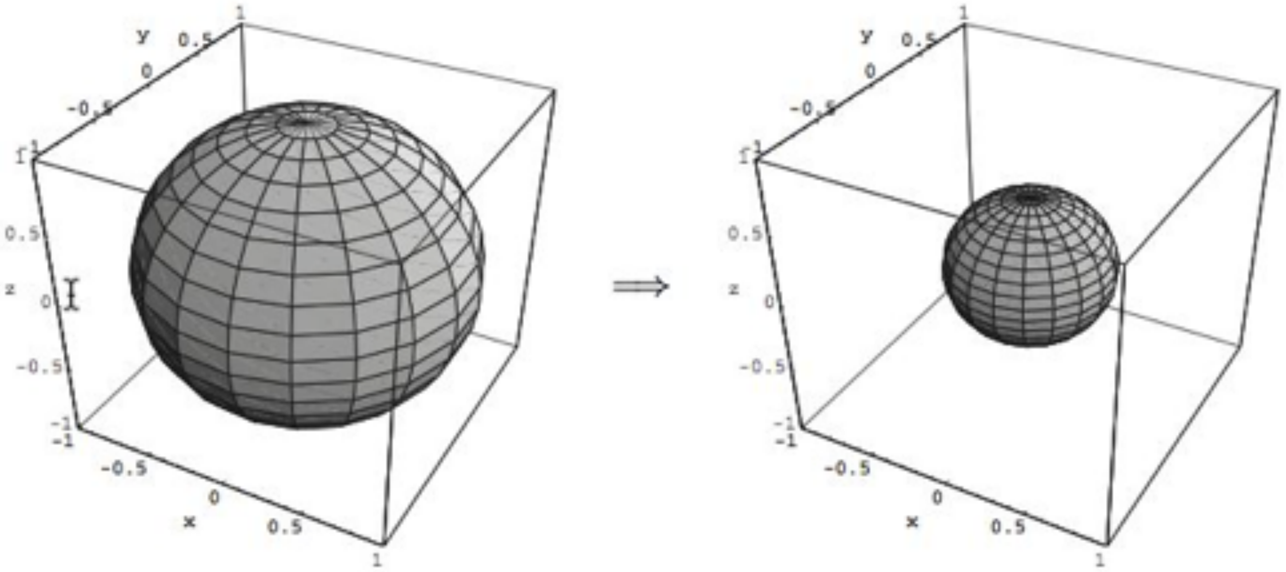
$$E_0 = \sqrt{p}I = \sqrt{p}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$E_1 = \sqrt{1-p}Y = \sqrt{1-p}\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$



Depolarizing channel

$$\mathcal{E}(\rho) = \frac{pI}{2} + (1-p)\rho$$



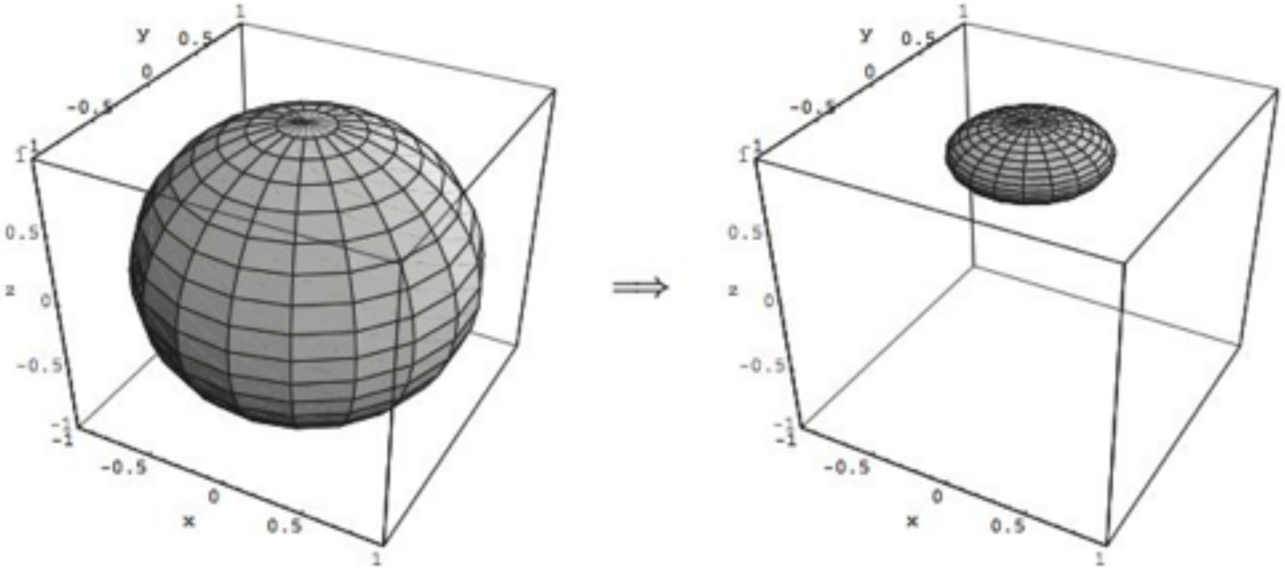
Nielsen & Chuang, QCQI (2000)

Modeling decoherence as quantum errors

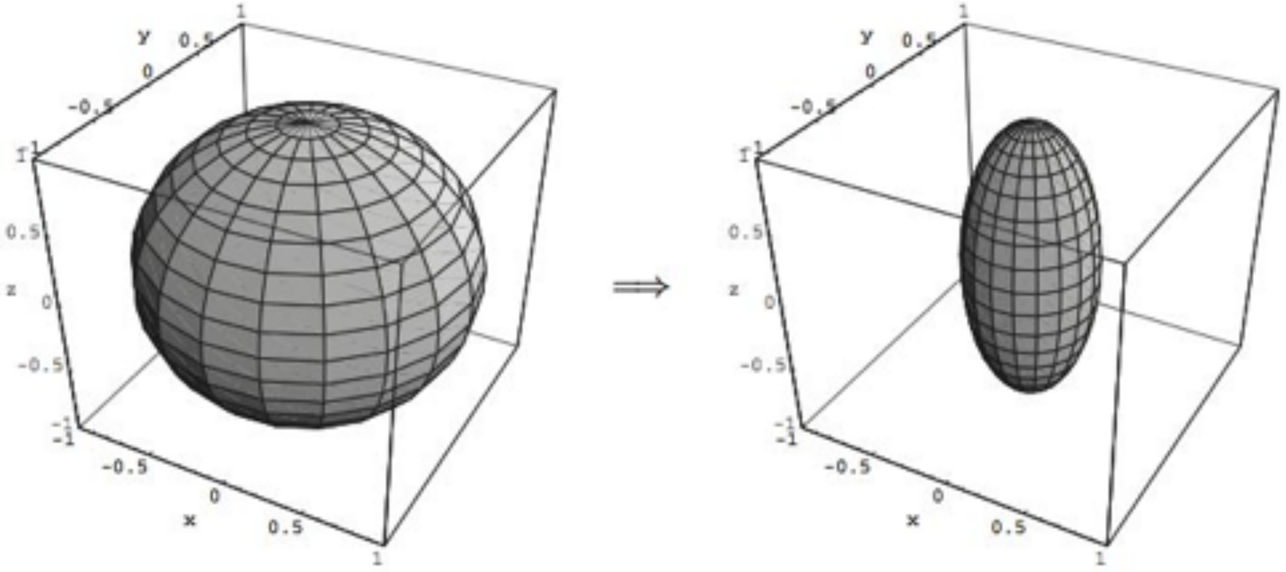
Amplitude damping

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}$$

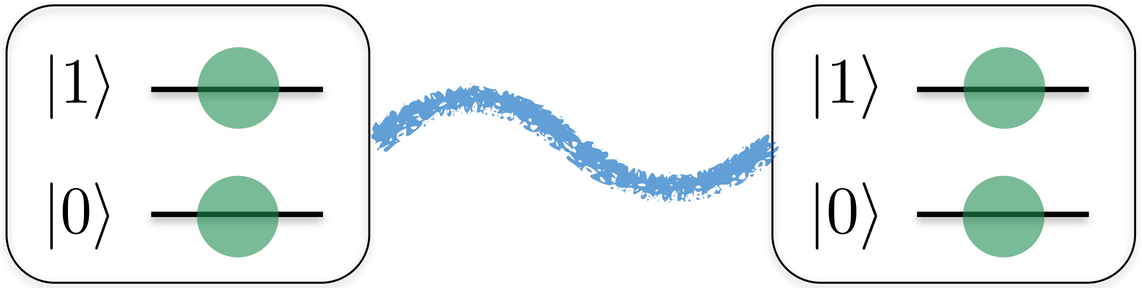


Phase damping = Phase flip



Nielsen & Chuang, QCQI (2000)

Multiple qubits can be entangled

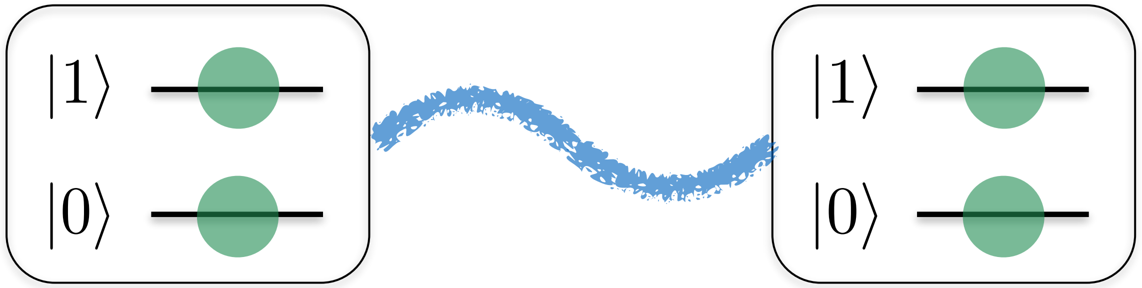


$$\rho = \begin{array}{cccccc} & |00\rangle & |01\rangle & |10\rangle & |11\rangle & \\ \left(\begin{array}{c} \\ \\ \\ \\ \end{array}\right. & \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} & \langle 00| \\ & \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} & \langle 01| \\ & \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} & \langle 10| \\ & \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} & \langle 11| \end{array}$$

Entangled States: quantum states that *cannot* be prepared by LOCC

$$\rho_{AB} \neq \sum_{ij} P_{AB}(a_i, b_j)\rho_i^{(A)} \otimes \rho_j^{(B)}$$

Concurrence



$$\rho = \begin{array}{cc} \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} & \\ \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & \rho_{22} & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & \rho_{33} & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & \rho_{44} \end{pmatrix} & \begin{array}{c} \langle 00| \\ \langle 01| \\ \langle 10| \\ \langle 11| \end{array} \end{array}$$

$$\mathcal{C}(\rho) \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

the square roots of the eigenvalues of $\rho\tilde{\rho}$
(descending order)

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

Property of separable states

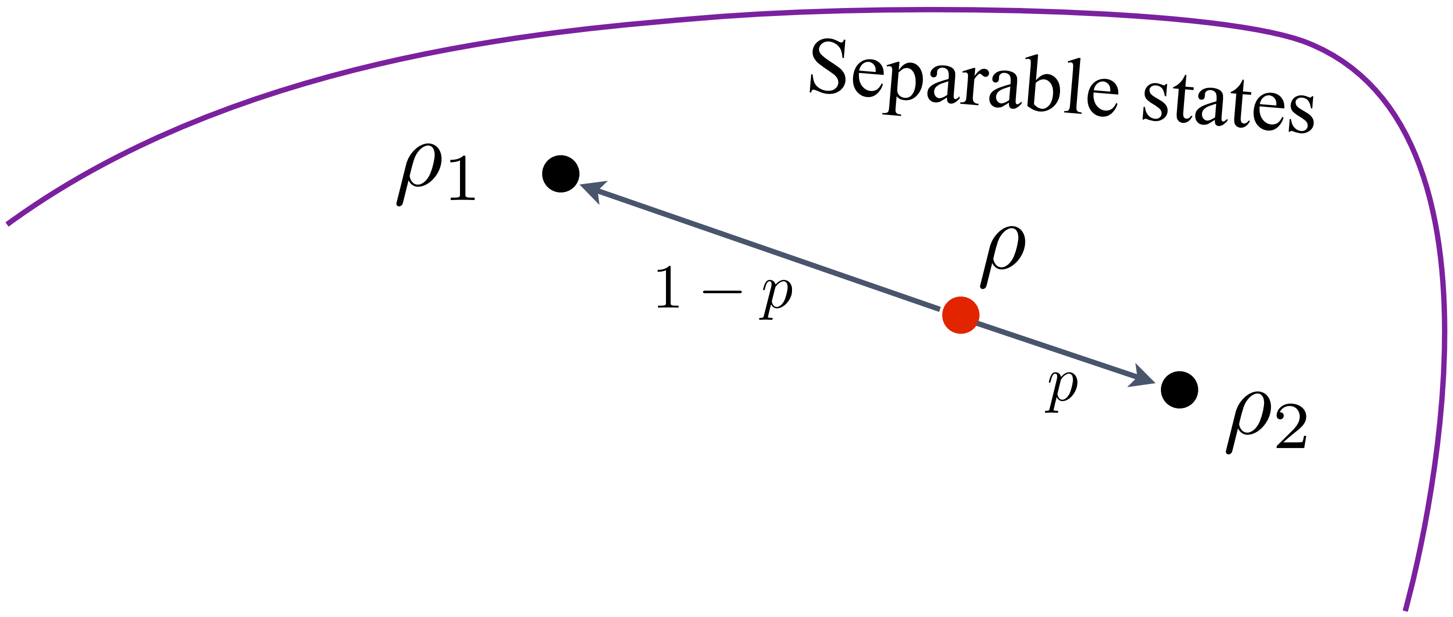
$$\rho_1 = |0\rangle_A\langle 0| \otimes |0\rangle_B\langle 0|$$

$$\rho_2 = |1\rangle_A\langle 1| \otimes |1\rangle_B\langle 1|$$

$$\Longrightarrow$$

$$\rho = p\rho_1 + (1-p)\rho_2$$

Separable!



Separable states form a convex set!

Entangled states

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$$

LOCC Measurement & State preparation

$$\rho =$$

$$\boxed{Tr[\rho M_x^a \otimes M_y^b] = Tr[\rho|a_x\rangle\langle a_x| \otimes |b_y\rangle\langle b_y|]}$$

Property of entangled states

$$\rho_1 = |\phi^+\rangle\langle\phi^+|$$

$$\rho_2 = |\phi^-\rangle\langle\phi^-|$$

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle)$$

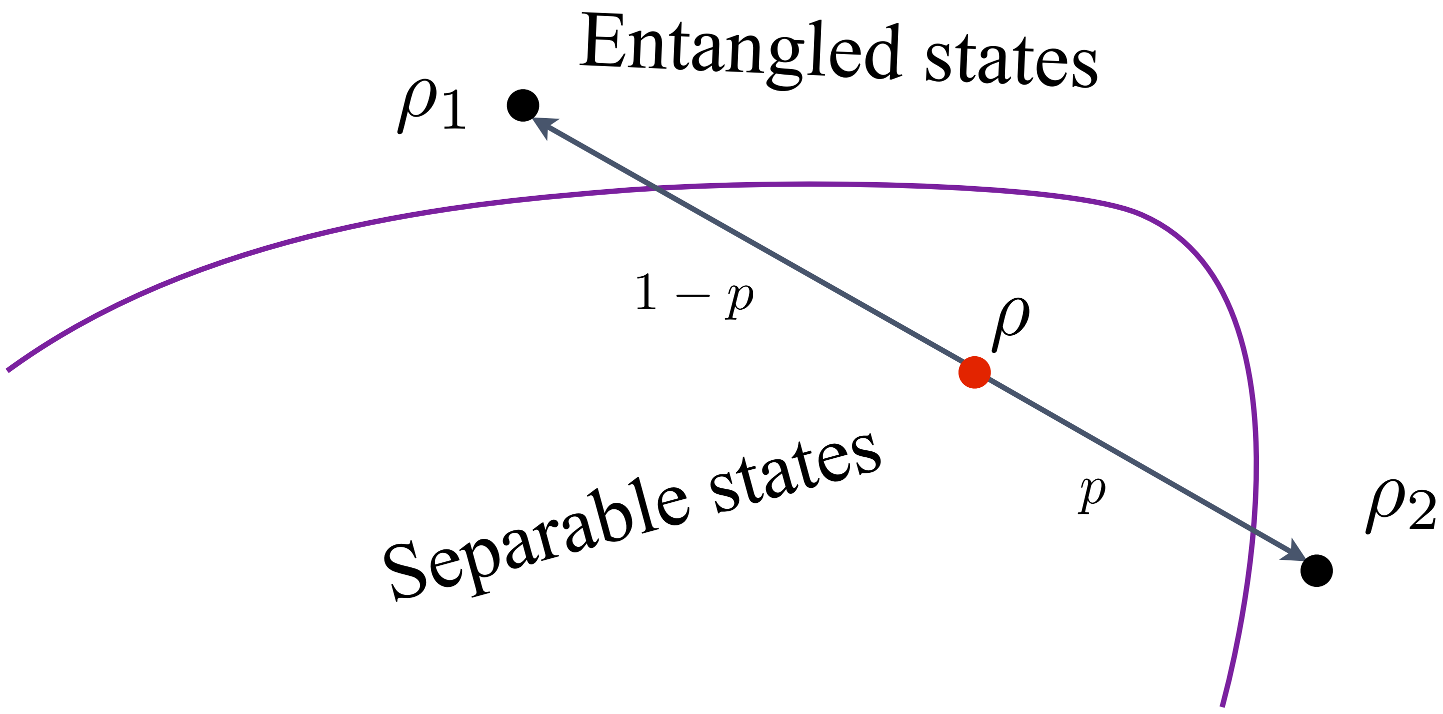
$$|\phi^-\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle - |1_A 1_B\rangle)$$

$\Rightarrow$

$$\rho = \frac{1}{2}\rho_1 + \frac{1}{2}\rho_2$$

Separable!

$$= \frac{1}{2}(|0_A 0_B\rangle\langle 0_A 0_B| + |1_A 1_B\rangle\langle 1_A 1_B|)$$



ρ_1, ρ_2 entangled $\not\rightarrow$ $\rho = p\rho_1 + (1-p)\rho_2$ is entangled

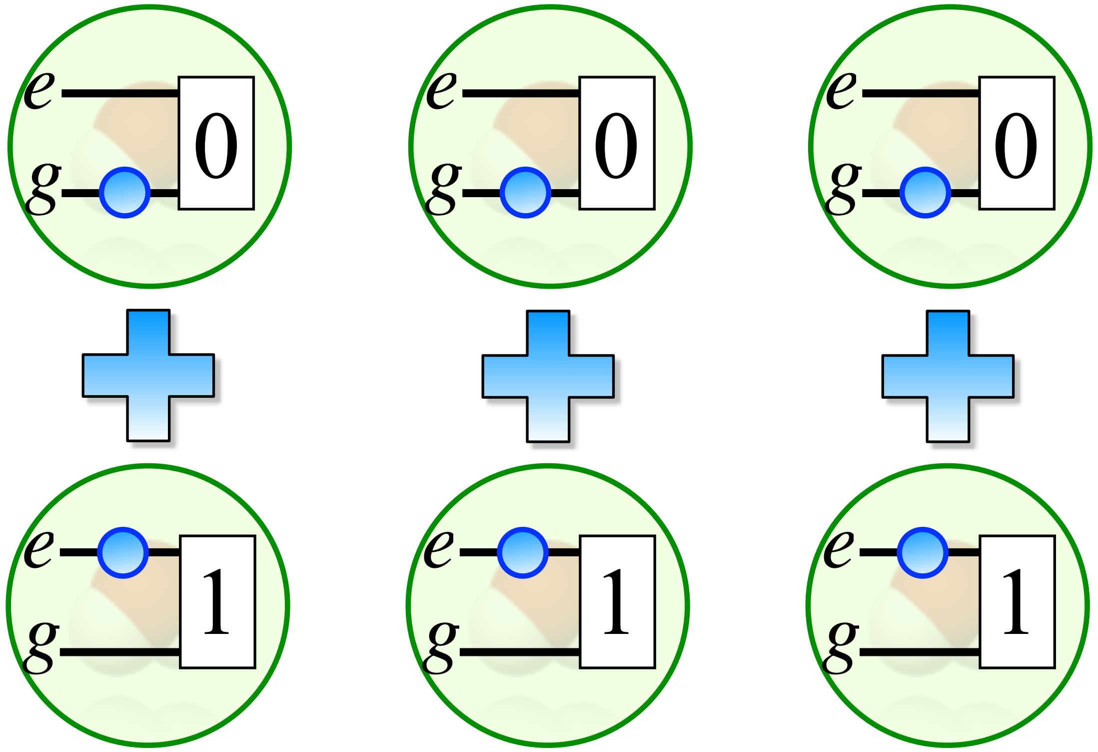
Entangled states do NOT form a convex set!

Information capacity increases with entanglement

Classical Information

101 → 1

Quantum Information



000, 001, 010, 011
100, 101, 110, 111 → 2^3

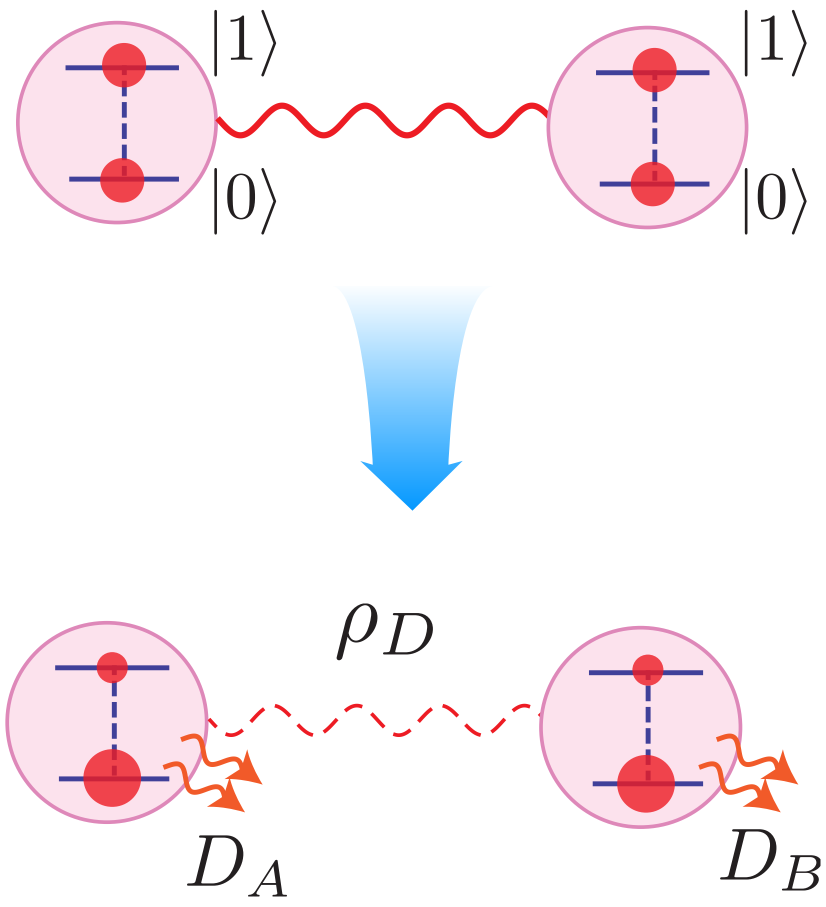
Entanglement gives rise to quantum parallelism

	qubit 1	qubit 2	qubit 3
C_0	0	0	0
C_1	0	0	1
C_2	0	1	0
C_3	0	1	
C_4	1	0	0
C_5	1	0	1
C_6	1	1	0
C_7	1	1	1

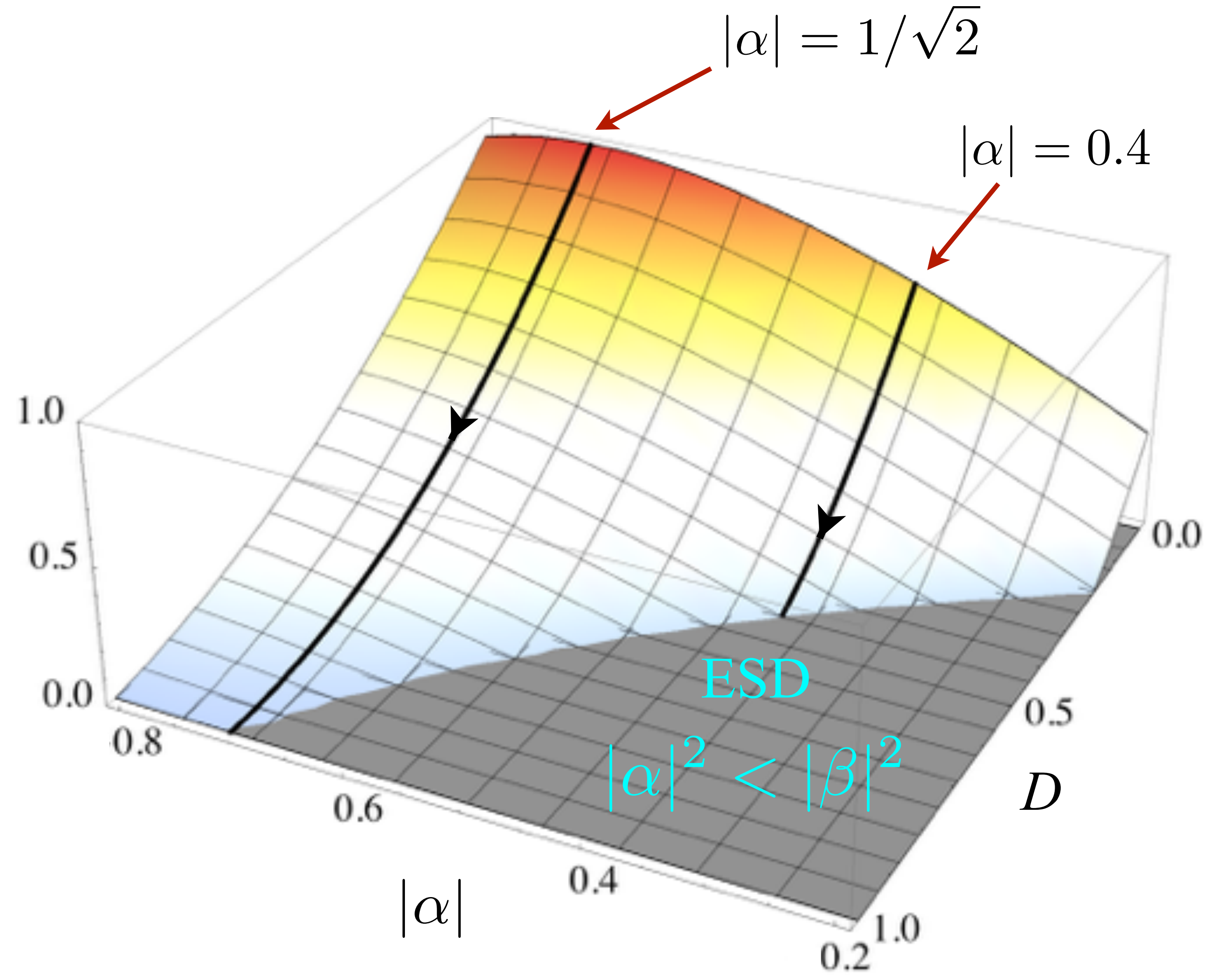
Quantum Coherence

	qubit 1	qubit 2	qubit 3
C_4	0	0	0
C_5	0	0	1
C_6	0	1	0
	0	1	1
C_0	1	0	0
C_1	1	0	1
C_2	1	1	0
C_3	1	1	1

Decoherence may cause “entanglement sudden death”



$$|\psi\rangle = \alpha|00\rangle + \beta|11\rangle$$



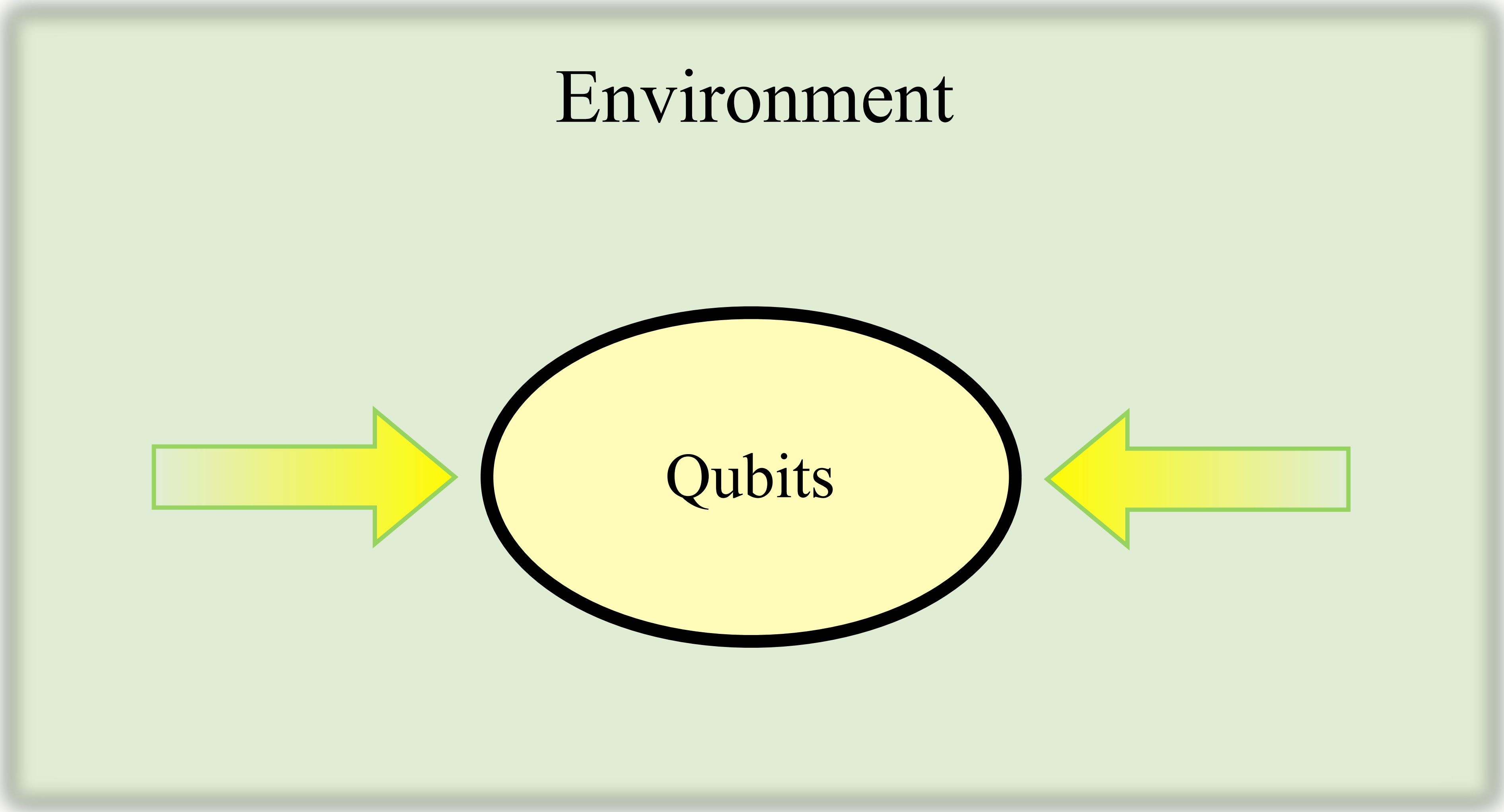
Yu & Eberly, PRL (2004)

Almeida *et al.*, Science (2007)

Yu & Eberly, Science (2007); Science (2009)

How to tackle decoherence?

⇒ Remove qubit-environment interaction



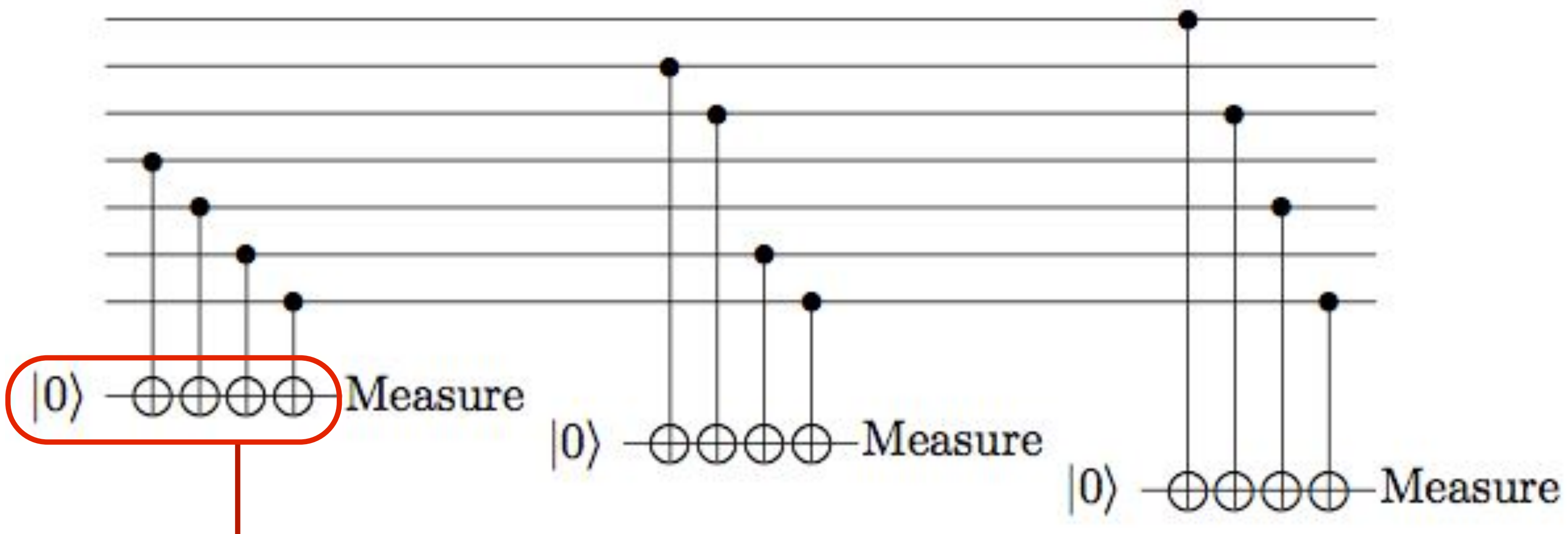
Not practical; Not useful

How to tackle decoherence?
$\Rightarrow$ Quantum error correction

Requires *many* ancilla qubits

One logical qubit

(level 1 encoding)



$g_1 = IIIXXXX$

$g_2 = IXXIIXX$

$g_3 = XIXIXIX$

$g_4 = IIIZZZZ$

$g_5 = IZZIIZZ$

$g_6 = ZIZIZIZ$

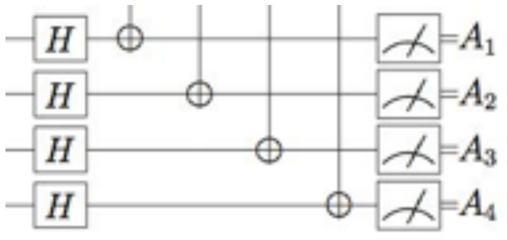
$$(4 \times 3) \times 6 = 72$$

$$6 \times 3 = 18$$

$$90 + 7 = 97$$

$|0000\rangle + |1111\rangle$

CAT(4)

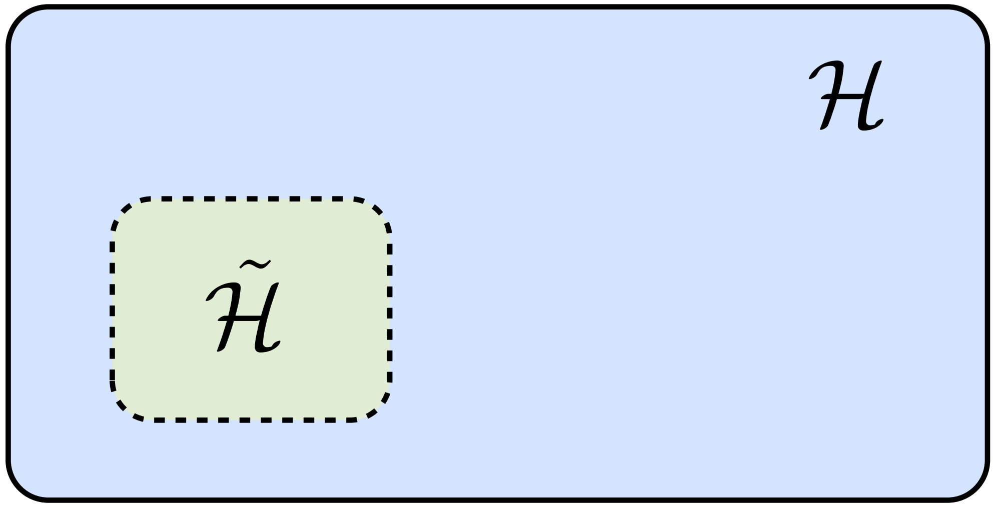


Fault-tolerant syndrome detection

Preskill, Proc. R. Roc. Lond. A (1998)

How to tackle decoherence?

$\Rightarrow$ Decoherence-free subspace



$$\tilde{\mathcal{H}} \subset \mathcal{H}$$

$\downarrow$

Evolution is purely unitary

The interaction Hamiltonian should have an appropriate symmetry.

Lidar, Chuang, & Whaley, PRL (1998)
Lidar & Whaley, quant-ph/0301032v1

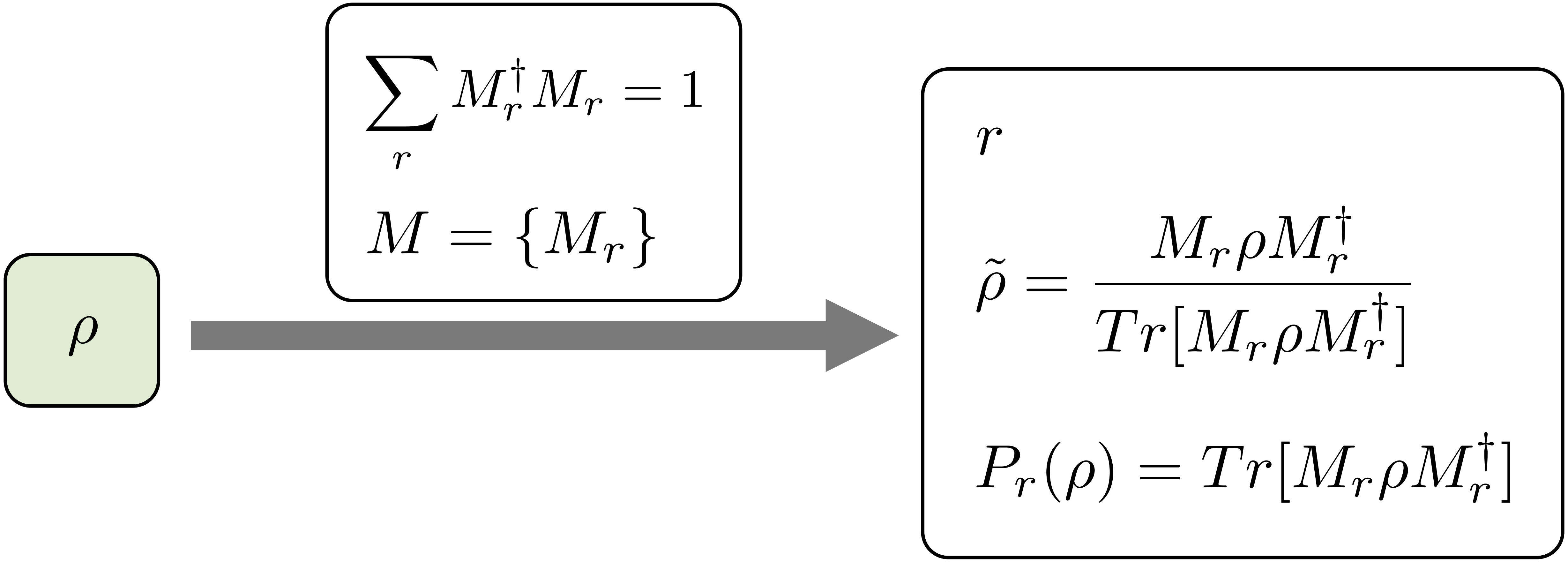
Kwiat *et al*., Science (2000)

D. Kielpinski *et al.,* Science (2001)

Ikuta *et al*., PRL (2011)

How to tackle decoherence?
⇒ Weak quantum measurement

Weak quantum measurement can be reversed!



$$\sum_r M_r^\dagger M_r = 1$$

$$M = \{M_r\}$$

$$\rho$$

$$r$$

$$\tilde{\rho} = \frac{M_r \rho M_r^\dagger}{Tr[M_r \rho M_r^\dagger]}$$

$$P_r(\rho) = Tr[M_r \rho M_r^\dagger]$$

Reversing Operator: $R_r = c_r M_r^{-1}$

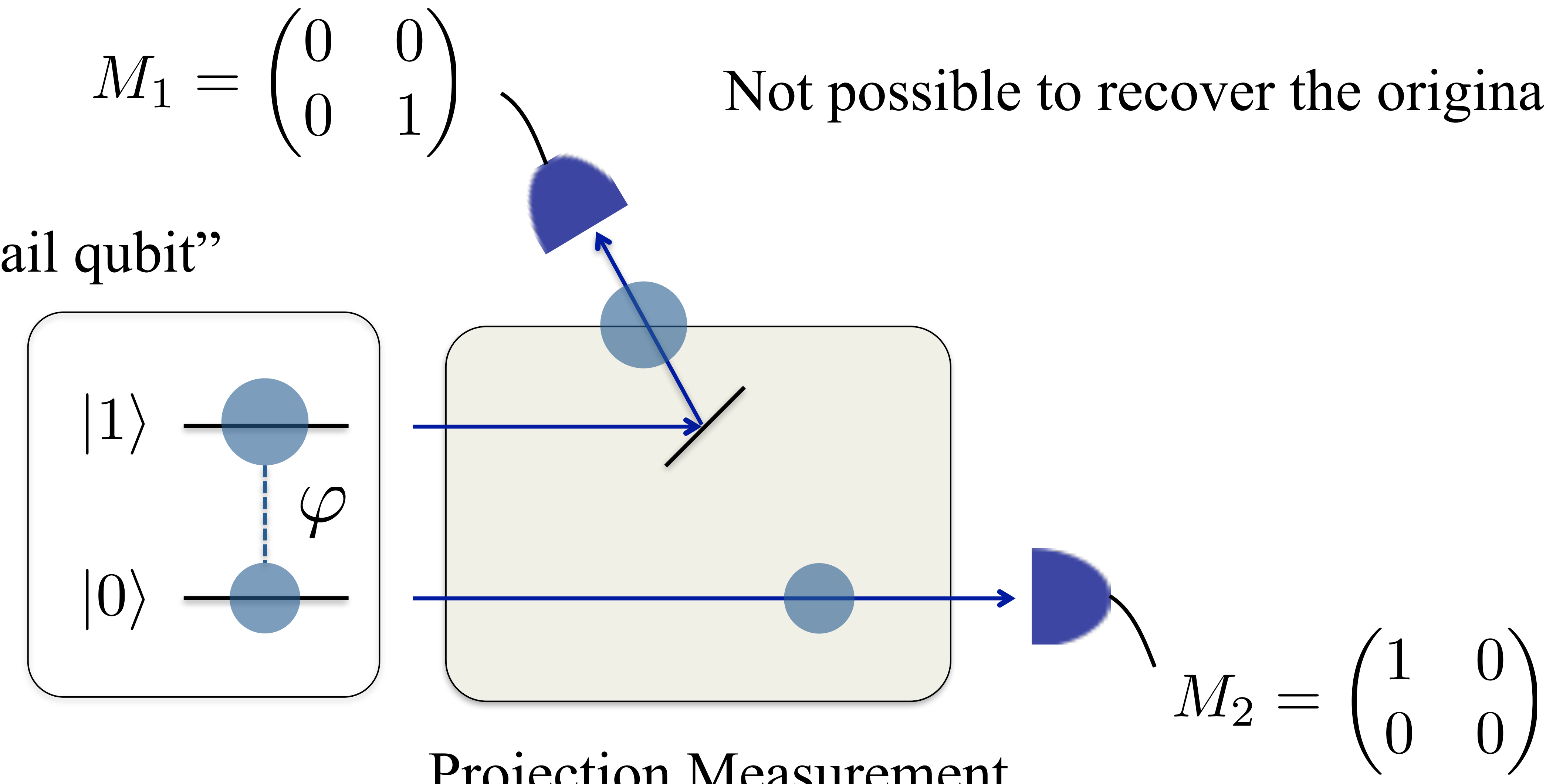
Koashi & Ueda, PRL 82, 2598 (1999)

Generalized quantum measurement

Projection measurement	Generalized measurement
Probabilities are observable quantities. $\hat{P}_m^\dagger = \hat{P}_m$	$\hat{E}_m^\dagger = \hat{E}_m$
The expectation value of the projector is a probability and, therefore, be positive or zero. $\langle\psi\|\hat{P}_m\|\psi\rangle \geq 0$	$\langle\psi\|\hat{E}_m\|\psi\rangle \geq 0$
They form a complete set so that the sum of the probabilities for all possible outcomes is unity. $\sum_m \hat{P}_m = \mathbb{1}$	$\sum_m \hat{E}_m = \mathbb{1}$
$\hat{P}_m \hat{P}_n = 0 \quad \text{unless } m = n$	

Barnett & Croke, Adv Opt Photonics **1**, 238 (2009)

Projection measurement - path qubit



$M_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

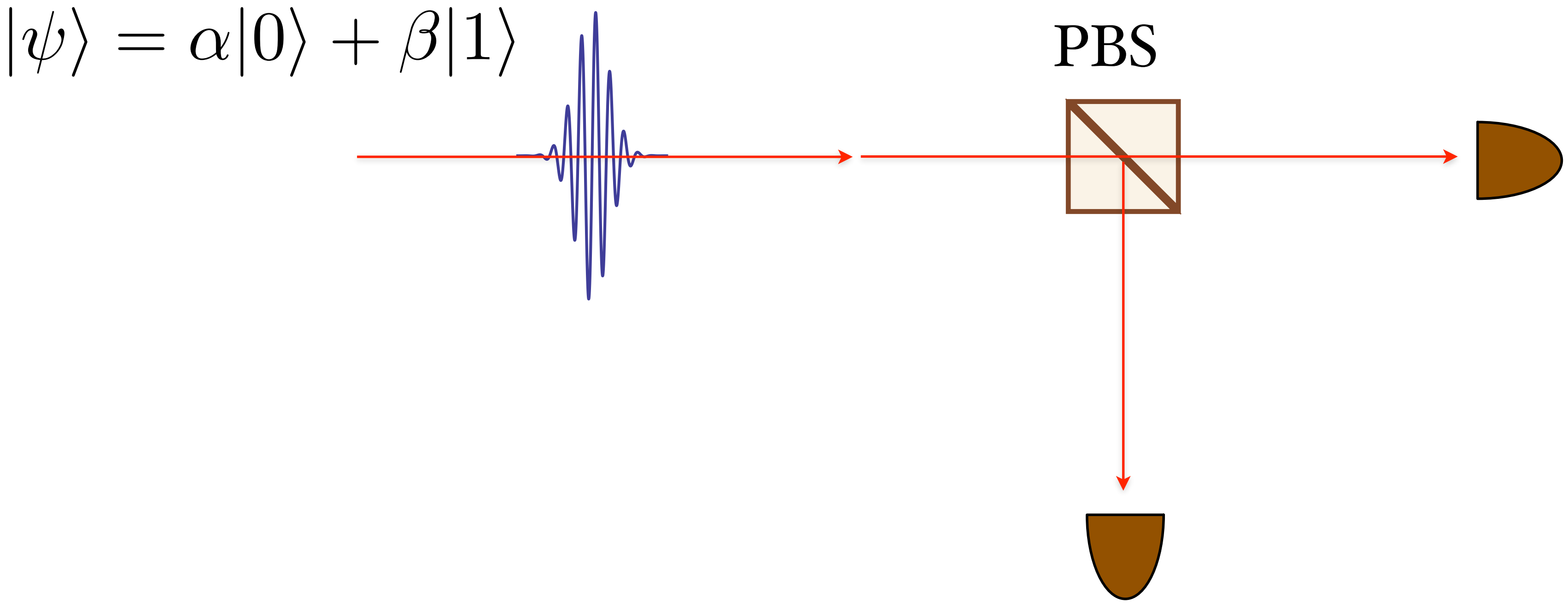
Not possible to recover the original state

“Dual-Rail qubit”

$M_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Projection Measurement

Projection Measurement - polarization qubit



$$\mathbb{P}_0 = |0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

No mathematical inverse!

$$\mathbb{P}_1 = |1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

The Schrödinger's cat: $|\Psi\rangle = |\text{alive}\rangle + |\text{dead}\rangle$

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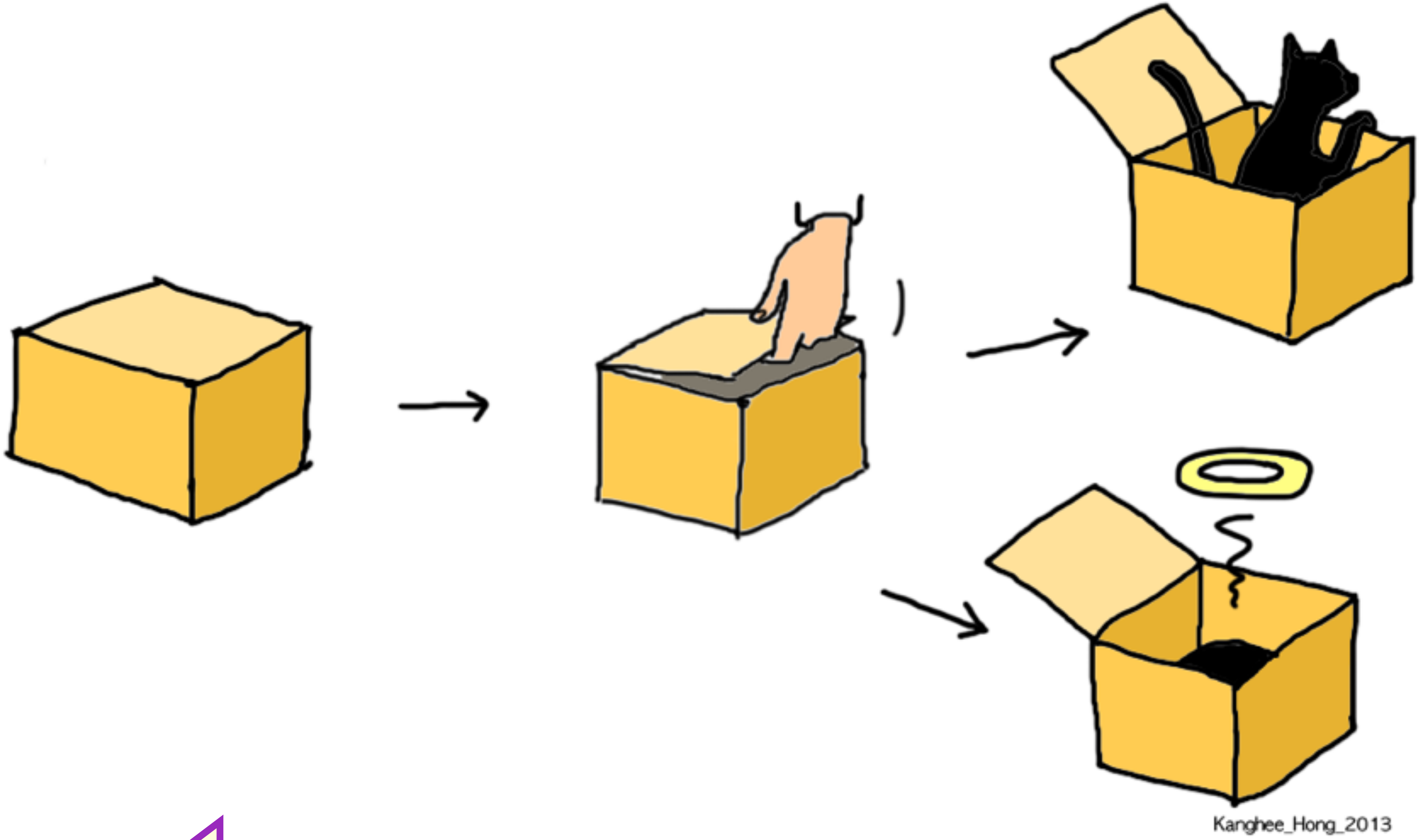
Projection measurement: opening the cat box

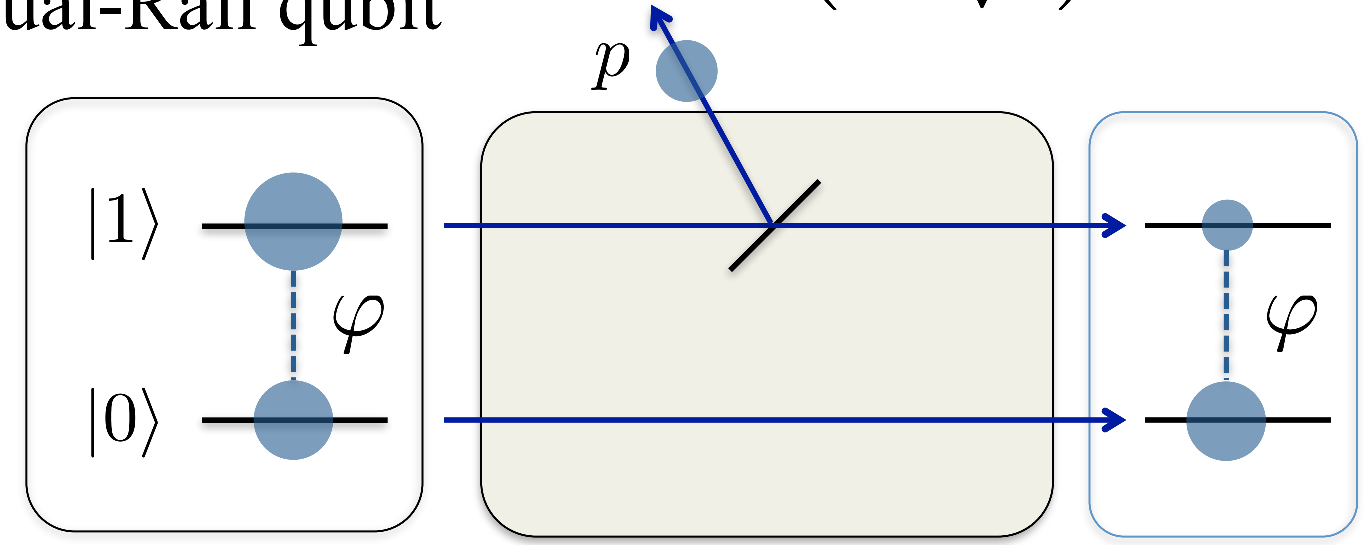


Impossible to go back to the Schrödinger's cat state

Weak measurement - path qubit

$$M_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}$$

“Dual-Rail qubit”



Weak Measurement

$$M_1^\dagger M_1 + M_2^\dagger M_2 = \mathbb{1}$$

$$M_2 = |0\rangle\langle 0| + \sqrt{1-p}|1\rangle\langle 1|$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$$

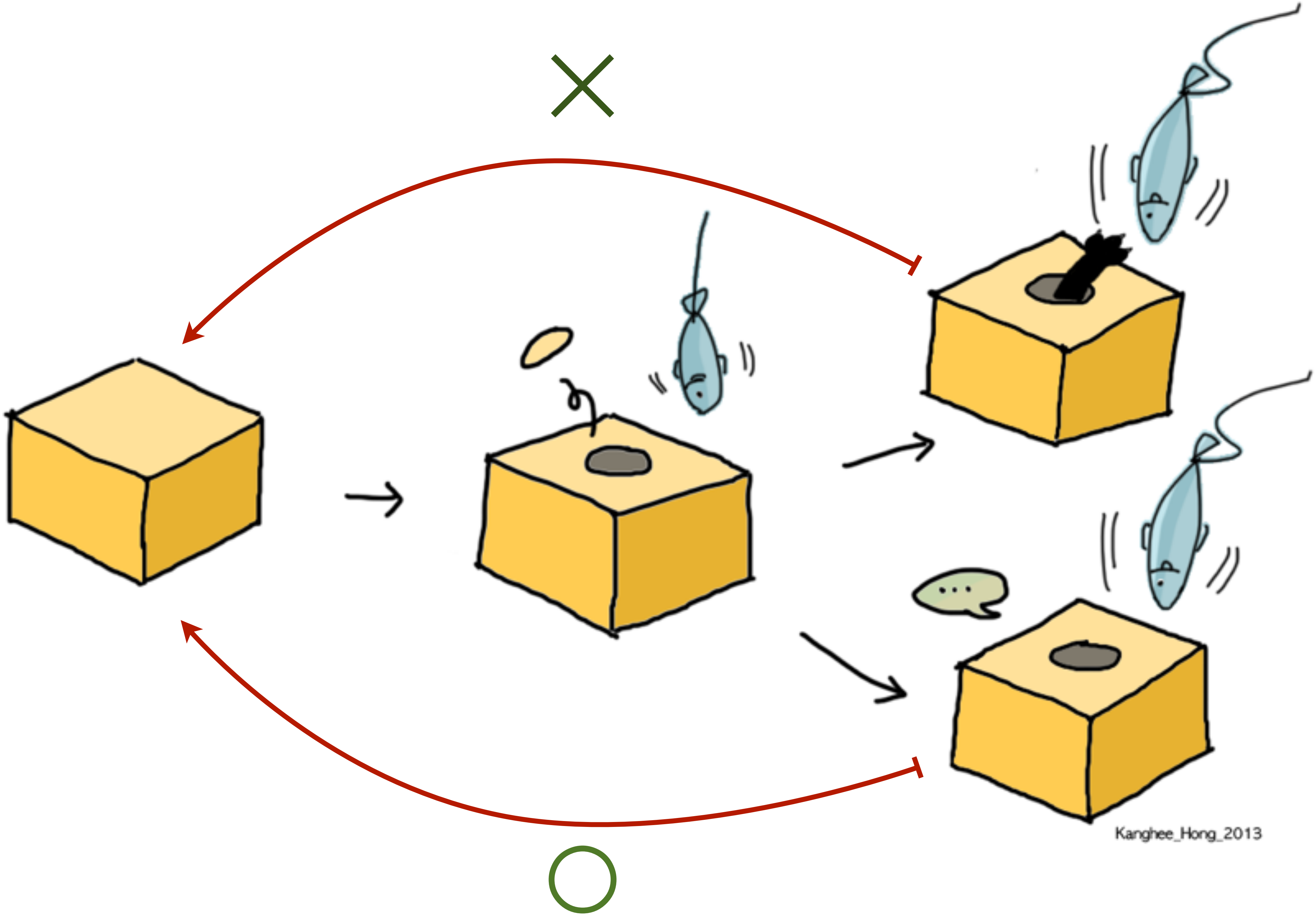
Moves the state closer to the ground state

Reversing Measurement:

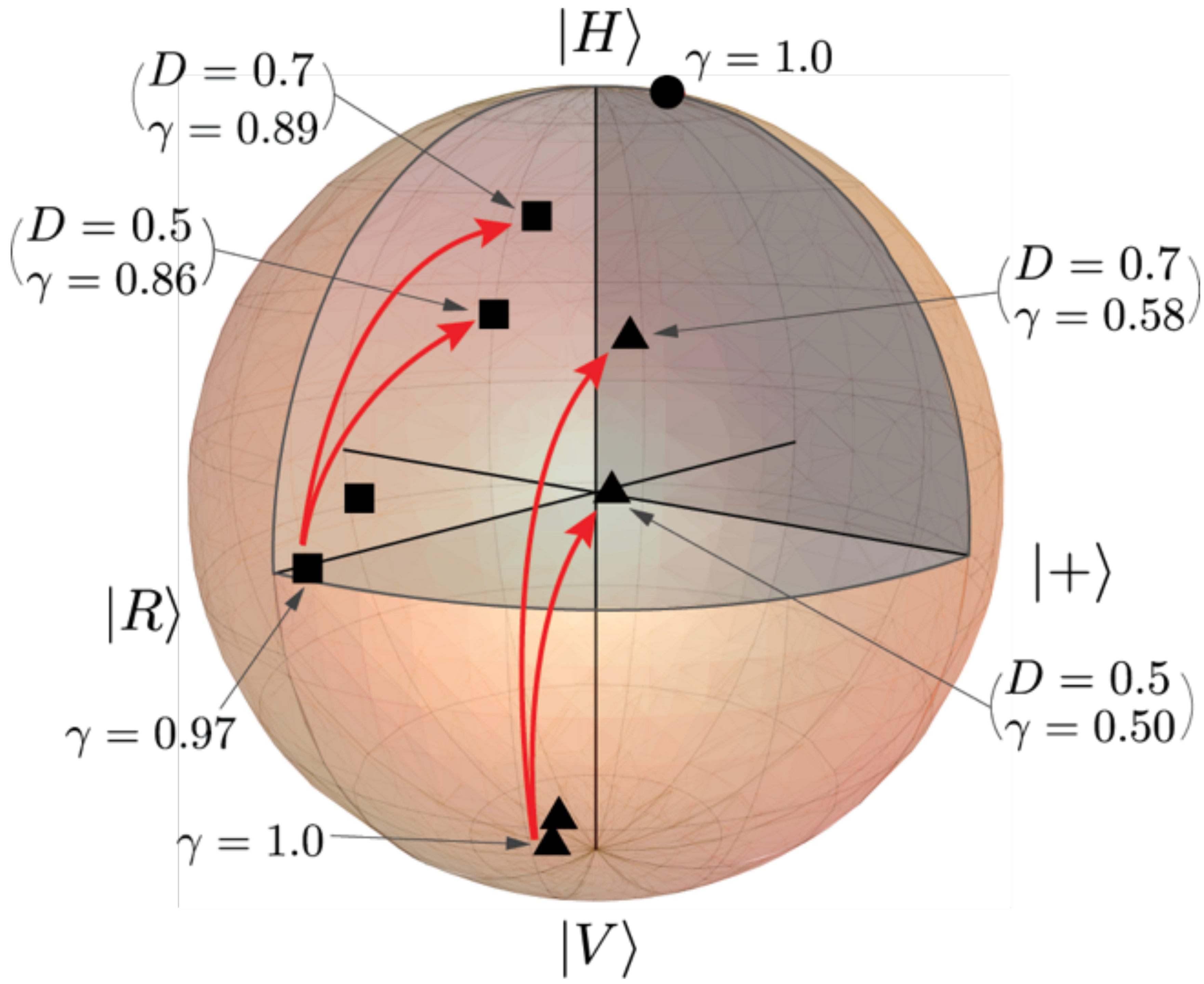
$$M_2^{-1} = \frac{1}{\sqrt{1-p}} \begin{pmatrix} \sqrt{1-p} & 0 \\ 0 & 1 \end{pmatrix}$$

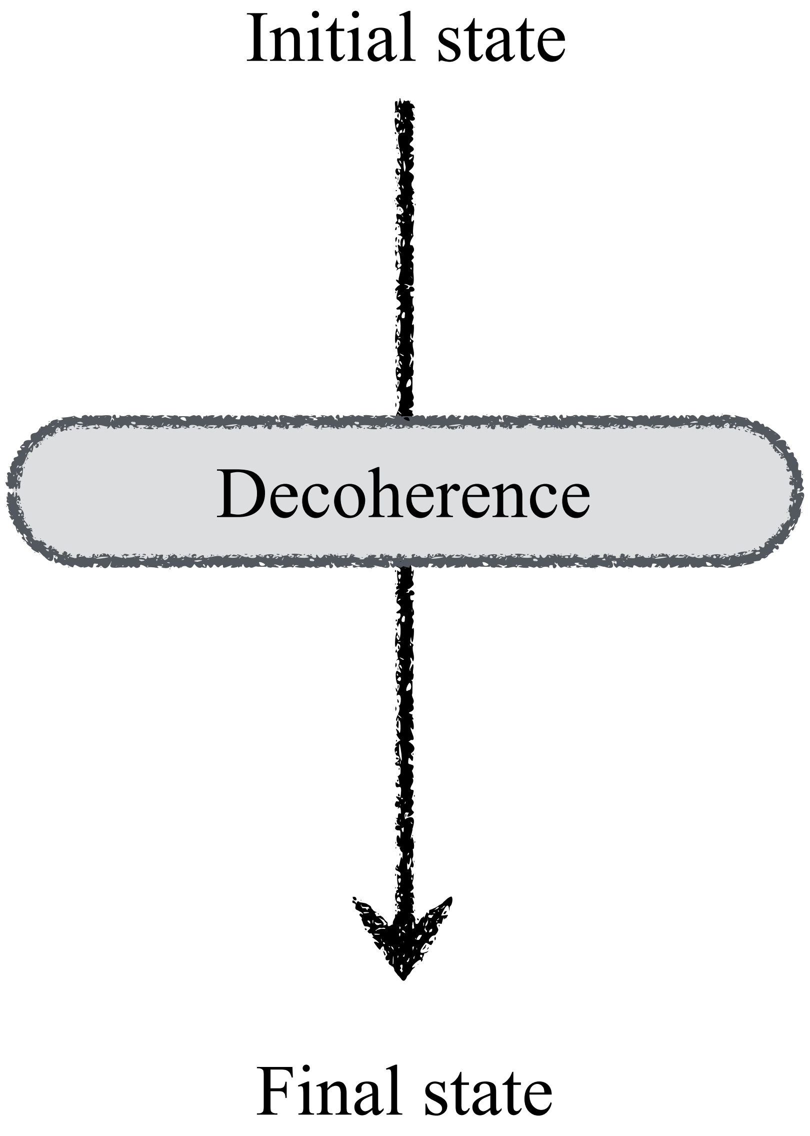
$$= \frac{1}{\sqrt{1-p}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \equiv \frac{1}{\sqrt{1-p}} M_2^{\mathrm{rev}}$$

Weak Measurement



Single-qubit decoherence suppression with weak and reversing measurement





Initial state

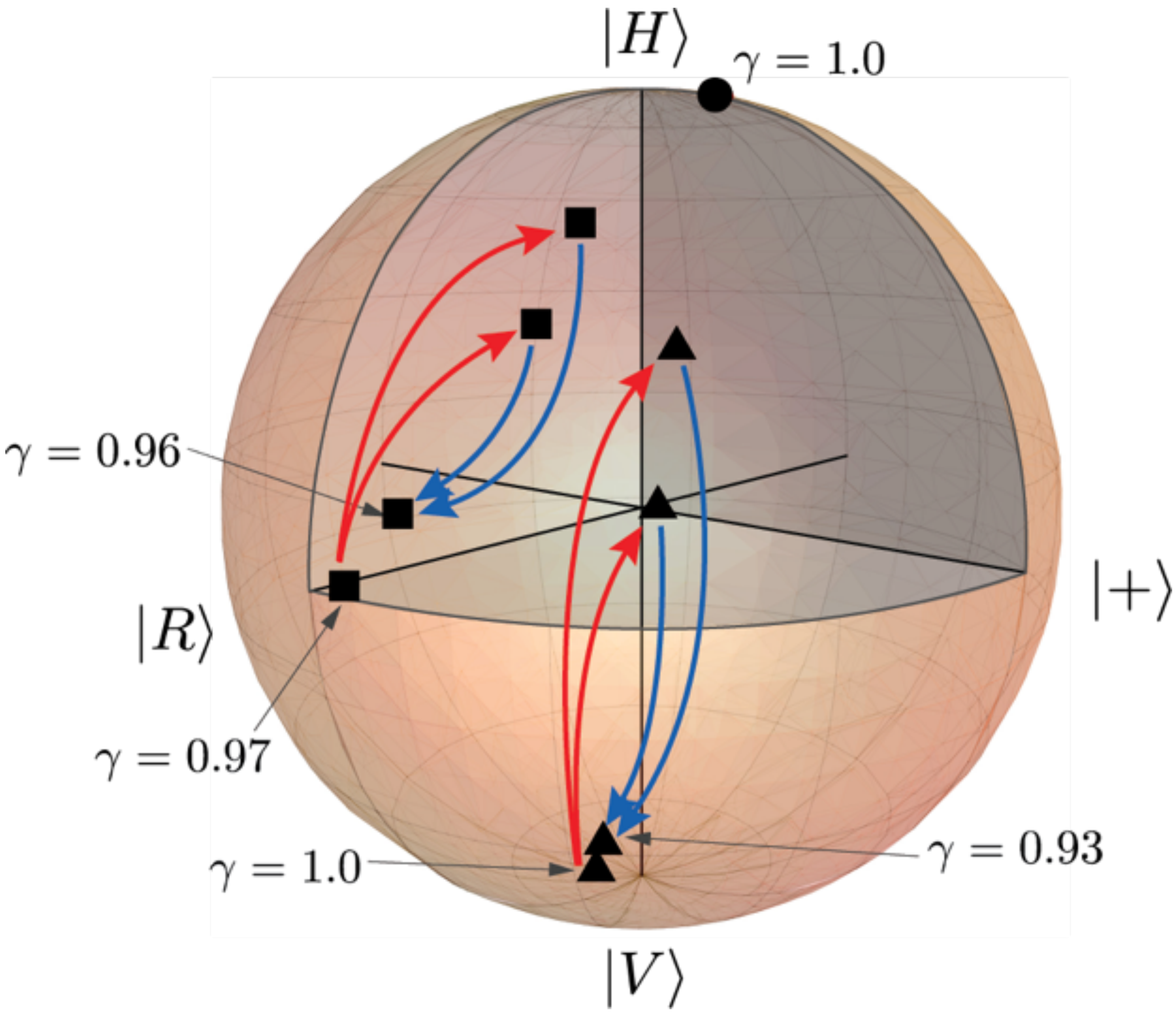
Decoherence

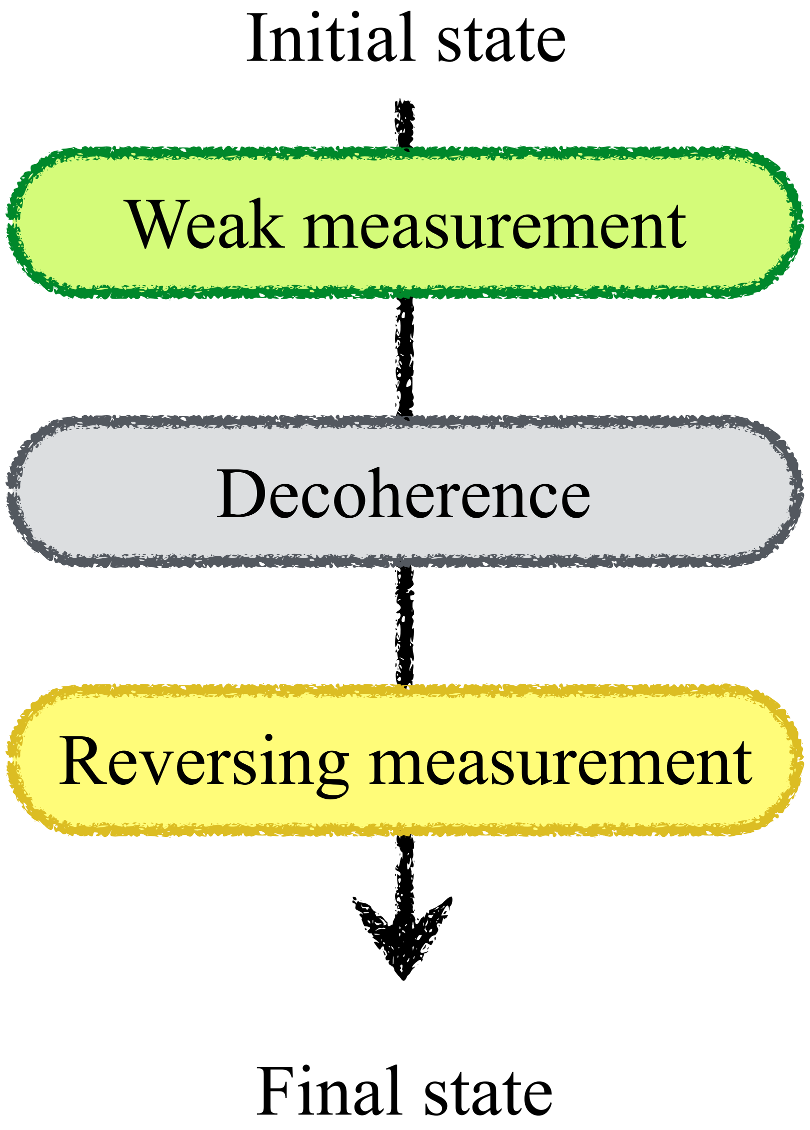
Final state

Kim *et al*., Opt Express **17**, 11978 (2009)

Lee *et al*., Opt Express **19**, 16309 (2011)

Single-qubit decoherence suppression with weak and reversing measurement





Initial state

Weak measurement

Decoherence

Reversing measurement

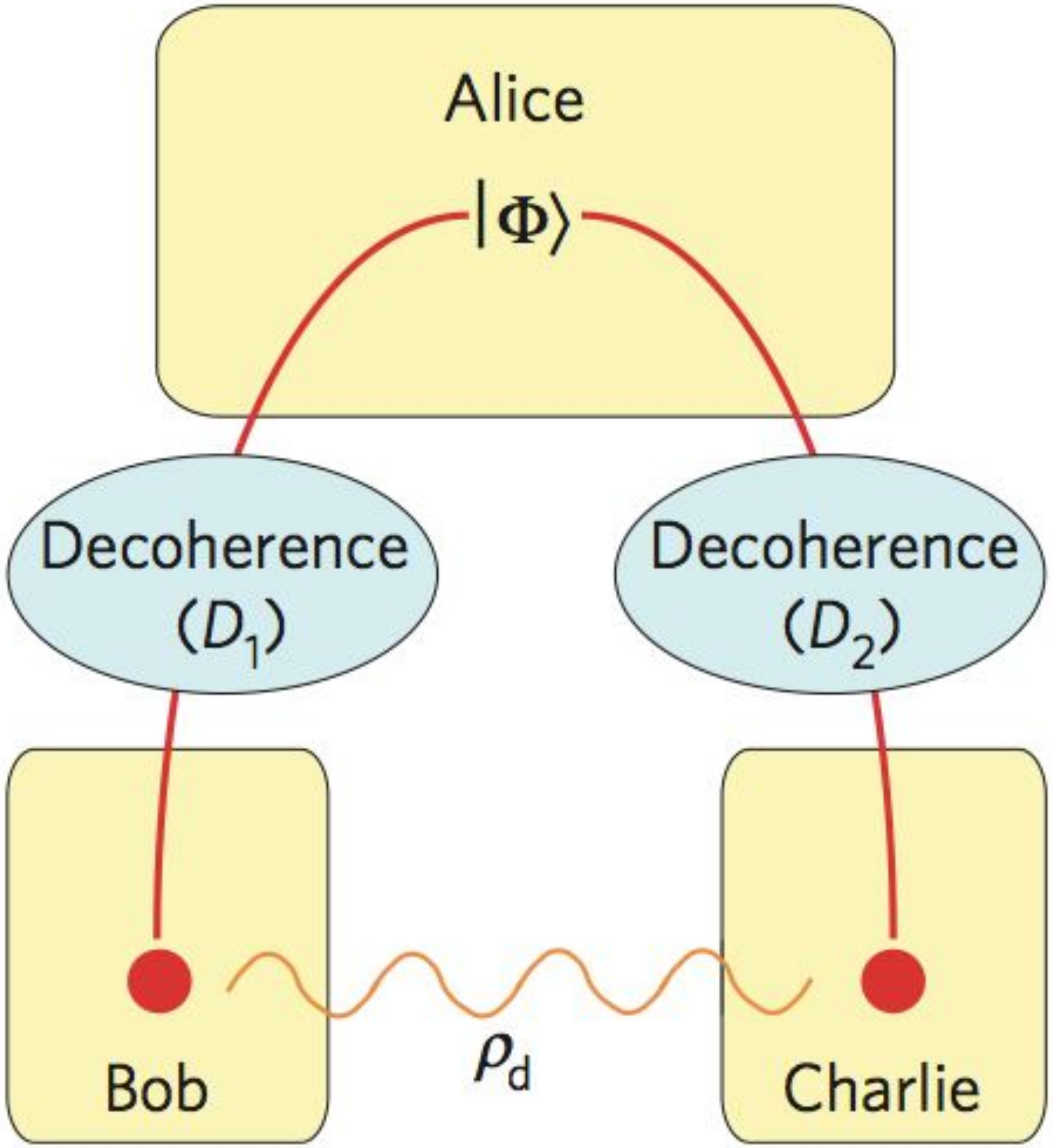
Final state

Kim *et al*., Opt Express **17**, 11978 (2009)

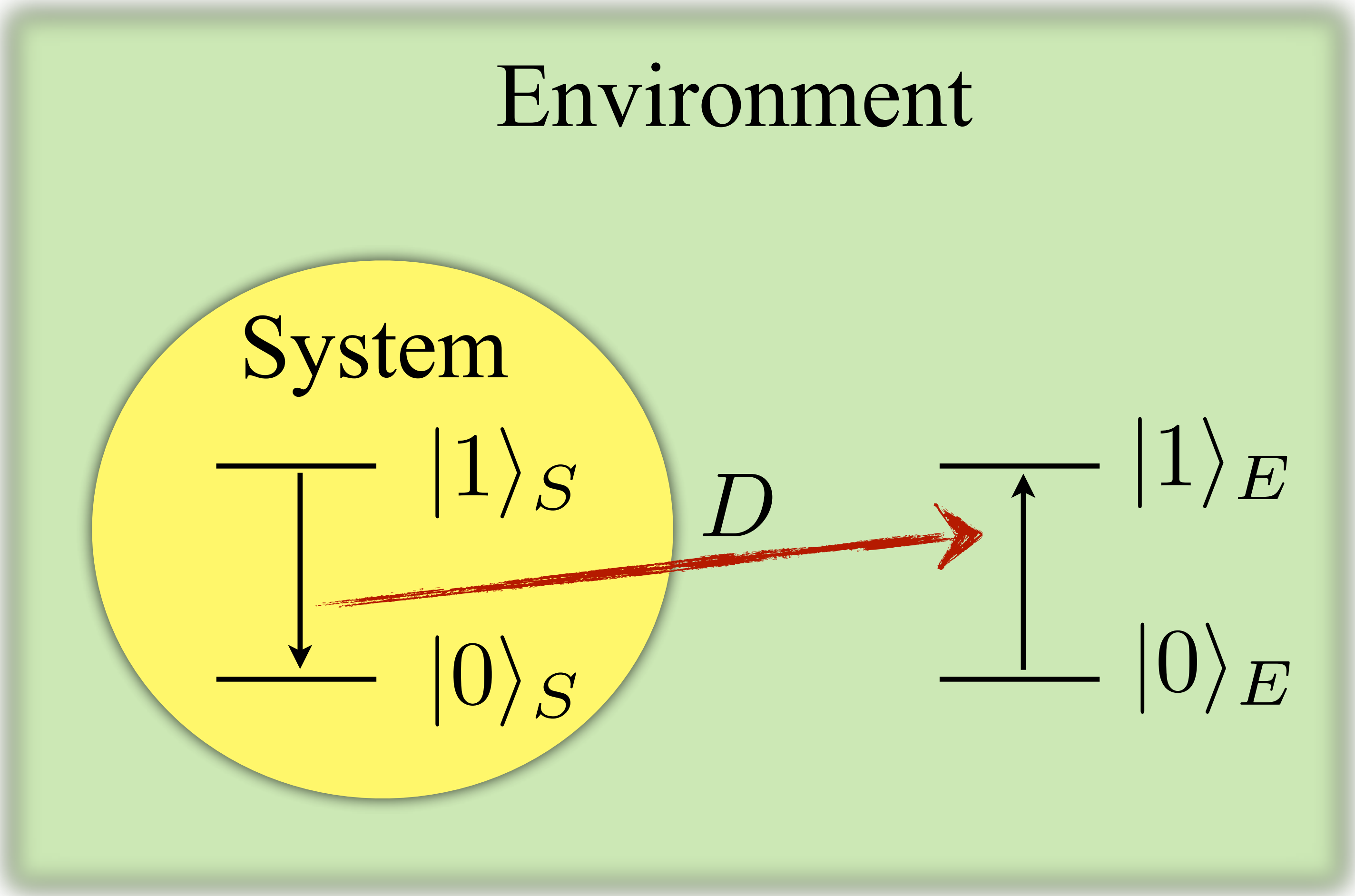
Lee *et al*., Opt Express **19**, 16309 (2011)

Two entangled qubits under amplitude damping decoherence

$$|\Phi\rangle = \alpha|00\rangle + \beta|11\rangle$$



Amplitude damping decoherence

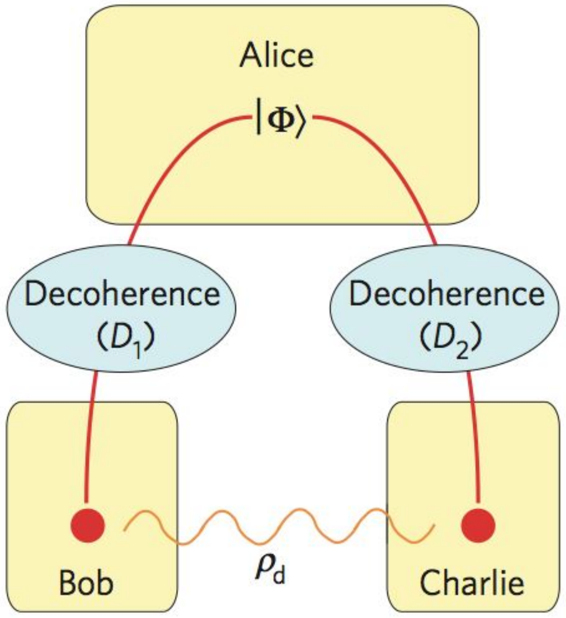


$$|0\rangle_S|0\rangle_E \longrightarrow |0\rangle_S|0\rangle_E$$

$$|1\rangle_S|0\rangle_E \longrightarrow \sqrt{1-D}|1\rangle_S|0\rangle_E + \sqrt{D}|0\rangle_S|1\rangle_E$$

- Photon loss for vacuum-single-photon qubit
- Spontaneous decay for the atomic/ion qubit
- Zero-temperature energy relaxation for the superconducting qubit
- etc

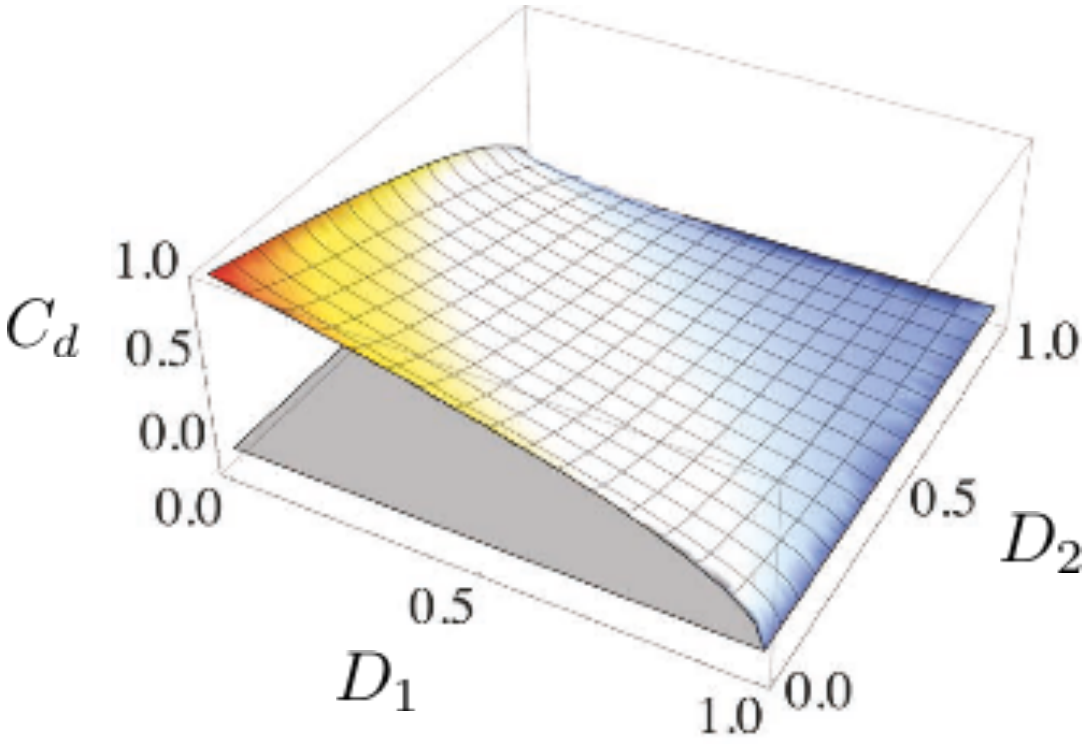
Two entangled qubits under amplitude damping decoherence



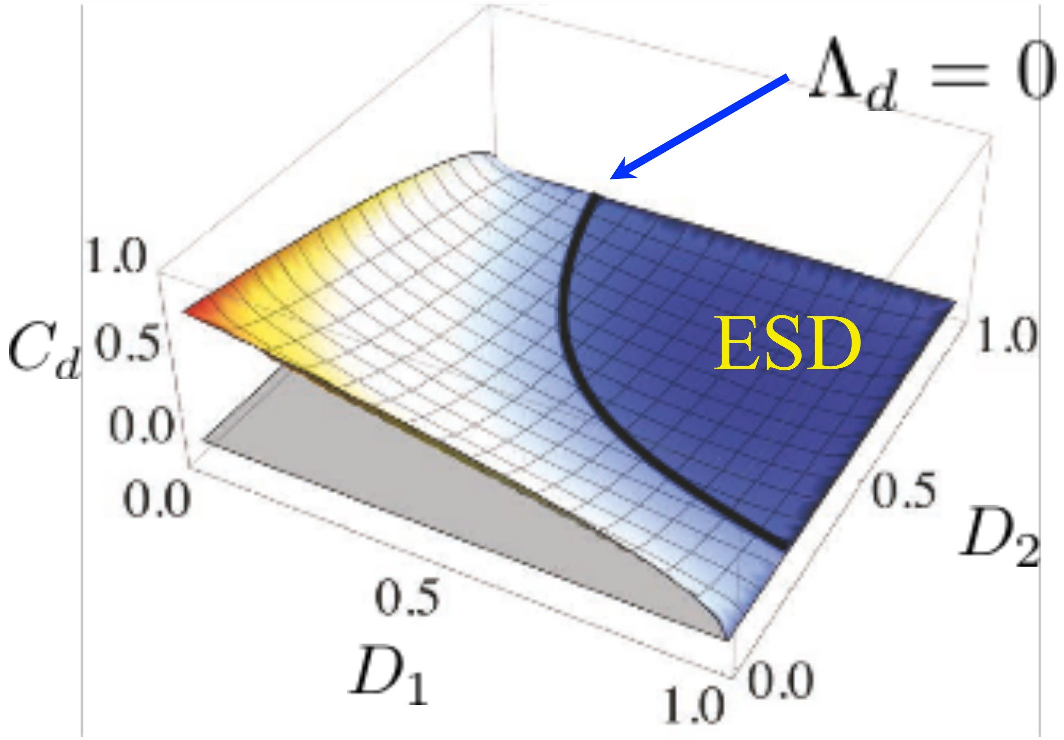
$$|\Phi\rangle = \alpha|00\rangle + \beta|11\rangle$$

$$\rho_d = \begin{pmatrix} |\alpha|^2 + |\beta|^2 D_1 D_2 & 0 & 0 & \alpha\beta^* \sqrt{\bar{D}_1 \bar{D}_2} \\ 0 & |\beta|^2 D_1 \bar{D}_2 & 0 & 0 \\ 0 & 0 & |\beta|^2 D_2 \bar{D}_1 & 0 \\ \alpha^* \beta \sqrt{\bar{D}_1 \bar{D}_2} & 0 & 0 & |\beta|^2 \bar{D}_1 \bar{D}_2 \end{pmatrix}$$

$$C_d = \max \left\{ 0, \Lambda_d \equiv 2\sqrt{\bar{D}_1 \bar{D}_2} |\beta| (|\alpha| - \sqrt{D_1 D_2} |\beta|) \right\}$$

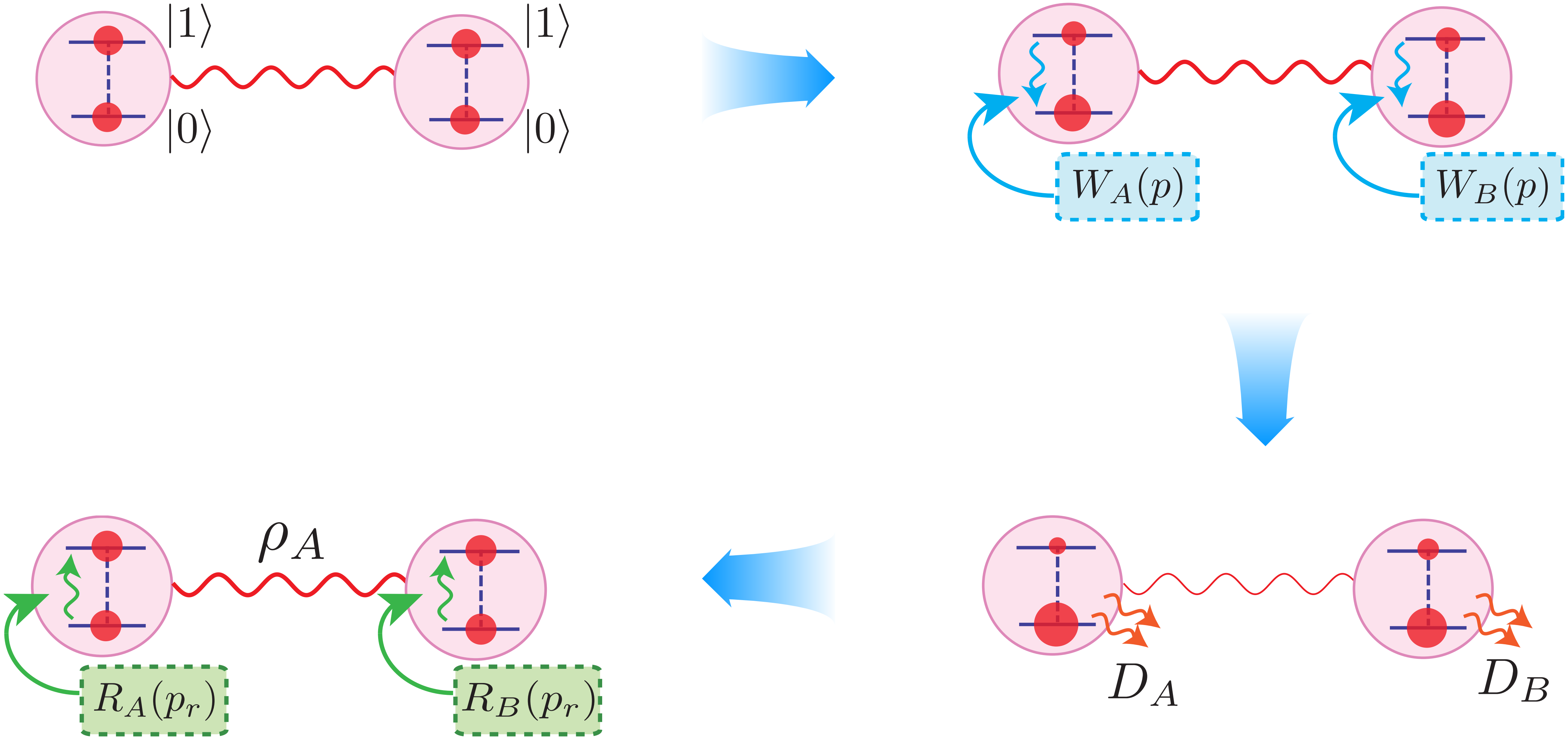


$|\alpha| = |\beta|$

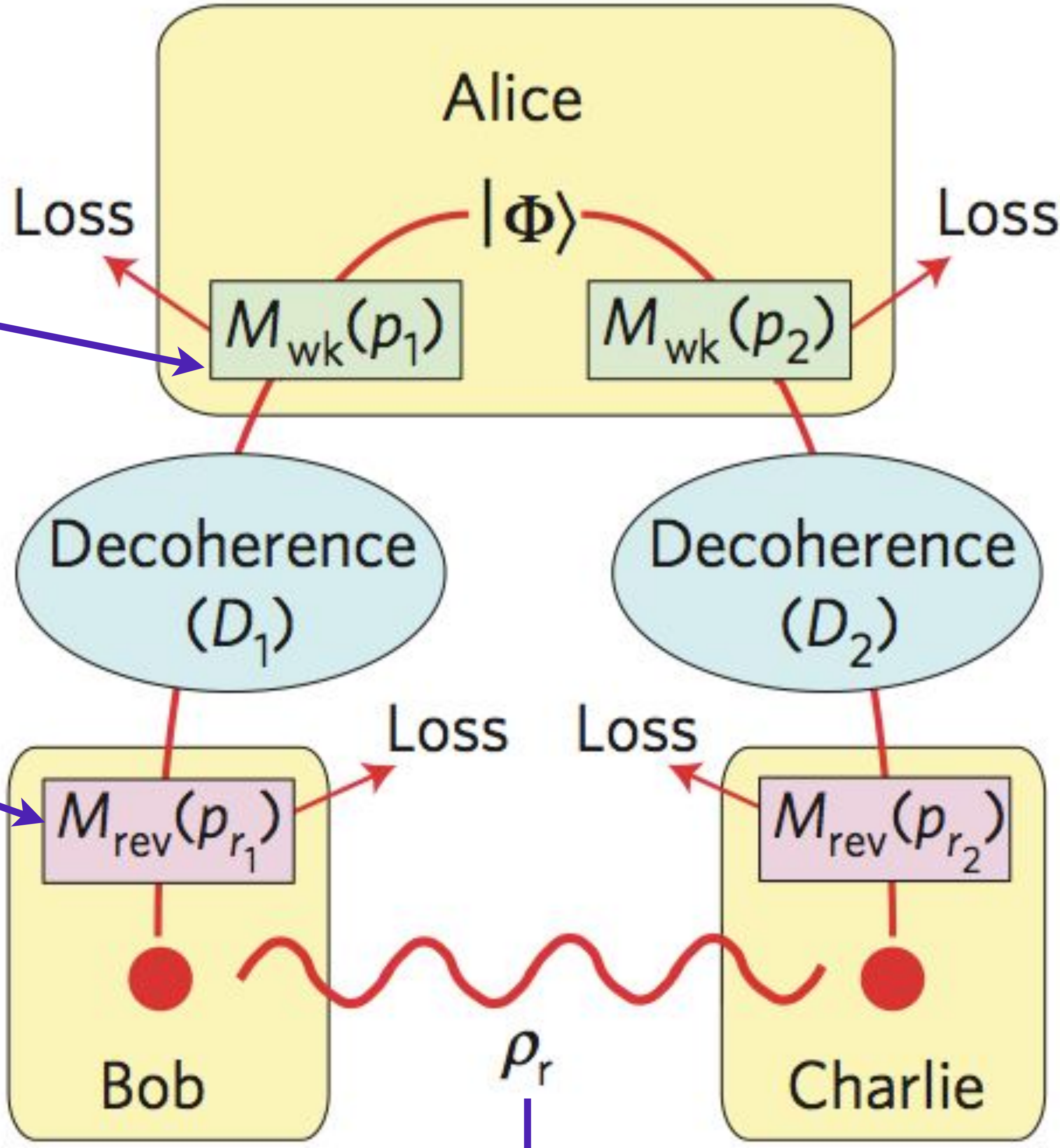


$|\alpha|^2 < |\beta|^2$ $(|\alpha| = 0.42)$

Decoherence suppression using weak measurement and reversing measurement



Applying weak measurement and reversing measurement



$$M_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}$$

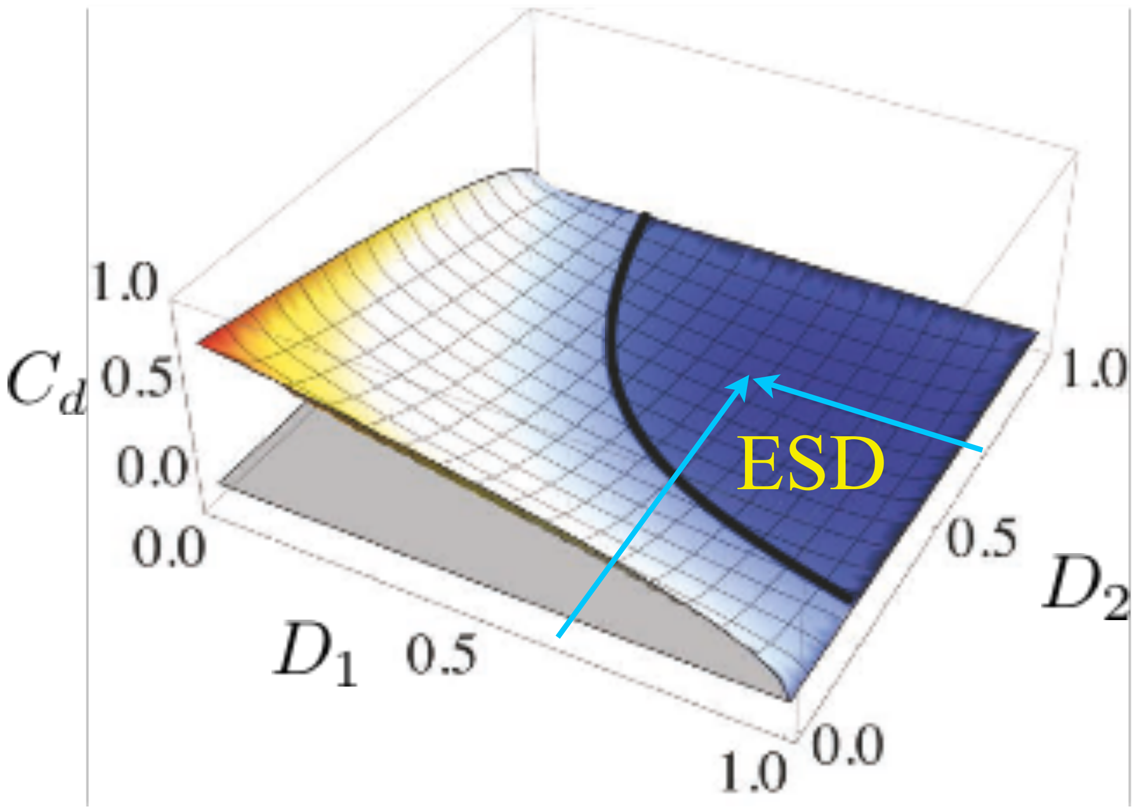
$$M_2^{\mathrm{rev}} = \begin{pmatrix} \sqrt{1-p_r} & 0 \\ 0 & 1 \end{pmatrix}$$

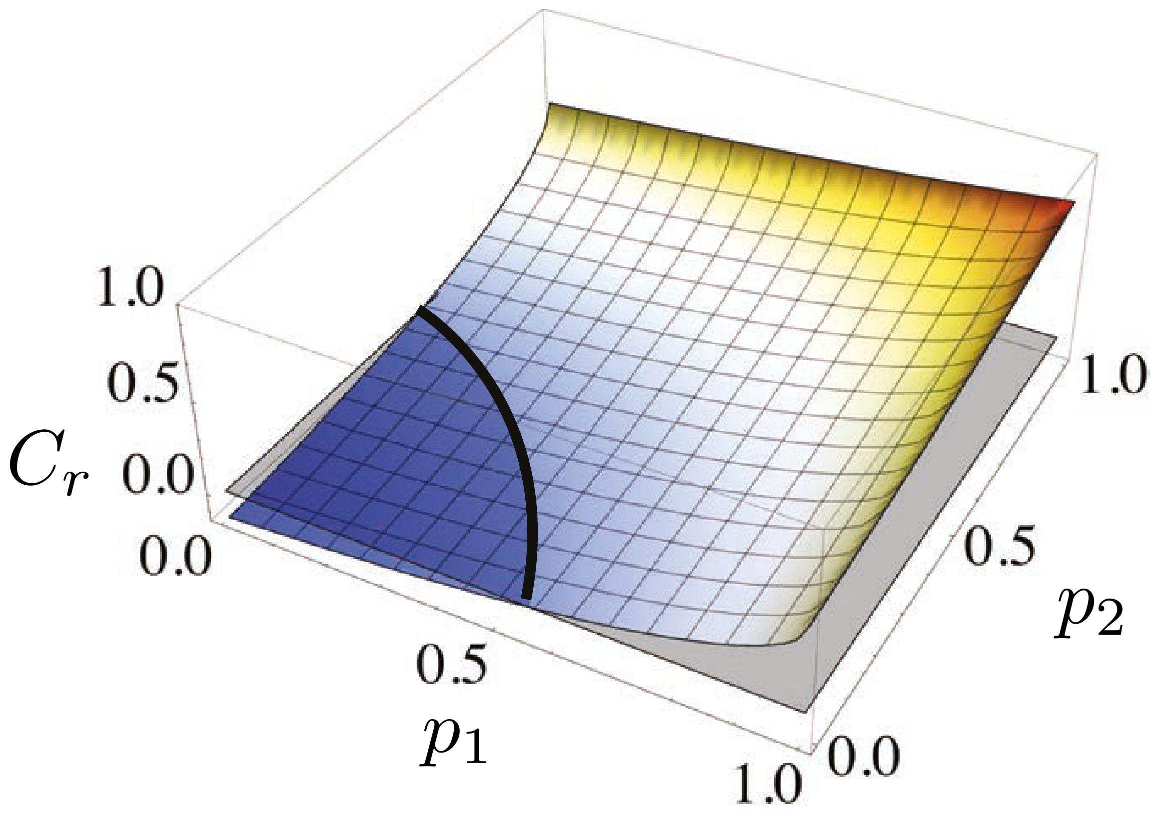
$$p_r = p + (1-p)D$$

(Optimal reversing strength)

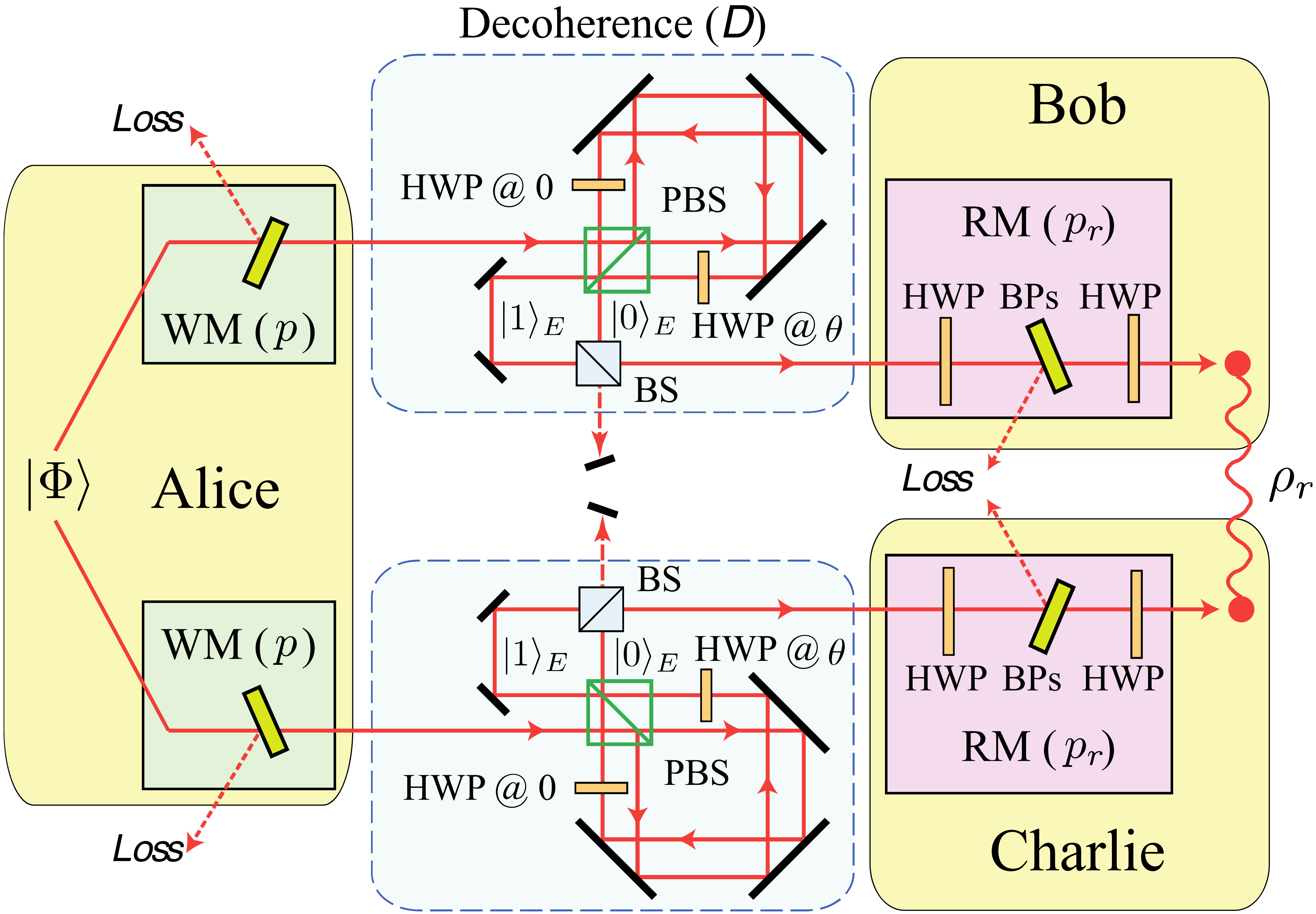
$$C_r = \max\left\{0, \Lambda_r \equiv \frac{2|\beta|(|\alpha| - \sqrt{D_1 D_2 \bar{p}_1 \bar{p}_2}|\beta|)}{1 + \{D_1 \bar{p}_1 (1 + D_2 \bar{p}_2) + D_2 \bar{p}_2\}|\beta|^2}\right\}$$

Decoherence suppression using weak measurement and reversing measurement





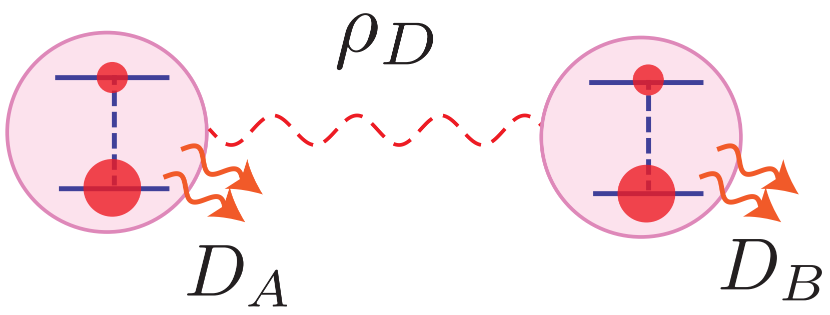
Experimental Scheme

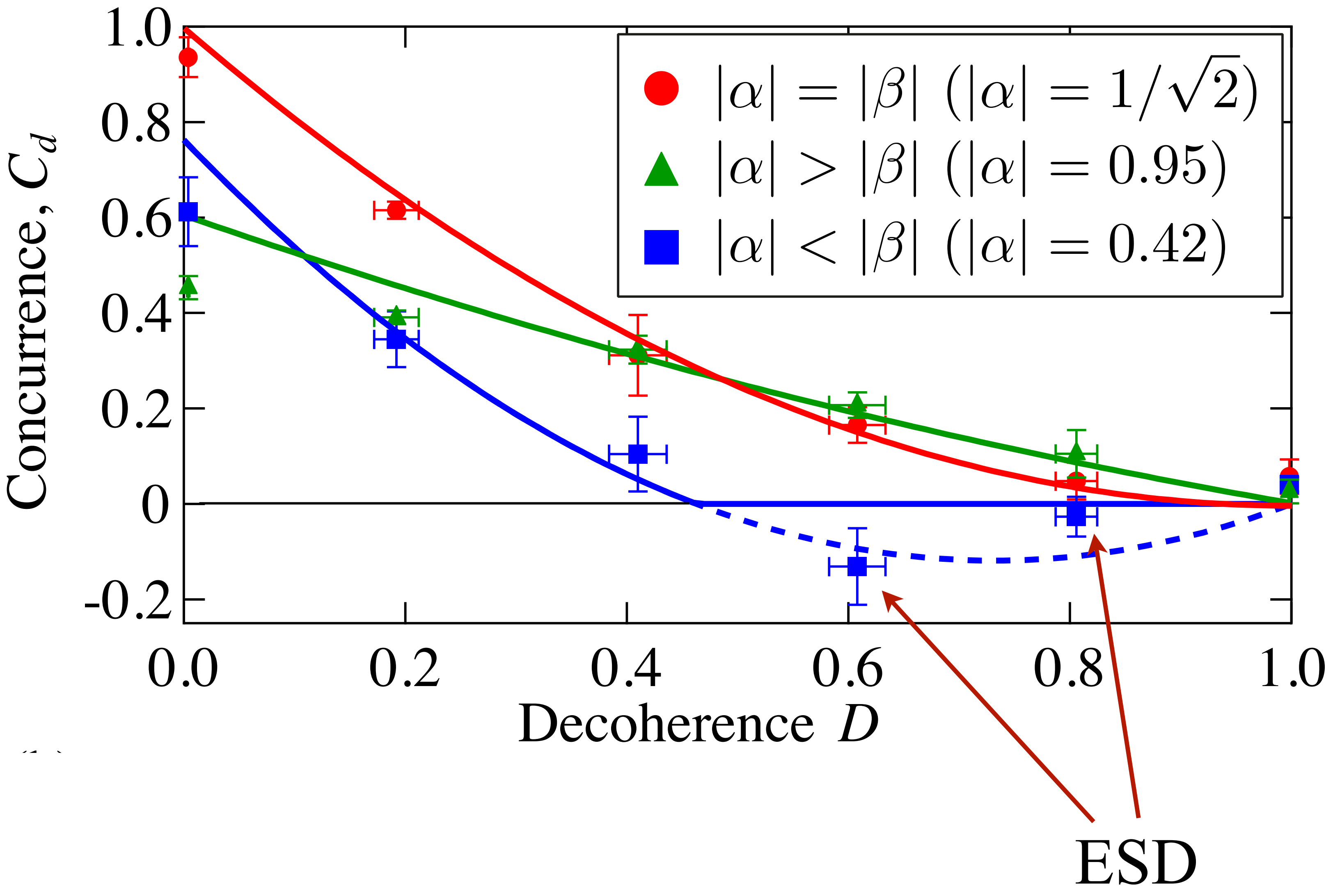


Kim *et al.*, Nature Phys **8**, 117 (2012)

Entanglement sudden death due to decoherence

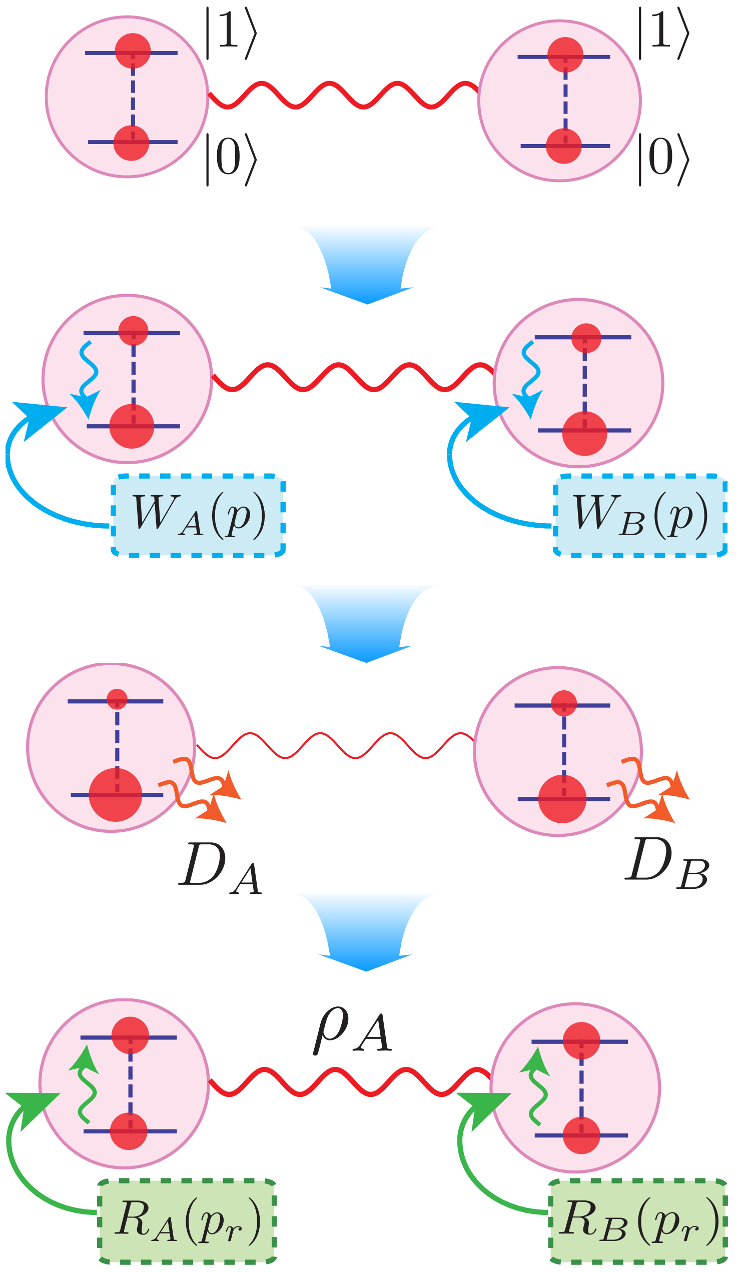
$$|\Phi\rangle = \alpha|00\rangle + \beta|11\rangle$$

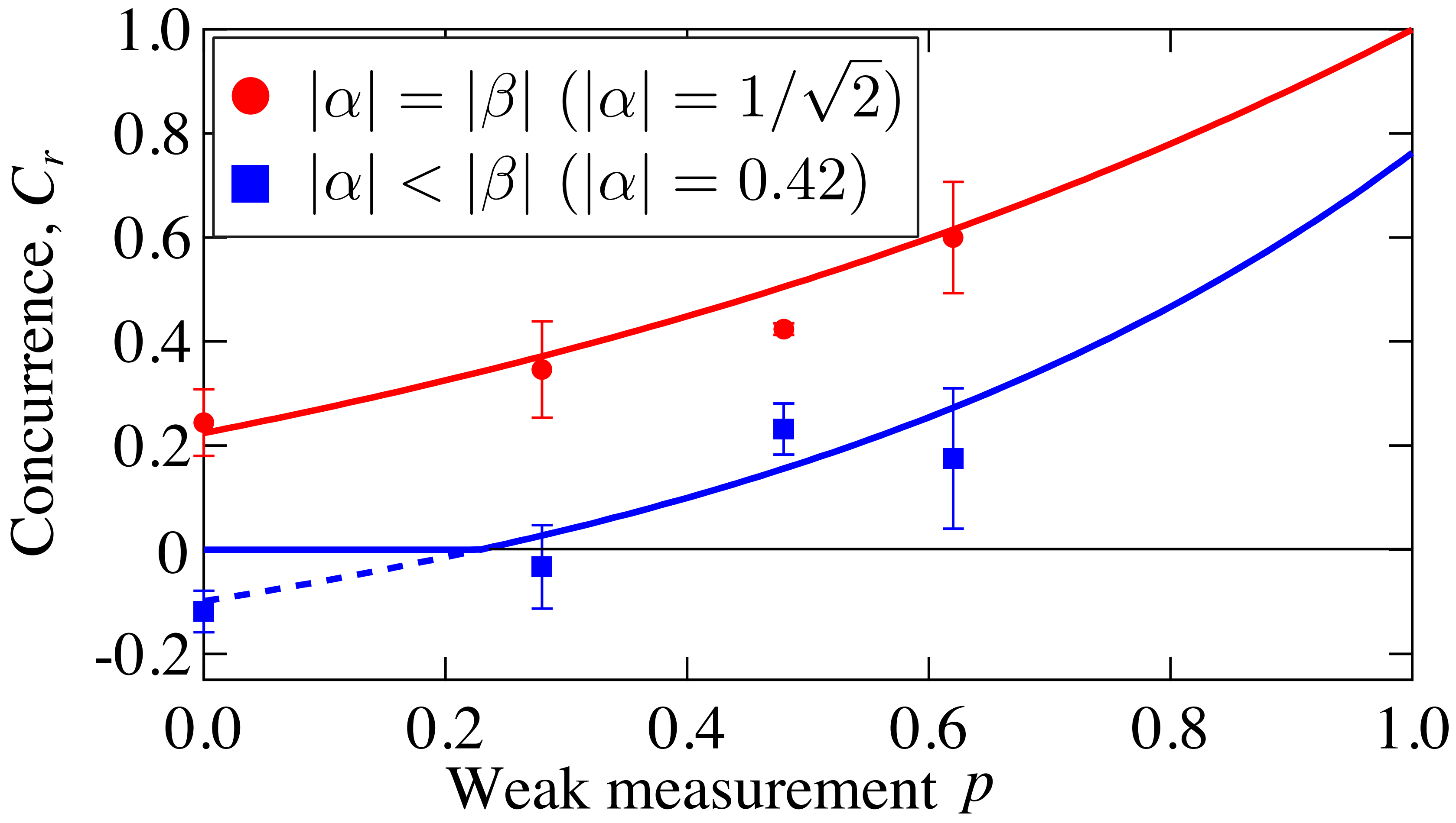




Kim *et al*., Nature Phys **8**, 117 (2012)

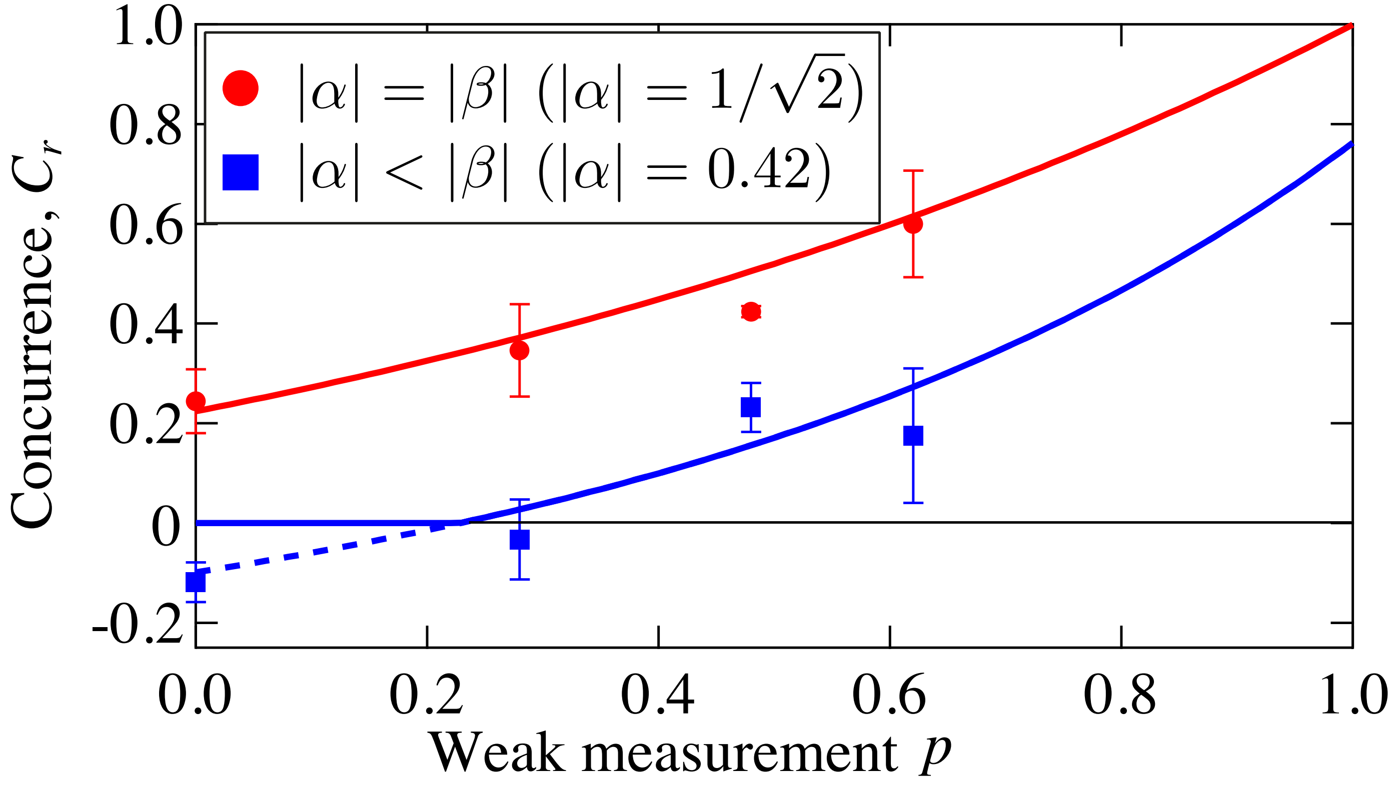
Decoherence suppression using weak measurement

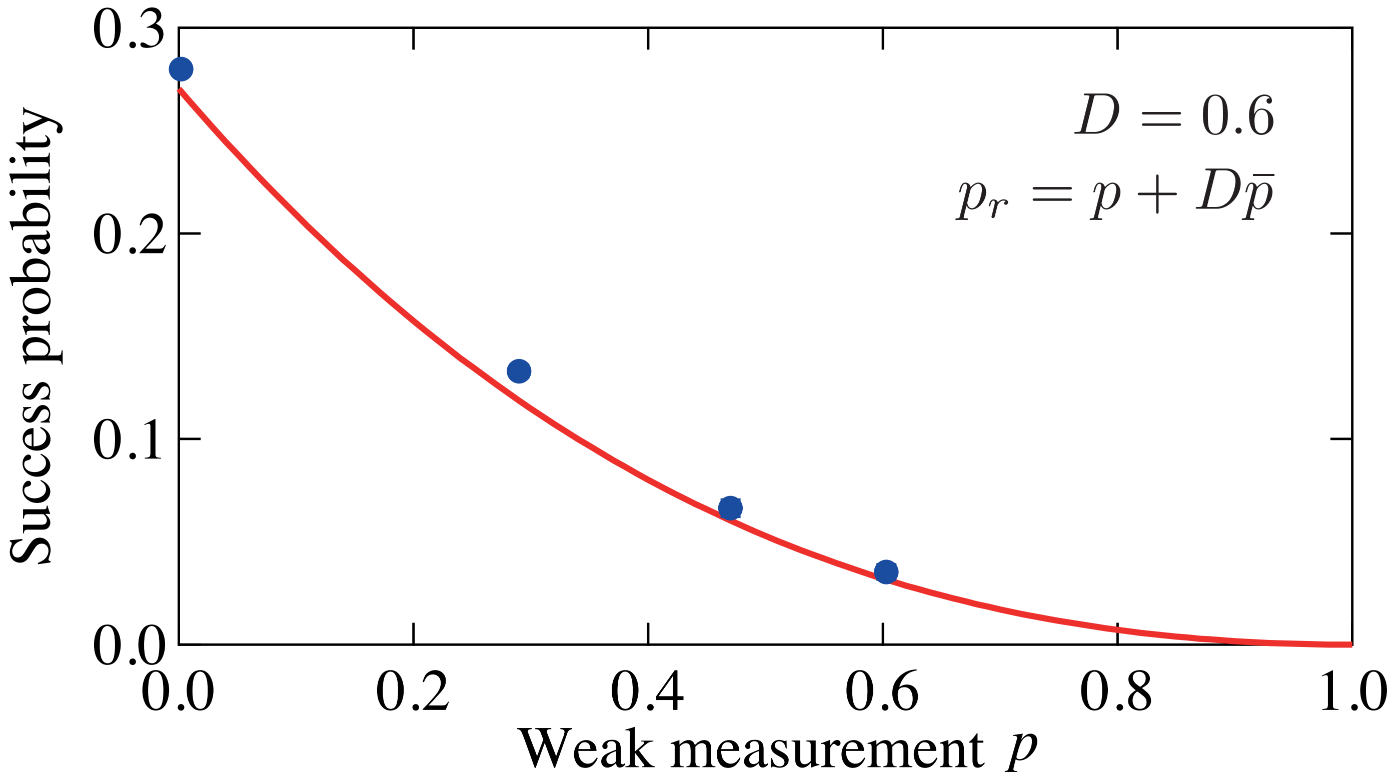




Kim *et al.*, Nature Phys **8**, 117 (2012)

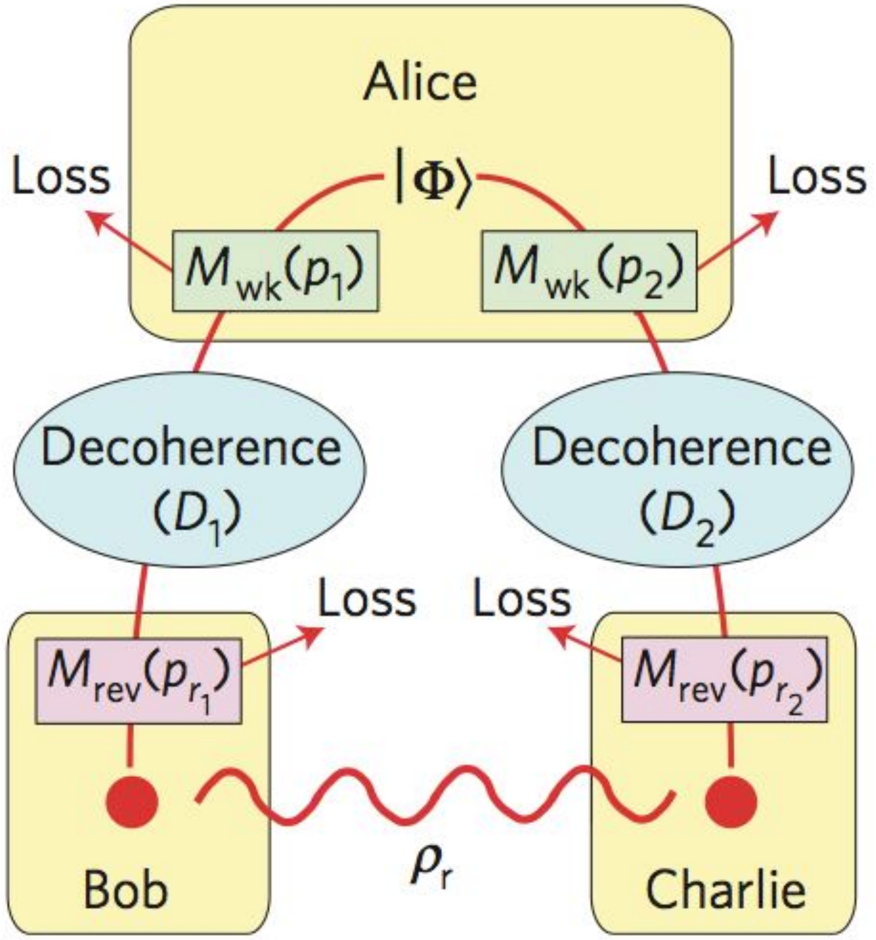
Trade-off





$p \uparrow \quad \Rightarrow$ Concurrence, $C_r \uparrow$

$\Rightarrow$ Success probability $\downarrow$



Take-home messages

1. Quantum states (even entanglement) can be protected from decoherence using weak measurement and quantum measurement reversal.

2. Allows entanglement distribution through noisy quantum channels.

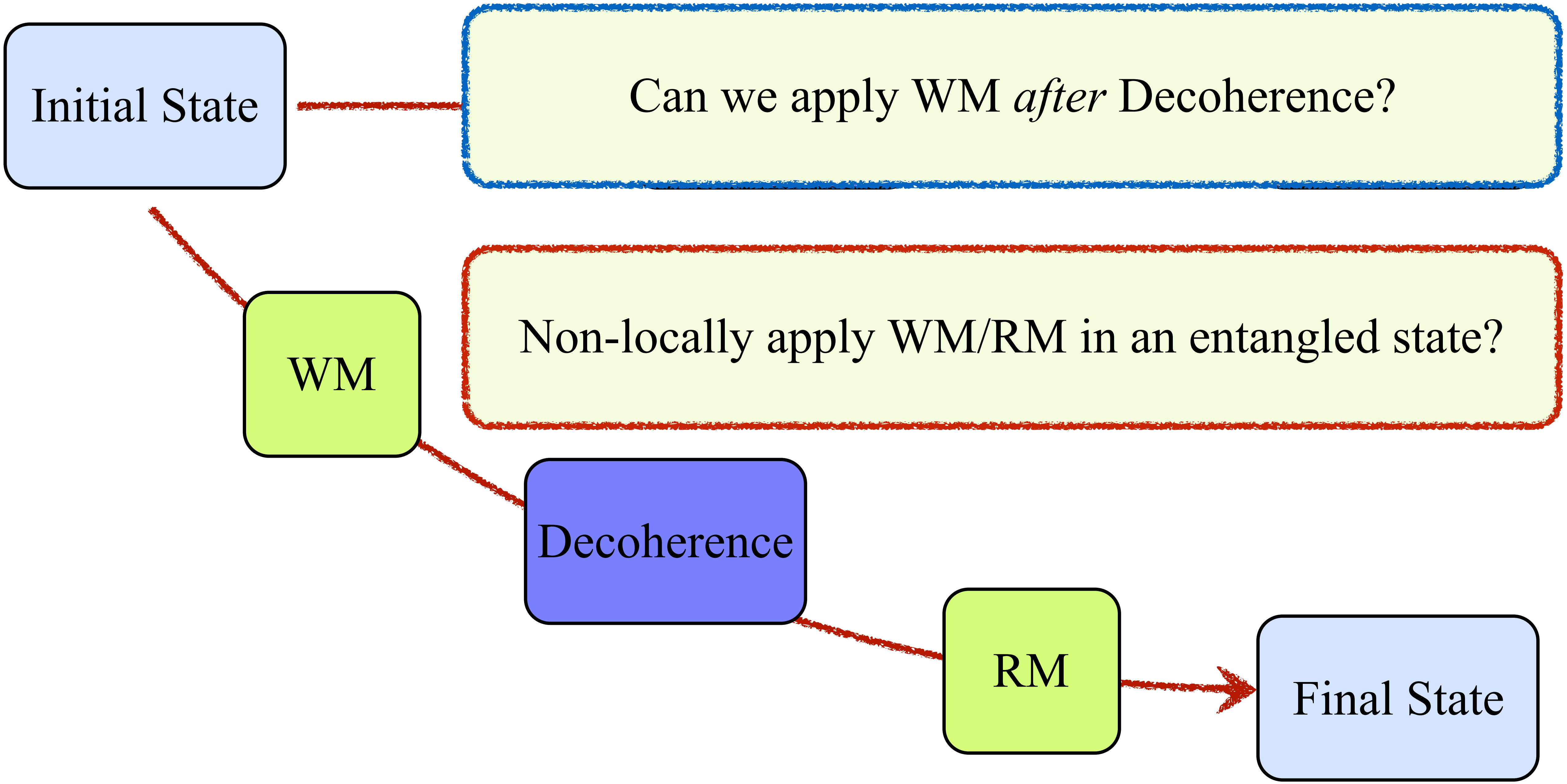
3. Kim *et al*., Nature Phys 8, 117 (2012).

4. Delayed-choice quantum walk
   Jeong *et al*., Nature Commun 4, 2471 (2013)

5. Delayed-choice decoherence suppression
   Lee *et al*., Nature Commun 5, 4522 (2014)

DELAYED-CHOICE WEAK MEASUREMENT AND DECOHERENCE SUPPRESSION

Decoherence suppression via weak quantum measurement and quantum measurement reversal



Initial State

WM

Decoherence

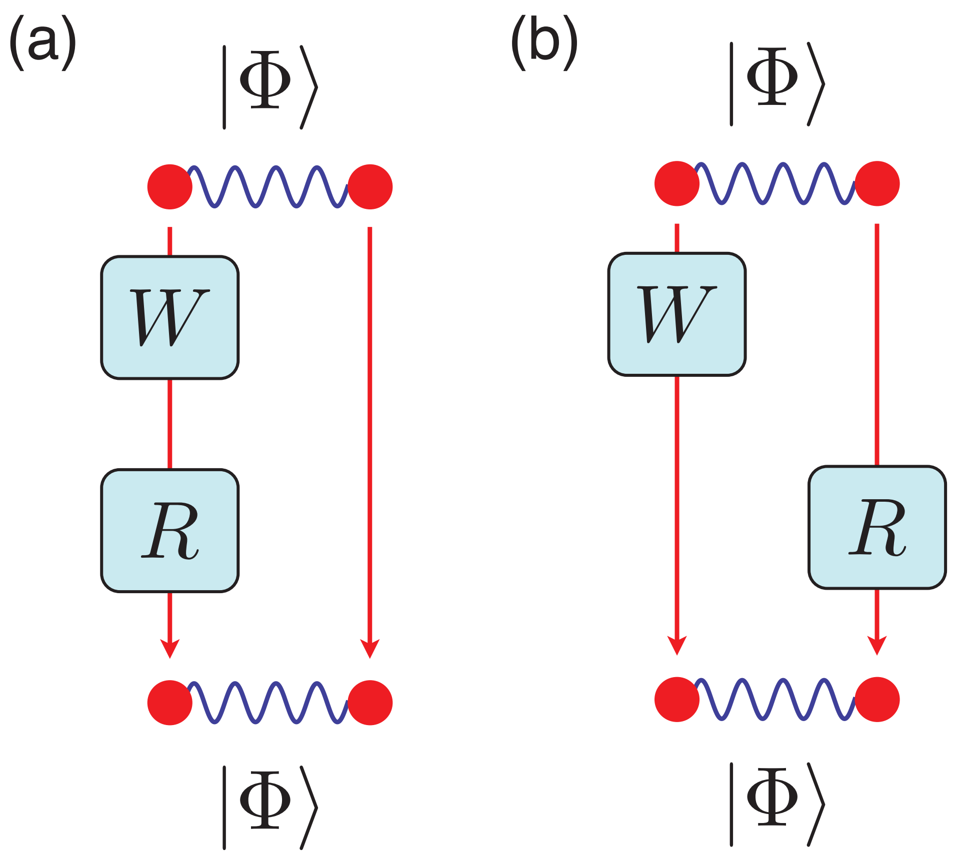
RM

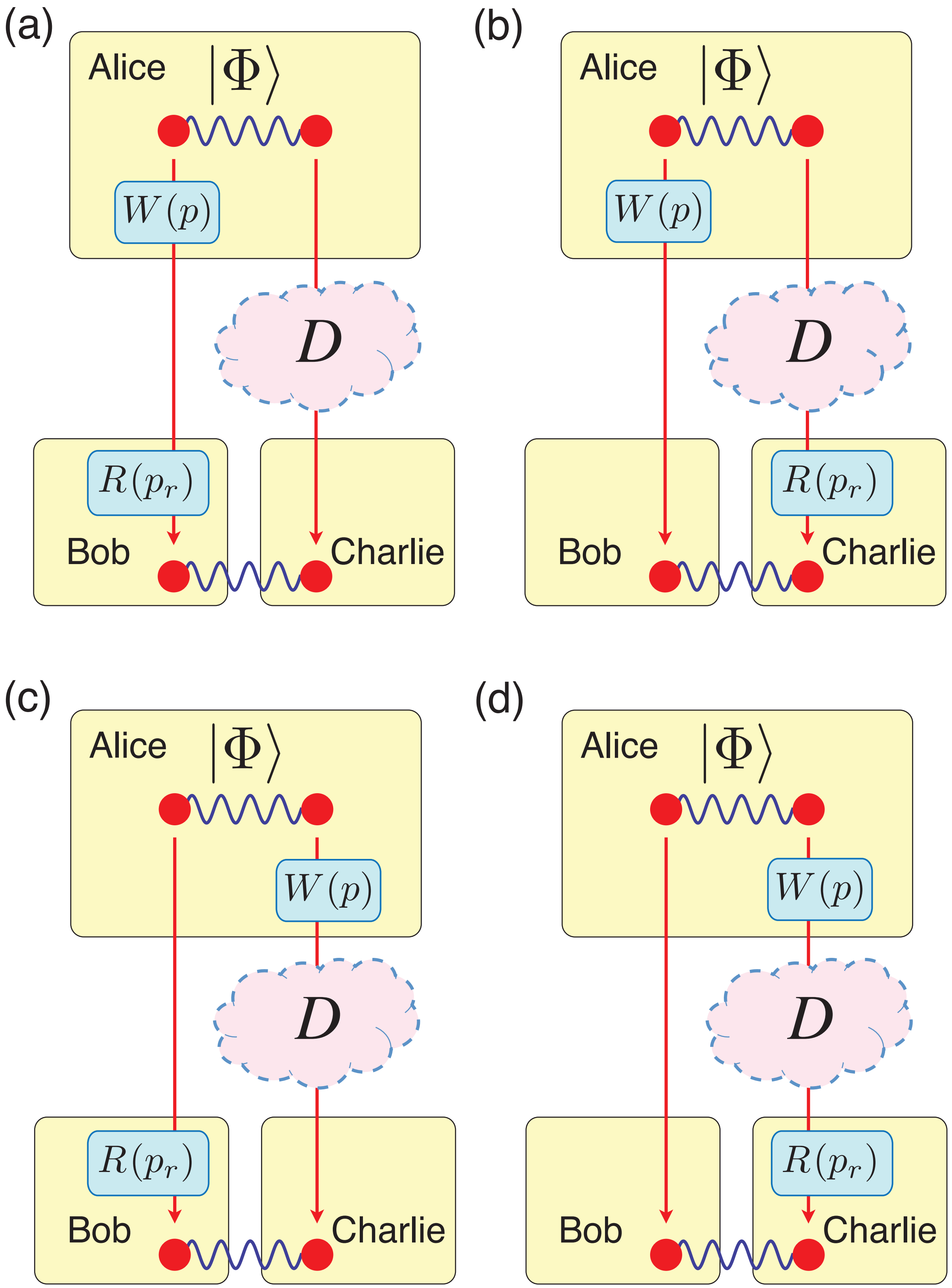
Final State

Can we apply WM *after* Decoherence?

Non-locally apply WM/RM in an entangled state?

Non-locally applying WM/RM in an entangled state

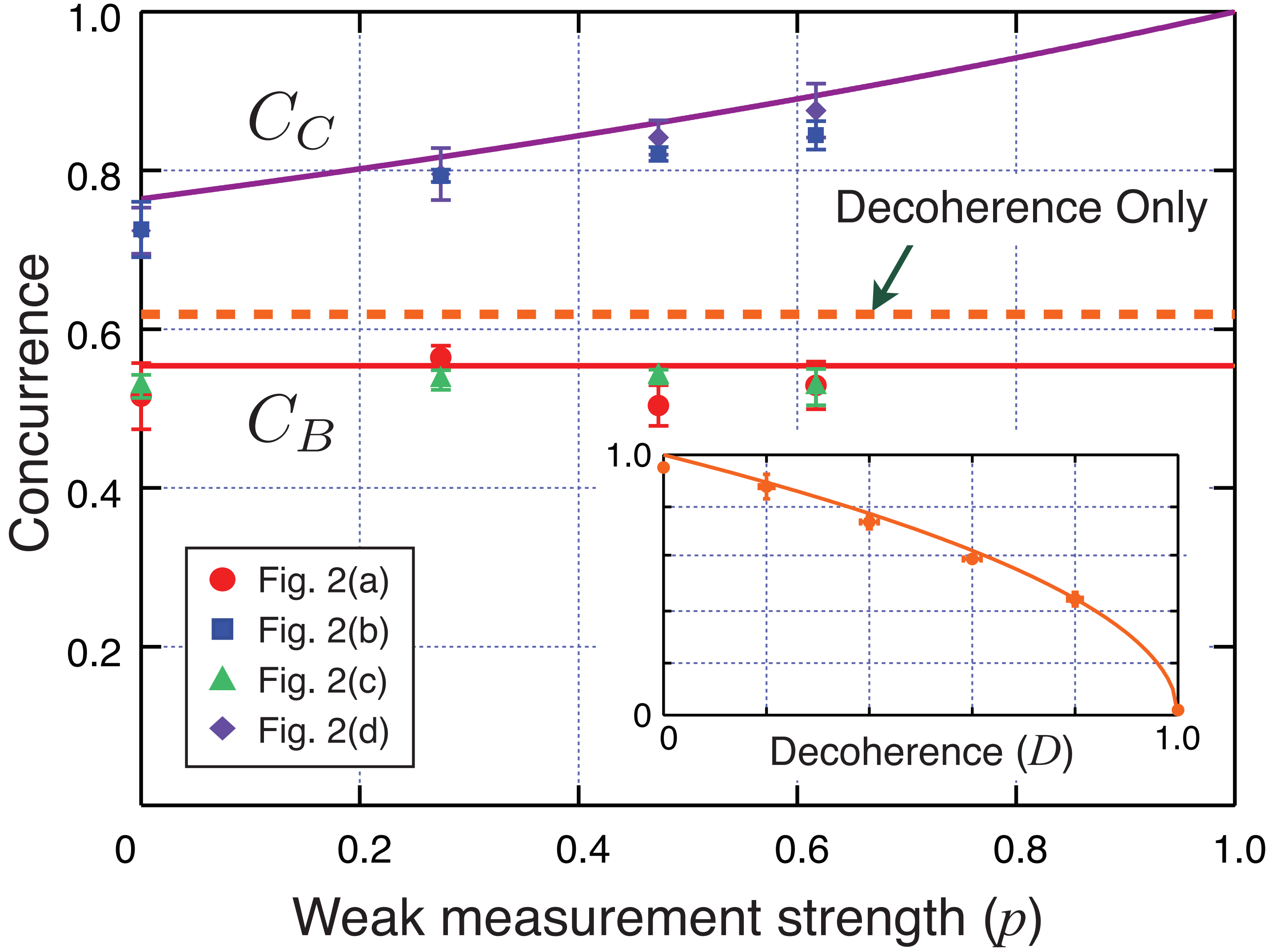




Lee et al., Nature Commun. 5, 4522 (2014)

Lim et al., Phys Rev A 90, 052328 (2014).

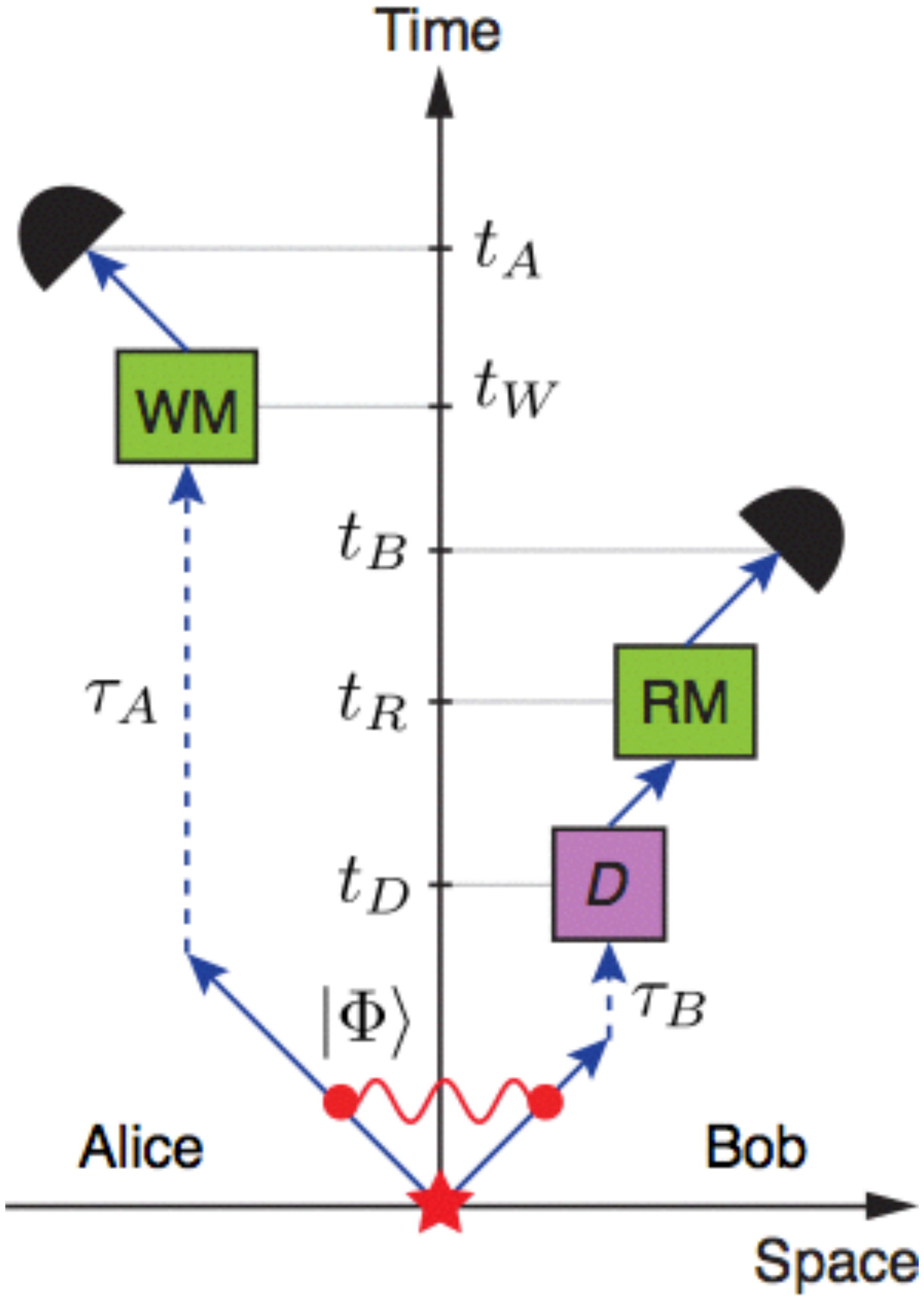
Decoherence causes breaking of exchange symmetry of quantum operations (weak measurement)



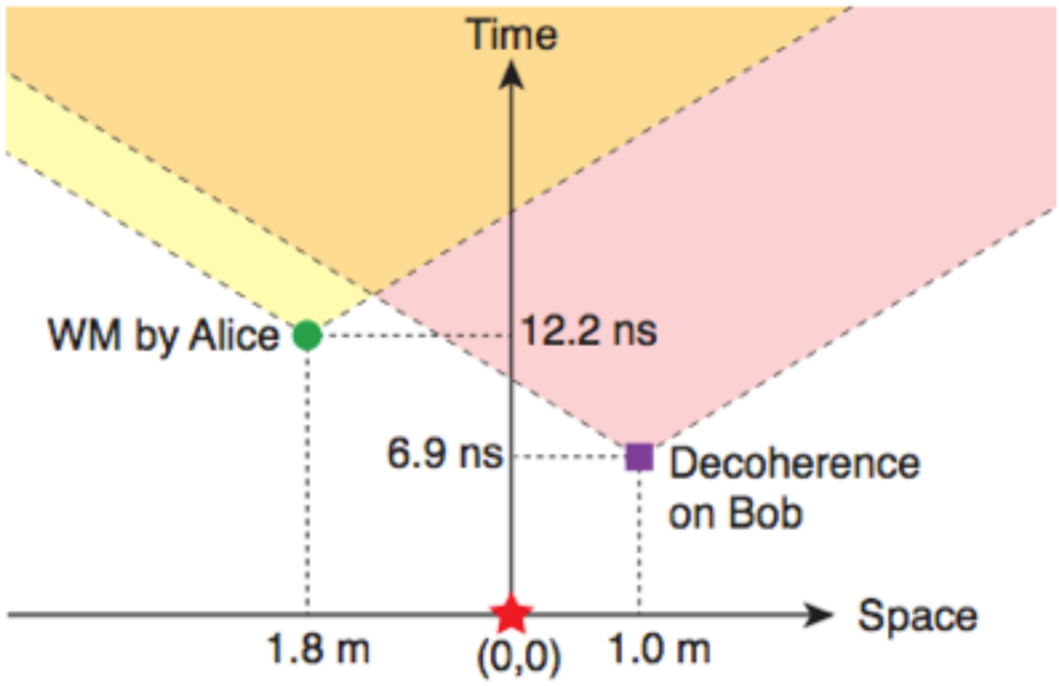
Lim et al., Phys Rev A 90, 052328 (2014).

Lee et al., Nature Commun. 5, 4522 (2014)

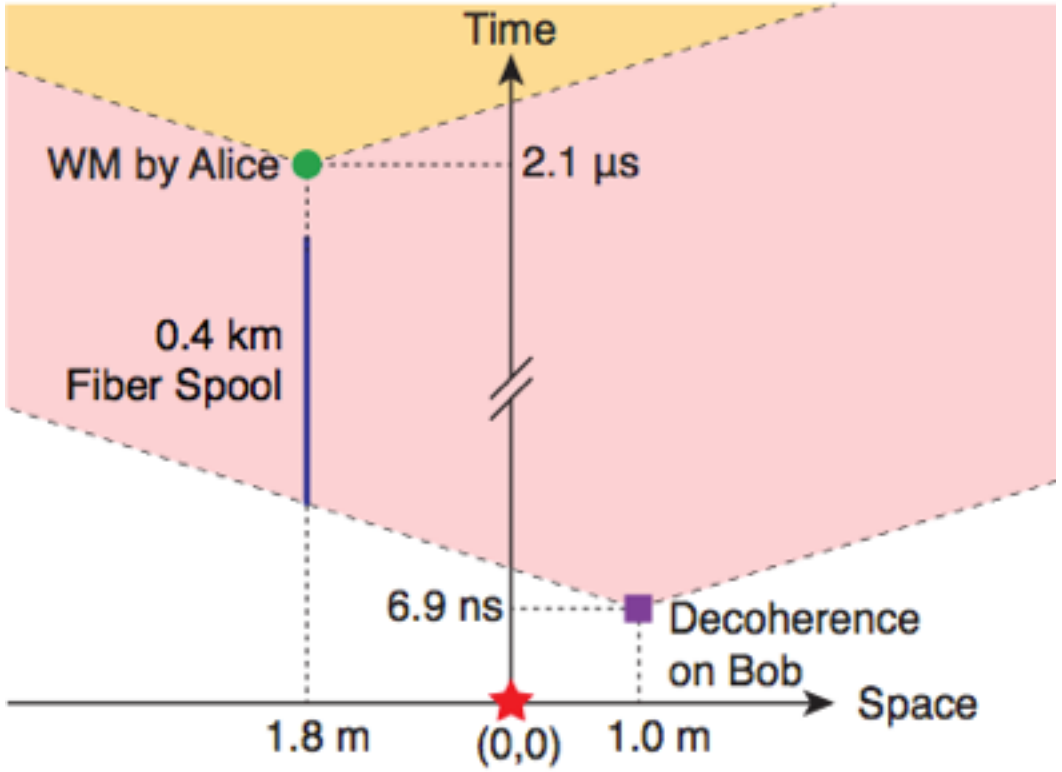
Delayed-choice decoherence suppression



(a) Space-like separation

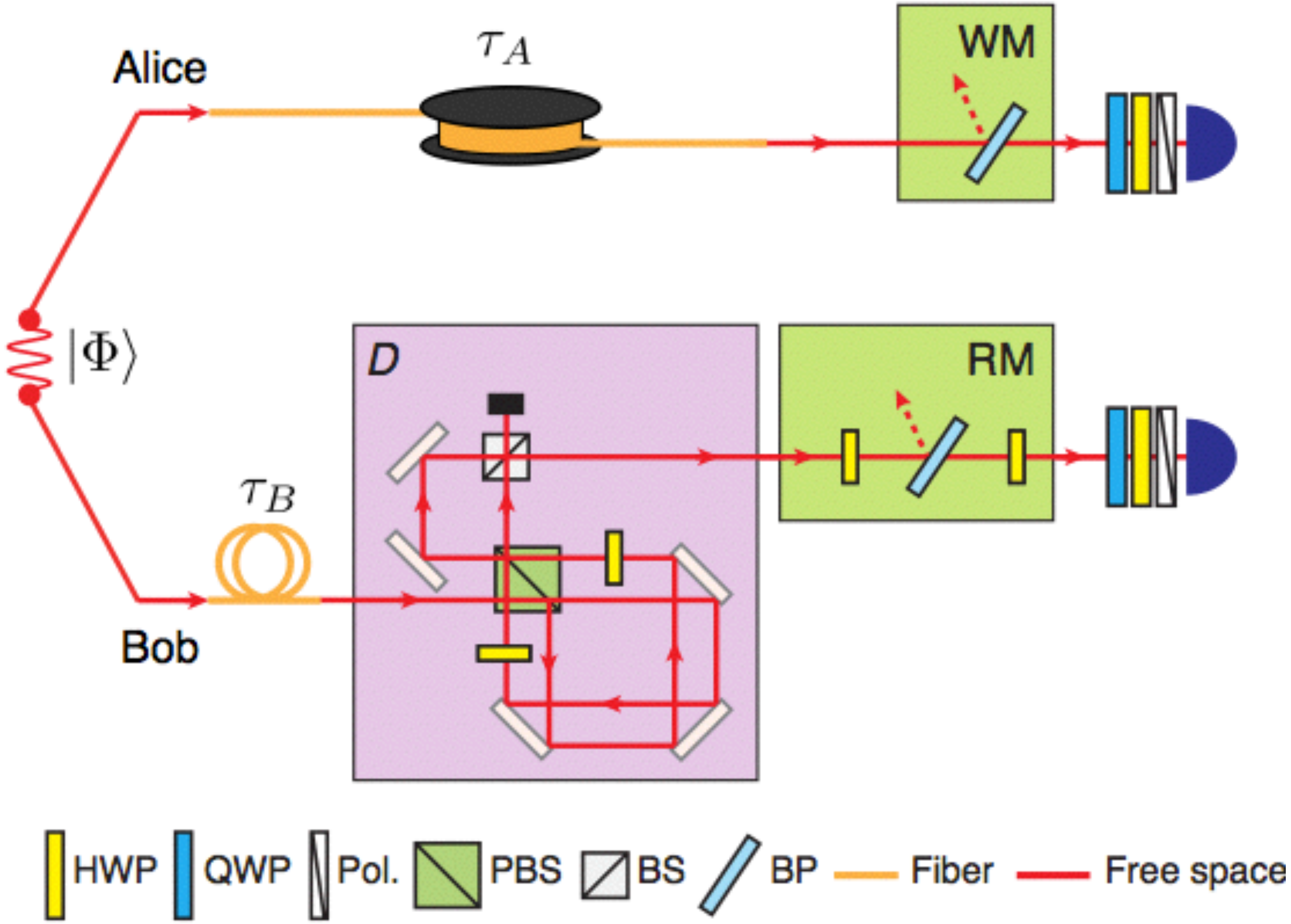


(b) Time-like separation



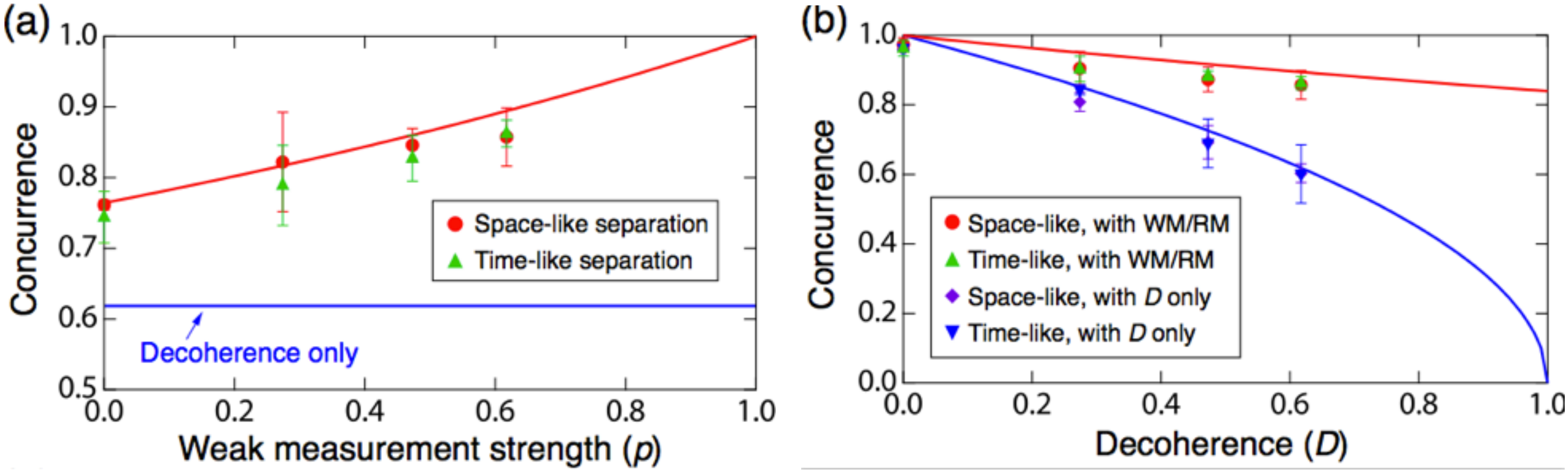
Lee et al., Nature Commun. 5, 4522 (2014)

Delayed-choice decoherence suppression



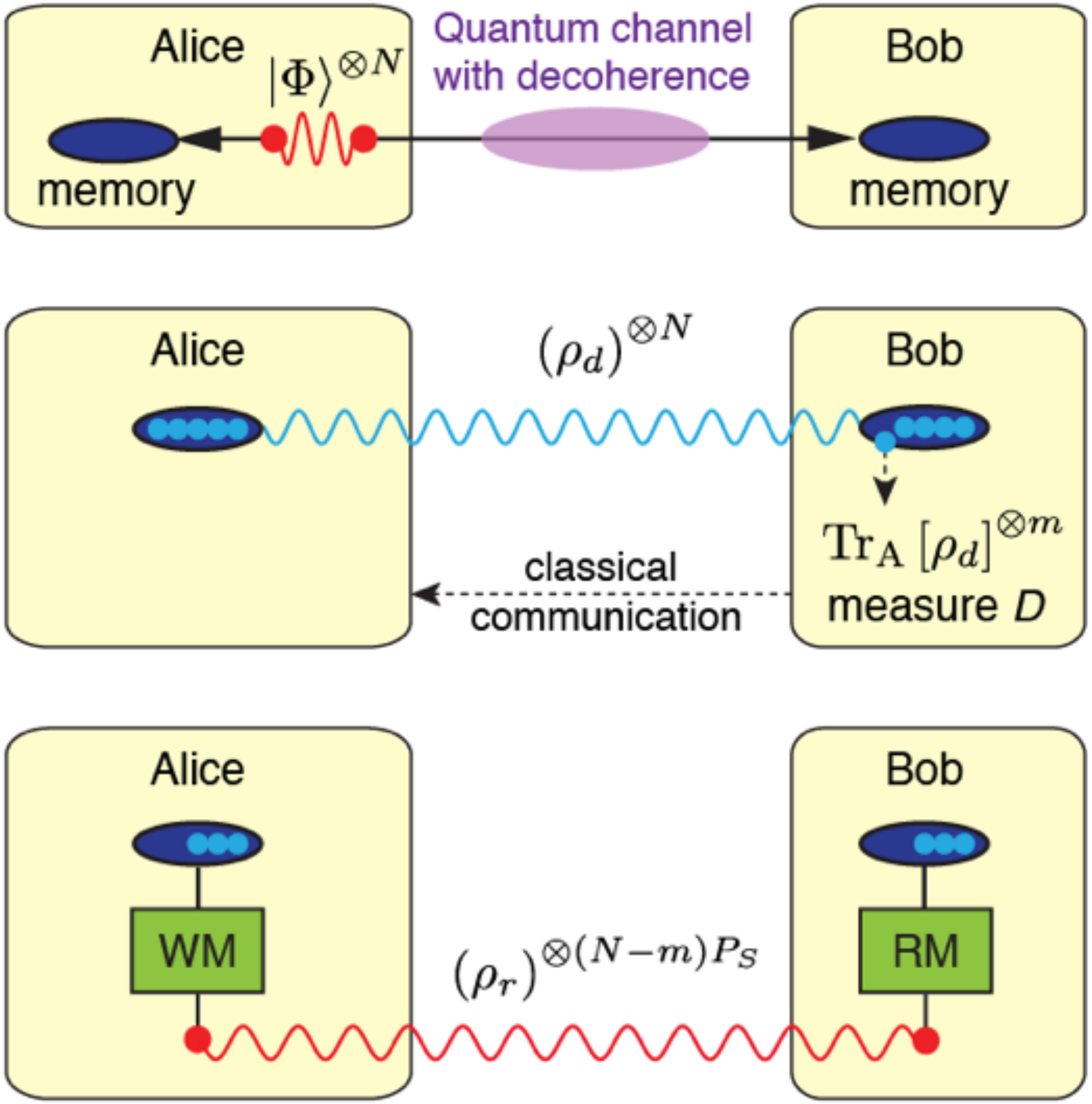
Lee et al., Nature Commun. 5, 4522 (2014)

Delayed-choice decoherence suppression



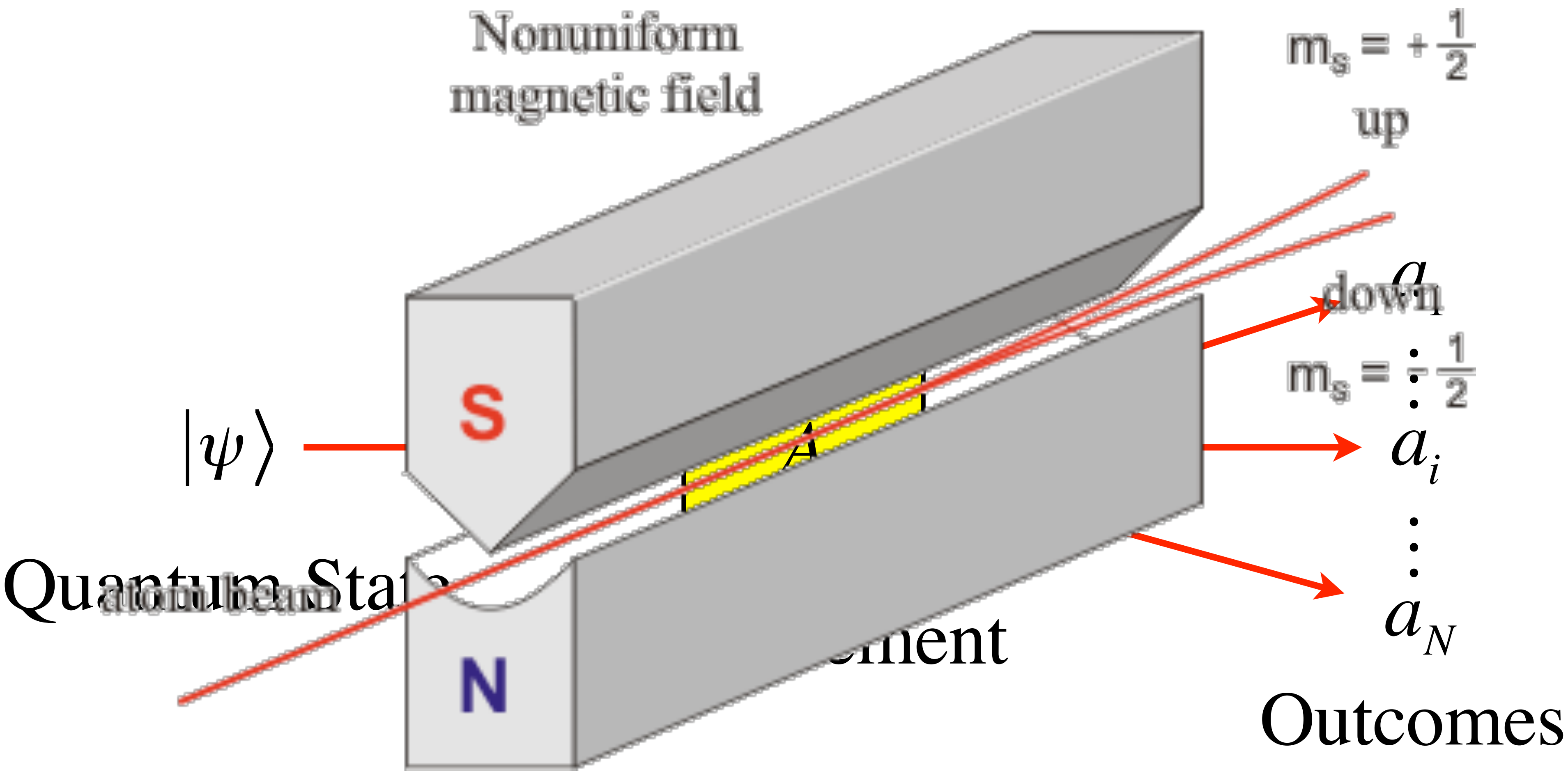
Lee et al., Nature Commun. 5, 4522 (2014)

Delayed-choice decoherence suppression for practical entanglement distribution over noisy channels



Lee et al., Nature Commun. 5, 4522 (2014)

Measurement in Quantum Physics



$|\psi\rangle$

Quantum State

A

ement

a_1

$\vdots$

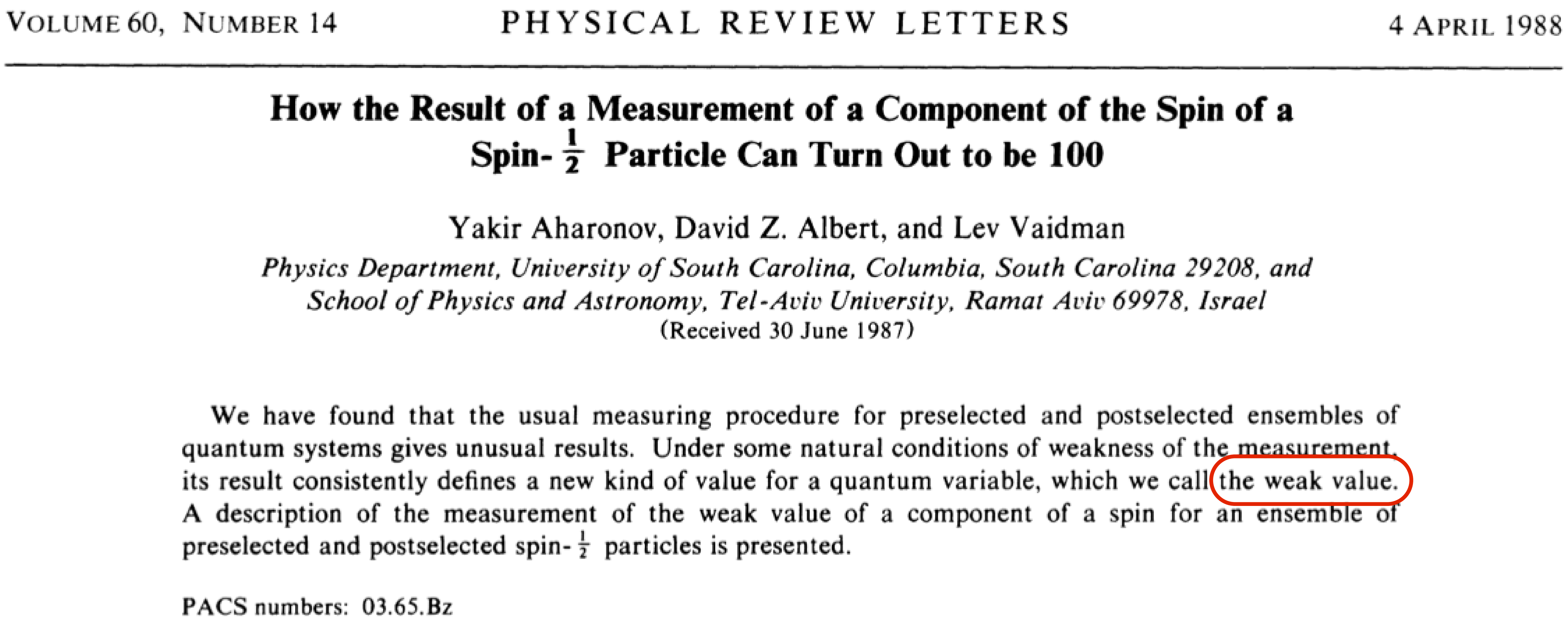
a_i

$\vdots$

a_N

Outcomes

“Weak Value” Measurement



VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

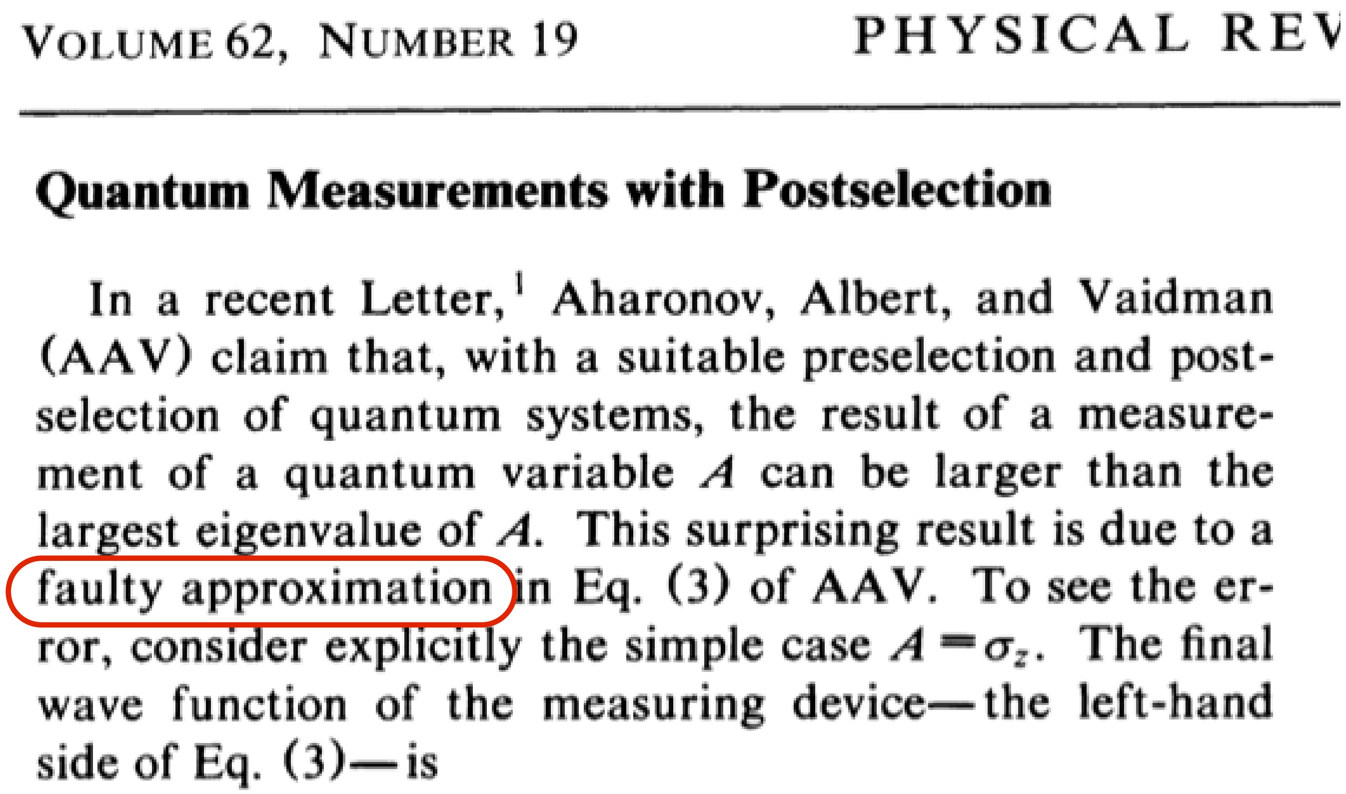
How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman
Physics Department, University of South Carolina, Columbia, South Carolina 29208, and School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel

(Received 30 June 1987)

We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin-$\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz



VOLUME 62, NUMBER 19 PHYSICAL REV

Quantum Measurements with Postselection

In a recent Letter,[1] Aharonov, Albert, and Vaidman (AAV) claim that, with a suitable preselection and postselection of quantum systems, the result of a measurement of a quantum variable A can be larger than the largest eigenvalue of A. This surprising result is due to a faulty approximation in Eq. (3) of AAV. To see the error, consider explicitly the simple case $A=\sigma_z$. The final wave function of the measuring device—the left-hand side of Eq. (3)—is

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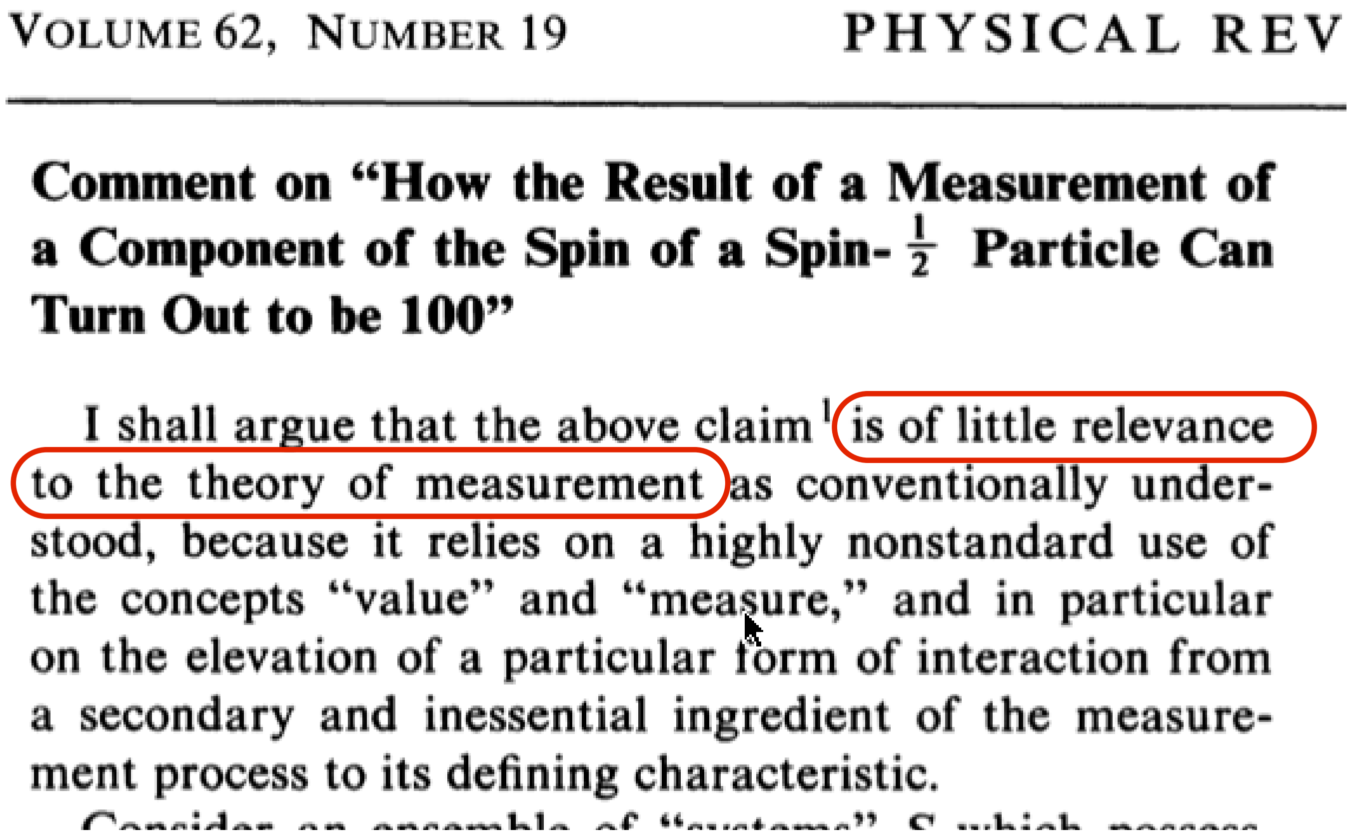

VOLUME 62, NUMBER 19 PHYSICAL REV

Comment on "How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100"

I shall argue that the above claim[1] is of little relevance to the theory of measurement as conventionally understood, because it relies on a highly nonstandard use of the concepts "value" and "measure," and in particular on the elevation of a particular form of interaction from a secondary and inessential ingredient of the measurement process to its defining characteristic.

Consider an ensemble of "systems" S which possess

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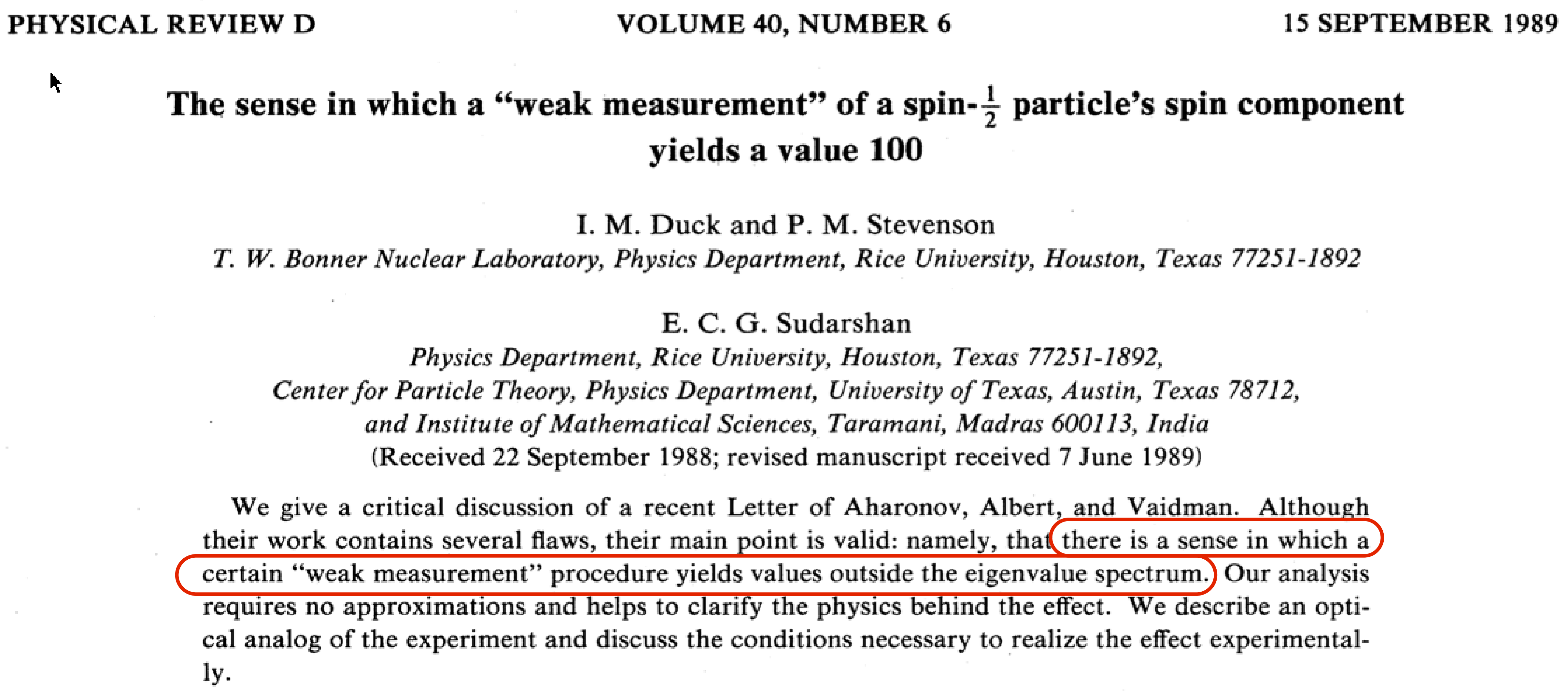
Aharonov and Vaidman Reply: In our recent Letter[1] we defined a *new* concept: *a weak value* of a quantum variable. We showed that a standard measuring procedure with weakened coupling, performed on an ensemble of both preselected *and postselected* systems, yields the weak value. The intuitive picture can be seen from our general approach[2] in which we consider two wave functions for a single system at a given time: the usual one evolving toward the future, and another evolving backward in time toward the past. Weak enough measurements do not disturb the above two wave functions and thus, the outcomes of such measurements should reflect properties of both states. The weakness of the interaction, therefore, is the essential requirement for the above measuring process. We claim that *for any measuring procedure of a physical variable the coupling can be made weak enough such that the effective value of the variable for a preselected and postselected ensemble will be its weak value.*



PHYSICAL REVIEW D VOLUME 40, NUMBER 6 15 SEPTEMBER 1989

The sense in which a "weak measurement" of a spin-$\frac{1}{2}$ particle's spin component yields a value 100

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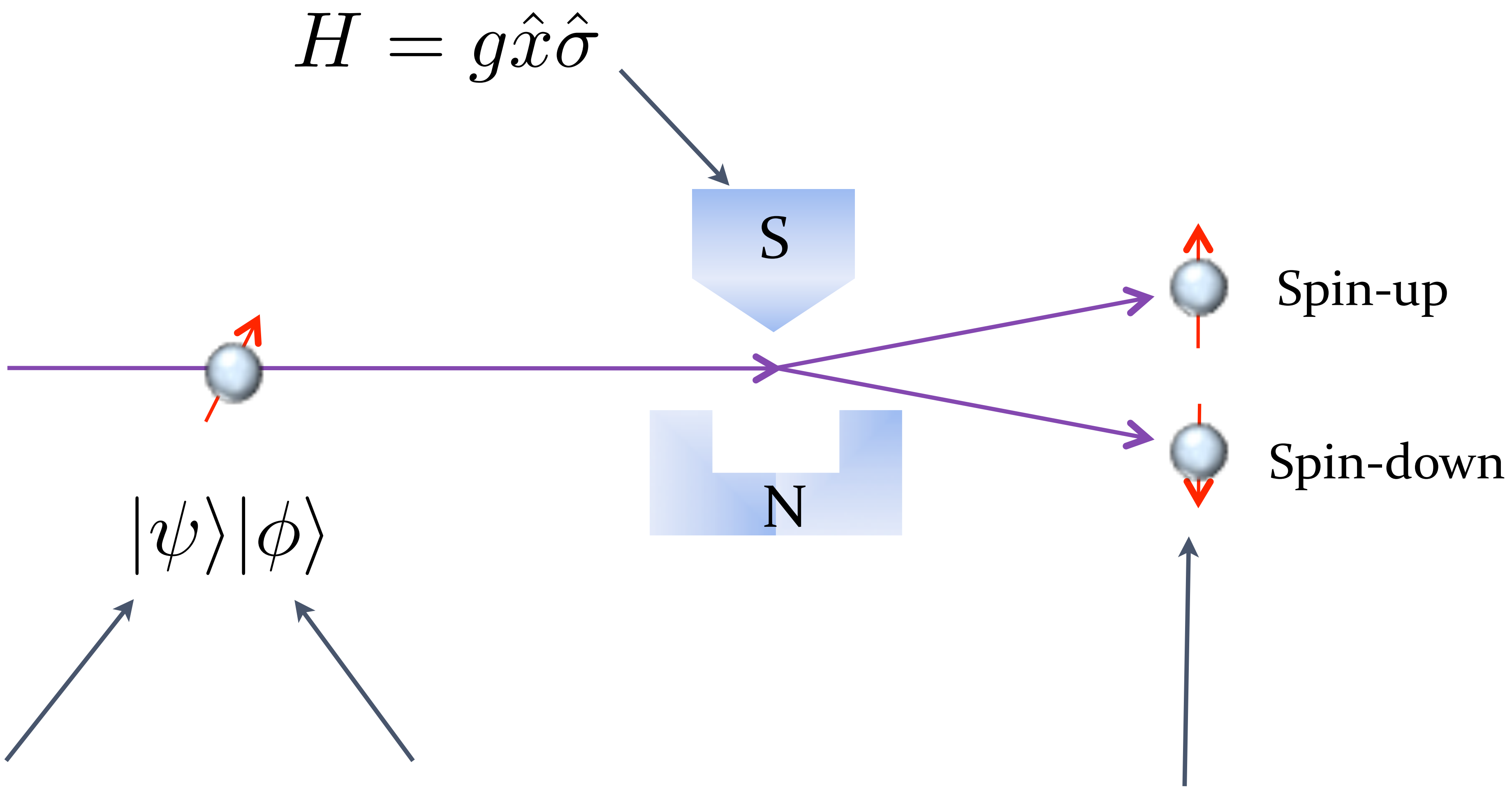
E. C. G. Sudarshan
Physics Department, Rice University, Houston, Texas 77251-1892,
Center for Particle Theory, Physics Department, University of Texas, Austin, Texas 78712,
and Institute of Mathematical Sciences, Taramani, Madras 600113, India

(Received 22 September 1988; revised manuscript received 7 June 1989)

We give a critical discussion of a recent Letter of Aharonov, Albert, and Vaidman. Although their work contains several flaws, their main point is valid: namely, that there is a sense in which a certain "weak measurement" procedure yields values outside the eigenvalue spectrum. Our analysis requires no approximations and helps to clarify the physics behind the effect. We describe an optical analog of the experiment and discuss the conditions necessary to realize the effect experimentally.

Spin measurement with a Stern-Gerlach device



$$H = g\hat{x}\hat{\sigma}$$

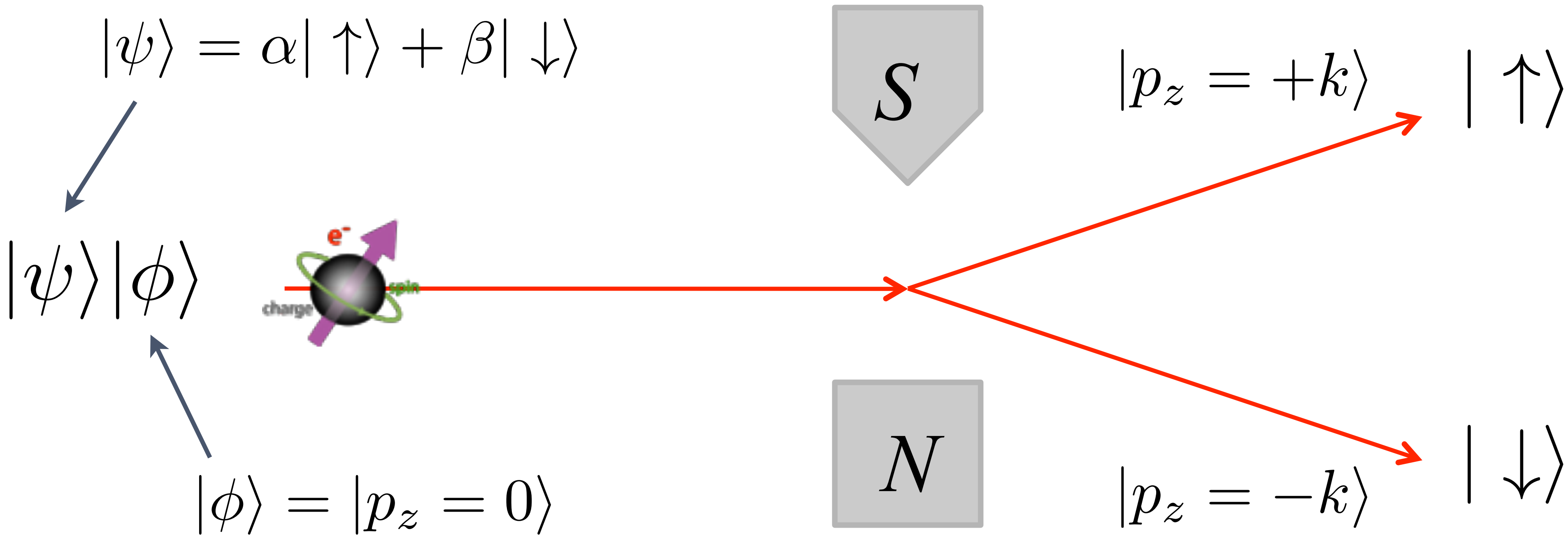
$$|\psi\rangle|\phi\rangle$$

Quantum System
(Spin state)

Pointer State
(Momentum)

Read the pointer state for measuring the spin value (+1/2 or -1/2)

The Pointer



$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

$|\psi\rangle|\phi\rangle$

$|\phi\rangle = |p_z = 0\rangle$

S

N

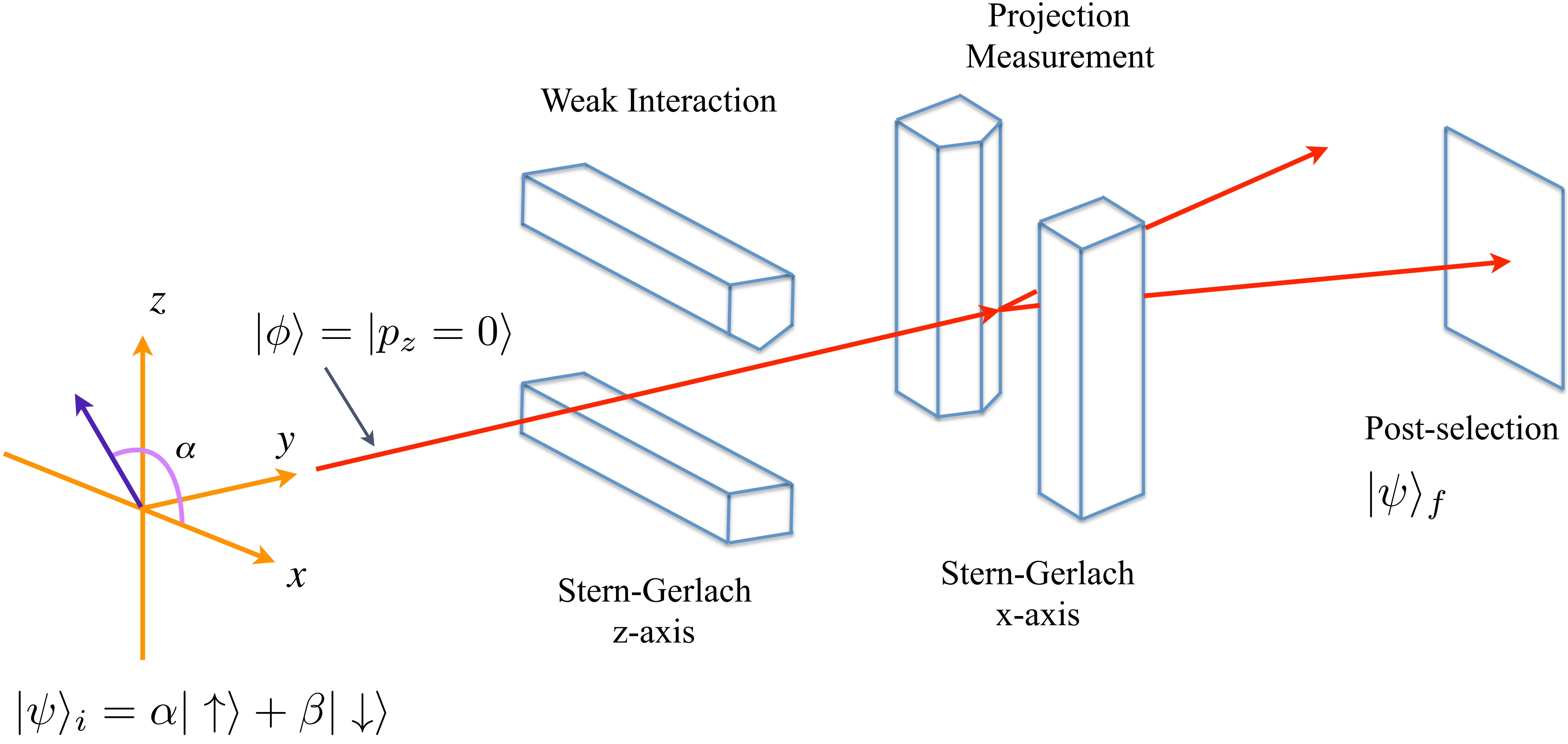
$|p_z = +k\rangle$ $|\uparrow\rangle$

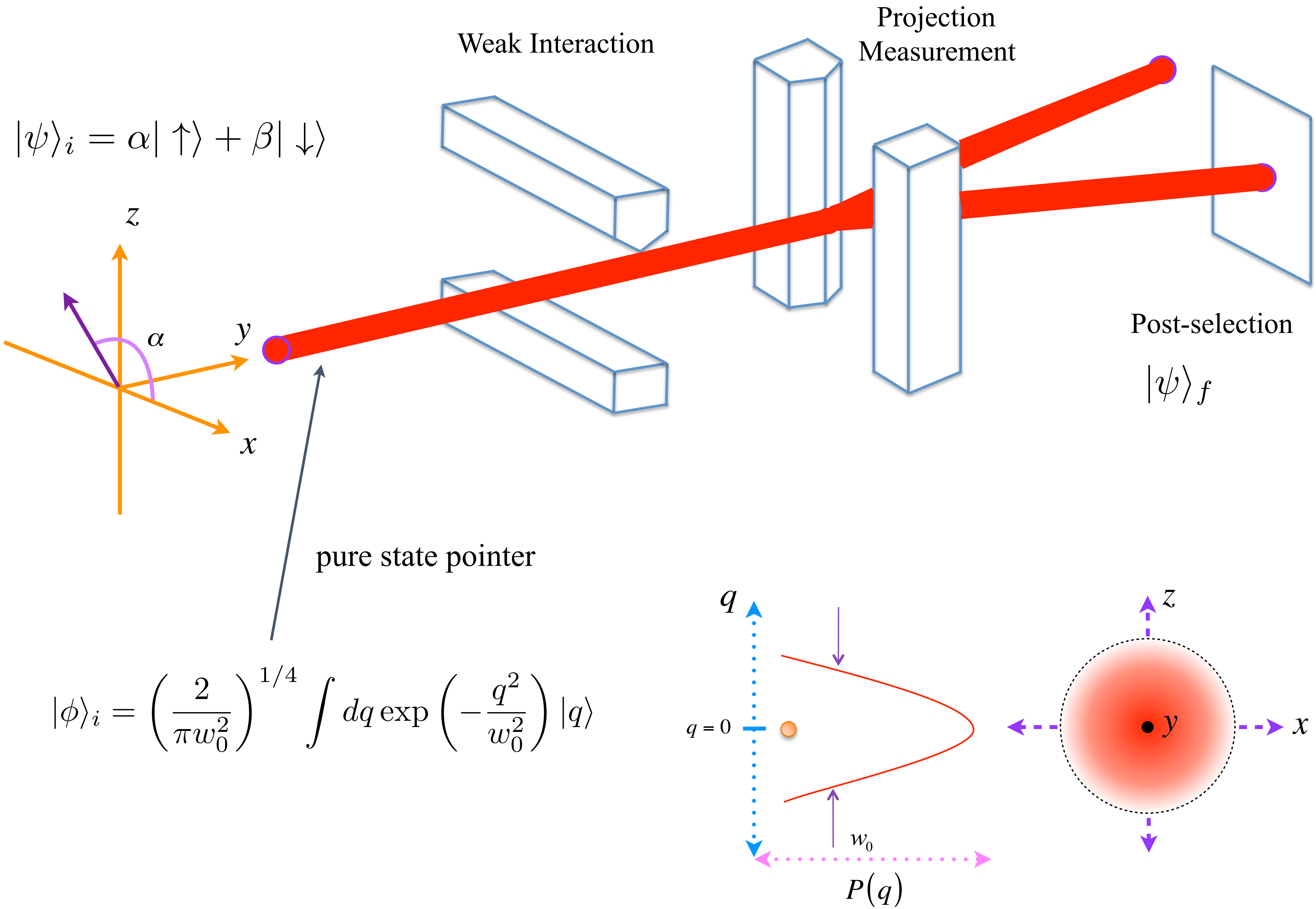
$|p_z = -k\rangle$ $|\downarrow\rangle$

- The momentum of the particle (the pointer state) is measured, not the spin state.
- The pointer state indicates the spin value.

Measuring Device = Pointer

AAV Weak Value Measurement




Weak Interaction
Projection Measurement
$|\psi\rangle_i = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$
z
α
y
x
Post-selection
$|\psi\rangle_f$
pure state pointer
$|\phi\rangle_i = \left(\frac{2}{\pi w_0^2}\right)^{1/4} \int dq \exp\left(-\frac{q^2}{w_0^2}\right)|q\rangle$
q
$q = 0$
w_0
$P(q)$
z
y
x

$$|\psi\rangle_i|\phi\rangle_i \xrightarrow[H = \delta(t-t_0)\hat{p}\hat{A}]{}$$

$$\boxed{\langle\psi_f|}\boxed{\exp(-i\hat{p}\hat{A}/\hbar)|\psi_{in}\rangle|\phi_{in}\rangle}$$

$$\simeq\left(\langle\psi_f|\psi_{in}\rangle - i\hat{p}\langle\psi_f|\hat{A}|\psi_{in}\rangle/\hbar+\ldots\right)|\phi_{in}\rangle$$

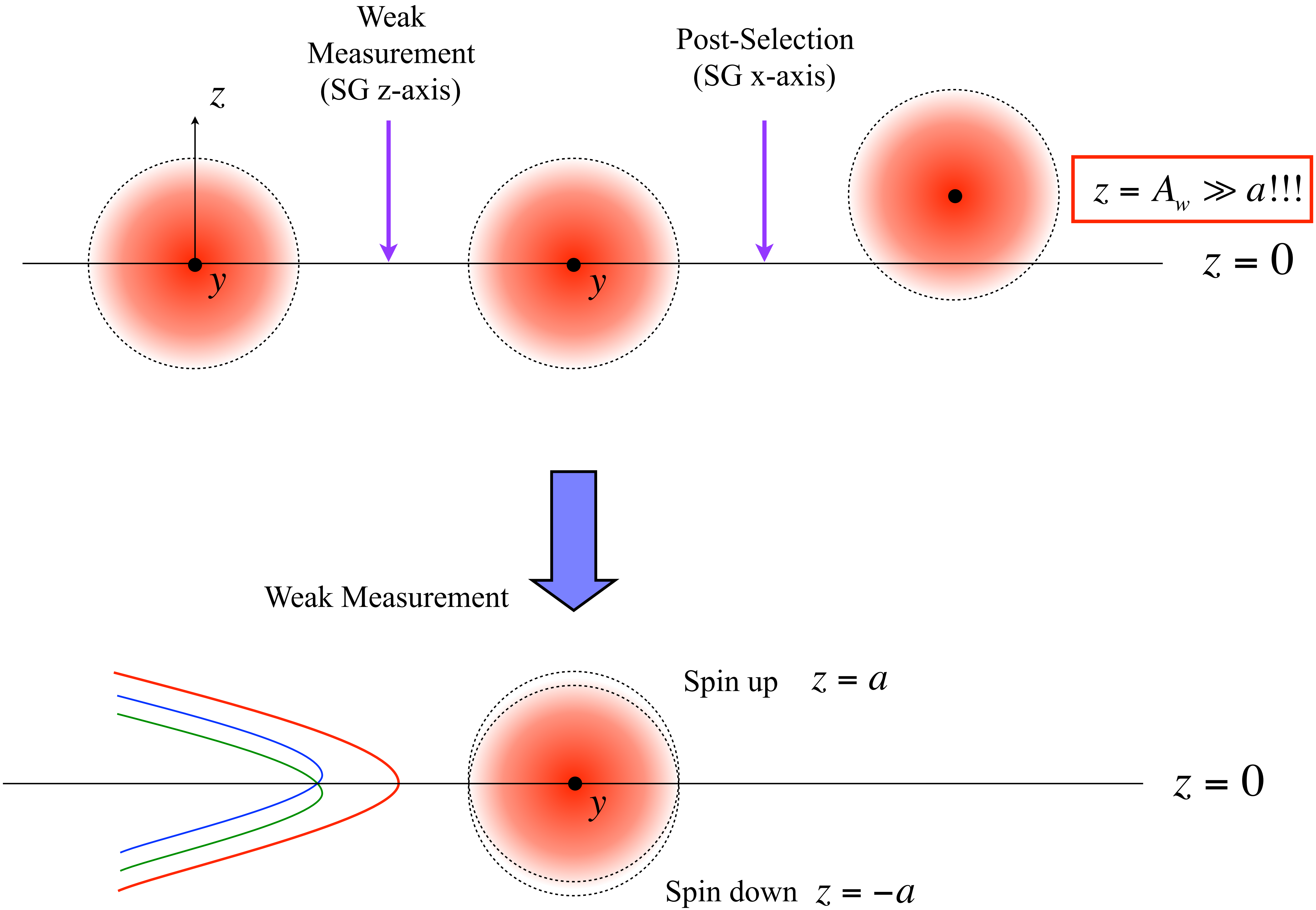
$$\simeq\left(\frac{w_0^2}{2\pi\hbar^2}\right)^{1/4}\langle\psi_f|\psi_{in}\rangle\int dp\exp\left(-\frac{w_0^2p^2+4iA_wp}{4\hbar^2}\right)|p\rangle$$

$$=\left(\frac{2}{\pi w_0^2}\right)^{1/4}\boxed{\langle\psi_f|\psi_{in}\rangle}\int\boxed{\exp\left[-\frac{(q-A_w)^2}{w_0^2}\right]|q\rangle}$$

1. If A_w >> by choosing $\langle\psi_f|\psi_{in}\rangle\approx 0$, the probability of the event becomes very small.

$$A_w \equiv \frac{\langle\psi_f|\hat{A}|\psi_{in}\rangle}{\langle\psi_f|\psi_{in}\rangle}$$

1. Pointer state is displaced by A_w
2. A_w can be well outside the eigenvalue spectrum if $\langle\psi_f|\psi_{in}\rangle\approx 0$.



Weak Measurement (SG z-axis)

Post-Selection (SG x-axis)

$z = A_w \gg a$!!!

$z = 0$

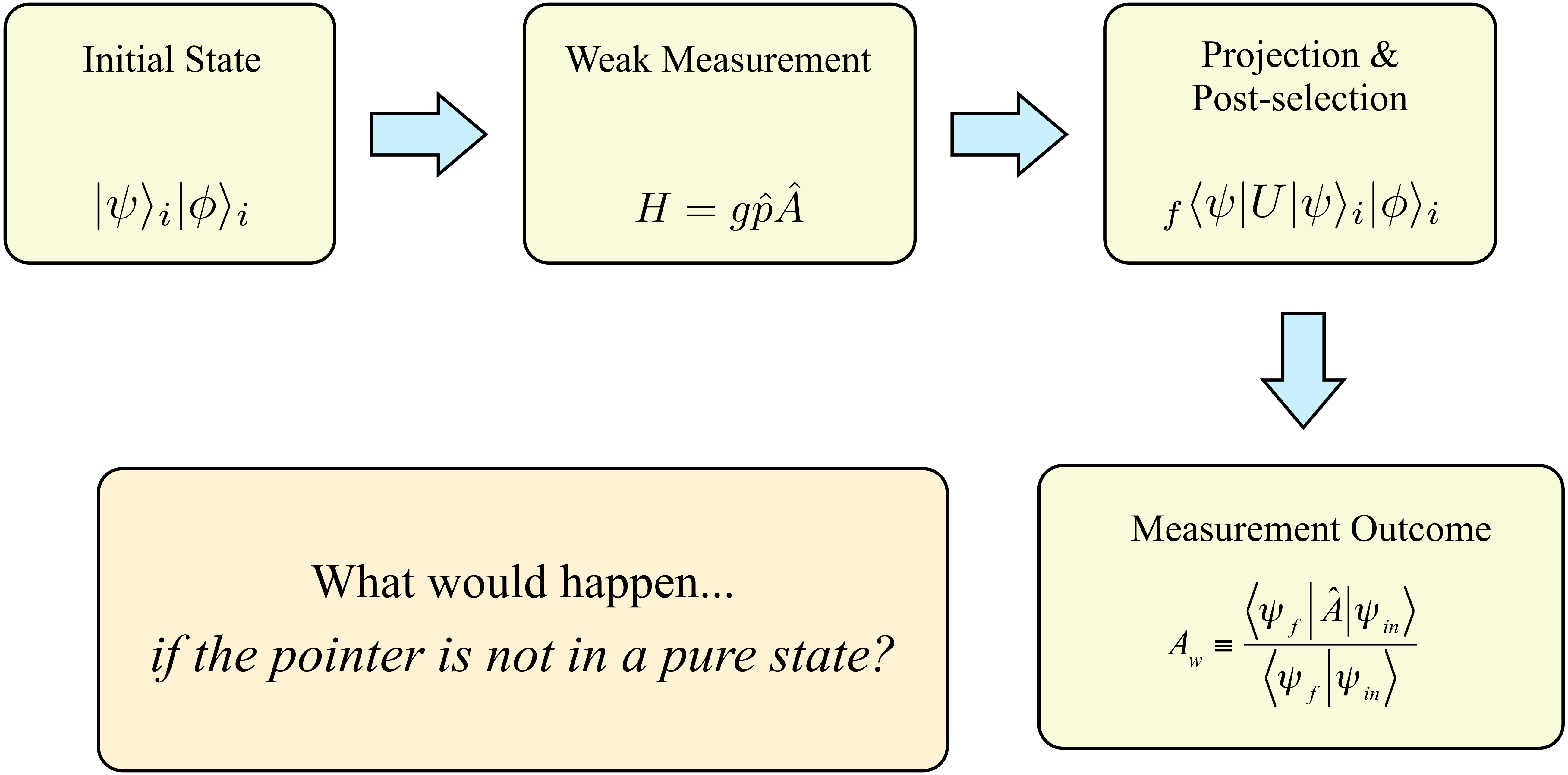
Weak Measurement

Spin up $z = a$

$z = 0$

Spin down $z = -a$

AAV Weak Value Measurement



Initial State

$$|\psi\rangle_i|\phi\rangle_i$$

Weak Measurement

$$H = g\hat{p}\hat{A}$$

Projection &
Post-selection

$${}_f\langle\psi|U|\psi\rangle_i|\phi\rangle_i$$

Measurement Outcome

$$A_w \equiv \frac{\langle\psi_f|\hat{A}|\psi_{in}\rangle}{\langle\psi_f|\psi_{in}\rangle}$$

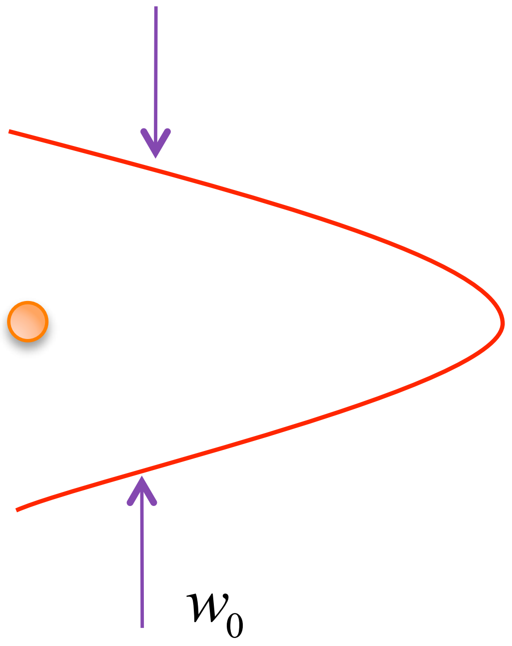
What would happen...

if the pointer is not in a pure state?

Cho *et al*, New J. Phys. 12, 023036 (2010)

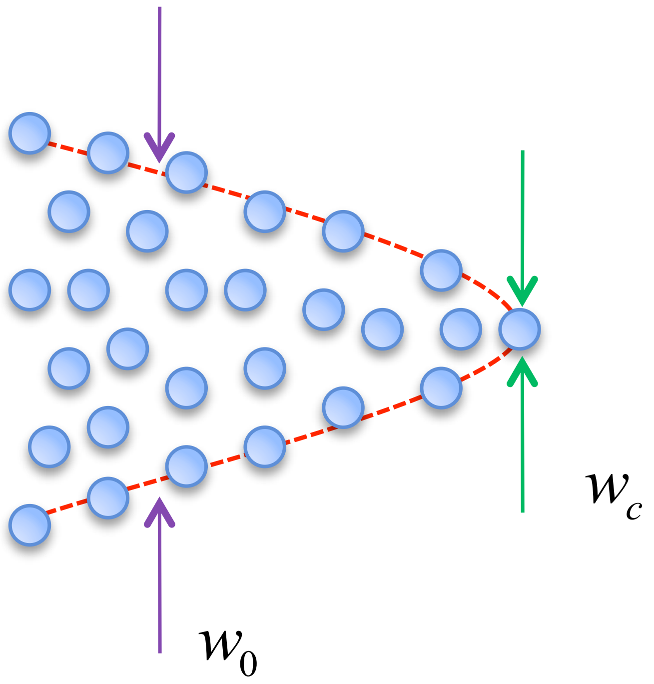
Incoherent pointer

Pure State Pointer



$$|\phi\rangle_i = \left(\frac{2}{\pi w_0^2}\right)^{1/4} \int dq \exp\left(-\frac{q^2}{w_0^2}\right) |q\rangle$$

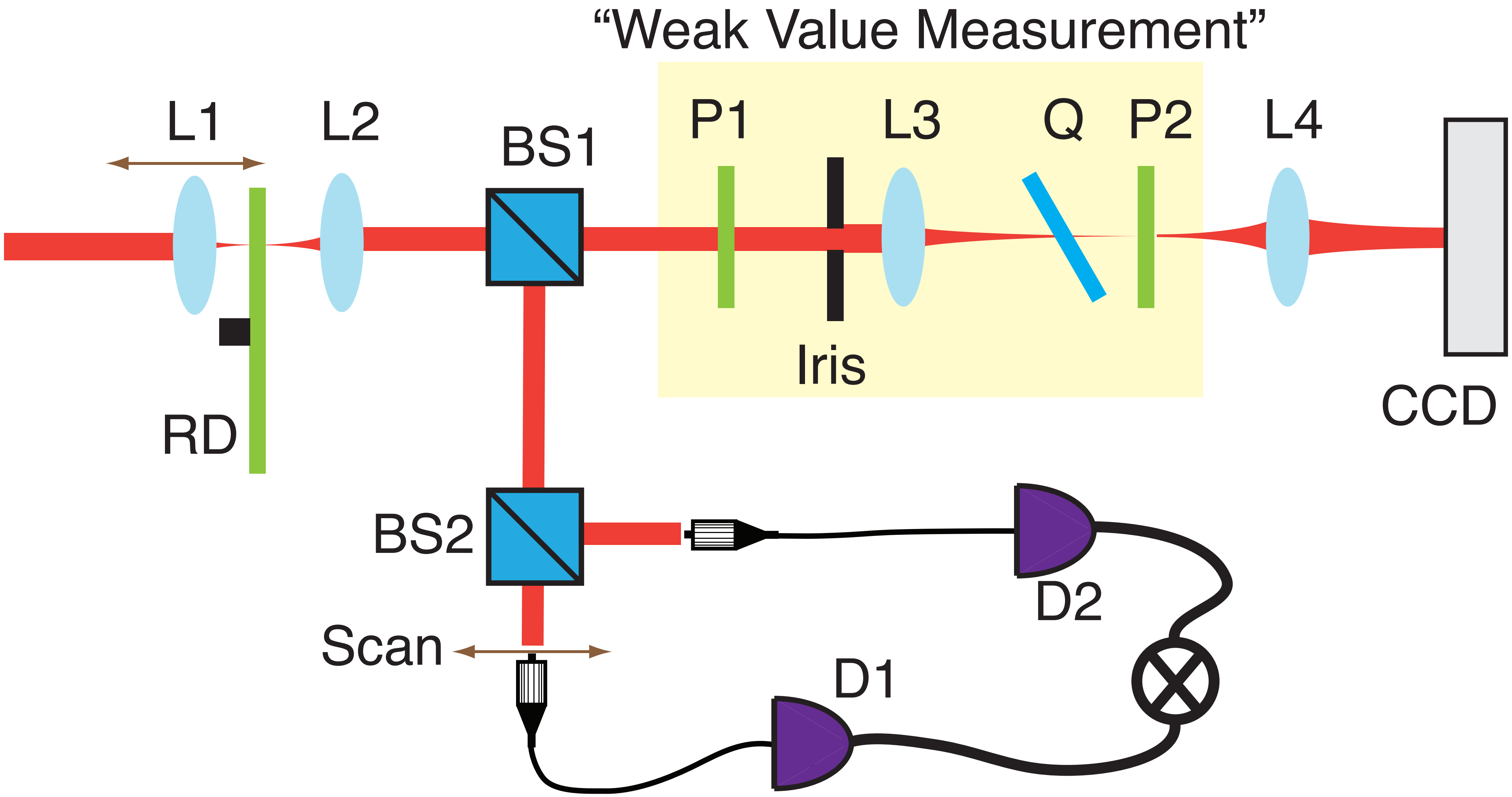
Mixed State Pointer



$$\rho_\phi = \frac{2}{\pi w_0 w_c}\int dq_0 dq' dq'' \exp\left[-\frac{q_0^2}{w_0^2}\right]\exp\left[-\frac{(q'-q_0)^2}{w_c^2}\right]\exp\left[-\frac{(q''-q_0)^2}{w_0^2}\right]|q'\rangle\langle q''|$$

Cho *et al*, New J. Phys. 12, 023036 (2010)

Optical "weak value" experiment



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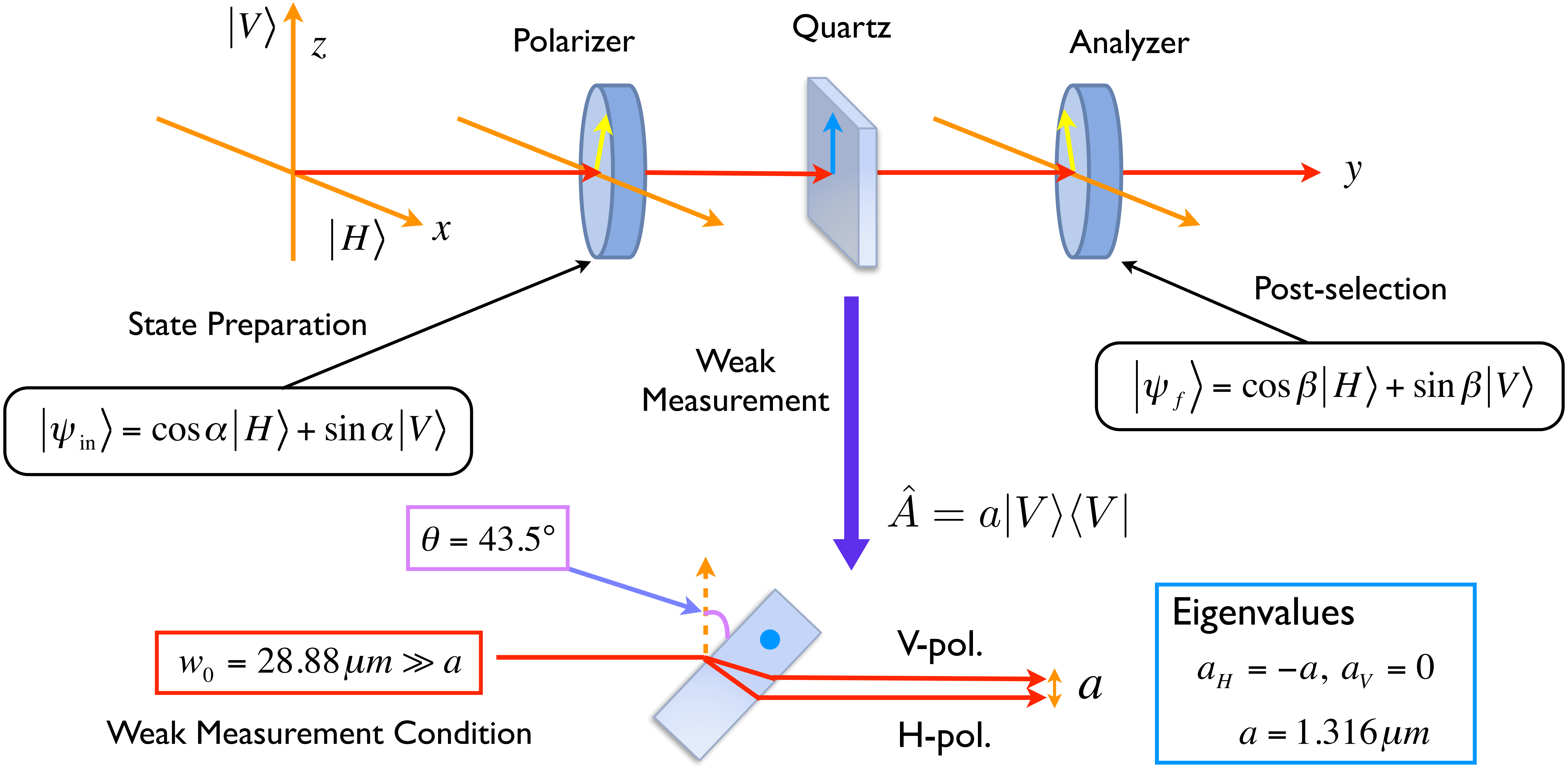
Optical “weak value” experiment



Polarizer

Quartz

Analyzer

State Preparation

$$|\psi_{\text{in}}\rangle = \cos\alpha|H\rangle + \sin\alpha|V\rangle$$

Weak Measurement

Post-selection

$$|\psi_f\rangle = \cos\beta|H\rangle + \sin\beta|V\rangle$$

$$\hat{A} = a|V\rangle\langle V|$$

$\theta = 43.5°$

$w_0 = 28.88\mu m \gg a$

Weak Measurement Condition

V-pol.

H-pol.

a

Eigenvalues

$a_H = -a,\ a_V = 0$

$a = 1.316\mu m$

Cho *et al*, New J. Phys. 12, 023036 (2010)