Primordial non-Gaussian signatures in CMB polarization

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Outline of talk

- Briefly about CMB Polarization
- Primordial non-Gaussianity and PDF of CMB polarization fields.
- Geometry and topology of CMB polarization field

CMB polarization

In the rest frame of a free electron

Quadrupole anisotropy of incoming photons

$+$

Thomson scattering off the electron

$\downarrow$

Linearly polarised scattered light

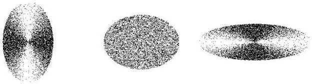
CMB polarization

Quadrupole anisotropy from density fluctuations

- Velocity gradients in the plasma due to photon pressure and gravitational attraction.
- Implies electron sees incident quadrupole anisotropy of incident radiation in its rest frame.

Quadrupole anisotropy from gravitational waves

- Presence of gravitational waves gives rise to quadrupole anisotropy.



http://quiet.uchicago.edu/capmap/A.htm

Polarization fields

Stokes parameters to E and B modes:

$$(Q, U) \quad \longleftrightarrow \quad (E, B)$$

Total polarization Intensity:

$$I \quad \equiv \quad \sqrt{Q^2 + U^2}$$

$$\tilde{I} \quad \equiv \quad I - \langle I \rangle$$

Density fluctuations produce only E modes.

Gravitational waves produce both E and B modes.

Primordial non-Gaussianity

- From primordial gravitational potential:

$$\Phi(\mathbf{x}) \;=\; \Phi^{G}(\mathbf{x}) + f_{\rm NL}\left((\Phi^{G}(\mathbf{x}))^2 - \langle(\Phi^{G})^2\rangle\right)$$

- to CMB fields

$$\Phi \quad \longrightarrow \quad \Delta T,\; E$$

$$a^{i}_{\ell m} \;=\; \int dr\, r^2\, \Phi^{\rm G}_{\ell m}(r)\, \Delta^{i}_{\ell}(r) + \int dr\, r^2\, \Phi^{\rm NG}_{\ell m}(r)\, \Delta^{i}_{\ell}(r)$$

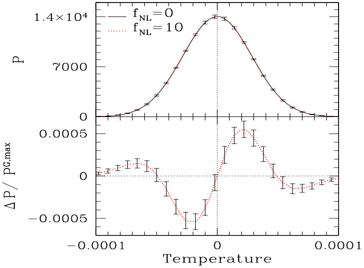
- Statistical nature of Φ is inherited by $\Delta T,\; E,\; I$.

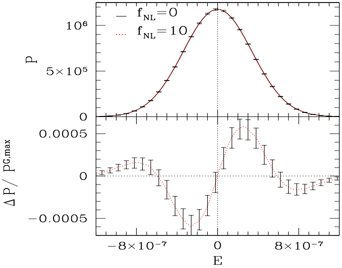
Used simulations of polarization with non-Gaussian initial conditions. **Elsner & Wandelt 2009.**

PDF for non-Gaussian fields: E mode

Consider generic random field y with local type non-Gaussian expression. Then

$$\mathrm{P}(y) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-y^2/2\sigma^2} \left(1 - f_{\mathrm{NL}}\sigma\, \frac{y(y^2 - 2\sigma^2)}{\sigma^3}\right)$$

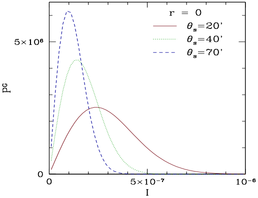




PDF for non-Gaussian fields: Polarization Intensity

If y and z Gaussian, then PDF of $r \equiv \sqrt{y^2 + z^2}$ is given by

$$\mathrm{P}^{\mathrm{G}}(r) \ = \ \frac{r}{\sigma^2}\, e^{-\frac{r^2}{2\sigma^2}}, \qquad r \geq 0$$



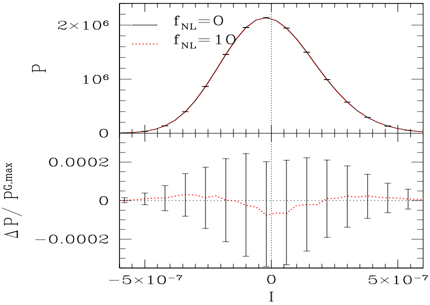
For local type non-Gaussian y and z the PDF is

$$\mathrm{P}(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \left(1 + \frac{f_{\mathrm{NL}}^2 \sigma^2}{16\sigma^{10}} (5r^6\,\sigma^4 - 54r^4\sigma^6 + 96r^2\sigma^8)\right).$$

Lowest order non-Gaussian correction is $(f_{\mathrm{NL}}\sigma)^2$. Hence small deviation and larger statistical fluctuations.

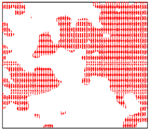
PDF for non-Gaussian fields: Polarization Intensity

Gaussian smoothing of I makes its PDF shape look close to Gaussian.

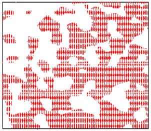


Excursion sets – geometry and topology

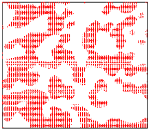
$$\nu_t = 0$$



Temperature



E mode



Intensity

Minkowski Functionals

- V_0 = Area fraction
- V_1 = contour length
- V_2 = genus

Betti numbers

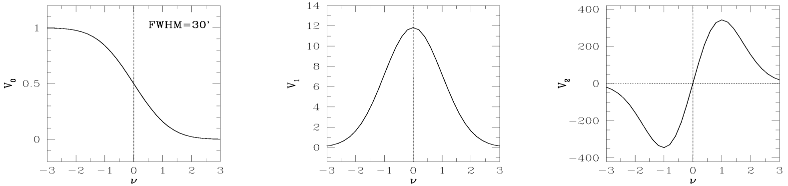
- β_0 = no of connected regions
- β_1 = no of holes

For Gaussian field

Tomita (1986)

Minkowski Functionals

$$V_0 \;=\; \tfrac{1}{2}\,\mathrm{erfc}\,(\nu_t/\sqrt{2}) \qquad V_1 \;=\; A_1\; e^{-\nu_t^2/2} \qquad V_2 \;=\; A_2\;\nu\; e^{-\nu_t^2/2}$$



Amplitudes $A_1,\; A_2$ encode the cosmological information.

Betti numbers

Analytic expressions not known.

Quantifying non-Gaussian deviations

- Any deviation from the known Gaussian shapes of the Minkowski Functionals and Betti numbers is a sign of non-Gaussian deviation.
- Quantify non-Gaussian deviations as

$$\Delta V_k \equiv V_k^{\rm NG} - V_k^{\rm G}$$

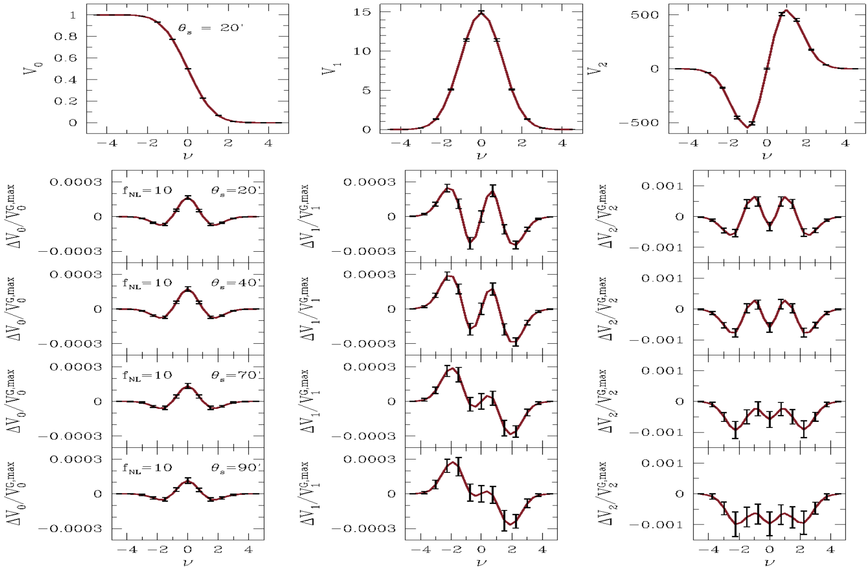
.

- Different physical origins lead to distinct shapes of ΔV_k. Hence can be distinguished from each other.

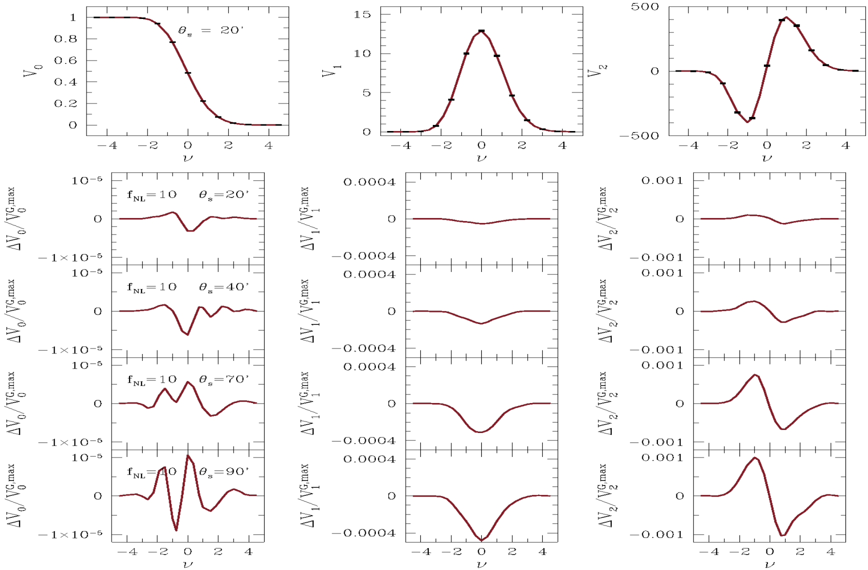
Extensively used to search for non-Gaussian deviations in temperature fluctuations.

Not yet used for non-Gaussian deviations in polarization data.

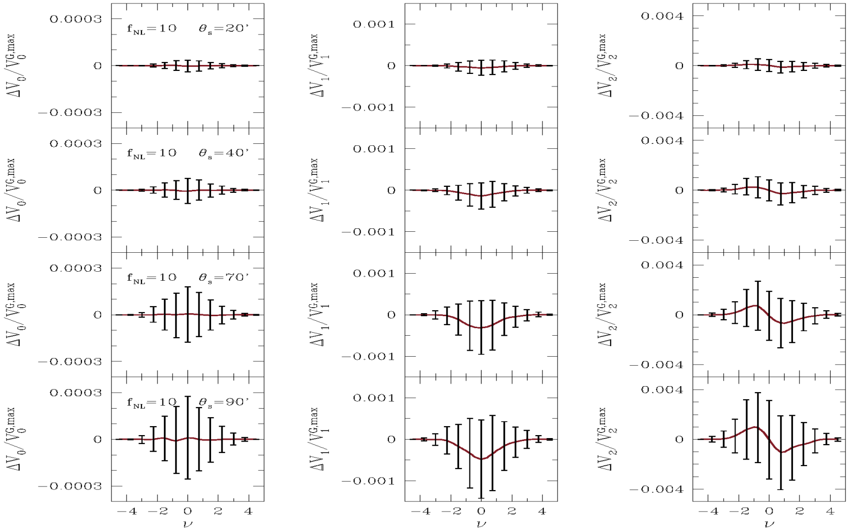
Minkowski functionals - E mode



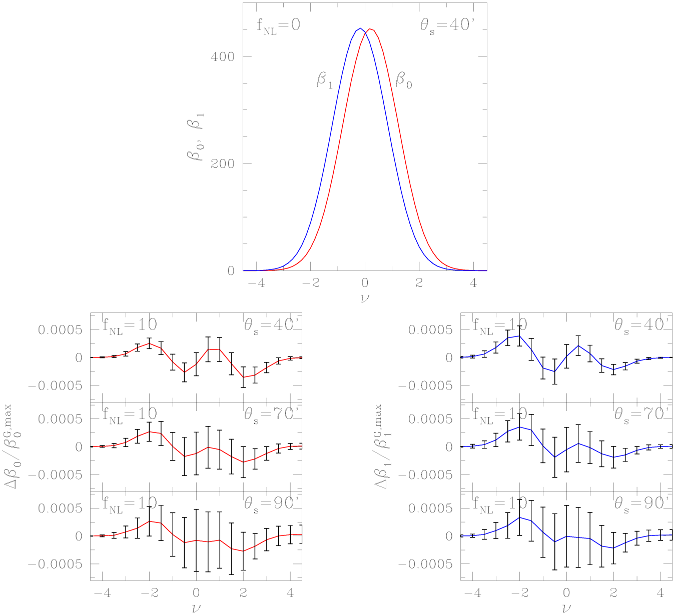
Minkowski functionals - Polarization intensity



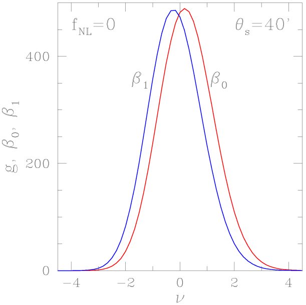
Minkowski functionals - Polarization intensity

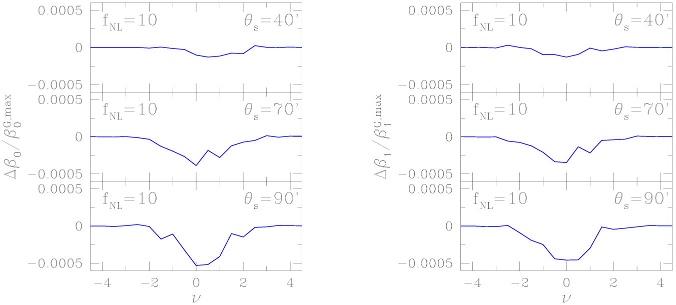


Betti numbers - E mode

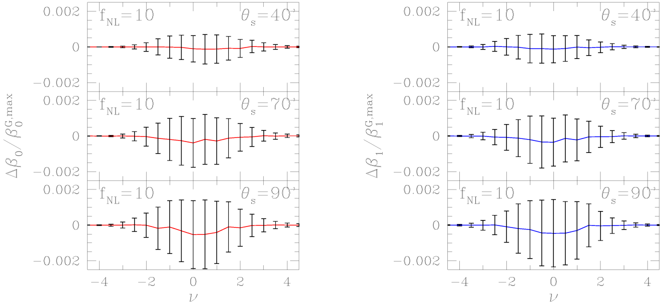


Betti numbers - Polarization intensity





Betti numbers - Polarization intensity



Statistical sensitivity of T, E and $\tilde{I}$

$$A = \Delta\nu \sum_{i=1}^{M} \left(|\Delta O(i)|/O^{\rm G,max}\right)/\sigma_s(i)$$

Field	θ_s	A for V_1	A for V_2	A for β_0	A for β_1
$\Delta T/T$	$40'$	27.3	22.8	3.8	3.8
	$70'$	21.3	14.5	2.3	2.3
	$90'$	18.4	26.0	1.8	1.8
E	$40'$	34.8	26.0	5.6	5.5
	$70'$	19.4	19.0	2.9	2.8
	$90'$	14.0	17.0	1.8	1.8
$\tilde{I}$	$40'$	1.2	1.2	0.3	0.3
	$70'$	1.8	1.6	0.5	0.5
	$90'$	1.7	1.5	0.4	0.5

Summary

- Analyzed statistical properties of CMB polarization for local type primordial non-Gaussianity.
- PDF for polarization intensity has non-Gaussian corrections at order $(f_{\rm NL}\sigma)^2$.
- Calculated non-Gaussian deviations for Minkowski Functional and Betti numbers for E mode and Intensity.
- The statistical significance of non-Gaussian deviations of E mode is comparable to temperature fluctuations. Hence equally useful in constraining non-Gaussianity.
- Intensity has smaller deviations and larger statistical fluctuations. Hence not as useful.