Test of Consistency between Planck and WMAP

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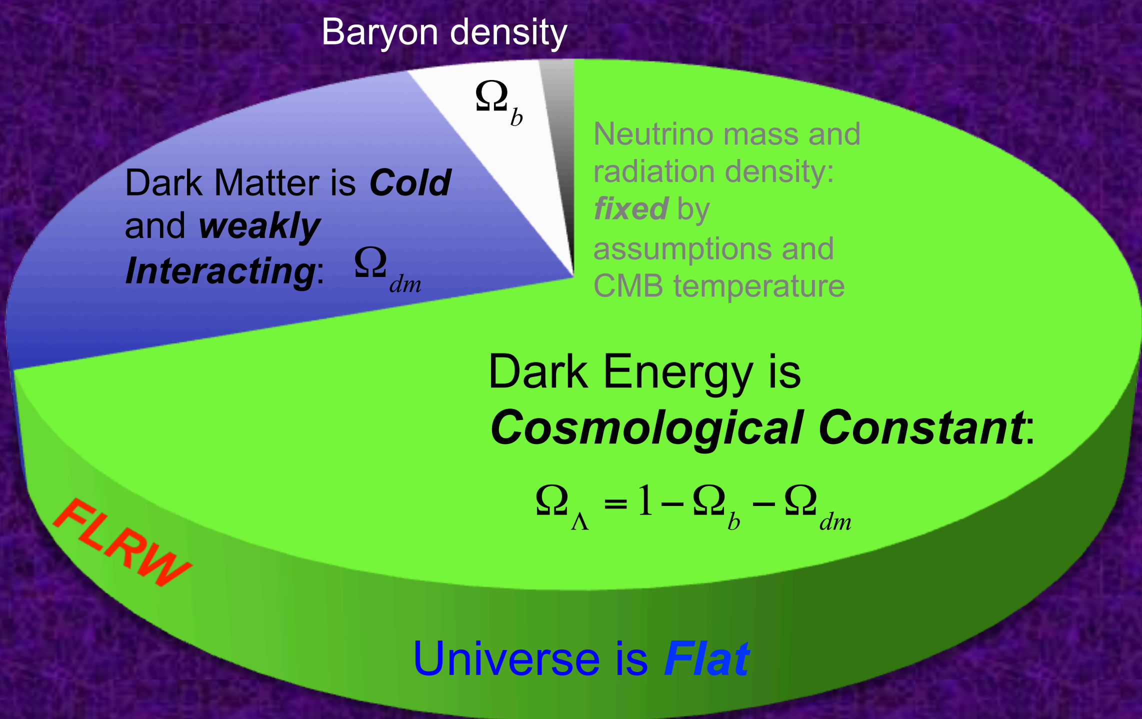
Standard Model of Cosmology

Using measurements and statistical techniques to place sharp constraints on parameters of the standard cosmological models.



Initial Conditions:
Form of the Primordial Spectrum is ***Power-law***

$$n_s, A_s$$

Epoch of reionization

$$\tau$$

Hubble Parameter and the Rate of Expansion

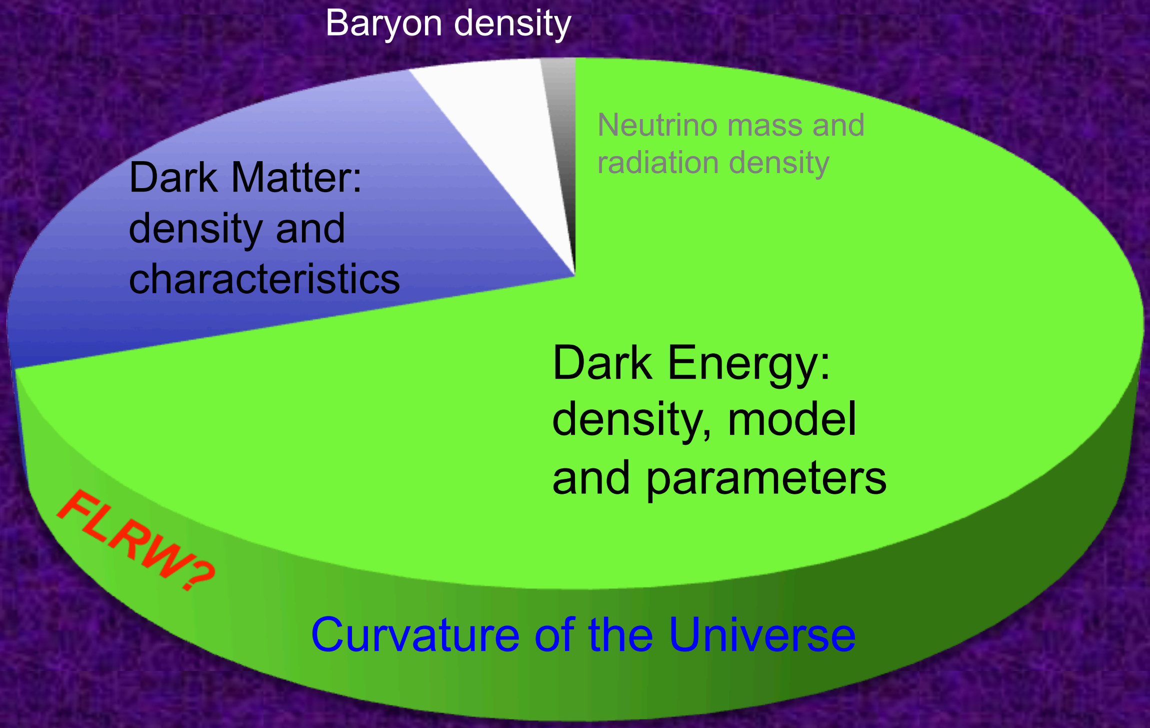
$$H_0$$

Beyond the Standard Model…

Reconstruction & Falsification

Reconstruction: Understanding the behavior

Falsification: Testing the Consistency



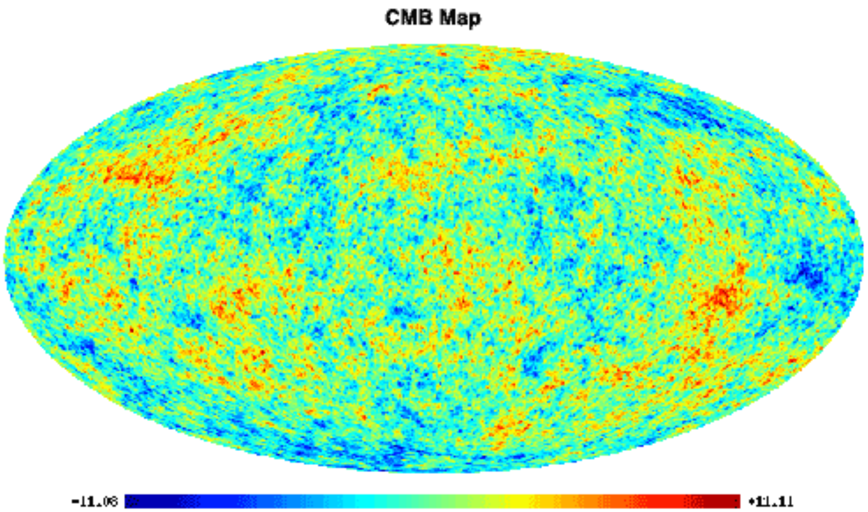
Initial Conditions:
Form of the Primordial Spectrum and Model of Inflation and its Parameters

Epoch of reionization

Hubble Parameter and the Rate of Expansion

Statistics of CMB

CMB Anisotropy Sky map => Spherical Harmonic decomposition



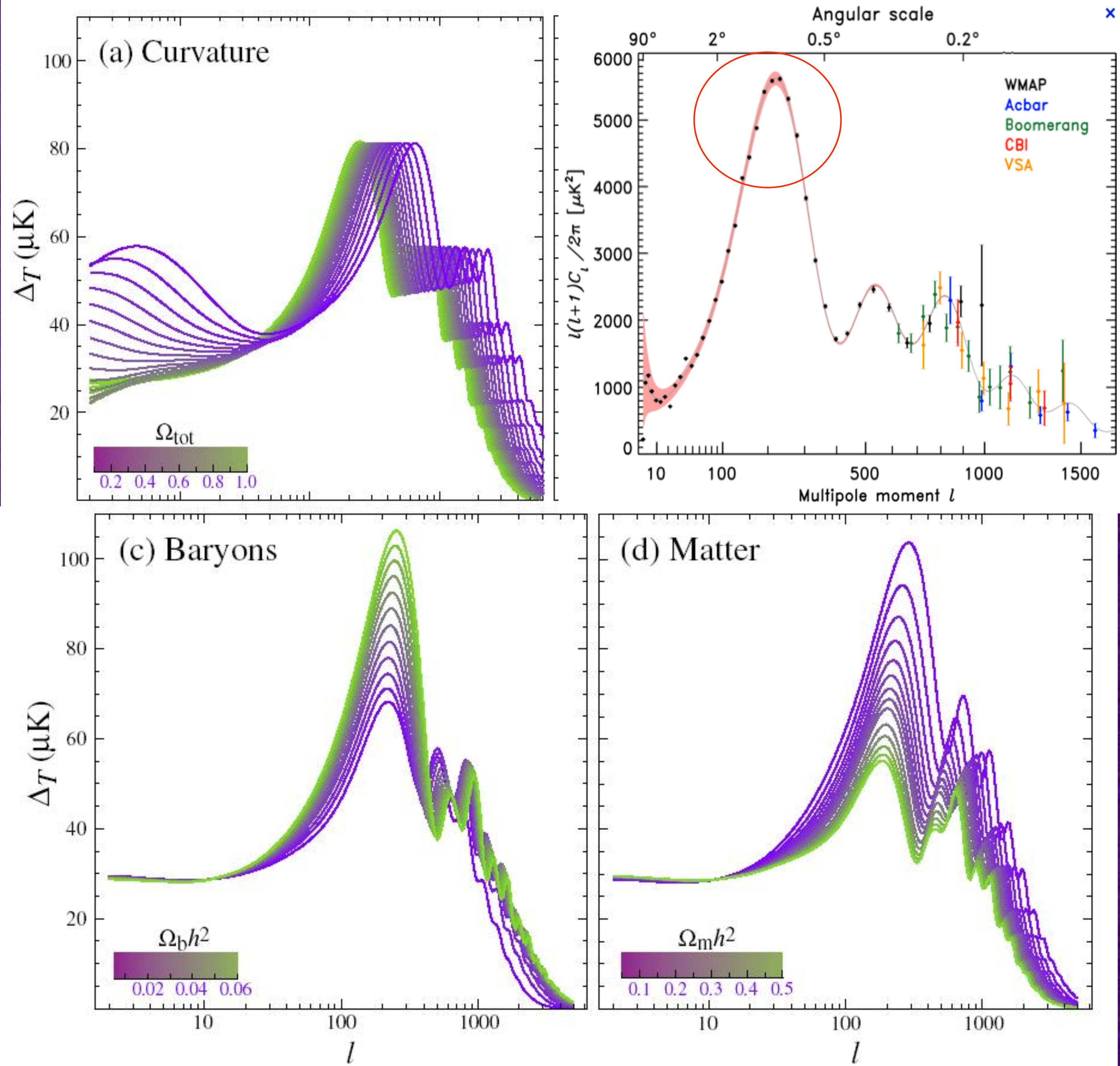
$$\Delta T(\theta,\phi) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi)$$

$$\boxed{\left\langle a_{lm}\, a^{*}_{l'm'} \right\rangle = C_l\ \delta_{ll'} \delta_{mm'}}$$

Gaussian Random field => Completely specified by

angular power spectrum $l(l+1)C_l$:

Power in fluctuations on angular scales of $\sim \pi/l$



Sensitivity of the CMB acoustic temperature spectrum to four fundamental cosmological parameters.

Total density

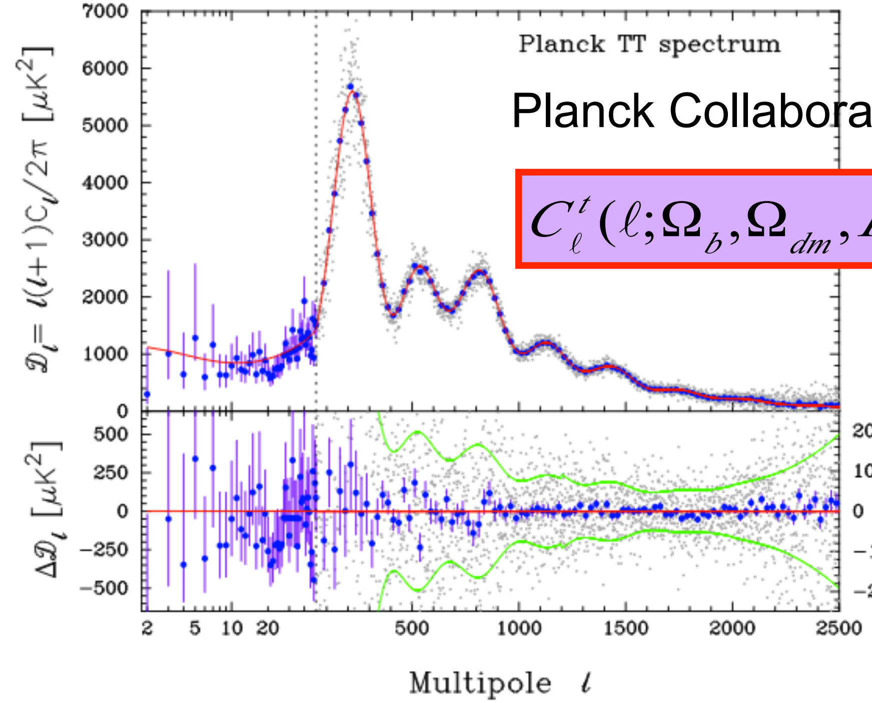
Dark Energy

Baryon density and

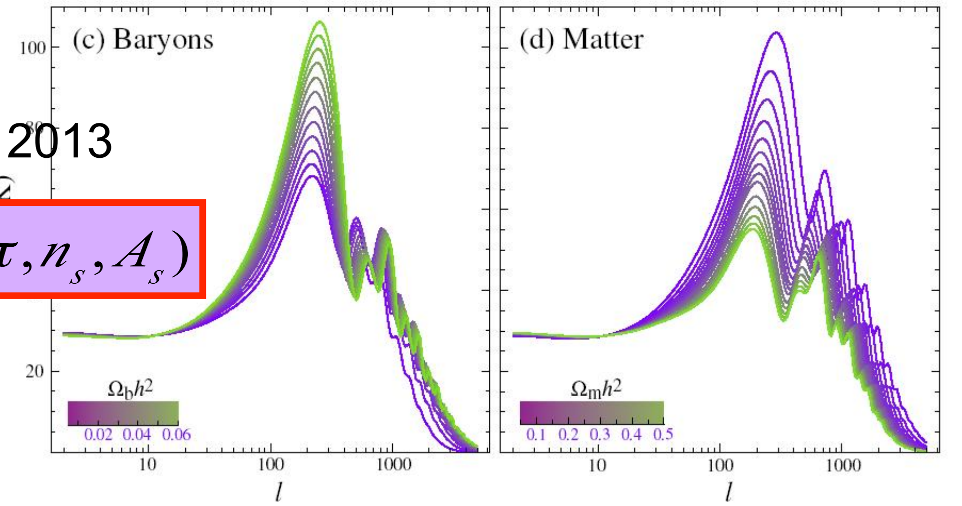
Matter density.

From Hu & Dodelson, 2002

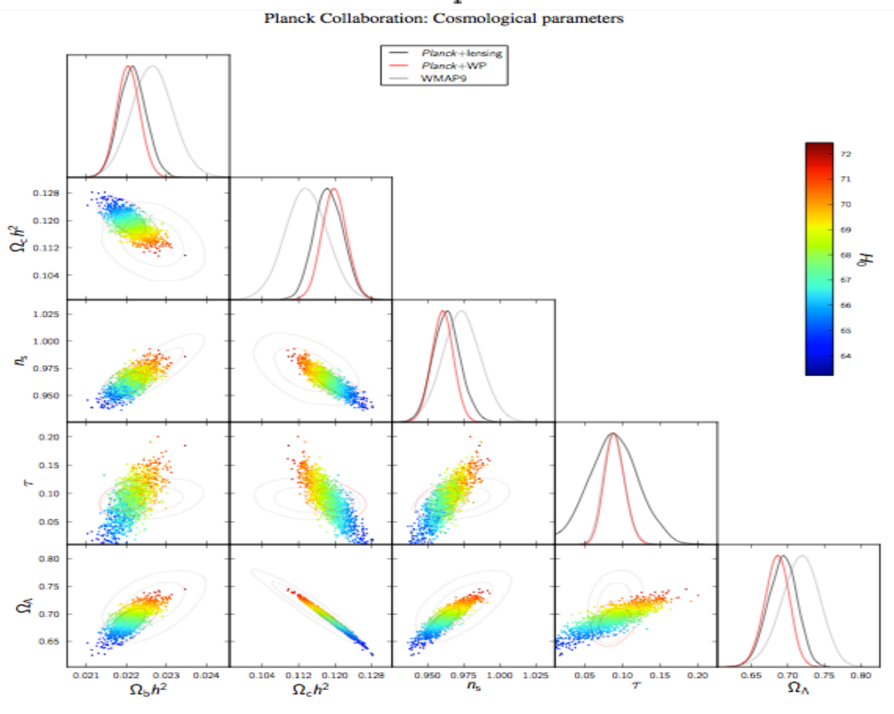
Planck Collaboration 2013

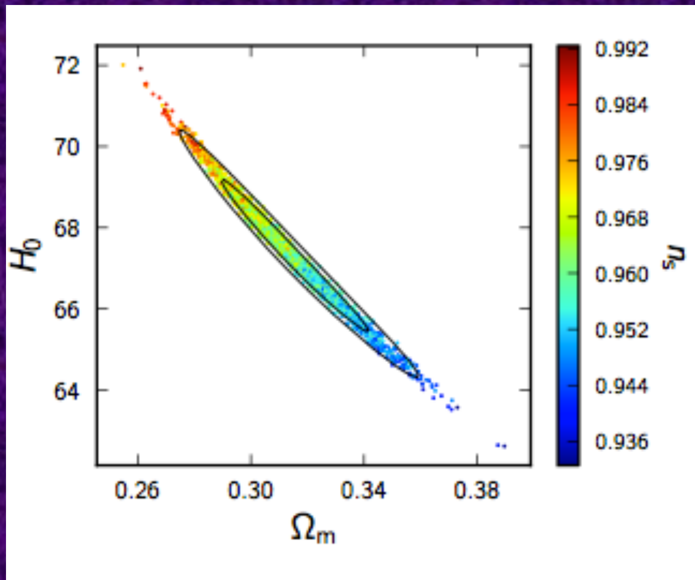


$$C_\ell^t(\ell;\Omega_b,\Omega_{dm},H_0,\tau,n_s,A_s)$$



$$\chi^2 = \sum_{\ell=2}^{N}(C_\ell^t - C_\ell^e)^T Cov^{-1}(C_\ell^t - C_\ell^e)$$





Planck temperature angular power spectrum,
Planck XV, Planck XVI.

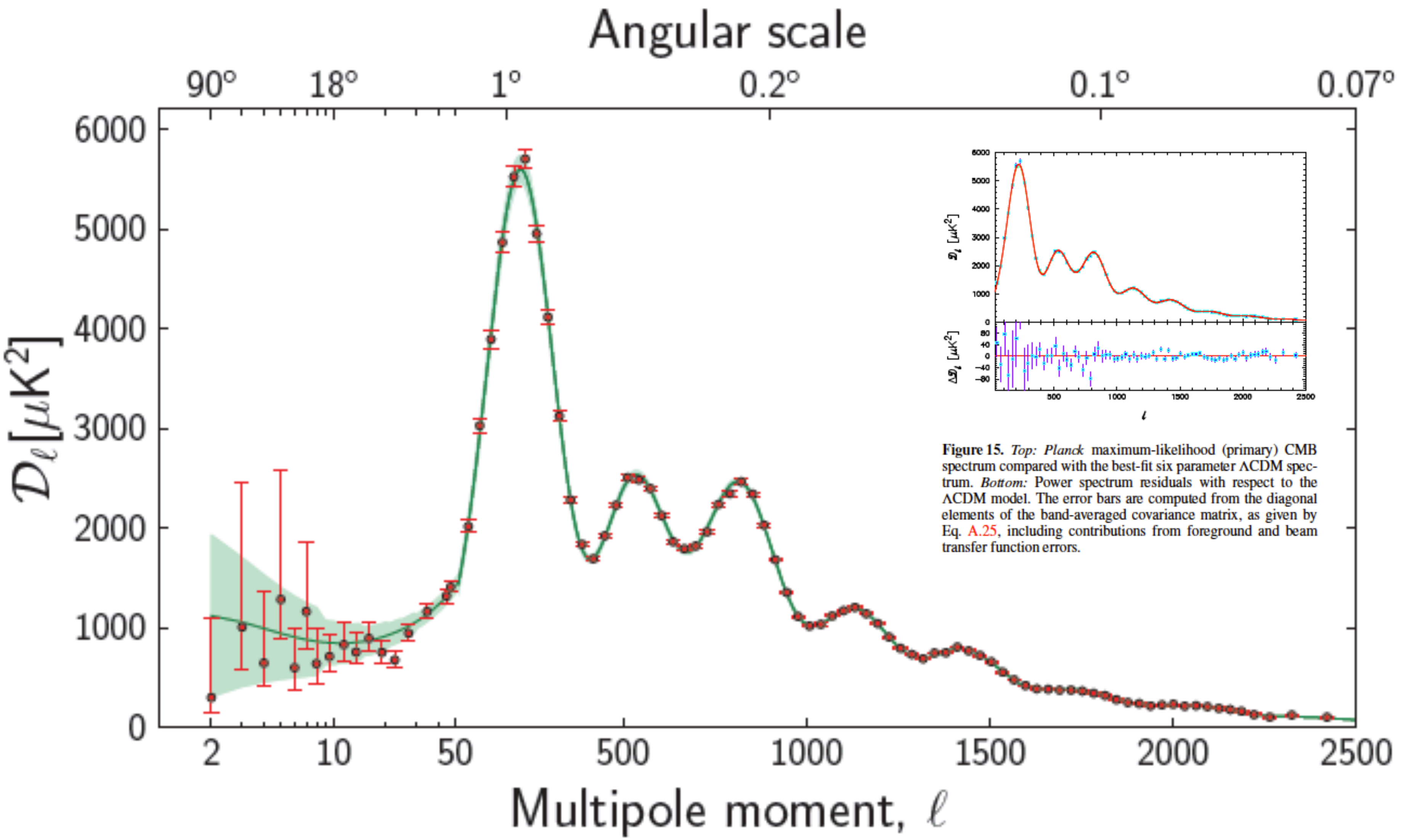
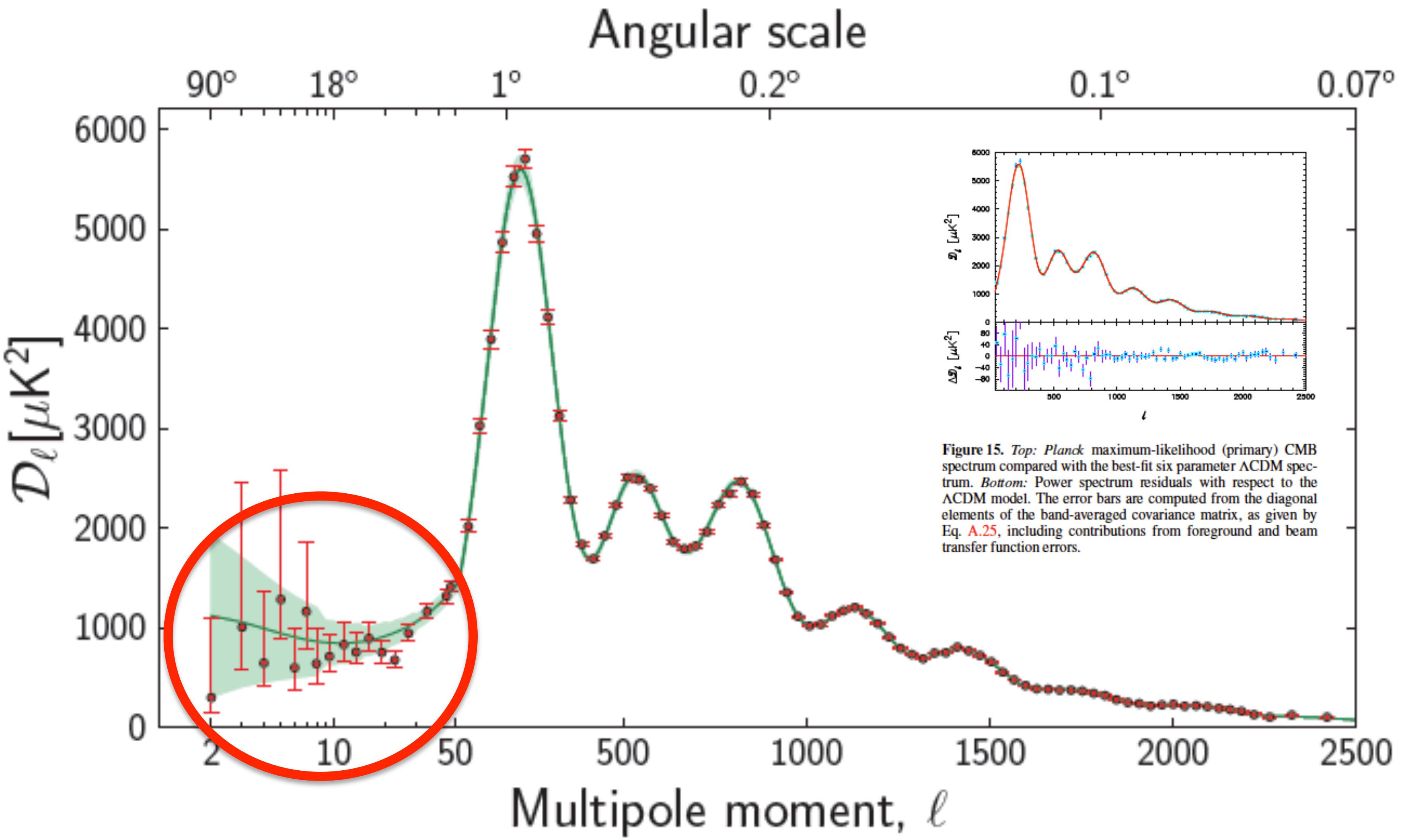
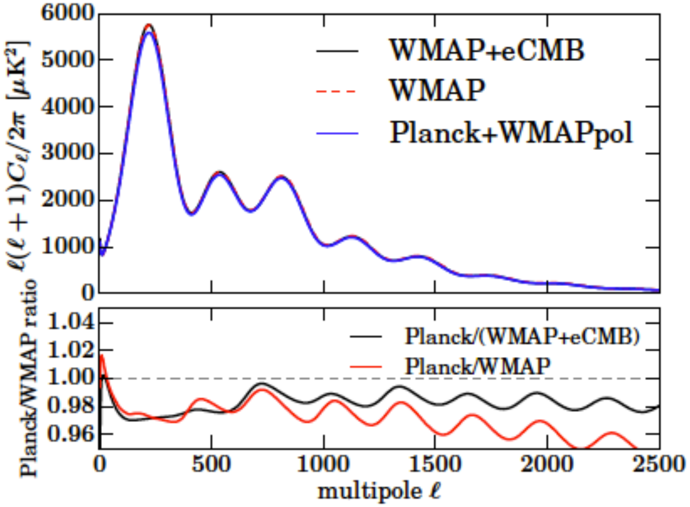


Figure 15. *Top: Planck* maximum-likelihood (primary) CMB spectrum compared with the best-fit six parameter ΛCDM spectrum. *Bottom:* Power spectrum residuals with respect to the ΛCDM model. The error bars are computed from the diagonal elements of the band-averaged covariance matrix, as given by Eq. A.25, including contributions from foreground and beam transfer function errors.

Planck temperature angular power spectrum,
Planck XV, Planck XVI.





Larsen et al, 1409.7718

Cosmological parameters from Planck and WMAP in the context of Vanilla LCDM model

	Planck		*Planck*+WP	
Parameter	Best fit	68% limits	Best fit	68% limits
$\Omega_b h^2$	0.022068	0.02207 ± 0.00033	0.022032	0.02205 ± 0.00028
$\Omega_c h^2$	0.12029	0.1196 ± 0.0031	0.12038	0.1199 ± 0.0027
$100\theta_{MC}$	1.04122	1.04132 ± 0.00068	1.04119	1.04131 ± 0.00063
τ	0.0925	0.097 ± 0.038	0.0925	$0.089^{+0.012}_{-0.014}$
n_s	0.9624	0.9616 ± 0.0094	0.9619	0.9603 ± 0.0073
$\ln(10^{10} A_s)$	3.098	3.103 ± 0.072	3.0980	$3.089^{+0.024}_{-0.027}$
Ω_Λ	0.6825	0.686 ± 0.020	0.6817	$0.685^{+0.018}_{-0.016}$
Ω_m	0.3175	0.314 ± 0.020	0.3183	$0.315^{+0.016}_{-0.018}$
σ_8	0.8344	0.834 ± 0.027	0.8347	0.829 ± 0.012
z_{re}	11.35	$11.4^{+4.0}_{-2.8}$	11.37	11.1 ± 1.1
H_0	67.11	67.4 ± 1.4	67.04	67.3 ± 1.2
$10^9 A_s$	2.215	2.23 ± 0.16	2.215	$2.196^{+0.051}_{-0.060}$
$\Omega_m h^2$	0.14300	0.1423 ± 0.0029	0.14305	0.1426 ± 0.0025
Age/Gyr	13.819	13.813 ± 0.058	13.8242	13.817 ± 0.048
z_*	1090.43	1090.37 ± 0.65	1090.48	1090.43 ± 0.54
$100\theta_*$	1.04139	1.04148 ± 0.00066	1.04136	1.04147 ± 0.00062
z_{eq}	3402	3386 ± 69	3403	3391 ± 60

Planck XV

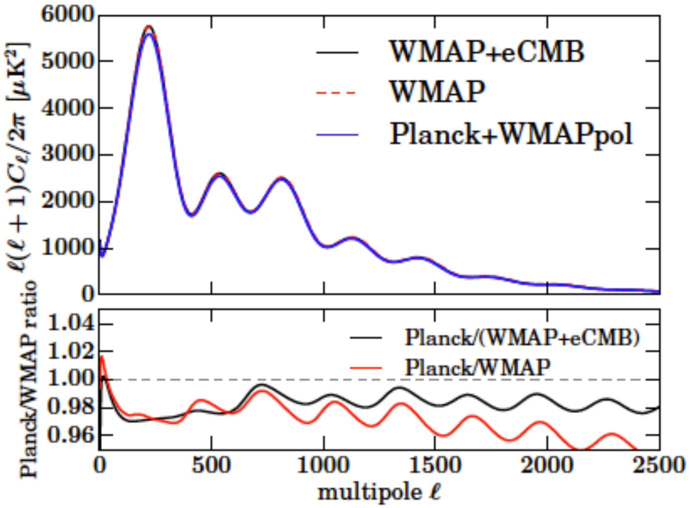
WMAP Cosmological Parameters

Model: lcdm

Data: wmap9

$10^9 \Delta^2_{\mathcal{R}}$	2.41 ± 0.10	H_0	70.0 ± 2.2 km/s/Mpc
$\ell(\ell+1)C_{220}/(2\pi)$	5746 ± 35 μK^2	$d_A(z_{eq})$	14194 ± 117 Mpc
$d_A(z_*)$	14029 ± 119 Mpc	$D_v(z=0.57)/r_s(z_d)$	13.28 ± 0.31
η	$(6.19 \pm 0.14) \times 10^{-10}$	k_{eq}	0.00996 ± 0.00032
ℓ_{eq}	139.7 ± 3.5	ℓ_*	302.35 ± 0.65
n_b	$(2.542 \pm 0.056) \times 10^{-7}$ cm^{-3}	n_s	0.972 ± 0.013
Ω_b	0.0463 ± 0.0024	$\Omega_b h^2$	0.02264 ± 0.00050
Ω_c	0.233 ± 0.023	$\Omega_c h^2$	0.1138 ± 0.0045
Ω_Λ	0.721 ± 0.025	Ω_m	0.279 ± 0.025
$\Omega_m h^2$	0.1364 ± 0.0044	$r_s(z_d)$	152.3 ± 1.3 Mpc
$r_s(z_d)/D_v(z=0.106)$	0.346 ± 0.012	$r_s(z_d)/D_v(z=0.2)$	0.1889 ± 0.0060
$r_s(z_d)/D_v(z=0.35)$	0.1135 ± 0.0032	$r_s(z_d)/D_v(z=0.44)$	0.0932 ± 0.0024
$r_s(z_d)/D_v(z=0.54)$	0.0787 ± 0.0019	$r_s(z_d)/D_v(z=0.57)$	$0.0753^{+0.0017}_{-0.0018}$
$r_s(z_d)/D_v(z=0.6)$	0.0724 ± 0.0016	$r_s(z_d)/D_v(z=0.73)$	0.0624 ± 0.0013
$r_s(z_*)$	145.8 ± 1.2	R	1.728 ± 0.016
σ_8	0.821 ± 0.023	$\sigma_8 \Omega_m^{0.5}$	0.434 ± 0.029
$\sigma_8 \Omega_m^{0.6}$	0.382 ± 0.029	A_{SZ}	< 2.0 (95% CL)
t_0	13.74 ± 0.11 Gyr	τ	0.089 ± 0.014
θ_*	0.010391 ± 0.000022	θ_*	0.5953 ± 0.0013 °
τ_{rec}	283.9 ± 2.4	t_{reion}	453^{+63}_{-64} Myr
t_*	376371^{+4115}_{-4111} yr	z_d	1020.7 ± 1.1
z_{eq}	3265^{+106}_{-105}	z_{rec}	1088.16 ± 0.79
z_{reion}	10.6 ± 1.1	z_*	$1090.97^{+0.85}_{-0.86}$

LAMBDA wensite



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Lets remember

WMAP 9 probes multipole l <1200 &

Planck probes multipole l <2500

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LAMBDA wensite

Testing deviations from an assumed model

(without comparing different models)

Gaussian Processes:

Modeling of the data around a mean function searching for likely features by looking at the the likelihood space of the hyperparameters.

Bayesian Interpretation of Crossing Statistic:

Comparing a model with its own possible variations.

Gaussian Process

➔Efficient in statistical modeling of stochastic variables
➔Derivatives of Gaussian Processes are Gaussian Processes
➔Provides us with all covariance matrices

Shafieloo, Kim & Linder, PRD 2012
Shafieloo, Kim & Linder, PRD 2013

Data **Mean Function**

$$\begin{bmatrix} \mathbf{y} \\ \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{m}(\mathbf{Z}) \\ \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m}'(\mathbf{Z_1}) \\ \mathbf{m}''(\mathbf{Z_1}) \end{bmatrix}, \begin{bmatrix} \Sigma_{00}(Z,Z) & \Sigma_{00}(Z,Z_1) & \Sigma_{01}(Z,Z_1) & \Sigma_{02}(Z,Z_1) \\ \Sigma_{00}(Z_1,Z) & \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z) & \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1) \\ \Sigma_{20}(Z_1,Z) & \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1) \end{bmatrix} \right),$$

$$\Sigma_{\alpha\beta} = \frac{d^{(\alpha+\beta)}K}{dz_i^\alpha dz_j^\beta},$$

$$\begin{bmatrix} \overline{\mathbf{f}} \\ \overline{\mathbf{f}'} \\ \overline{\mathbf{f}''} \end{bmatrix} = \begin{bmatrix} \mathbf{m}(\mathbf{Z_1}) \\ \mathbf{m}'(\mathbf{Z_1}) \\ \mathbf{m}''(\mathbf{Z_1}) \end{bmatrix} + \begin{bmatrix} \Sigma_{00}(Z_1,Z) \\ \Sigma_{10}(Z_1,Z) \\ \Sigma_{20}(Z_1,Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\,\mathbf{y}$$

Kernel

$$k(z,z') = \underline{\sigma_f^2} \exp\left(-\frac{|z-z'|^2}{\underline{2l^2}}\right),$$

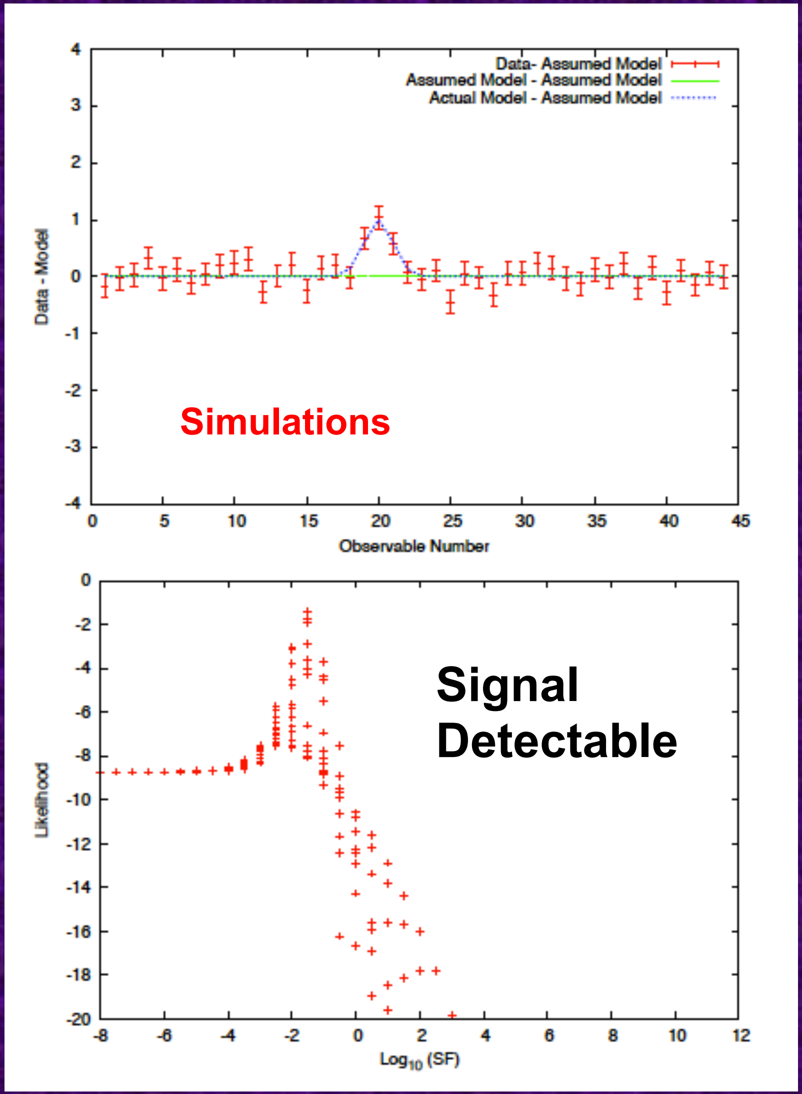
GP Hyper-parameters

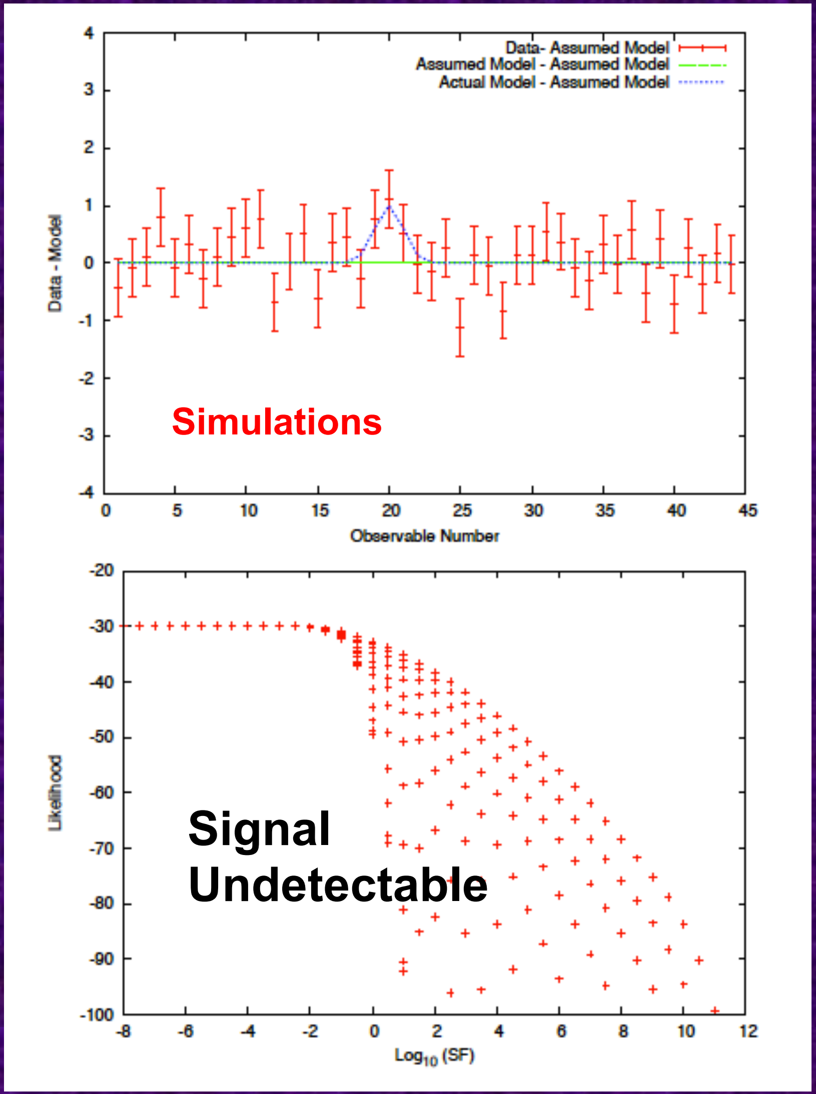
$$\mathrm{Cov}\left(\begin{bmatrix} \mathbf{f} \\ \mathbf{f}' \\ \mathbf{f}'' \end{bmatrix}\right) = \begin{bmatrix} \Sigma_{00}(Z_1,Z_1) & \Sigma_{01}(Z_1,Z_1) & \Sigma_{02}(Z_1,Z_1) \\ \Sigma_{10}(Z_1,Z_1) & \Sigma_{11}(Z_1,Z_1) & \Sigma_{12}(Z_1,Z_1) \\ \Sigma_{20}(Z_1,Z_1) & \Sigma_{21}(Z_1,Z_1) & \Sigma_{22}(Z_1,Z_1) \end{bmatrix} - \begin{bmatrix} \Sigma_{00}(Z_1,Z) \\ \Sigma_{10}(Z_1,Z) \\ \Sigma_{20}(Z_1,Z) \end{bmatrix} \Sigma_{00}^{-1}(Z,Z)\,[\Sigma_{00}(Z,Z_1), \Sigma_{01}(Z,Z_1), \Sigma_{02}(Z,Z_1)].$$

$$2\ln p(y|f) = -y^T \Sigma_{00}(Z,Z)^{-1} y - \ln\det\Sigma_{00}(Z,Z) - n\ln(2\pi),$$

GP Likelihood

Detection of the features in the residuals



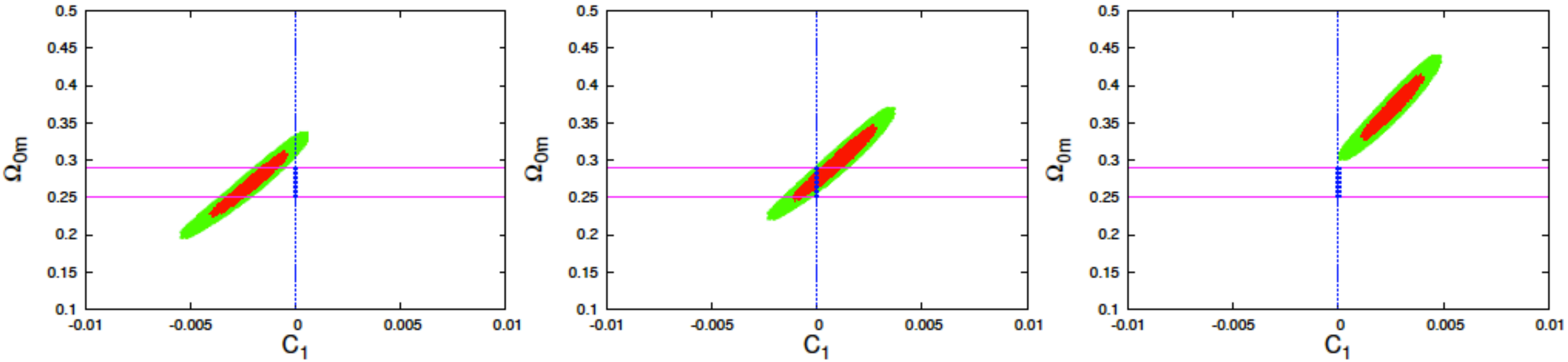


Crossing Statistic (Bayesian Interpretation)

Theoretical model Crossing function

$$\mu_M^{T_N}(z) = \mu_M(p_i, z) \times T_N(C_1, \ldots, C_N, z)$$

Comparing a model with its own variations



$$T_I(C_1, z) = 1 + C_1\left(\frac{z}{z_{max}}\right)$$

Chebishev Polynomials as Crossing Functions

$$T_{II}(C_1, C_2, z) = 1 + C_1\left(\frac{z}{z_{max}}\right) + C_2\left[2\left(\frac{z}{z_{max}}\right)^2 - 1\right],$$

Shafieloo. JCAP 2012 (a)
Shafieloo, JCAP 2012 (b)

Bayesian Interpretation of Crossing Statistics

Shafieloo, JCAP 2012b

$$C_\ell^{\mathrm{TT}} \,\Big|^{N}_{\mathrm{modified}} = C_\ell^{\mathrm{TT}} \,\Big|_{\mathrm{best\ fit\ model}} \times T_{\mathrm{N}}(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\mathrm{II}}(C_0, C_1, C_2, x) = C_0 + C_1\, x + C_2(2x^2 - 1)$$

$$T_{\mathrm{III}}(C_0, C_1, C_2, C_3, x) = C_0 + C_1\, x + C_2(2x^2 - 1) + C_3(4x^3 - 3x)$$

Chebychev Polynomials

$$T_{\mathrm{IV}}(C_0, C_1, C_2, C_3, C_4, x) = C_0 + C_1\, x + C_2(2x^2 - 1) + C_3(4x^3 - 3x) + C_4(8x^4 - 8x^2 + 1)$$

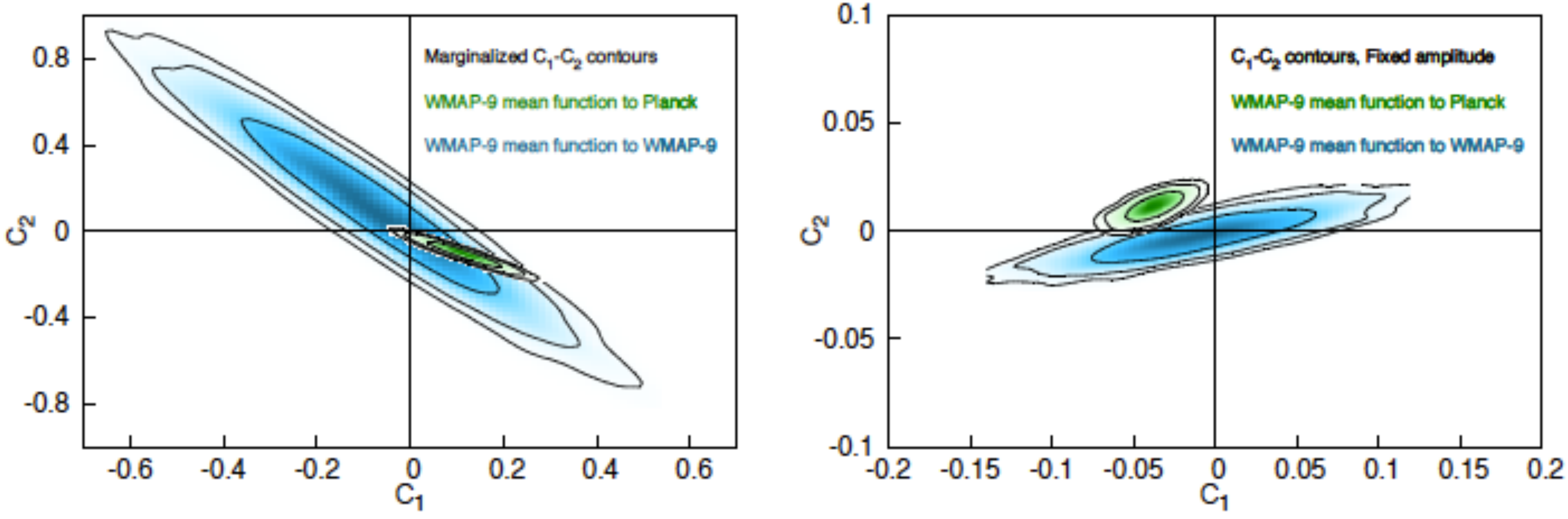
$x = \ell/\ell_{\mathrm{max}}$ $\ell_{\mathrm{max}} = 2500.$

Chebychev polynomials have the properties of orthogonality and convergence within the limited range of -1 < x < 1.

$$C_\ell^{\rm TT}\,|_{\rm modified}^{N} = C_\ell^{\rm TT}\,|_{\rm best\ fit\ model} \times T_{\rm N}(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\rm II}(C_0, C_1, C_2, x) = C_0 + C_1\, x + C_2(2x^2 - 1)$$

$$x = \ell/\ell_{\rm max}$$



Fitting the best fit WMAP LCDM model along with a Crossing function to Planck and WMAP 9-year data.

Hazra & Shafieloo, PRD 2014

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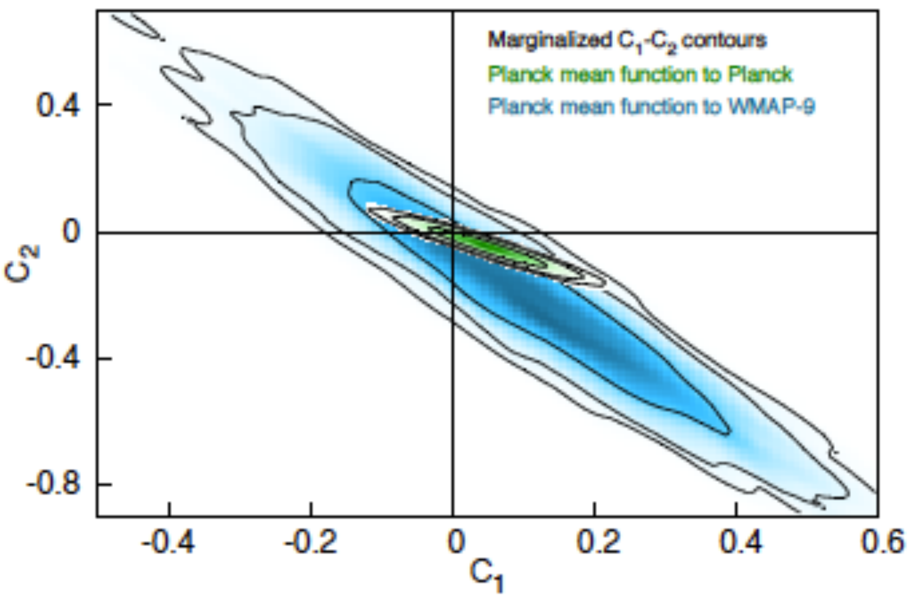

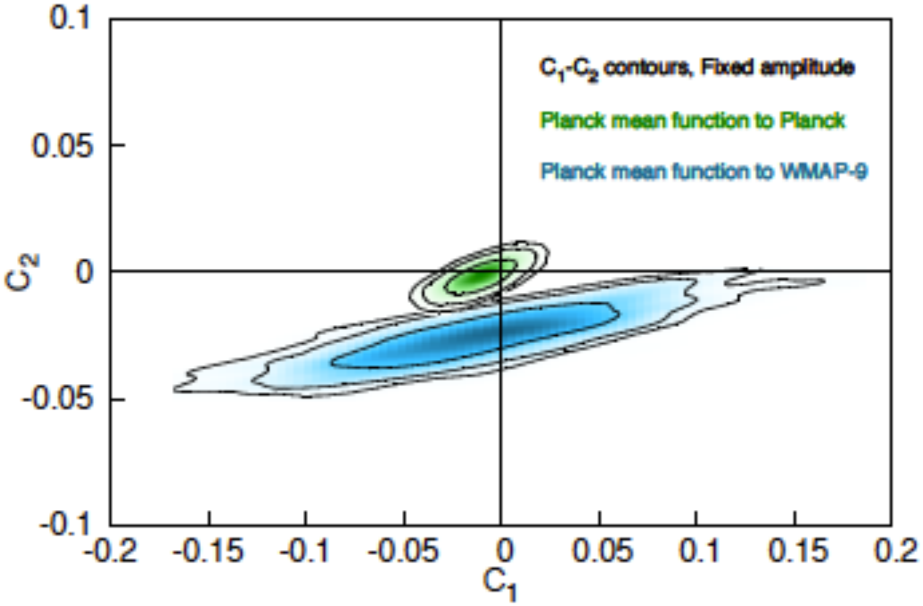

Fitting the best fit Planck LCDM model along with a Crossing function to Planck and WMAP 9-year data.

Hazra & Shafieloo, PRD 2014

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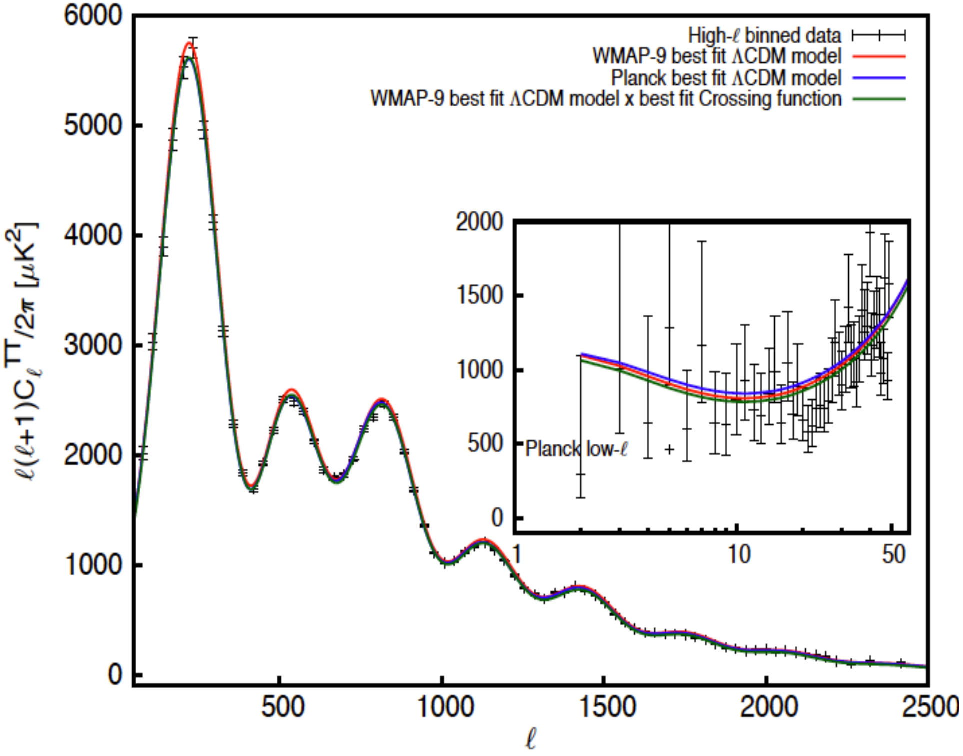
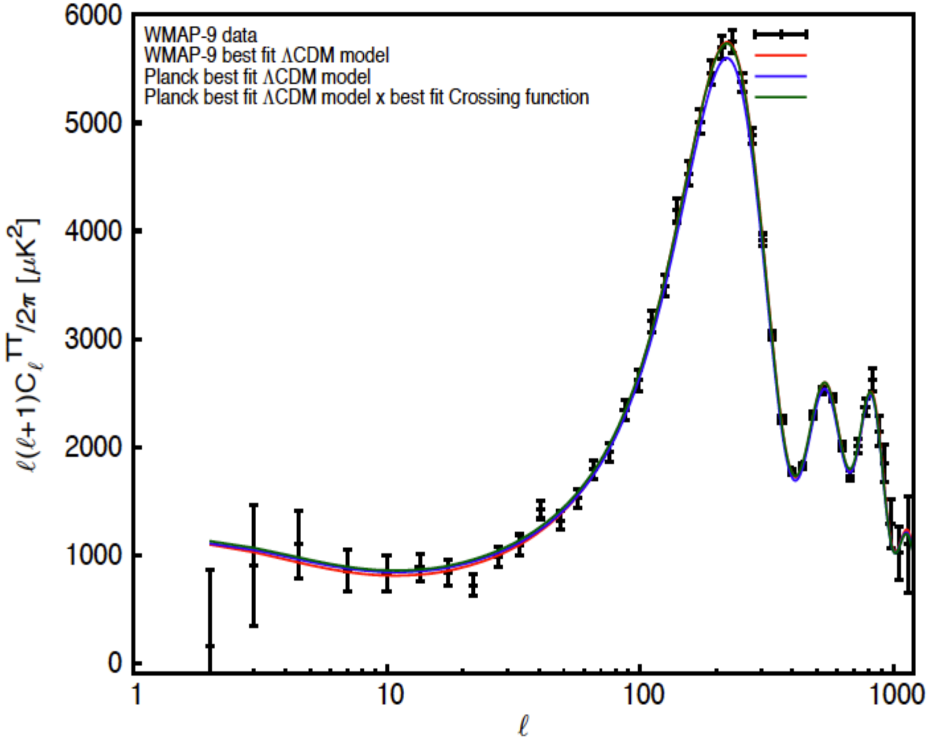
$$T_{\rm II}(C_0, C_1, C_2, x) = C_0 + C_1\, x + C_2(2x^2 - 1)$$

Fitting Planck and WMAP data using best fit LCDM models from Planck and WMAP as mean functions along with second order Crossing functions





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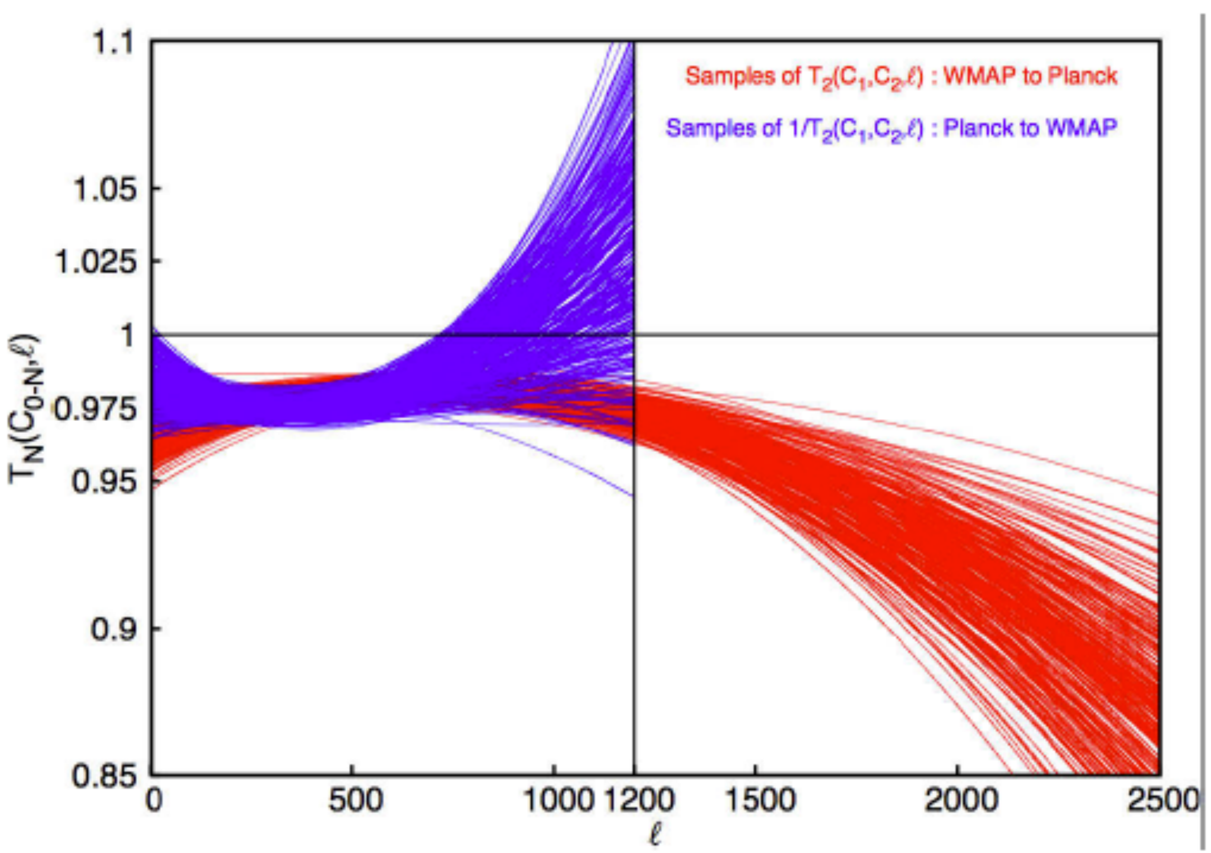
Recovered Crossing functions



With overall amplitude shift two data are consistent.

Without amplitude shift, they are not consistent at more than 3 sigma confidence.

Hazra & Shafieloo, PRD 2014

Crossing Statistic (Bayesian Interpretation)

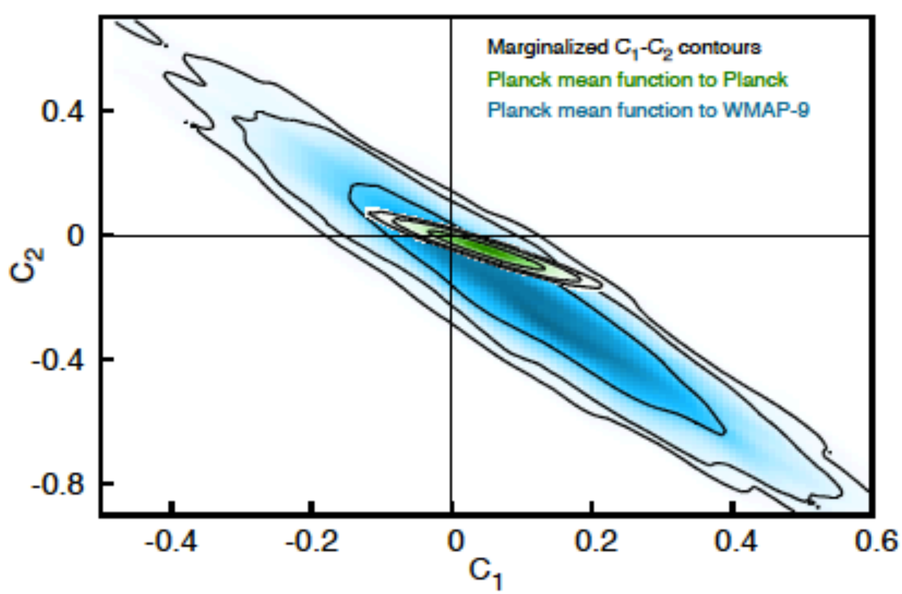
Theoretical model Crossing function

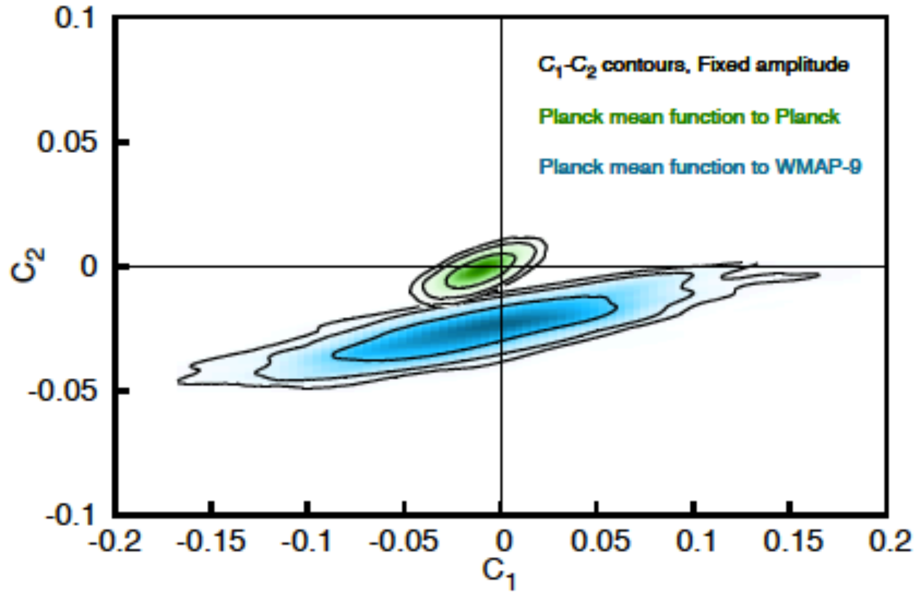
$$C_\ell^{\mathrm{TT}}\,\Big|^{N}_{\mathrm{modified}} = C_\ell^{\mathrm{TT}}\,\Big|_{\mathrm{best\ fit\ model}} \times T_{\mathrm{N}}(C_0, C_1, C_2, \ldots, C_N, \ell).$$

Test of consistency between Planck and WMAP,

Hazra and Shafieloo, PRD 2014

Consistent only by allowing amplitude shift





Bayesian Interpretation of Crossing Statistics

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\mathrm{TT}} \big|^{N}_{\mathrm{modified}} = \mathcal{C}_\ell^{\mathrm{TT}} \big|_{\Omega_\mathrm{b},\Omega_\mathrm{CDM},\mathrm{H_0},\tau,\mathrm{A_s},\mathrm{n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$\begin{aligned}
T_0(C_0,x) &= C_0 \\
T_\mathrm{I}(C_0,C_1,x) &= T_0(C_0,x) + C_1\ x \\
T_\mathrm{II}(C_0,C_1,C_2,x) &= T_\mathrm{I}(C_0,C_1,x) + C_2(2x^2-1) \\
T_\mathrm{III}(C_0,C_1,C_2,C_3,x) &= T_\mathrm{II}(C_0,C_1,C_2,x) + C_3(4x^3-3x) \\
T_\mathrm{IV}(C_0,C_1,C_2,C_3,C_4,x) &= T_\mathrm{III}(C_0,C_1,C_2,C_3,x) + C_4(8x^4-8x^2+1) \\
T_\mathrm{V}(C_0,C_1,C_2,C_3,C_4,C_5,x) &= T_\mathrm{IV}(C_0,C_1,C_2,C_3,C_4,x) + C_5(16x^5-20x^3+5x).
\end{aligned}$$

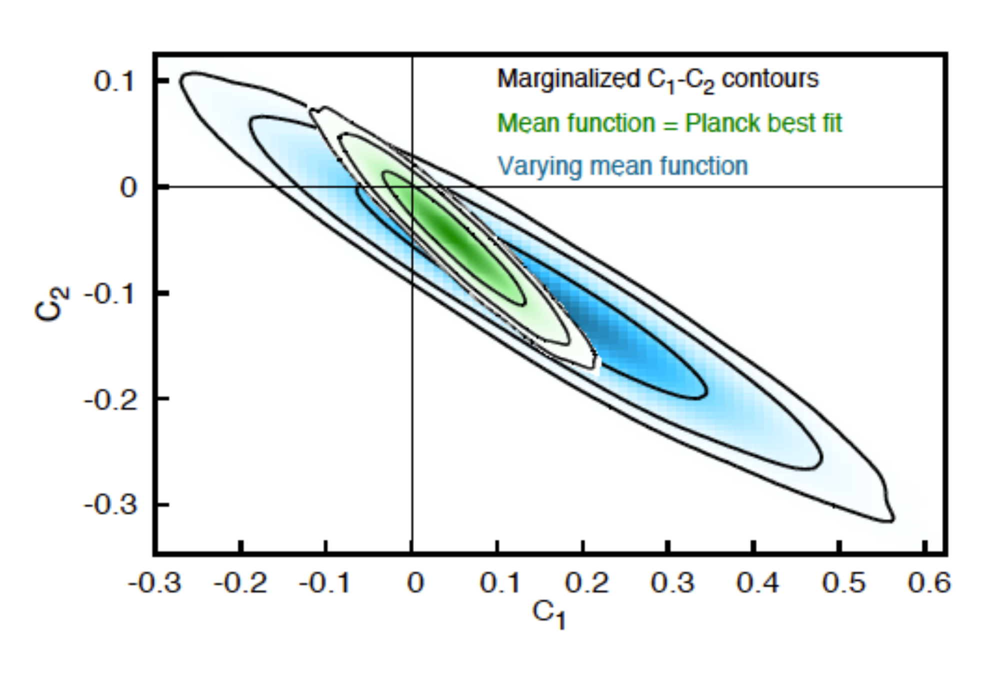
$x = \ell/\ell_{\mathrm{max}}$ $\ell_{\mathrm{max}} = 2500.$

Is standard model consistent to Planck temperature data?

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\mathrm{TT}}\Big|^{N}_{\mathrm{modified}} = \mathcal{C}_\ell^{\mathrm{TT}}\Big|_{\Omega_{\mathrm{b}},\Omega_{\mathrm{CDM}},\mathrm{H_0},\tau,\mathrm{A_s},\mathrm{n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\mathrm{II}}(C_0, C_1, C_2, x) = C_0 + C_1\, x + C_2(2x^2 - 1)$$

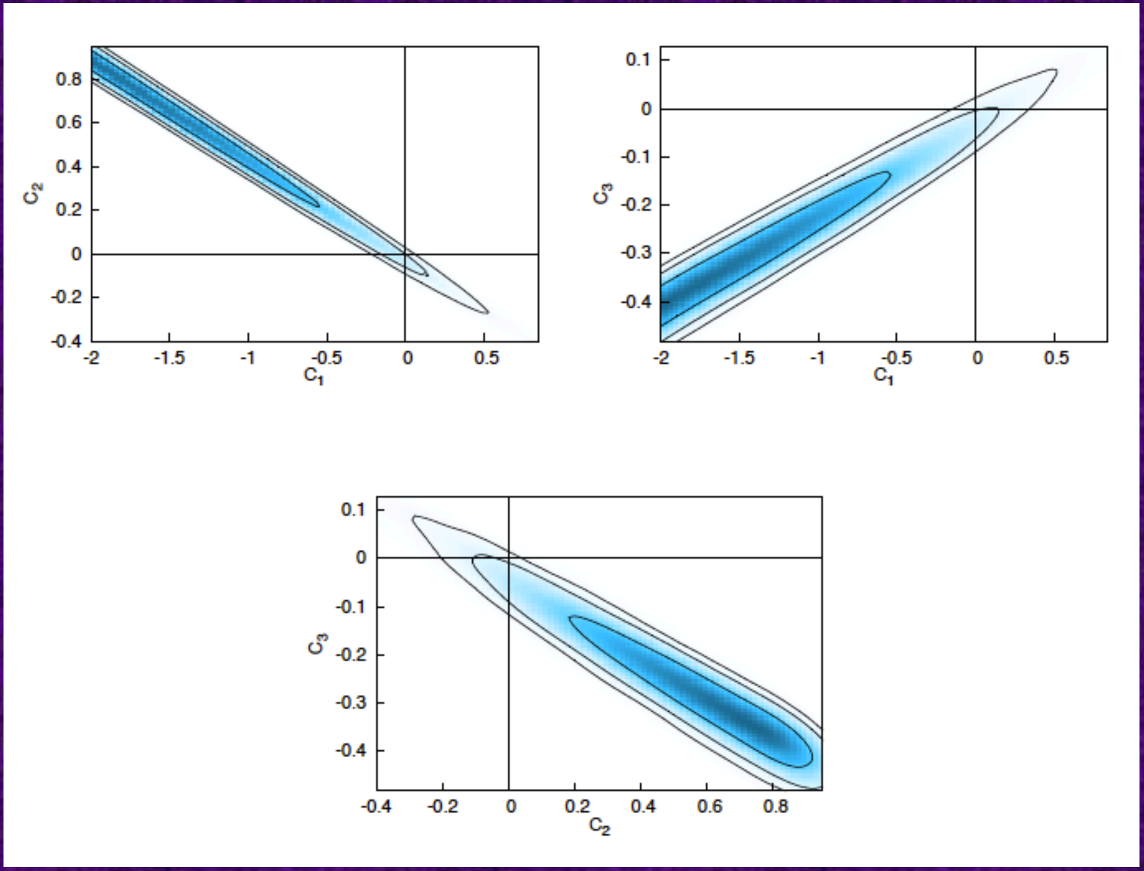


Hazra & Shafieloo, JCAP 2014

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,\Big|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,\Big|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H_0},\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

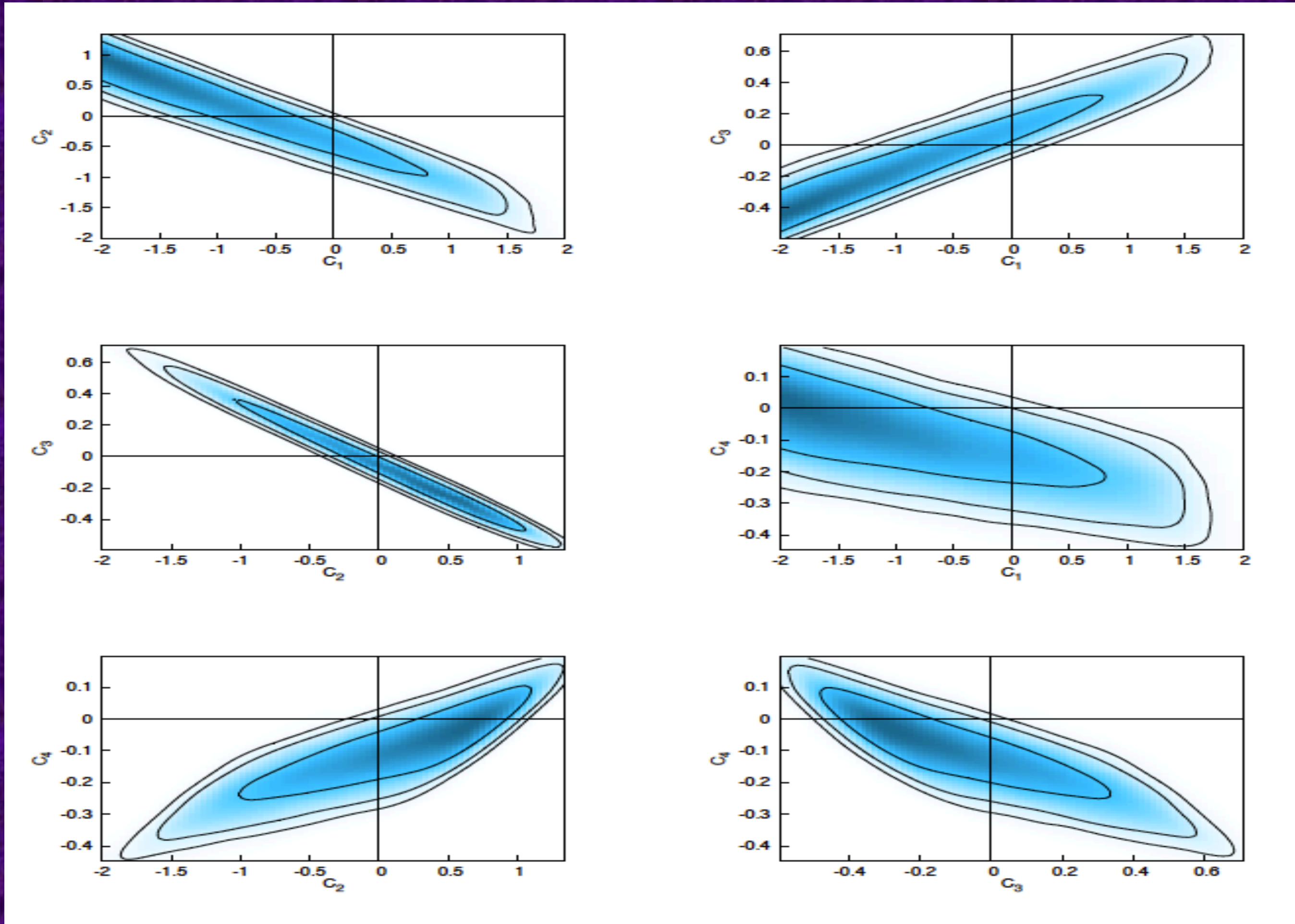
$$T_{\rm III}(C_0, C_1, C_2, C_3, x) = C_0 + C_1\, x + C_2(2x^2 - 1) + C_3(4x^3 - 3x)$$



Theoretical Model   Crossing Function

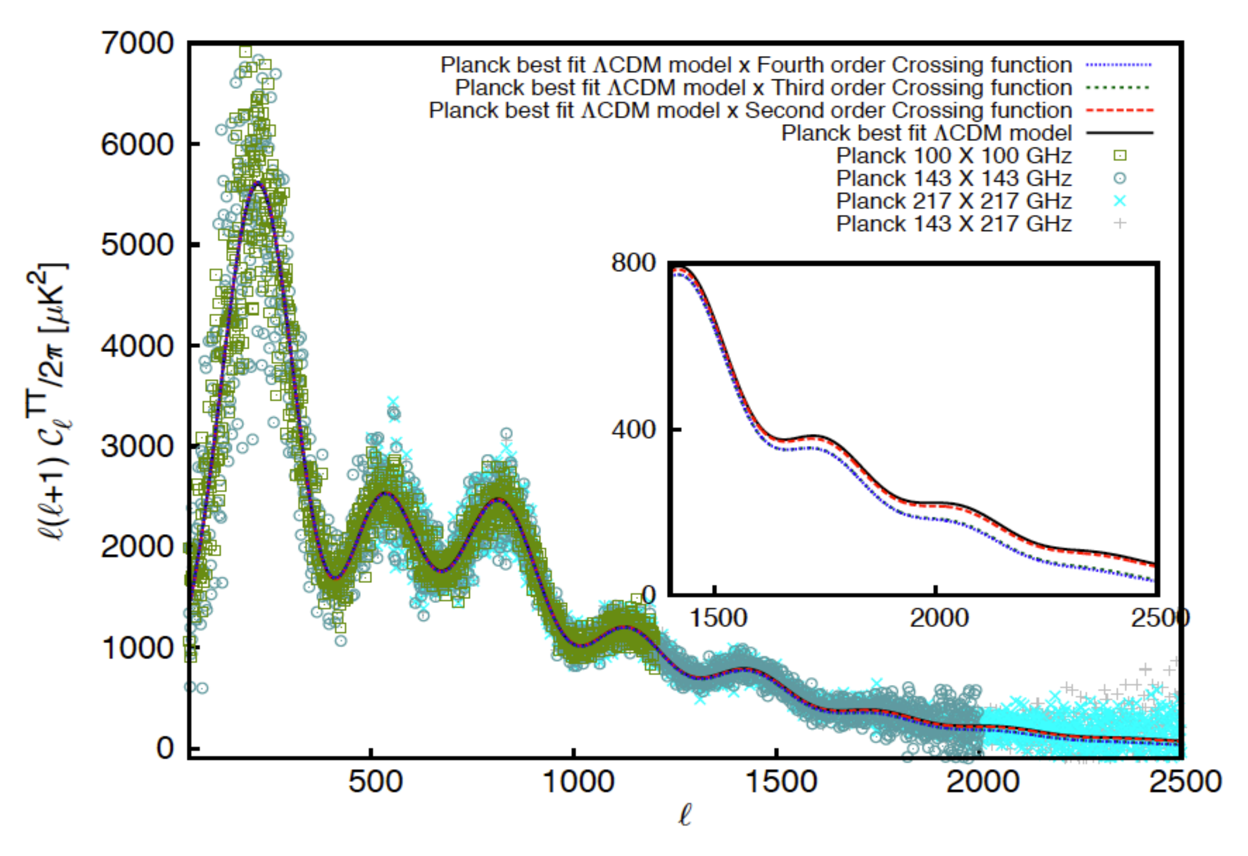
$$\mathcal{C}_\ell^{\rm TT}\,|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

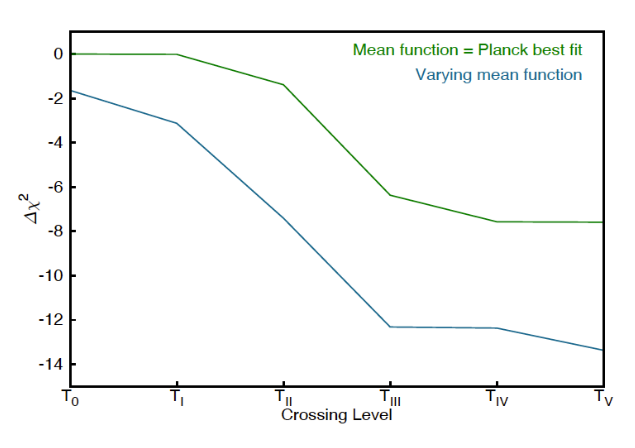
$$T_{\rm IV}(C_0, C_1, C_2, C_3, C_4, x) = C_0 + C_1\, x + C_2(2x^2-1) + C_3(4x^3-3x) + C_4(8x^4-8x^2+1)$$



Theoretical Model　　　Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,\Big|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,\Big|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A}_{\rm s},{\rm n}_{\rm s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

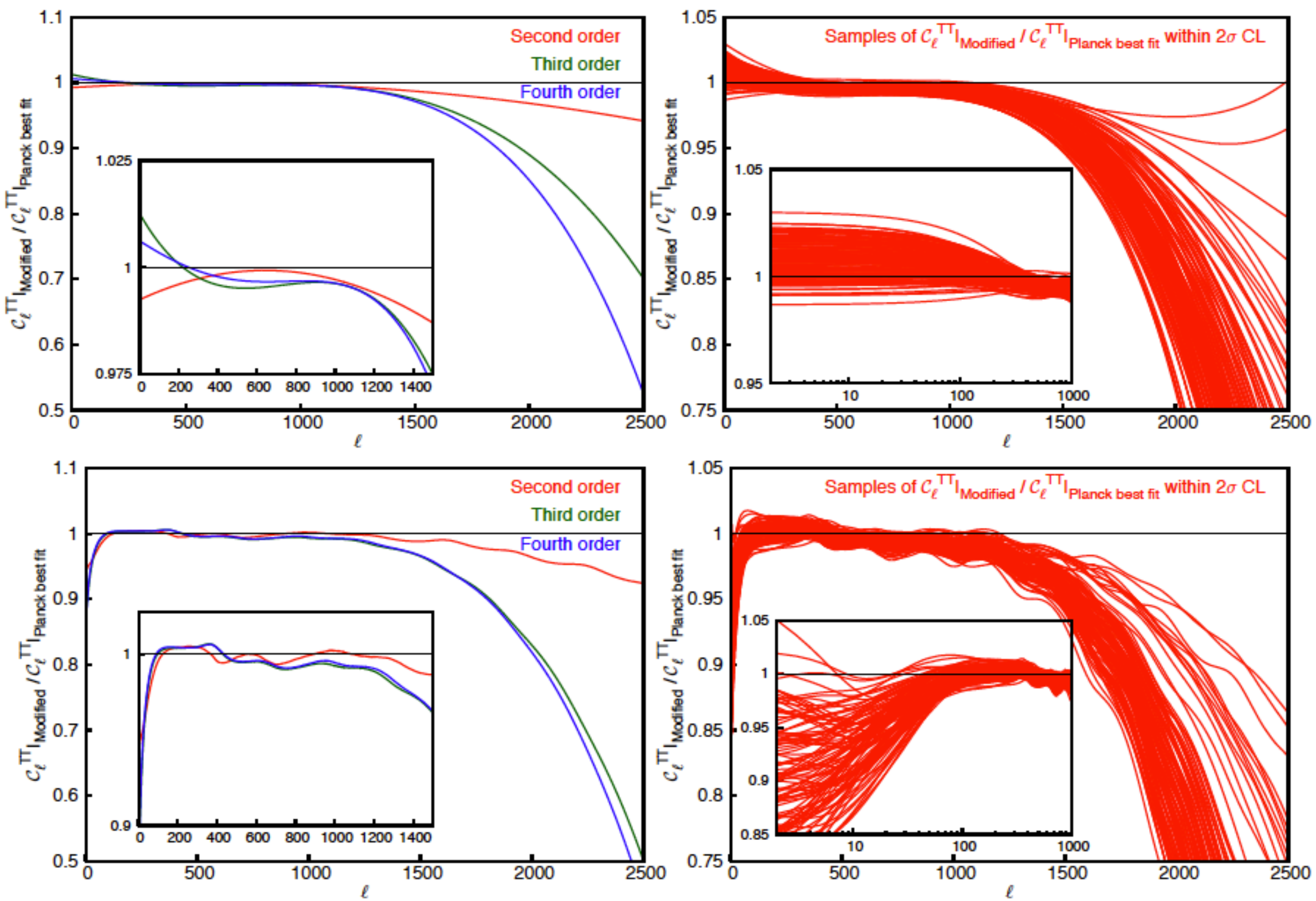




Data	ΛCDM	T_0	$T_{\rm I}$	$T_{\rm II}$	$T_{\rm III}$	$T_{\rm IV}$	$T_{\rm V}$
Planck low-ℓ (ℓ=2-49)	-6.3	-7	-8.5	-8.6	-9.8	-9.7	-9.7
Planck high-ℓ (ℓ=50-2500)	7794.9	7793.8	7793.8	7789.6	7785.9	7785.7	7784.7
Total	7788.6	7786.8	7785.3	7781	7776.1	7776	7775
$\chi^2_{\rm Model}$ - $\chi^2_{\Lambda{\rm CDM}}$	-	-1.8	-3.3	-7.6	-12.5	-12.6	-13.6

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,\Big|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,\Big|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

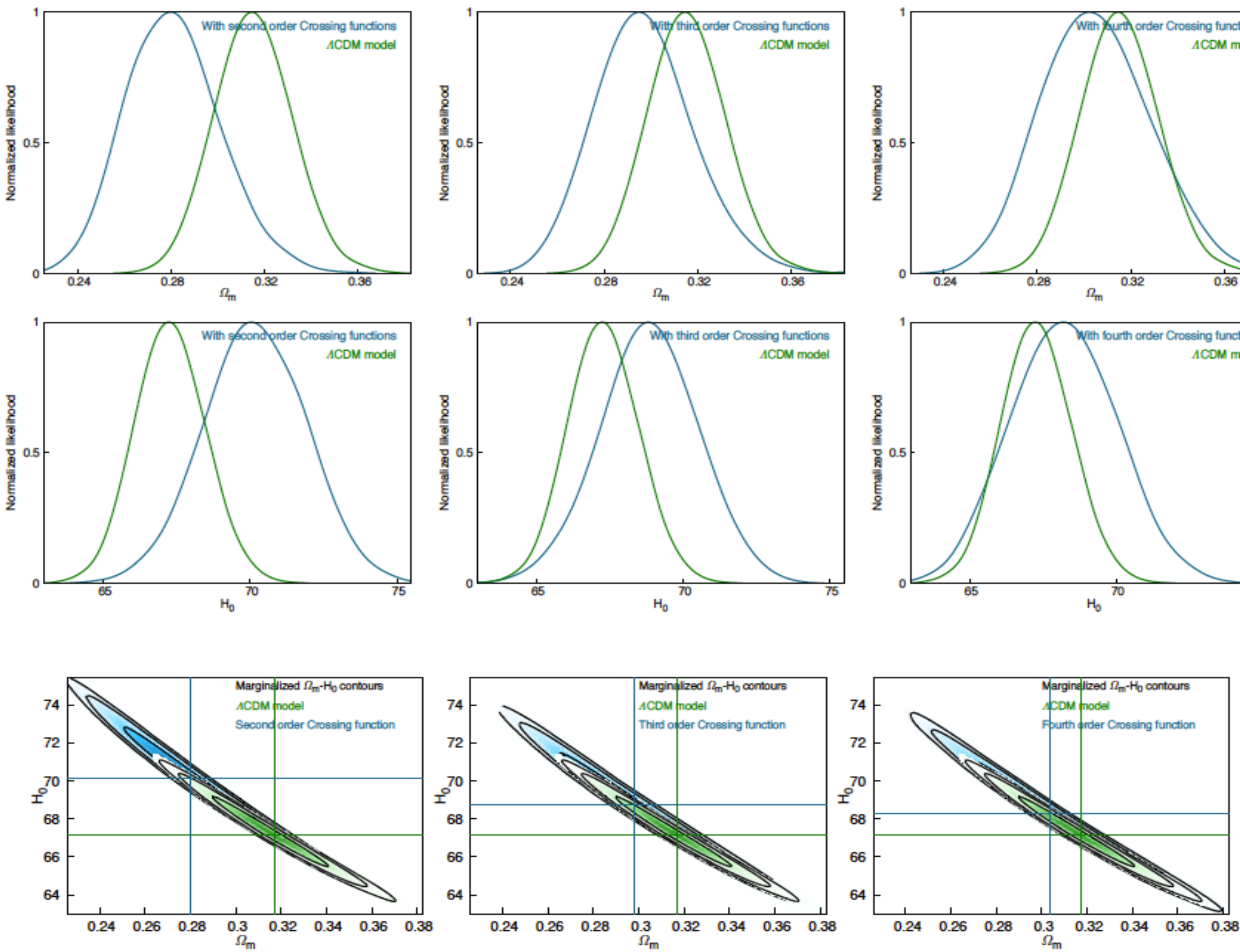


Data suggests substantial suppressions are required at both low and high multiples.

Hazra & Shafieloo, JCAP 2014

Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,\Big|^{N}_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,\Big|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H}_0,\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$



Cosmological parameters while considering Crossing functions.

Hazra & Shafieloo, JCAP 2014

Crossing Statistic (Bayesian Interpretation)

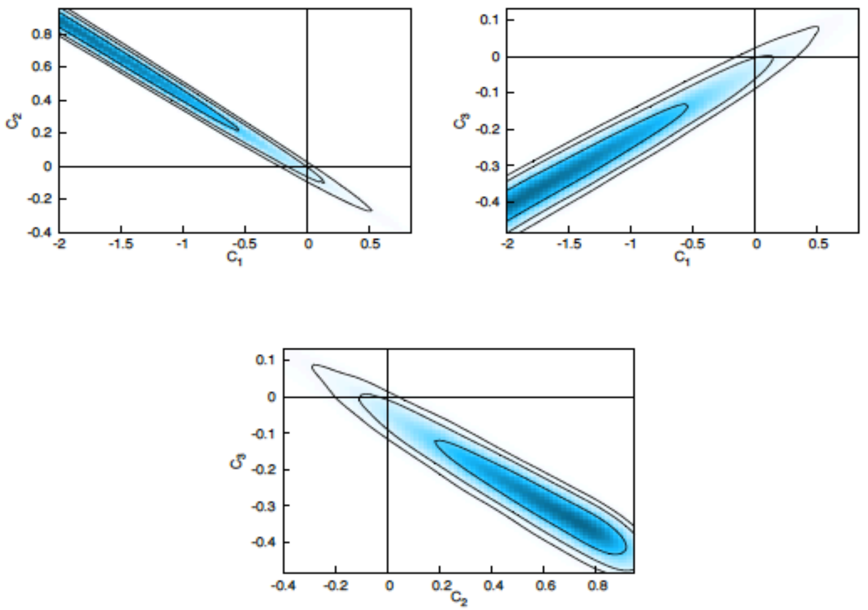
Theoretical model Crossing function

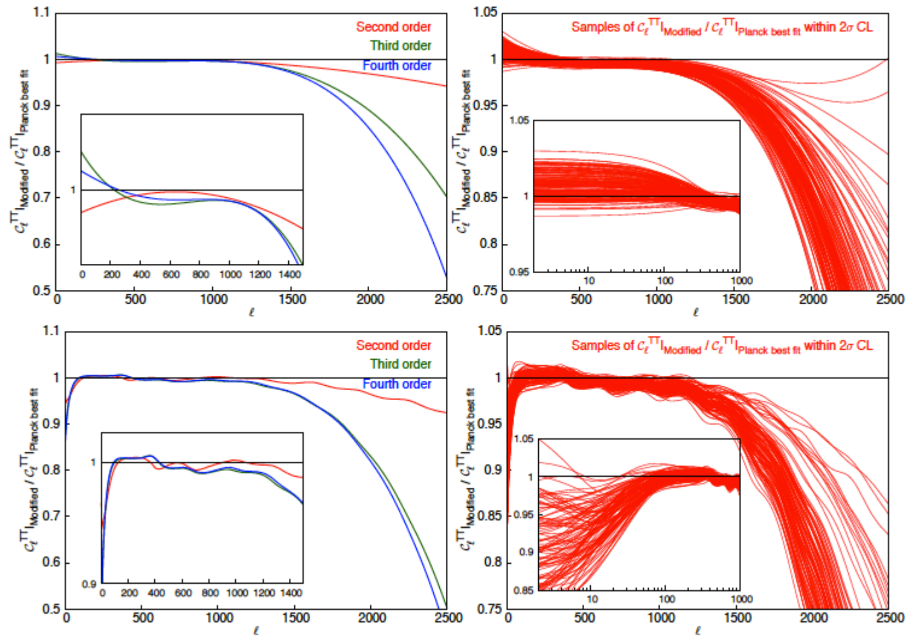
$$\mathcal{C}_\ell^{\mathrm{TT}}\,\Big|^{N}_{\mathrm{modified}} = \mathcal{C}_\ell^{\mathrm{TT}}\,\Big|_{\Omega_{\mathrm{b}},\Omega_{\mathrm{CDM}},\mathrm{H_0},\tau,\mathrm{A_S},\mathrm{n_S},\ell} \times T_i(C_0, C_1, C_2, \ldots, C_N, \ell).$$

Confronting the concordance model of cosmology with Planck data

Hazra and Shafieloo, JCAP 2014

Consistent only at 2~3 sigma CL





Dates

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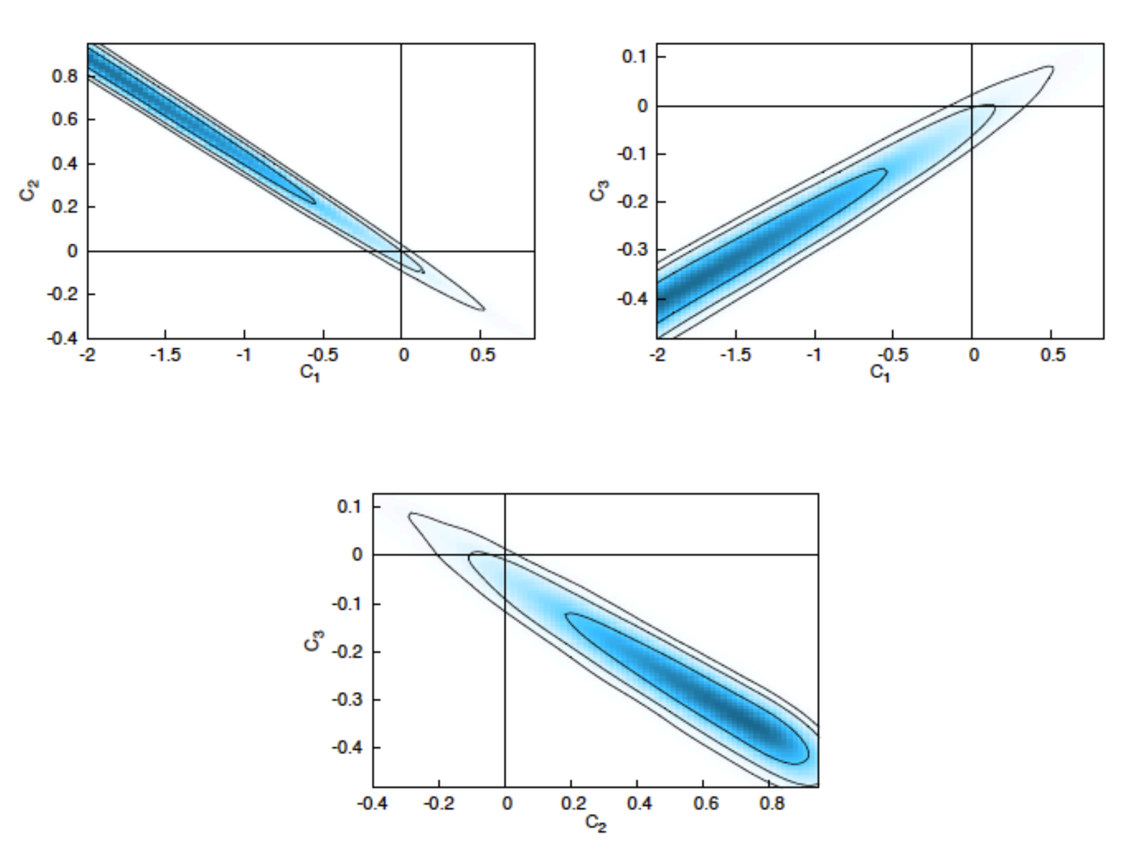
Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\,\Big|^N_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\,\Big|_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H_0},\tau,{\rm A_s},{\rm n_s}} \times T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\rm III}(C_0, C_1, C_2, C_3, x) = C_0 + C_1\,x + C_2(2x^2-1) + C_3(4x^3-3x)$$

With 217 GHz x 217 GHz

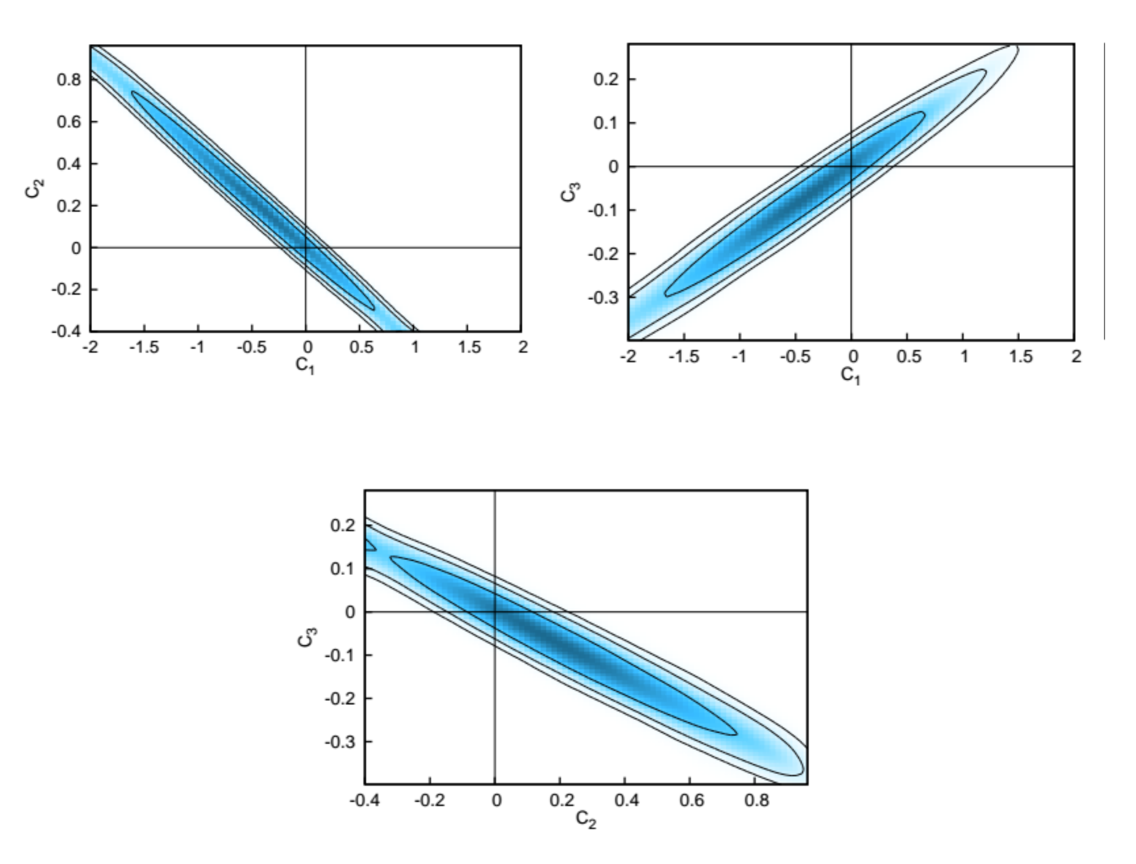


Theoretical Model Crossing Function

$$\mathcal{C}_\ell^{\rm TT}\ |^N_{\rm modified} = \mathcal{C}_\ell^{\rm TT}\ |_{\Omega_{\rm b},\Omega_{\rm CDM},{\rm H_0},\tau,{\rm A_s},{\rm n_s}} \quad \times\ T_N(C_0, C_1, C_2, ..., C_N, \ell).$$

$$T_{\rm III}(C_0, C_1, C_2, C_3, x) = C_0 + C_1\ x + C_2(2x^2-1) + C_3(4x^3-3x)$$

Without 217 GHz x 217 GHz

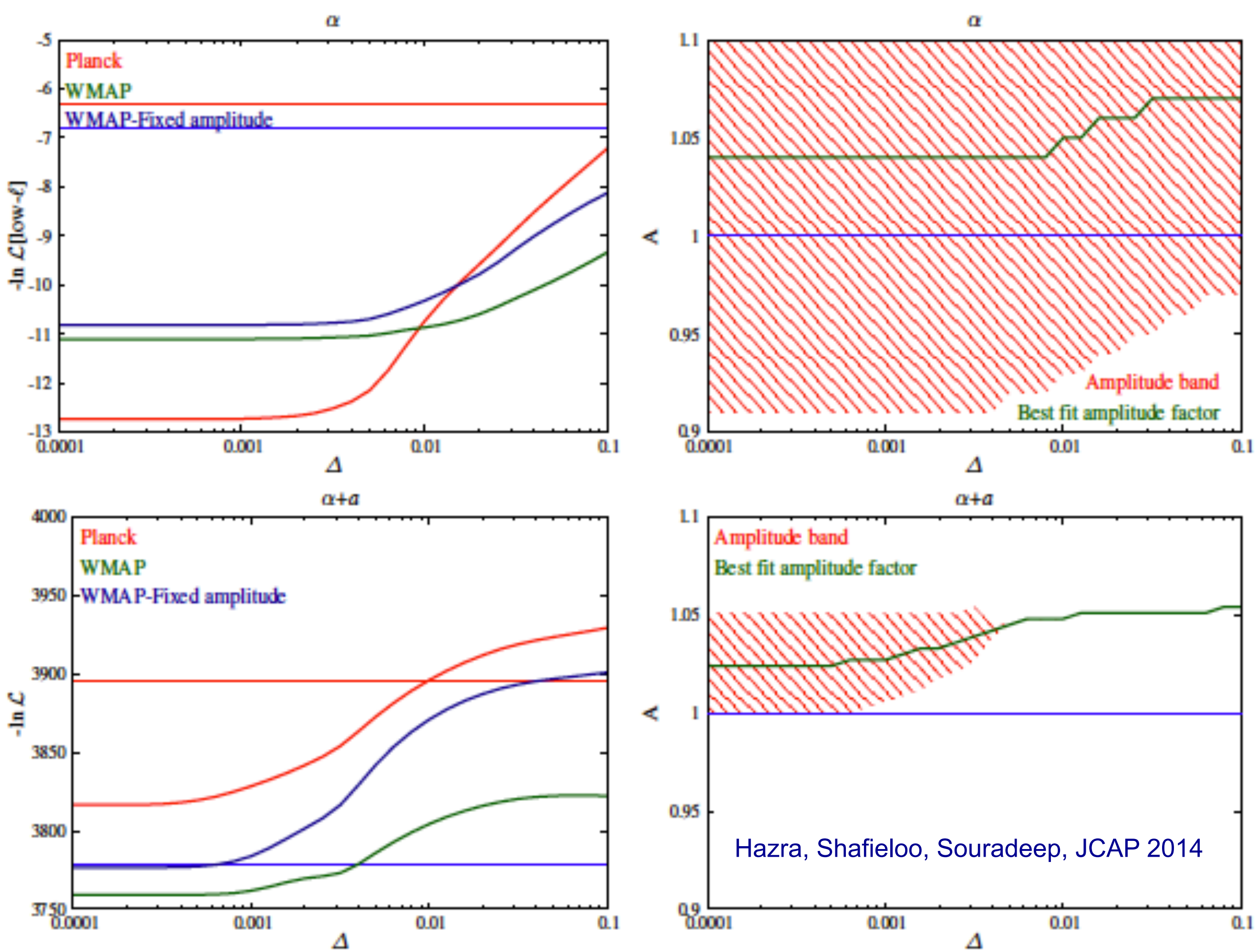


$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \left[\sum_{\ell=\ell_{\rm min}^{\nu}}^{\ell_{\rm max}^{\nu}(\leq 1900)} \frac{1}{g_\nu(\ell)} \widetilde{G}_{\ell k} \left\{ \left(\frac{\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}}{\mathcal{C}_\ell^{\mathrm{T}(i)}} \right) \tanh^2 \left[Q_\ell (\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}) \right] \right\} \right.$$

$$\left. + \sum_{\ell=\ell_{\rm min}^{\nu}(>1900)}^{\ell_{\rm max}^{\nu}} \frac{1}{g'_\nu(\ell)} \widetilde{G'}_{\ell k} \left\{ \left(\frac{\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}}{\mathcal{C}_\ell^{\mathrm{T}(i)}} \right) \tanh^2 \left[\frac{\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}}{\sigma_\ell^{\mathrm{D}_\nu}} \right]^2 \right\}_{\rm binned} \right]$$

$$Q_\ell = \sum_{\ell'} (C_{\ell'}^{\mathrm{D}'_\nu} - C_{\ell'}^{\mathrm{T}(i)}) \mathrm{COV}^{-1}(\ell, \ell')$$

Modified Richardson-Lucy Deconvoloution



Comparing the reconstructed form of PPS from Planck data with WMAP

Hazra Shafieloo, Souradeep, JCAP 2014

Our symbol	Spectra	Multipoles(ℓ)	Scales
α	low-ℓ	2-49	Largest scales
a	100 GHz × 100 GHz	50-1200	Intermediate scales
b	143 GHz × 143 GHz	50-2000	Intermediate scales
1	217 GHz × 217 GHz	500-2500	Small scales
2	143 GHz × 217 GHz	500-2500	Small scales

$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \Bigg[\sum_{\ell=\ell_{\rm min}^{\nu}}^{\ell_{\rm max}^{\nu}(\leq 1900)} \frac{1}{g_\nu(\ell)} \widetilde{G}_{\ell k} \Bigg\{ \left(\frac{\mathcal{C}_\ell^{D'_\nu} - \mathcal{C}_\ell^{T(i)}}{\mathcal{C}_\ell^{T(i)}} \right) \tanh^2 \left[Q_\ell (\mathcal{C}_\ell^{D'_\nu} - \mathcal{C}_\ell^{T(i)}) \right] \Bigg\}$$

$$+ \sum_{\ell=\ell_{\rm min}^{\nu}(>1900)}^{\ell_{\rm max}^{\nu}} \frac{1}{g'_\nu(\ell)} \widetilde{G'}_{\ell k} \Bigg\{ \left(\frac{\mathcal{C}_\ell^{D'_\nu} - \mathcal{C}_\ell^{T(i)}}{\mathcal{C}_\ell^{T(i)}} \right) \tanh^2 \left[\frac{\mathcal{C}_\ell^{D'_\nu} - \mathcal{C}_\ell^{T(i)}}{\sigma_\ell^{D_\nu}} \right]^2 \Bigg\}_{\rm binned} \Bigg]$$

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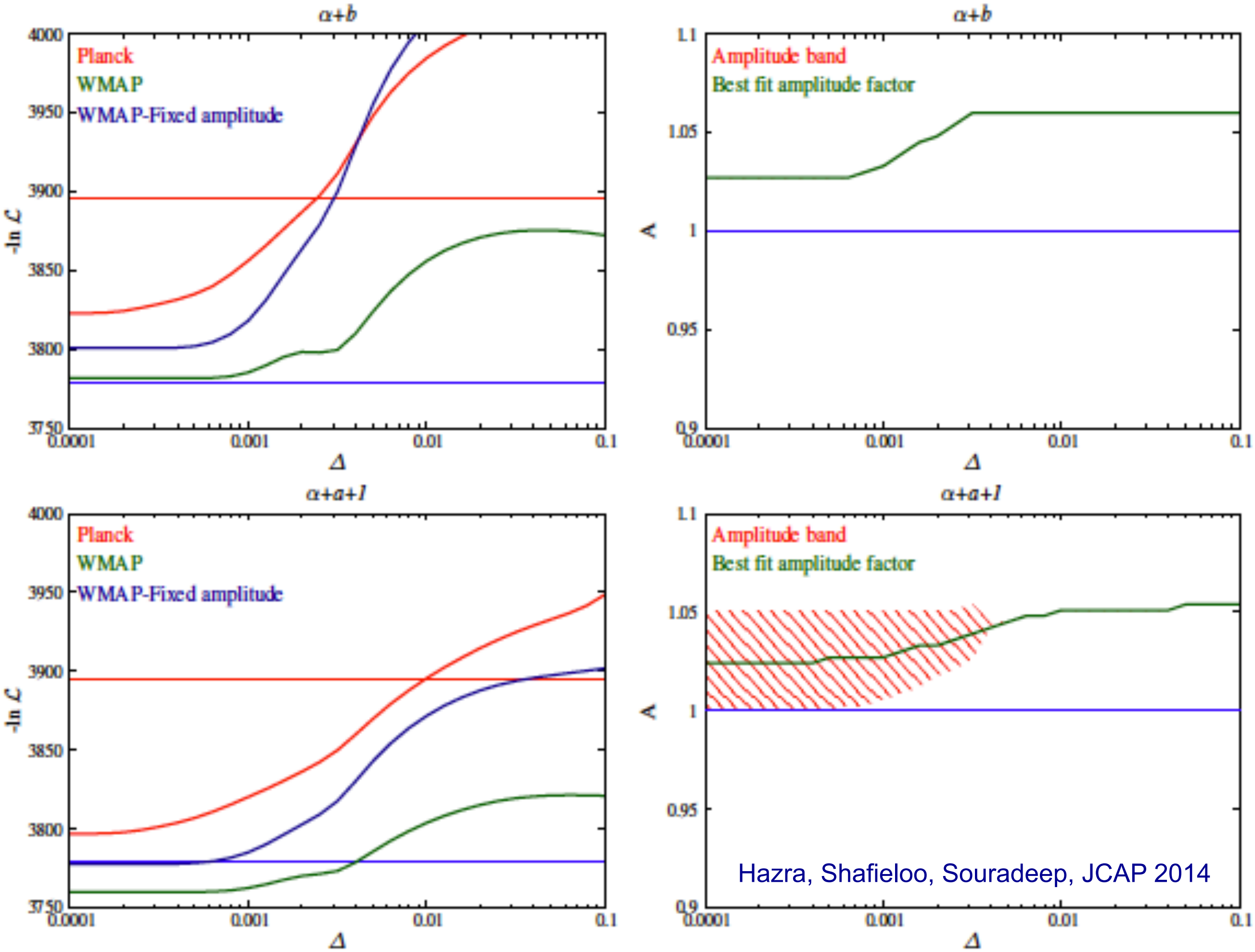

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$$P_k^{(i+1)} - P_k^{(i)} = P_k^{(i)} \times \left[\sum_{\ell=\ell^{\nu}_{\min}}^{\ell^{\nu}_{\max}(\leq 1900)} \frac{1}{g_{\nu}(\ell)} \widetilde{G}_{\ell k} \left\{ \left(\frac{\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}}{\mathcal{C}_\ell^{\mathrm{T}(i)}} \right) \tanh^2 \left[Q_\ell (\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}) \right] \right\} \right.$$

$$\left. + \sum_{\ell=\ell^{\nu}_{\min}(>1900)}^{\ell^{\nu}_{\max}} \frac{1}{g'_{\nu}(\ell)} \widetilde{G'}_{\ell k} \left\{ \left(\frac{\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}}{\mathcal{C}_\ell^{\mathrm{T}(i)}} \right) \tanh^2 \left[\frac{\mathcal{C}_\ell^{\mathrm{D}'_\nu} - \mathcal{C}_\ell^{\mathrm{T}(i)}}{\sigma_\ell^{\mathrm{D}_\nu}} \right]^2 \right\}_{\mathrm{binned}} \right]$$

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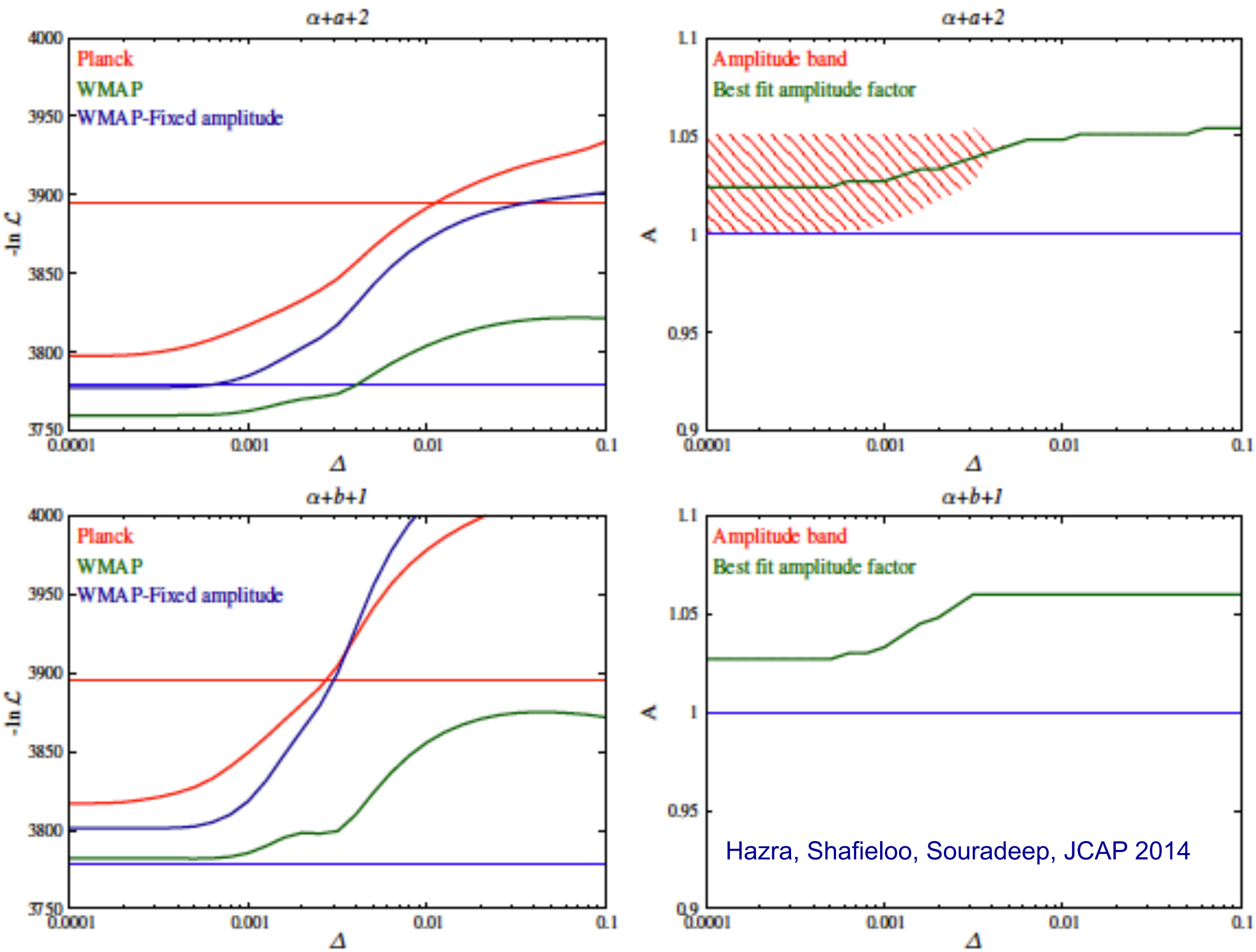

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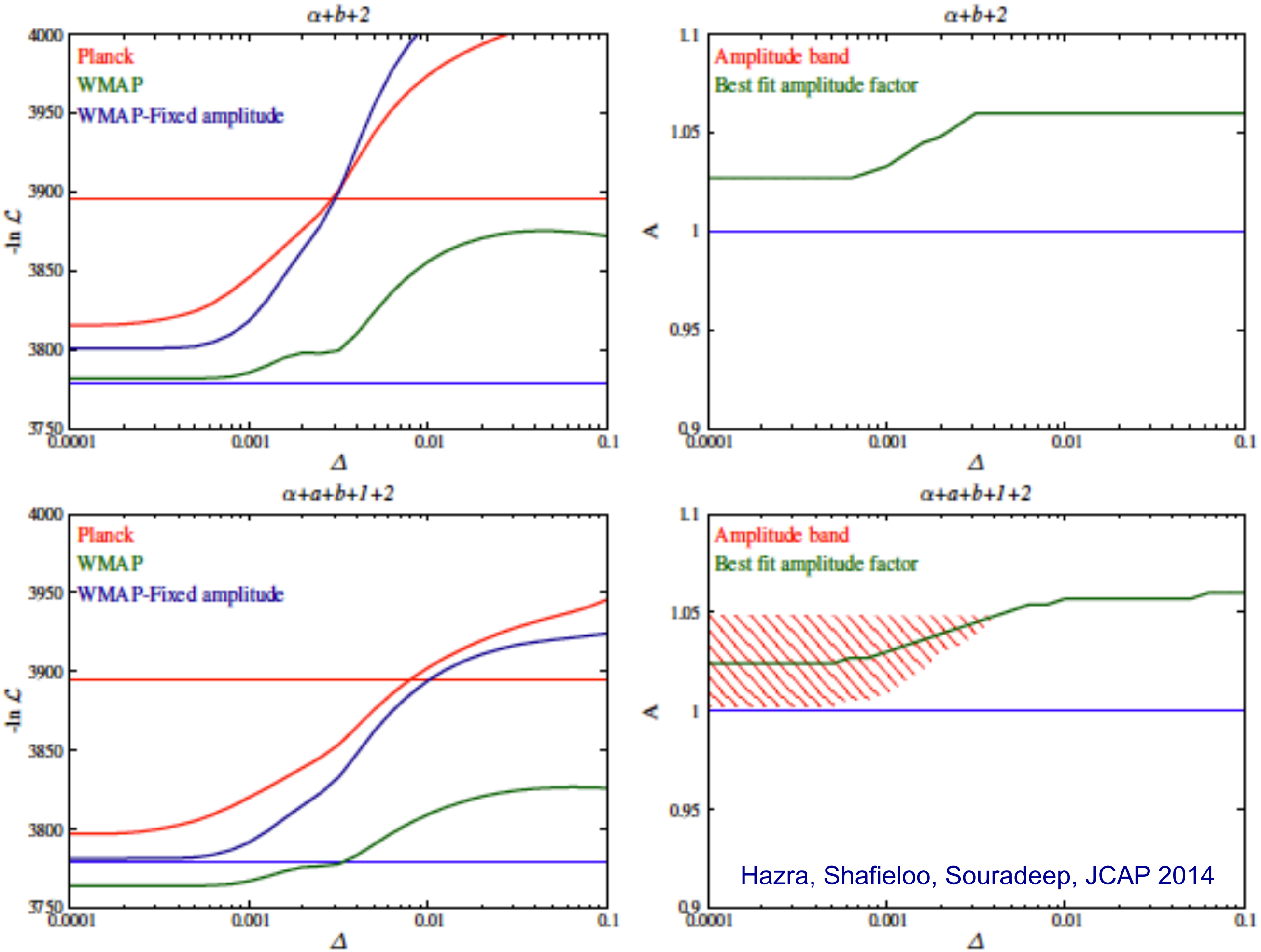
Modified Richardson-Lucy Deconvoloution



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Hazra Shafieloo, Souradeep
JCAP 2014

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Summary

We conclude that there is no clear tension between Planck and WMAP 9 year angular power spectrum data ***allowing the overall amplitude shift***.

While the angular power spectrum from CMB observations is a function of various cosmological parameters, comparing individual parameters might be misleading in the presence of cosmographic degeneracies (they are not orthogonal).

Fixing the amplitudes at the reported values by Planck and WMAP results in an unresolvable tension between the two observations at more than 3σ level which can be a hint towards a serious systematic.

Crossing functions suggest the presence of some broad features in angular spectrum beyond the expectations of the concordance model.

Best fit Crossing functions indicate that there are lack of power in the data at both low-ℓ and high-ℓ with respect to the concordance model. Concordance model of cosmology is consistent to the Planck data only at 2 to 3σ confidence level.

This might be due to random fluctuations or may hint towards smooth features in the primordial spectrum or departure from another aspect of the standard model. This hints that we may need some modifications in the foreground modeling to resolve the significant inconsistency at high-ℓ. However, presence of some systematics at high-ℓ might be another reason for the deviation we found in our analysis (such as the feature at multipole l ~ 1800).

Planck 2013 results. XV. CMB power spectra and likelihood

Planck intermediate results. XVI. Profile likelihoods for cosmological parameters

Planck 2013 results. XXXI. Consistency of the Planck data

On the Coherence of WMAP and Planck TemperatureMaps
A. Kov´acs, J. Carron, I. Szapudi, MNRAS, 436, 1422 (2013) [arXiv:1307.1111]

Test of consistency between Planck and WMAP
D. Hazra, A. Shafieloo, Phys. Rev. D 89, 043004 (2014) [arXiv:1308.2911]

PLANCK DATA RECONSIDERED
D. Spergel, R. Flauger, R. Hlozek, [arXiv:1312.3313]

Confronting the concordance model of cosmology with Planck data
D. Hazra, A. Shafieloo, JCAP 01, 043 (2014) [arXiv:1401.0595]

Primordial power spectrum from Planck
D. Hazra, A. Shafieloo, T. Souradeep, JCAP to be published (2014) [arXiv:1406.4827]

COMPARING PLANCK AND WMAP: MAPS, SPECTRA, AND PARAMETERS
D. Larson, J. L. Weiland, G. Hinshaw , C. L. Bennett, [arXiv:1409.7718]