# Neural Islands: An analytical model of the late stage of reionization

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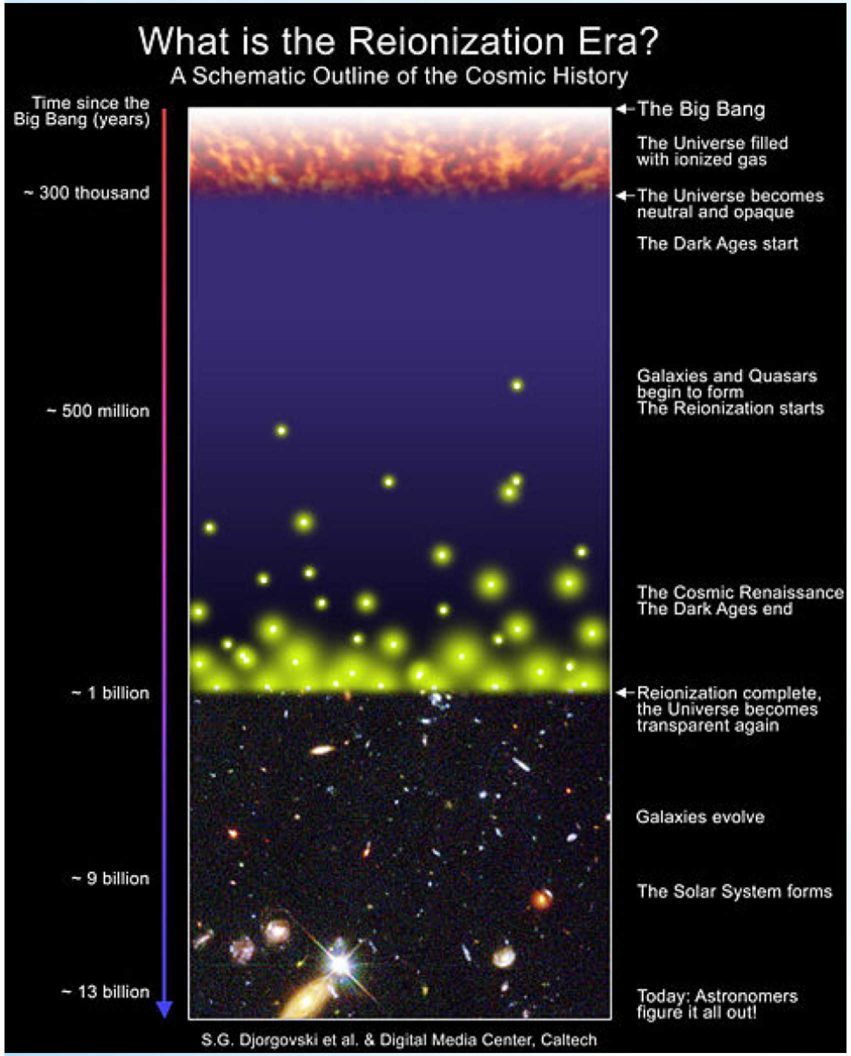
Based on Yidong Xu, Bin Yue, Meng Su, Zuhui Fan, Xuelei Chen:

ApJ 781, 97 (2014)

# Epoch of Reionization




Current Observational Probes

CMB polarization → z ~ 11;

Gunn-Peterson tests → complete at z ~ 6;

Deep Field high-z galaxies

Upcoming & Future:

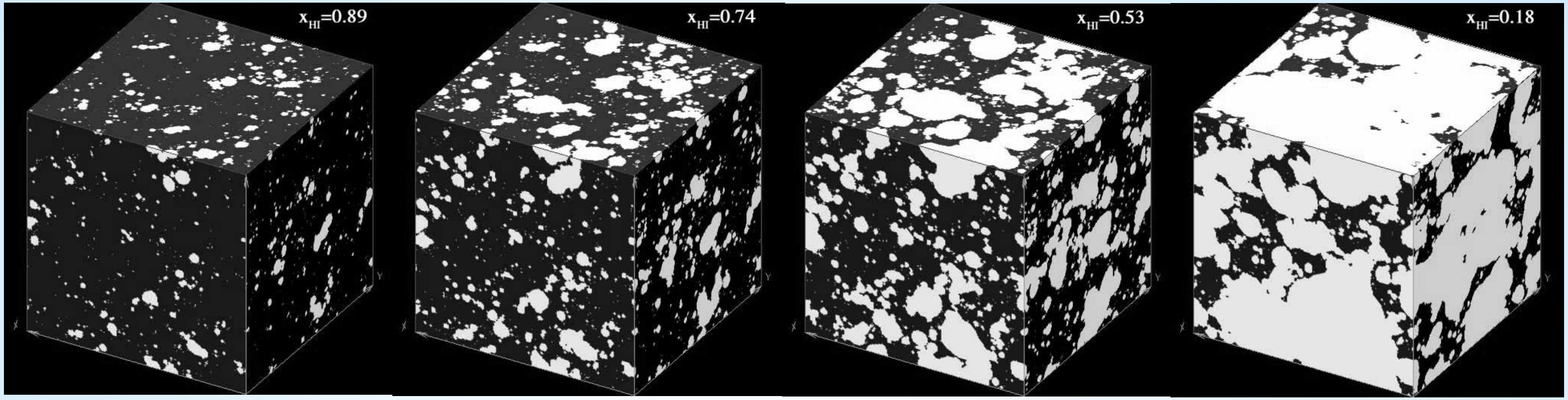
21cm experiments

EDGES, 21CMA, LOFAR< PAPER, LWA, MWA,

LOFAR, HERA, SKA

high-z galaxy observations: JWST, ...

# Theoretical Understanding




(From Mesinger & Furlanetto 2007 ApJ, 669, 663)

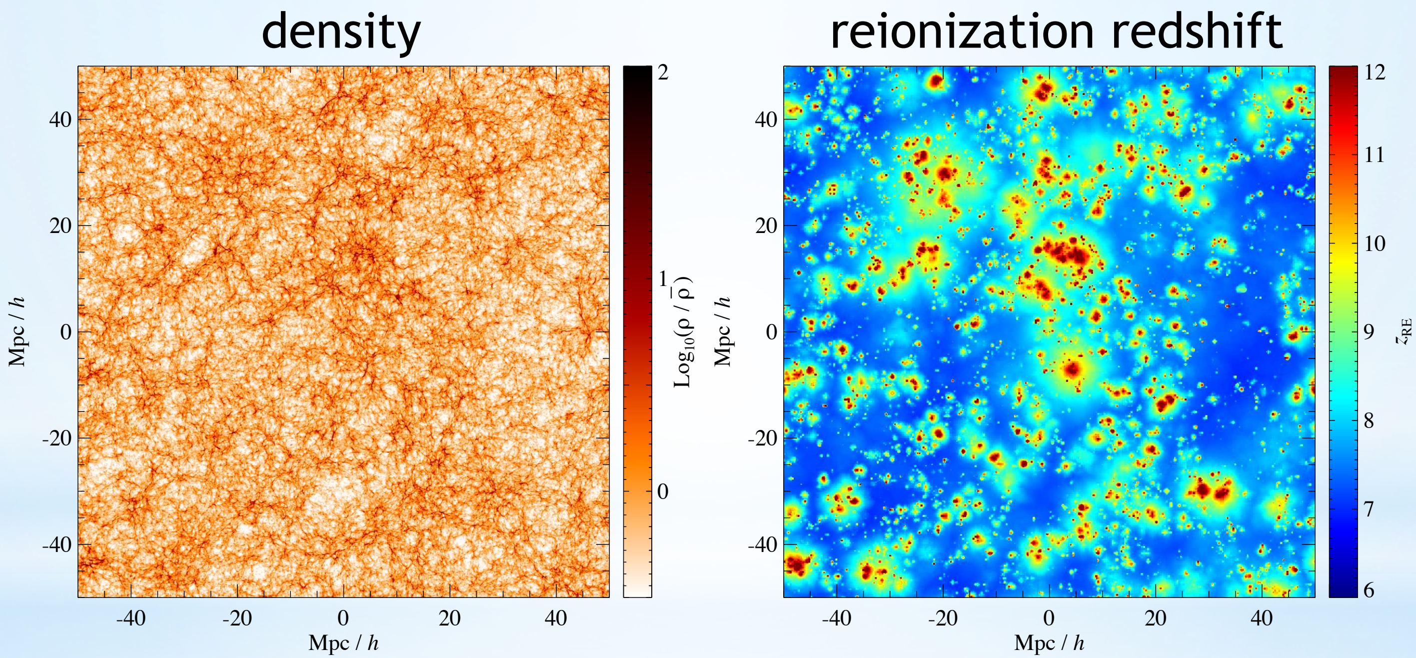
- formation of first stars
- feedback
- formation of first galaxies and blackholes
- subsequent galaxy formation
- radiative transfer

Tools of Investigation:

- Numerical Simulations
- Analytical Model
  bubble model (Furlanetto et al 2004)
- Semi-Numerical Model

# Basis of Analytical Model

***The reionization field follows the density field on large scales***



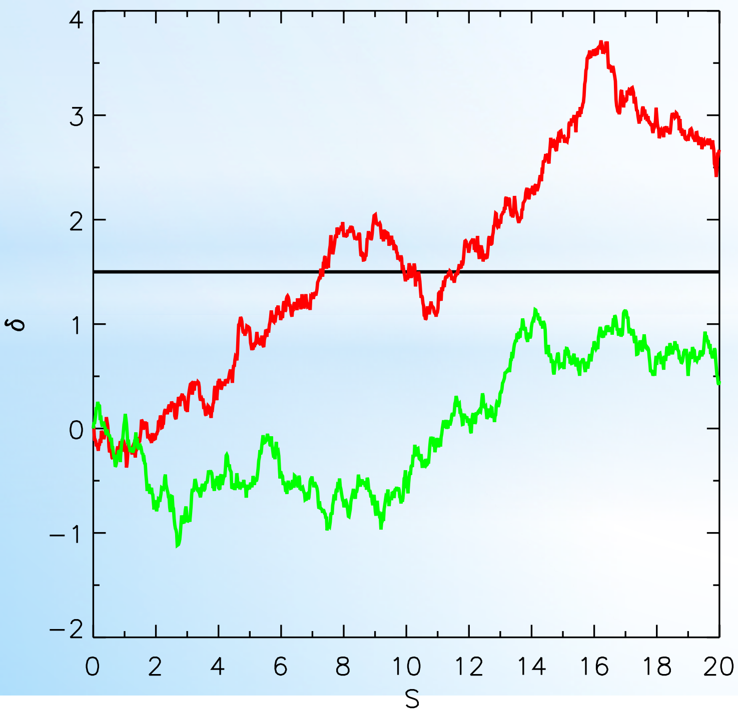

(From Battaglia et al. 2013 ApJ, 776, 81)

# Modeling sturecture growth and halo formation: Excursion Set Theory

(Bond et al. 1991, Lacey & Cole 1993)

The linearly extrapolated density contrast field δ (x, R), $S=\sigma^2$ (R)

k-space top-hat window function




- Each trajectory of δ(S) executes a random walk, halo identified when up crossing a preset barrier
- To solve the *cloud-in-cloud* problem, first up-crossing distribution

$$\Pi(\delta, S+\Delta S) = \int \mathrm{d}(\Delta\delta)\ \Psi(\Delta\delta; \Delta S)\Pi(\delta - \Delta\delta, S).$$

$$\Psi(\Delta\delta; \Delta S)\ \mathrm{d}(\Delta\delta) = \frac{1}{\sqrt{2\pi\Delta S}} \exp\left(-\frac{(\Delta\delta)^2}{2(\Delta S)^2}\right) \mathrm{d}(\Delta\delta)$$

$$\frac{\partial\Pi}{\partial S} = \frac{1}{2}\frac{\partial^2\Pi}{\partial\delta^2}$$

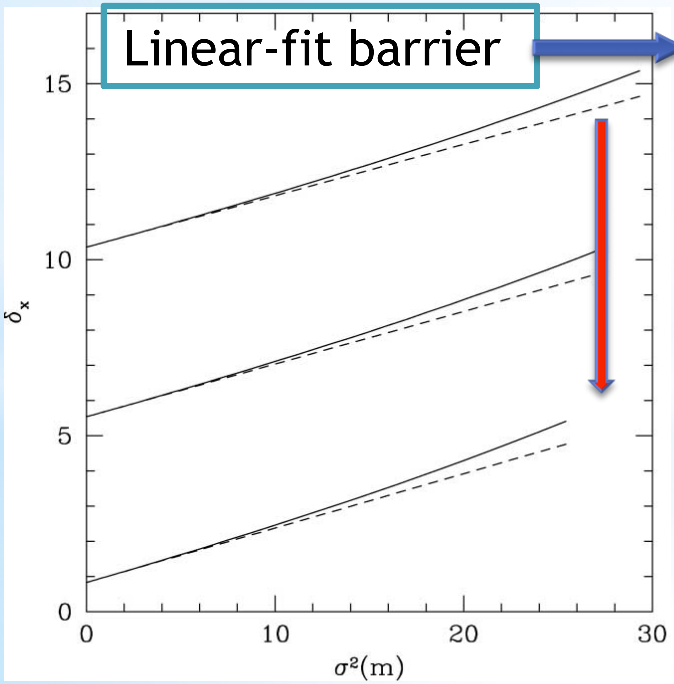
$$\Pi(\delta, S) = \frac{1}{\sqrt{2\pi\Delta S}}\left[\exp\left(-\frac{(\Delta\delta)^2}{2\Delta S}\right) - \exp\left(-\frac{[2(\delta_c - \delta_0) - \Delta\delta]^2}{2\Delta S}\right)\right],$$

# The Excursion Set Approach for ionized bubbles - The bubble model of reionization

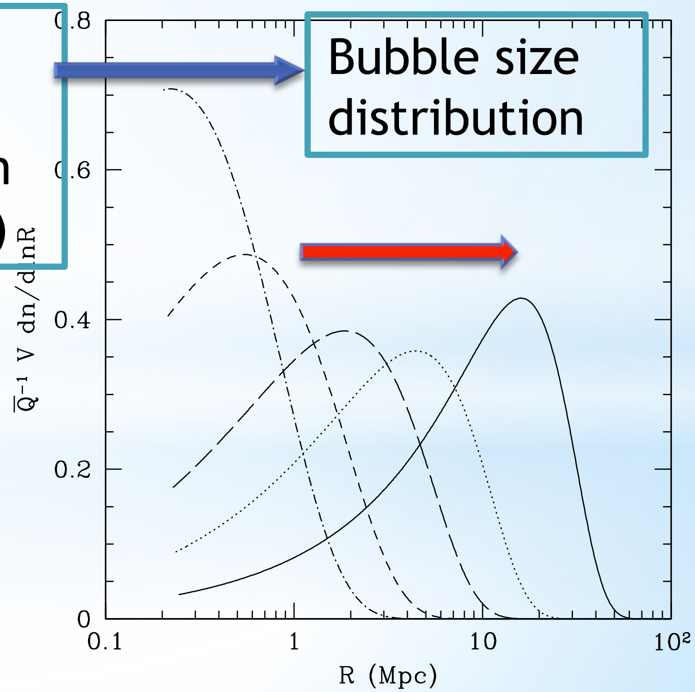
(Furlanetto et al. 2004)

* Relate the ionization field to the initial density field
* Ask whether an isolated region of mass M can be fully self-ionized.

$$f_{\rm coll} \geq f_x \equiv \zeta^{-1}. \longrightarrow \delta_m \geq \delta_x(m,z) \equiv \delta_c(z) - \sqrt{2}K(\zeta)[\sigma^2_{\rm min} - \sigma^2(m)]^{1/2}$$




First-up-crossing distribution (analytical)



# However, after percolation of bubbles…

Late Stage of EoR is interesting, and it may be easier for the upcoming instruments to probe the signal at the late reionization stages.

But: The isolated and spherical assumption for the ionized bubbles breaks down

➔ the neutral islands are more isolated

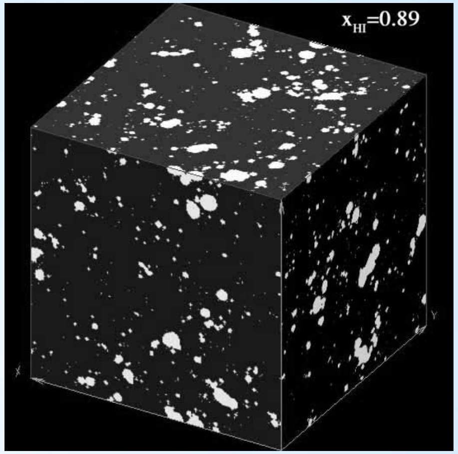
2. The existence of an ionizing background

➔ the shape of barriers could be changed

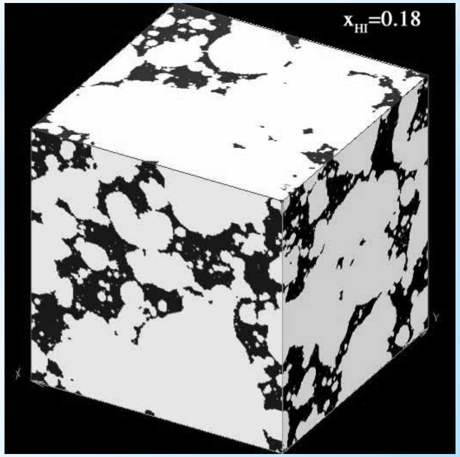
**The island model**

Note: By islands we mean large, uncollapsed regions, the minihalos and galaxies are smaller neutral regions

early




late



# The Island Model

- Negative island barrier ("inside-out" reionization)
- Island mass scales are identified by ***first-down-crossings*** through the island barrier (but not the "never-up-crossing" distribution).
- With the inclusion of an ionizing background, the condition of keeping from being ionized:

$$\xi f_{\rm coll}(\delta_{\rm M}; M, z) + \frac{\Omega_m}{\Omega_b}\frac{N_{\rm back}\, m_{\rm H}}{M X_{\rm H}(1+\bar{n}_{\rm rec})} < 1,$$

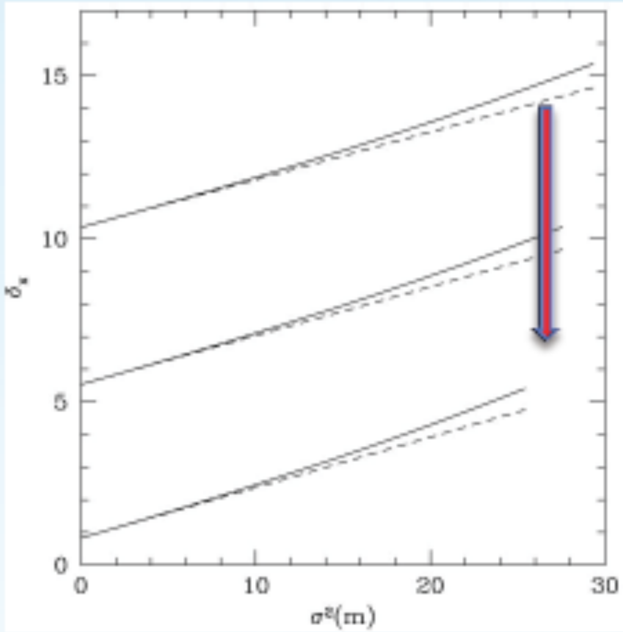
➔ The island barrier:

$$\delta_{\rm M} < \delta_{\rm I}(M,z) \equiv \delta_c(z) - \sqrt{2[S_{\rm max} - S(M)]}\,{\rm erfc}^{-1}\left[K(M,z)\right],$$

$$K(M,z) = \xi^{-1}\left[1 - N_{\rm back}(1+\bar{n}_{\rm rec})^{-1}\frac{m_{\rm H}}{M(\Omega_b/\Omega_m)X_{\rm H}}\right].$$

$N_{\rm back}$: the integral number of background ionizing photons consumed by an island during the time interval between the setup of an ionizing background and the redshift under consideration.

# The Island Model




* Define the ***"background onset time"*** as the time at which the barrier curve passes through the origin point on the δ – S plane

$$\delta_{\rm I}(S=0; z=z_{\rm back}) = \delta_c(z_{\rm back}) - \sqrt{2\,S_{\rm max}(z_{\rm back})}\,{\rm erfc}^{-1}(\xi^{-1}) = 0.$$

* We take $\{f_{\rm esc}, f_\star, N_{\gamma/{\rm H}}, \bar{n}_{\rm rec}\} = \{0.2, 0.1, 4000, 1\}$ as the fiducial set of parameters, so that $\xi = 40$ and $z_{back} = 8.6$.

* Solving for the first-down-crossing distribution (Zhang & Hui 2006):

(the "island-in-island" problem is naturally solved)

$$f_{\rm I}(S_{\rm I}) = -g_1(S_{\rm I}) - \int_0^{S_{\rm I}} {\rm d}S'\, f_{\rm I}(S') \left[g_2(S_{\rm I}, S')\right],$$

$$g_1(S_{\rm I}) = \left[\frac{\delta_{\rm I}(S_{\rm I})}{S_{\rm I}} - 2\frac{{\rm d}\delta_{\rm I}}{{\rm d}S_{\rm I}}\right] P_0[\delta_{\rm I}(S_{\rm I}), S_{\rm I}], \quad P_0(\delta, S) = \frac{1}{\sqrt{2\pi S}} \exp\left(-\frac{\delta^2}{2S}\right)$$

$$g_2(S_{\rm I}, S') = \left[2\frac{{\rm d}\delta_{\rm I}}{{\rm d}S_{\rm I}} - \frac{\delta_{\rm I}(S_{\rm I}) - \delta_{\rm I}(S')}{S_{\rm I} - S'}\right] P_0[\delta_{\rm I}(S_{\rm I}) - \delta_{\rm I}(S'), S_{\rm I} - S'],$$

* The mass function of islands:

$$\frac{{\rm d}n}{{\rm d}\ln M_{\rm I}}(M_{\rm I}, z) = \bar{\rho}_{\rm m,0} f_{\rm I}(S_{\rm I}, z) \left|\frac{{\rm d}S_{\rm I}}{{\rm d}M_{\rm I}}\right|.$$

* The volume fraction of neutral regions:

$$\mathbf{Q}_{\rm V}^{\rm I} = \int {\rm d}M_{\rm I}\, \frac{{\rm d}n}{{\rm d}M_{\rm I}}\, V(M_{\rm I}).$$

# A toy model - island-V

- The ionizing photons permeated through the neutral islands with a uniform density (e.g. all X-rays)
  - Extremely large mean free path
  - Neglecting the absorption by dense clumps

- The averaged number density of the background ionizing photons

$$n_\gamma = \bar{n}_{\rm H}\, f_{\rm coll}(z)\, f_\star\, N_{\gamma/{\rm H}}\, f_{\rm esc} - (1 - Q_{\rm V}^{\rm I})\, \bar{n}_{\rm H}\, (1 + \bar{n}_{\rm rec}),$$

# A toy model - island-V

$$\delta_{\rm M} < \delta_{\rm I}(M,z) \equiv \delta_c(z) - \sqrt{2[S_{\rm max} - S(M)]}\,{\rm erfc}^{-1}\left[K(M,z)\right],$$

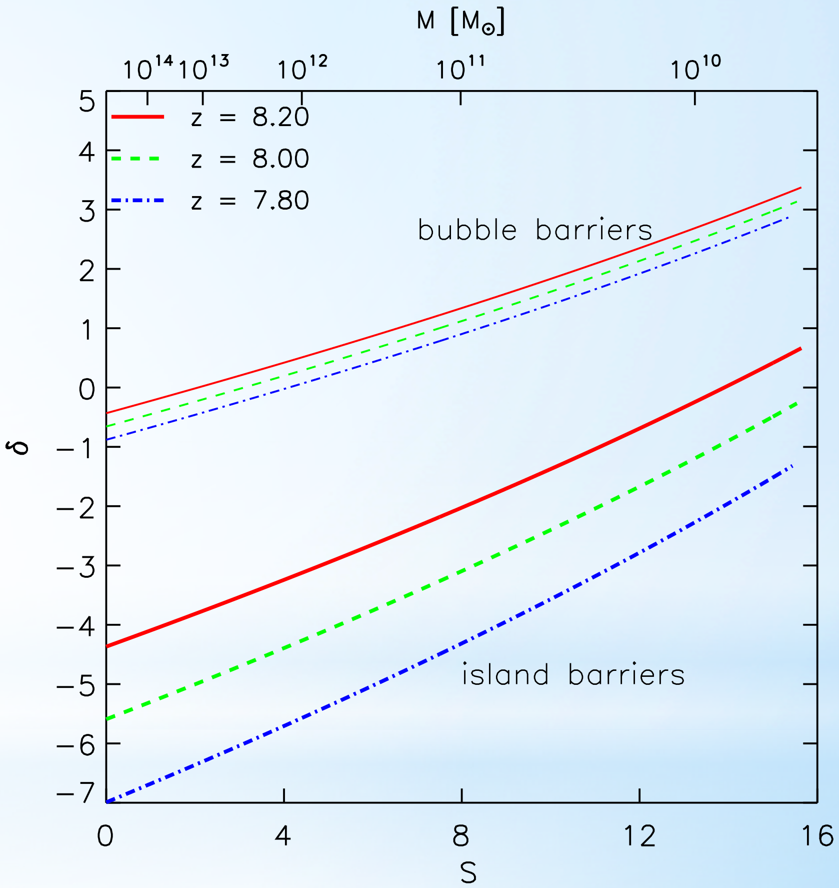
$$K(M,z) = \xi^{-1}\left[1 - N_{\rm back}(1+\bar{n}_{\rm rec})^{-1}\frac{m_{\rm H}}{M(\Omega_b/\Omega_m)X_{\rm H}}\right].$$

- $N_{back}$ proportional to V

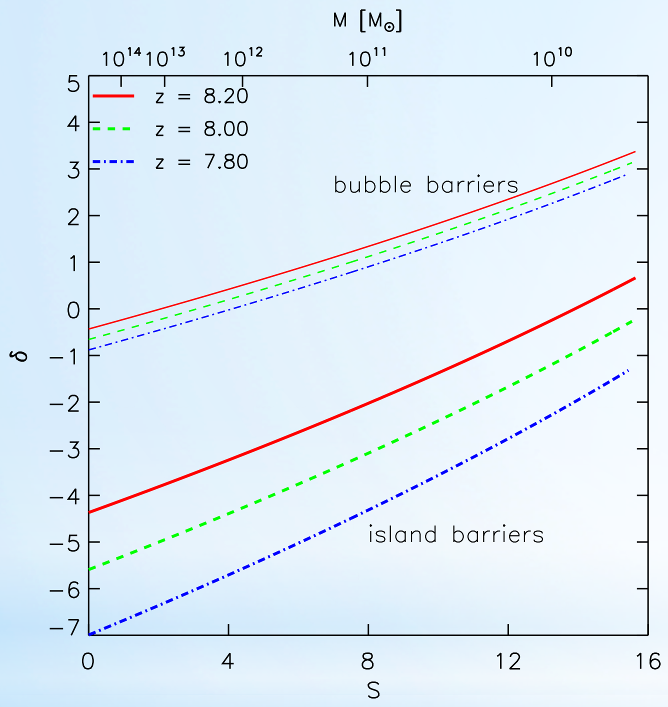
$$N_{\rm back}/M \;=\; n_\gamma/\bar{\rho}_{\rm m}.$$

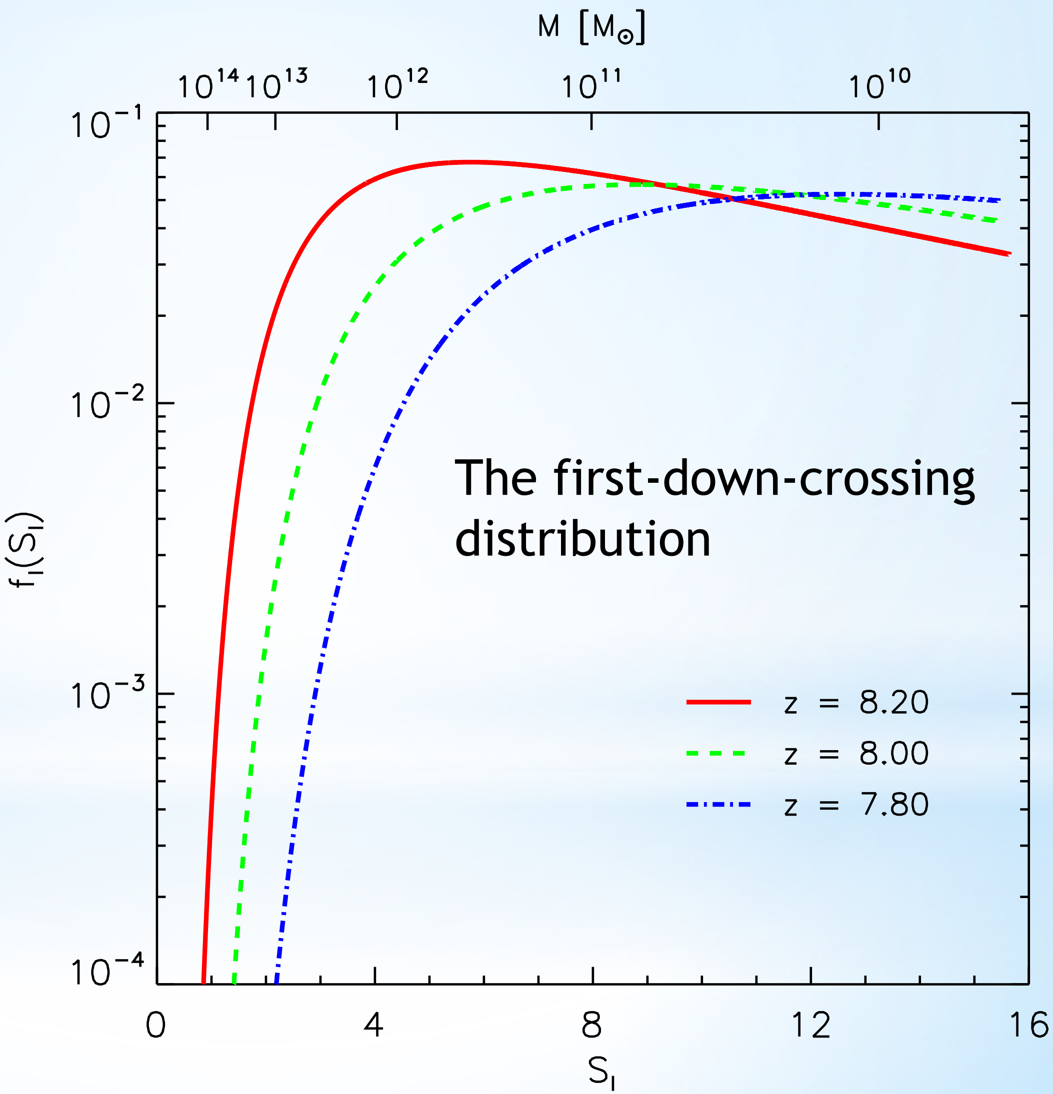
↓

$$\delta_{\rm I}(M,z) = \delta_c(z) - \sqrt{2\,[S_{\rm max} - S(M)]}\,{\rm erfc}^{-1}\left[K(z)\right]$$



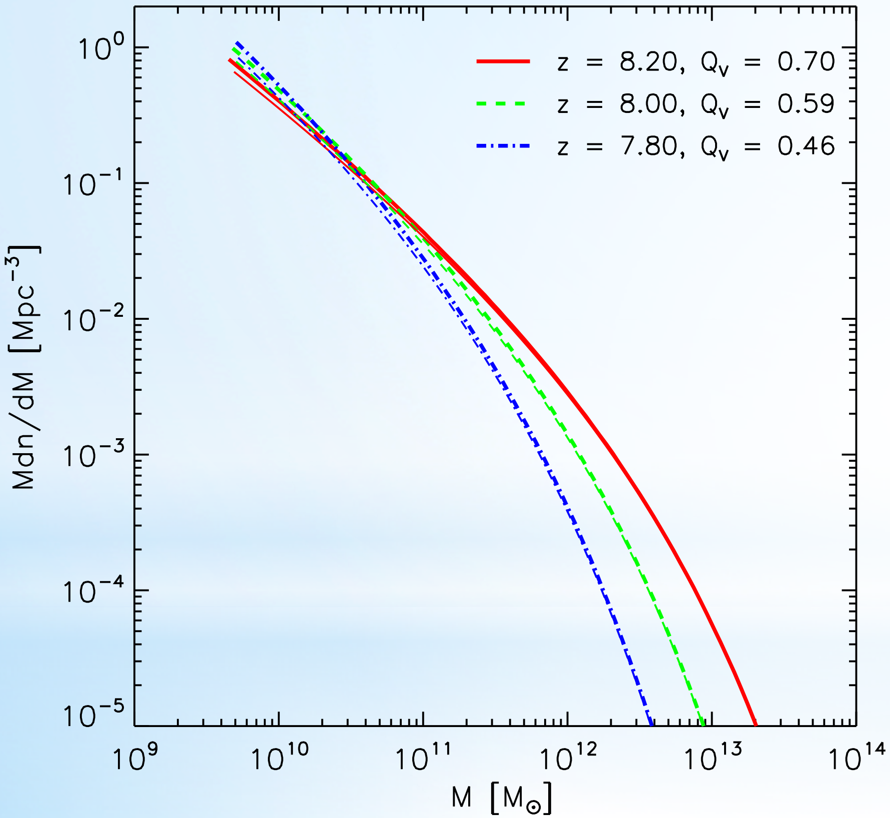
# A toy model - island-V



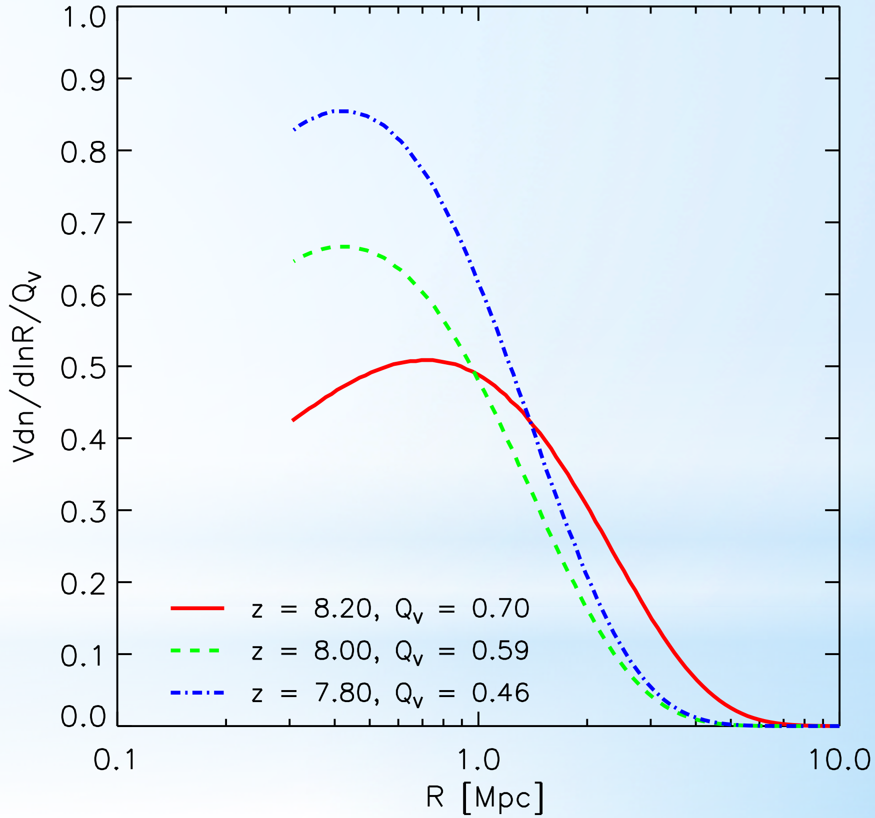



# A toy model - island-V model

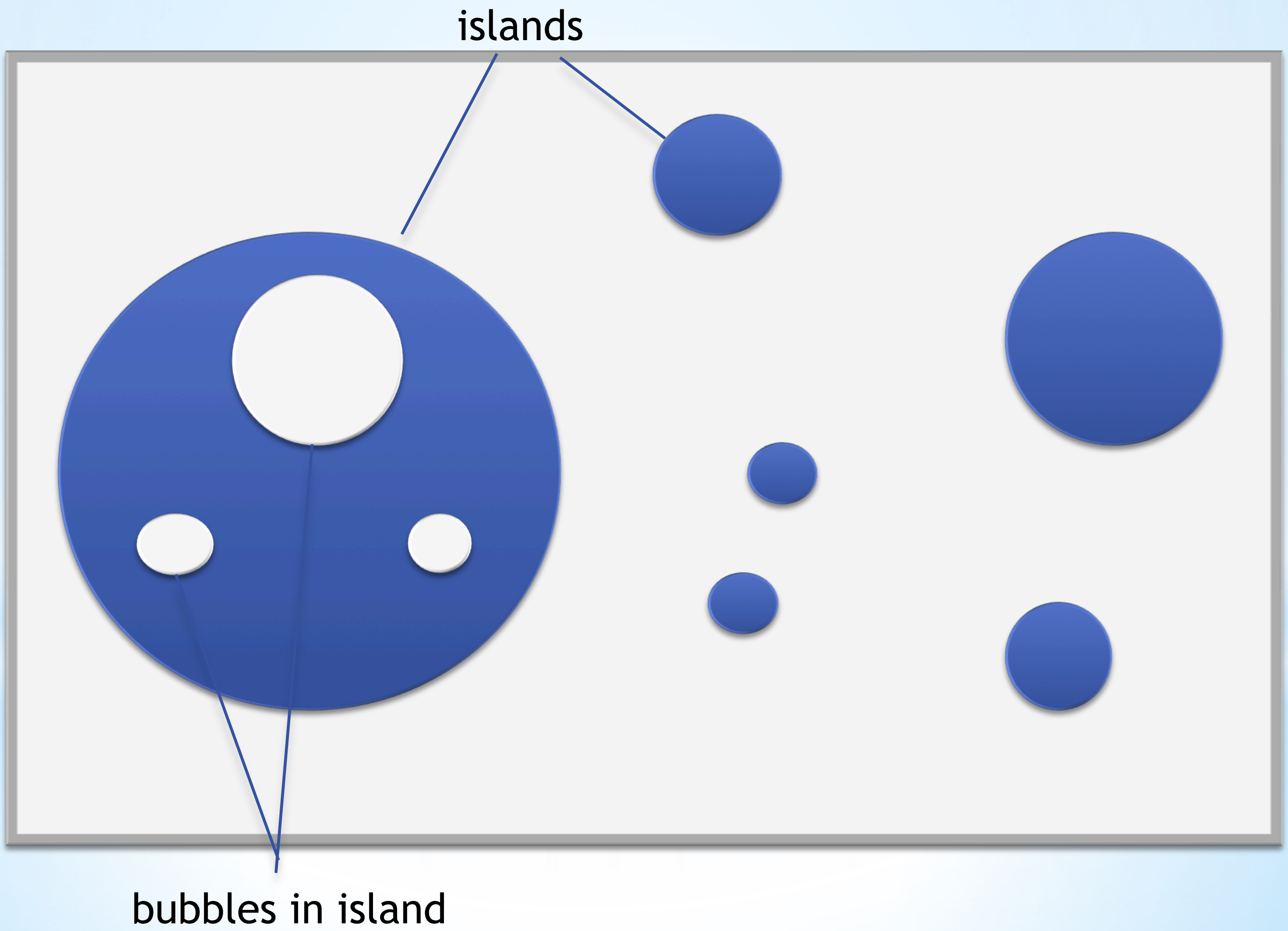
Mass function of islands



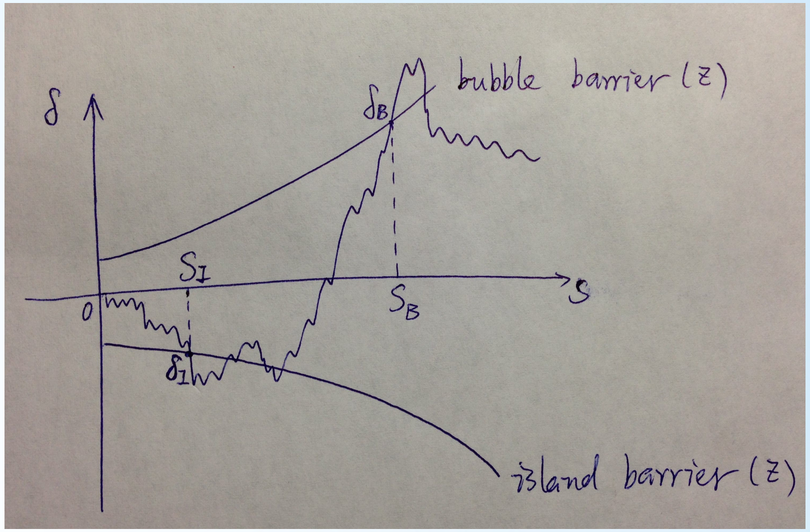

Size distribution of islands



# Complication: bubbles in island



# Modeling the bubbles-in-island effect




* Solving for a two-barrier problem:

1 - The first down-crossing distribution of random walks w.r.t. ***island barrier***:

$$f_{\rm I}(S_{\rm I}, z)$$

2 - The conditional first up-crossing distribution w.r.t. ***bubble barrier***: $f_{\rm B}[S_{\rm B}, \delta_{\rm B}|S_{\rm I}, \delta_{\rm I}]$

* The effective bubble barrier:

$$\delta'_{\rm B} = \delta_{\rm B}(S + S_{\rm I}) - \delta_{\rm I}(S_{\rm I}) \qquad S = S_{\rm B} - S_{\rm I}.$$

# The bubbles-in-island effect

* The bubbles-in-island fraction:

$$q_{\rm B}(S_{\rm I},\delta_{\rm I};z) = \int_{S_{\rm I}}^{S_{\rm max}(\xi\cdot M_{\rm min})} [1+\delta_{\rm I}\, D(z)]\;\; f_{\rm B}[S_{\rm B},\delta_{\rm B}|S_{\rm I},\delta_{\rm I}]\;{\rm d}S_{\rm B}.$$

* The neutral island mass function:

$$\frac{{\rm d}n}{{\rm d}M}(M,z) = \frac{{\rm d}n}{{\rm d}M_{\rm I}}\frac{{\rm d}M_{\rm I}}{{\rm d}M} = \frac{\bar{\rho}_{\rm m,0}}{M_{\rm I}} f_{\rm I}(S_{\rm I},z)\left|\frac{{\rm d}S_{\rm I}}{{\rm d}M_{\rm I}}\right|\frac{{\rm d}M_{\rm I}}{{\rm d}M}.$$

$$M = M_{\rm I}(S_{\rm I})\,[1 - q_{\rm B}(S_{\rm I},\delta_{\rm I};z)]$$

# The ionizing background

* Considering the effect of ***Lyman limit systems*** on the mean free path of ionizing photons, the comoving number density of background ionizing photons is

$$n_\gamma(z) \;=\; \int_z \bar{n}_{\rm H} \left| \frac{{\rm d}f_{\rm coll}(z')}{{\rm d}z'} \right| f_\star\, N_{\gamma/{\rm H}}\, f_{\rm esc}\, \exp\left[ -\frac{l(z,z')}{\lambda_{\rm mfp}(z)} \right] {\rm d}z',$$

* With the MHR00 model for the volume-weighted density distribution of the IGM (Miralda-Escude et al. 2000),

$$P_{\rm V}(\Delta)\, d\Delta = A_0 \exp\left[ -\frac{(\Delta^{-2/3} - C_0)^2}{2\,(2\delta_0/3)^2} \right] \Delta^{-\beta}\, d\Delta$$

the mean free path of ionizing photons can be written as

$$\lambda_{\rm mfp} \;=\; \frac{\lambda_0}{[1 \,-\, F_{\rm V}(\Delta_{\rm crit})]^{2/3}},$$

# The ionizing background

* The critical relative density for a clump to self-shield

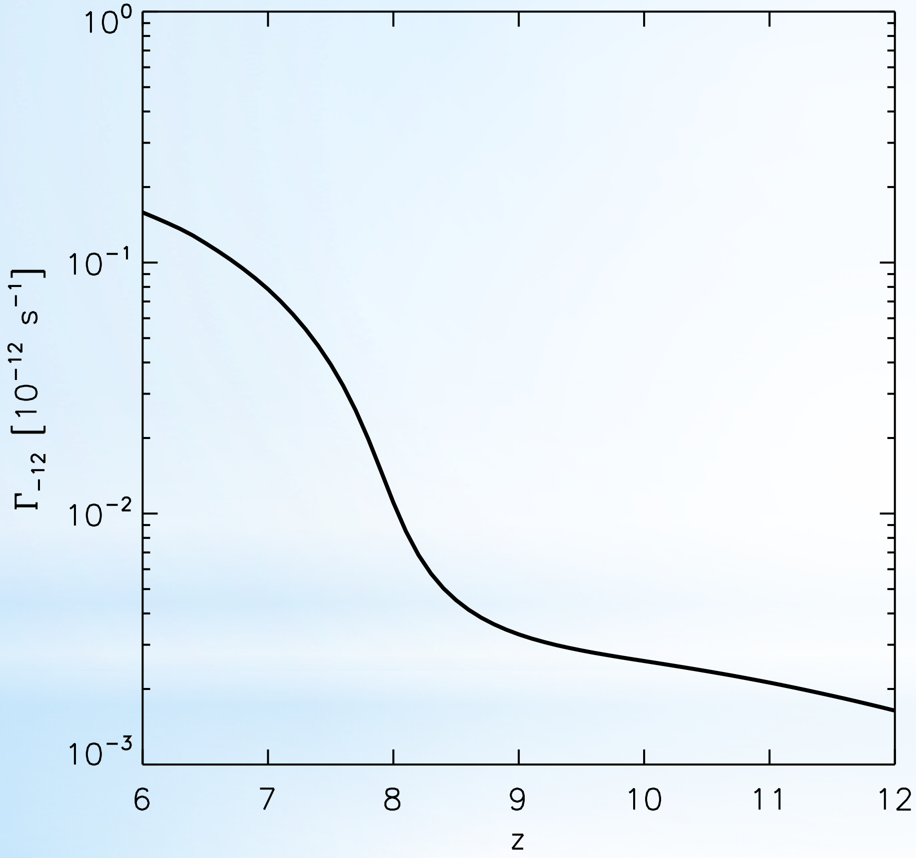
$$\Delta_{\rm crit} = 36\,\Gamma_{-12}^{2/3}\,T_4^{2/15}\left(\frac{\mu}{0.61}\right)^{1/3}\left(\frac{f_e}{1.08}\right)^{-2/3}\left(\frac{1+z}{8}\right)^{-3}$$

* The HI photoionization rate $\Gamma_{HI}$ is related to the total number density of ionizing photons $n_\gamma$ by

$$\Gamma_{\rm HI} = \int \frac{{\rm d}n_\gamma}{{\rm d}\nu}(1+z)^3\,c\,\sigma_\nu\,{\rm d}\nu,$$

* Scaling the hydrogen photoionization rate to be $\Gamma_{HI} = 10^{-12.8}\ s^{-1}$ at redshift 6, as suggested by recent measurements from the Ly-α forest (Wyithe & Bolton 2011; Calverley et al. 2011)

# The ionizing background




Consistent with our definition of the "background onset time"

# The island-vS model (nearly realistic)

* We assume that the photons consumed by an island at any instant is proportional to its surface area, then

$$N_{\rm back} = \int F(z)\,\Sigma_{\rm I}(t)\,{\rm d}t, \qquad F(z) = n_\gamma(z)\,(1+z)^3\,c/4.$$

* For a spherical island,

$$n_{\rm H}(R)(1+\bar{n}_{\rm rec})\,4\pi R^2\,(-{\rm d}R) = F(z)\,\frac{4\pi R^2}{(1+z)^2}\,{\rm d}t,$$

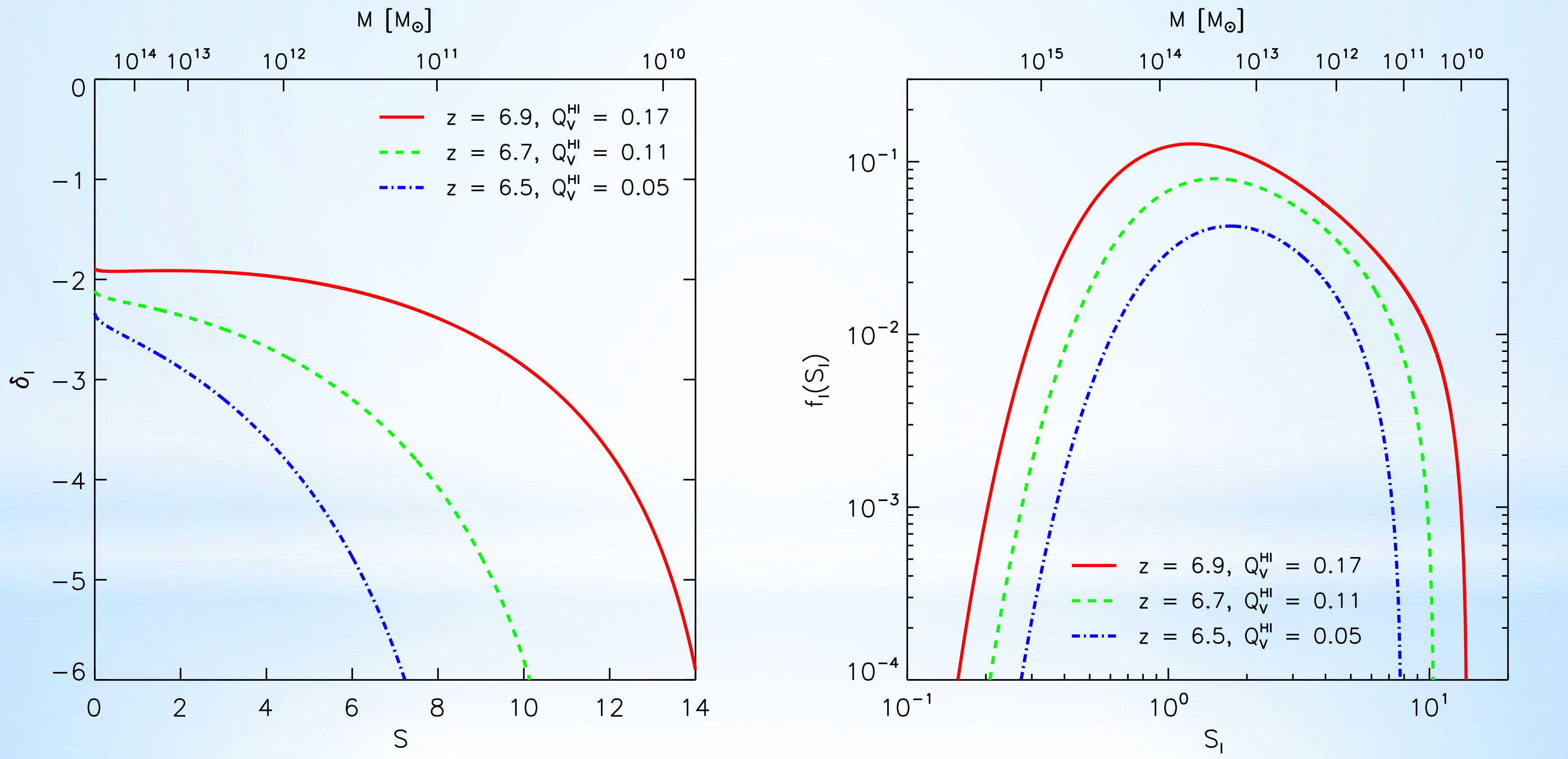
$$\Delta R \equiv R_i - R_f = \int_z^{z_{\rm back}} \frac{F(z)}{\bar{n}_{\rm H}(1+\bar{n}_{\rm rec})}\,\frac{{\rm d}z}{H(z)(1+z)^3}.$$

* The total number of background ionizing photons consumed is

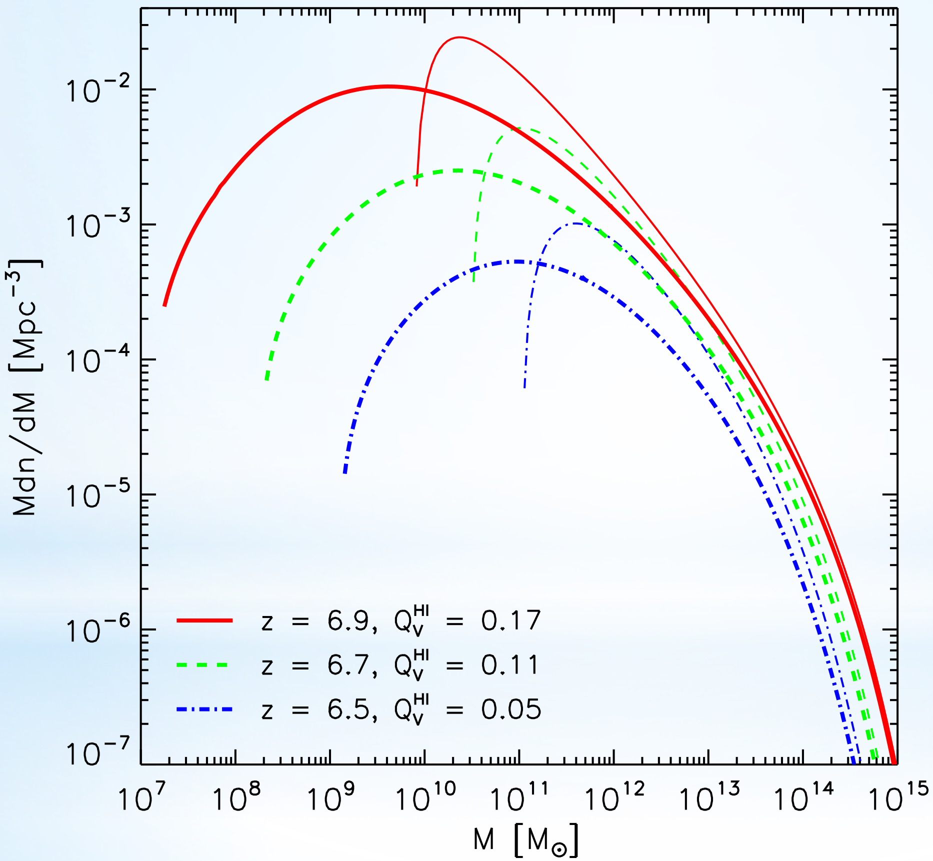
$$N_{\rm back} = \frac{4\pi}{3}\left(R_i^3 - R_f^3\right)\bar{n}_{\rm H}(1+\bar{n}_{\rm rec}),$$

# The island-vS model

## - the island barrier and first down-crossing distribution



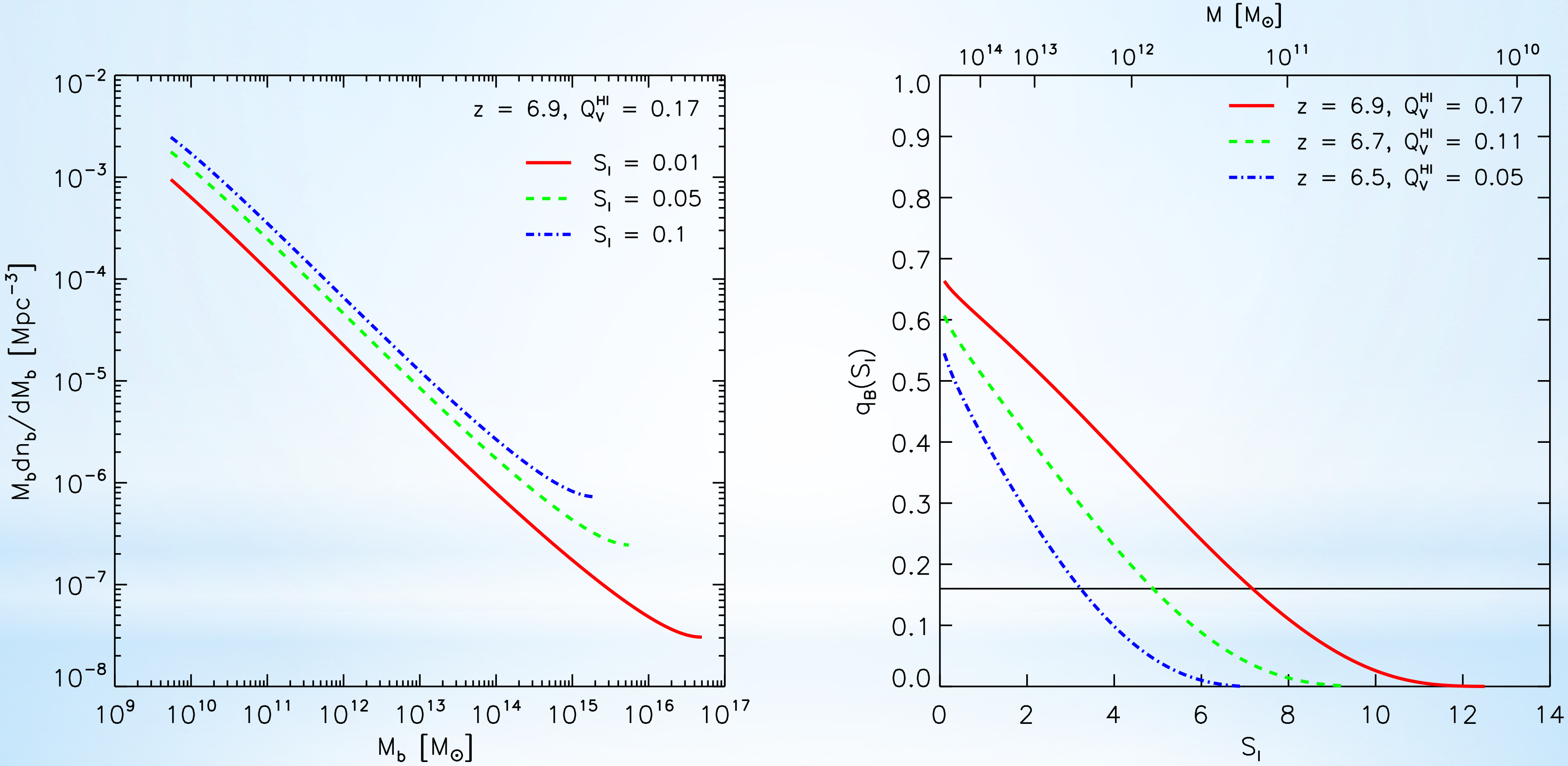
# The island-vS model
## - the host island (including bubbles) mass function



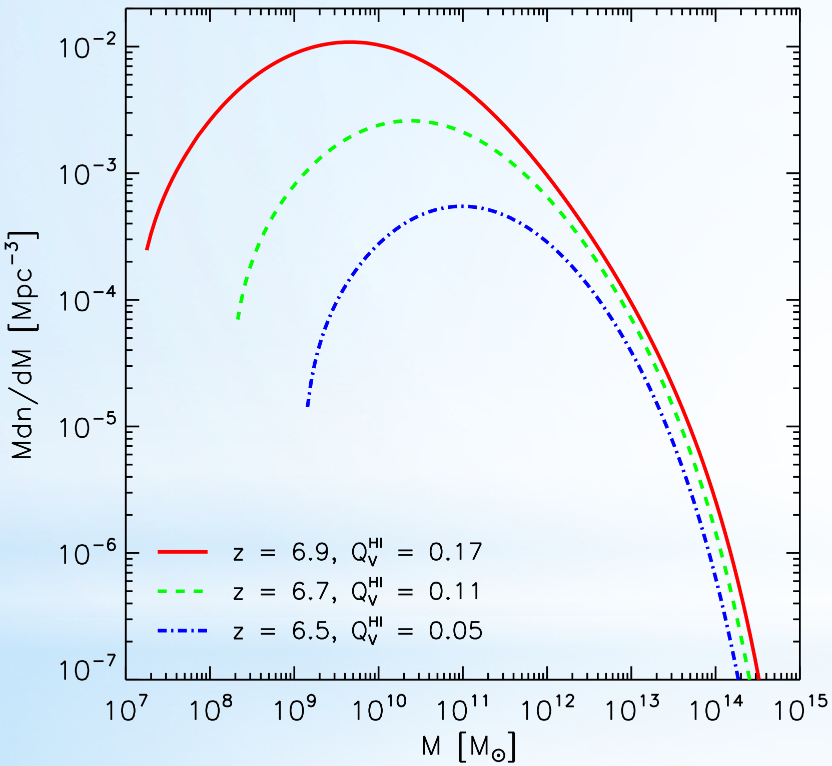

- The shrinking hosts

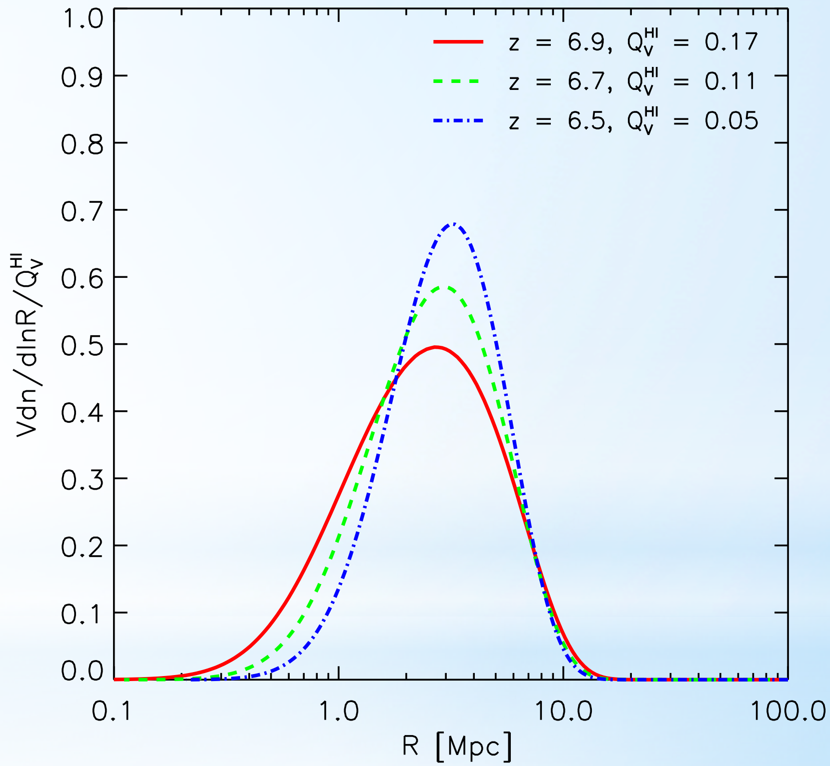
$$\frac{M_f}{M_i} = (1 - \frac{\Delta R}{R_i})^3$$

# The island-vS model
## - the bubbles-in-island



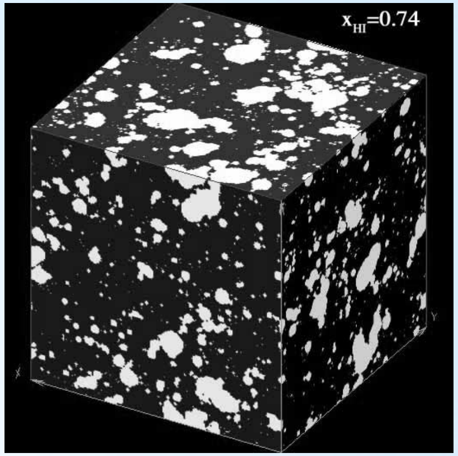
# The island-vS model
## - the mass function and size distribution

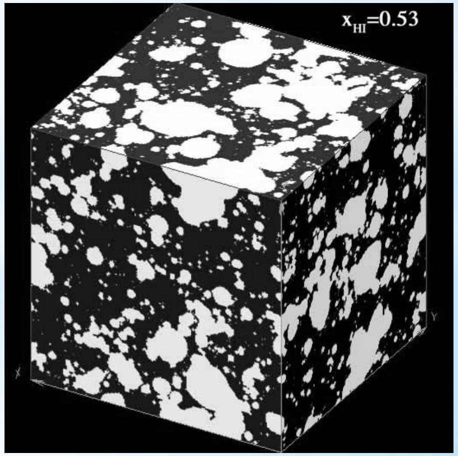


# The problem of large bubbles-in-island fraction

* Host islands → overestimate the neutral fraction
* Neutral island → atoll or smaller islands?
* Difficult to visually identify the host islands
* Break down of bubble model inside islands
* Percolation of islands



# percolation problem

percolation threshold $p_c$

* The bubble model regime:

  $z > z_{Bp}$ ($x_{HII} < p_c$)

* The island model regime:

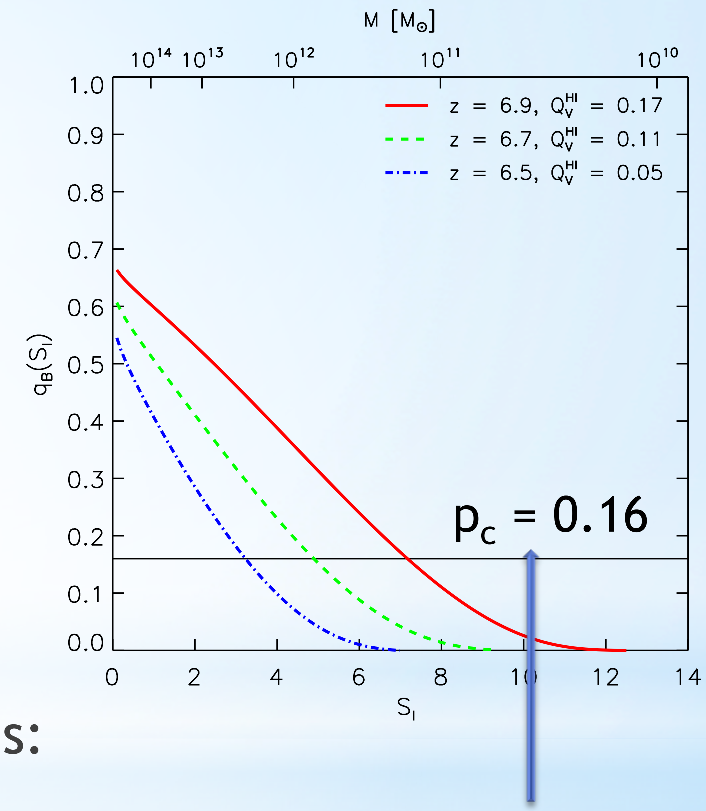
  $z < z_{Ip}$ ($x_{HI} < p_c$)

* The background onset redshift:

  $z_{Bp} > z_{back} > z_{Ip}$

* The definition of bona fide neutral islands:

  $q_B < p_c$



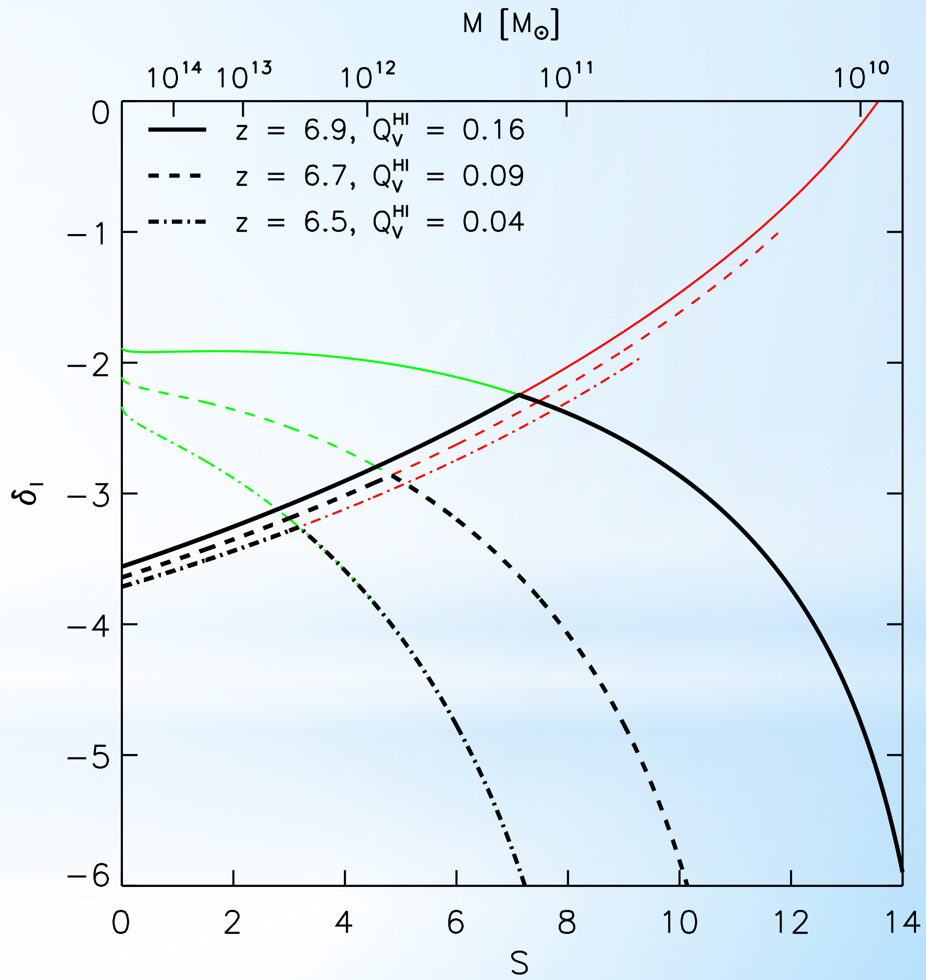

for Gaussian random fields

# The percolation criterion:

* Limit ourselves to the valid (not percolated) case
* Find bona fide neutral islands
* The additional barrier is obtained by solving

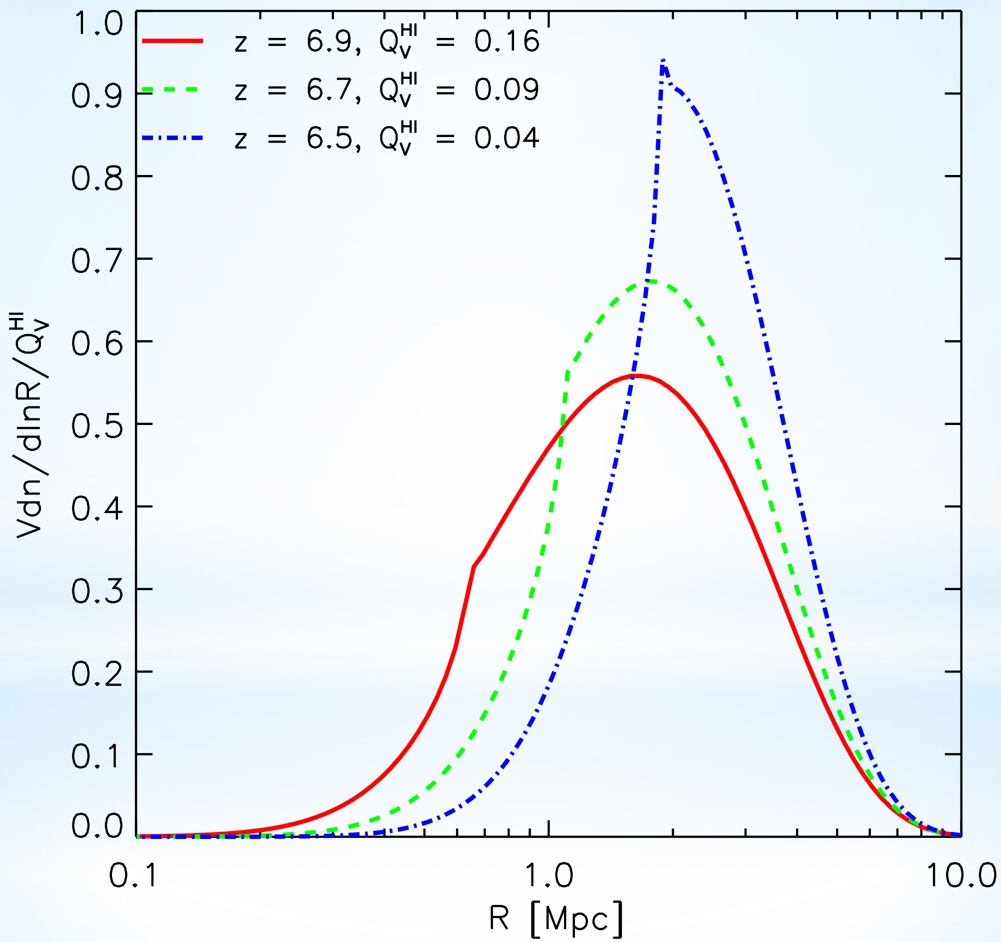
  $q_B(S_I, \delta_I; z) < p_c$

* $p_c = 0.16$ for Gaussian random fields



# Results

## - the size distribution with $p_c$ cutoff



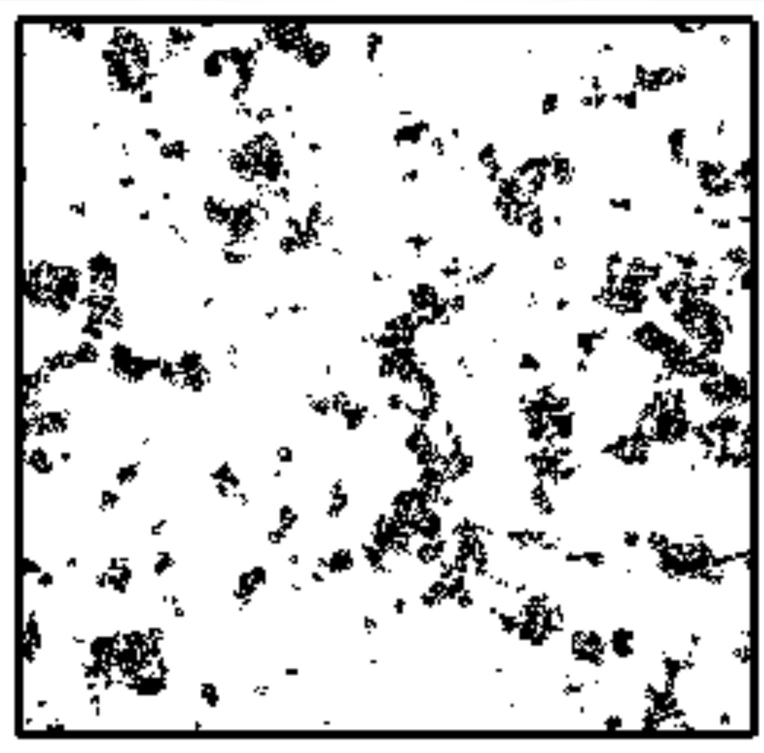
# Summary

An analytical model of neutral islands during the late EoR based on the excursion set theory, to help understand reionization process

* An island barrier on the density contrast for the islands to remain neutral, with the inclusion of an ionizing background.
* An island was identified when the random walk first-down-crosses the island barrier.
* We took into account the effect of bubbles-in-island by computing the conditional first up-crossing distribution.
* An semi-empirical way to determine the intensity of the ionizing background self-consistently.
* A percolation criterion was applied to find bona fide neutral islands

* The size distribution of neutral islands shows a peak indicating a characteristic scale of the islands, and it does not change much with redshift.

# THANK YOU!



512 Mpc, $1024^3$