Searching for general relativistic signatures on large scales

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6th KIAS Workshop on Cosmology and Structure Formation
KIAS, Seoul, Korea
3rd November, 2014

Based on

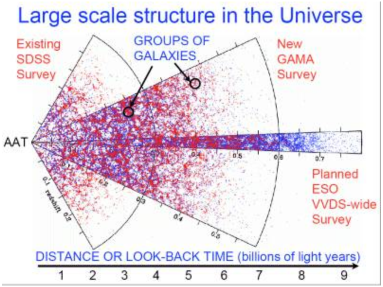
- D. Jeong, JG, H. Noh and J.-c. Hwang, Astrophys. J. 727, 22 (2011)
- S. G. Biern, JG and D. Jeong, Phys. Rev. D89, 103523 (2014)
- J.-c. Hwang, H. Noh, D. Jeong, JG and S. G. Biern, arXiv:1408.4656 [astro-ph.CO]]

Outline

1. Introduction
2. Non-linear correlation functions
   - Setup
   - Comoving gauge
   - Synchronous gauge
3. Conclusions

Why GR in LSS?

Planned galaxy surveys: DESI, HETDEX, LSST, Euclid...



Larger and larger volumes, eventually accessing the scales comparable to the horizon: beyond Newtonian gravity, fully general relativistic approach (or any modification) is necessary

Why non-linearity and gauge in LSS?

- Non-linearity is prominent in large scale structure thus accurate modeling of of non-linearity is very important
- GR is a gauge theory, thus observational quantities only make sense after choosing the coordinate systems

On large scales where non-linearity can be probed by observations with improved accuracy, density contrast $\delta \equiv (\rho - \rho_0)/\rho_0$ deviates the Newtonian prediction

Q: how the deviations appear on large scales at non-linear level?

Setup and strategy

We consider scalar metric pert in Einstein-de Sitter universe

$$g_{00} = -(1+2\alpha)dt^2\,, \quad g_{0i} = -a\beta_{,i}\,, \quad g_{ij} = a^2\left[(1+2\varphi)\delta_{ij} + \gamma_{,ij}\right]$$

The dynamical equations to be solved are:

Energy conservation eq → Continuity eq

Trace of the Einstein eq → Euler eq

We identify the perturbation variables as

$$\delta \equiv \frac{\rho - \rho_0}{\rho_0} \quad \text{with} \quad \rho \equiv -T^0{}_0$$

$$\theta \equiv \frac{\nabla \cdot \boldsymbol{u}}{a} = 3H - K^i{}_i$$

Non-linear perturbations

With the linear solution the same as the standard one

$$\delta_1(\boldsymbol{k},t) = D(t)\delta_1(\boldsymbol{k},t_0)$$

we expand $\delta = \delta_1 + \delta_2 + \cdots$ using symmetric kernels

$$\delta(\boldsymbol{k},t) = \sum_{n=1}^{\infty} D^n(t) \int \frac{d^3d_1 \cdots d^3q_n}{(2\pi)^{3(n-1)}} \delta^{(3)}\left(\boldsymbol{k} - \boldsymbol{q}_1 - \cdots \boldsymbol{q}_n\right)$$
$$\times F_n(\boldsymbol{q}_1, \cdots \boldsymbol{q}_n)\delta_1(\boldsymbol{q}_1)\cdots\delta_1(\boldsymbol{q}_n)$$

Then correlation functions are

$$\langle\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{12})P(k_1) \quad \text{with} \quad P = P_{11} + \underbrace{P_{22} + P_{13}}_{\text{1-loop}} + \cdots$$

$$\langle\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\delta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})B(k_1,k_2,k_3)$$
$$\text{with} \quad B = B_{112} + \underbrace{B_{222} + B_{123} + B_{114}}_{\text{1-loop}} + \cdots$$

Comoving gauge

We set the gauge condition as

$$\gamma = 0 \text{ (fixing the spatial gauge) and } T^0{}_i = 0 \text{ (temporal gauge)}$$

Kernels are found to be:

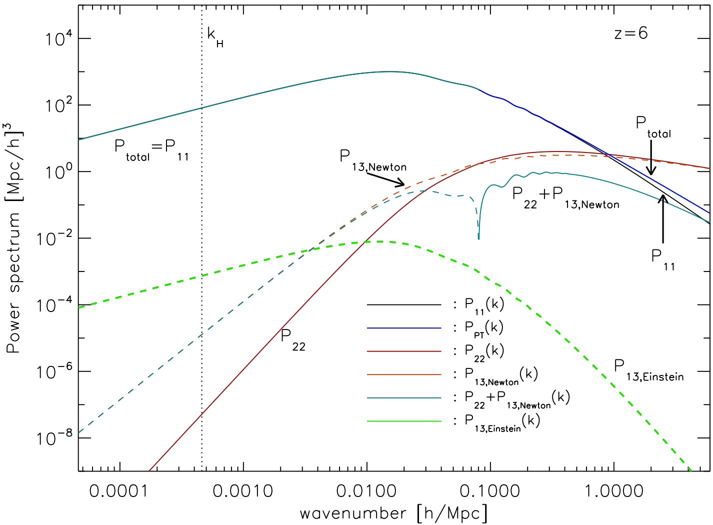
$$F_2 = \frac{5}{7} + \frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{2q_1 q_2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{2}{7}\left(\frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1 q_2}\right)^2$$

$$F_3 = F_{3\mathrm{N}} + F_{3\mathrm{GR}} \quad \text{where} \quad F_{3\mathrm{GR}} \propto k_H^2 \quad \text{with} \quad k_H \equiv aH$$

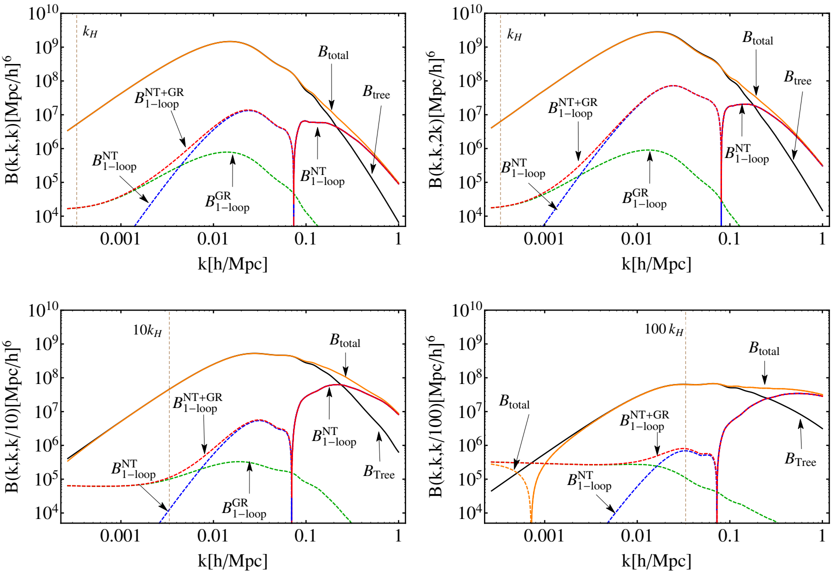
$$F_4 = F_{4\mathrm{N}} + (\cdots)k_H^2 + (\cdots)k_H^4$$

- Those w/o φ are identical to the Newtonian kernels
- Newtonian kernels are the same as those found in the standard perturbation theory based on the Newtonian gravity
- GR contributions appear from 3rd order, prop to $k_H \equiv aH$

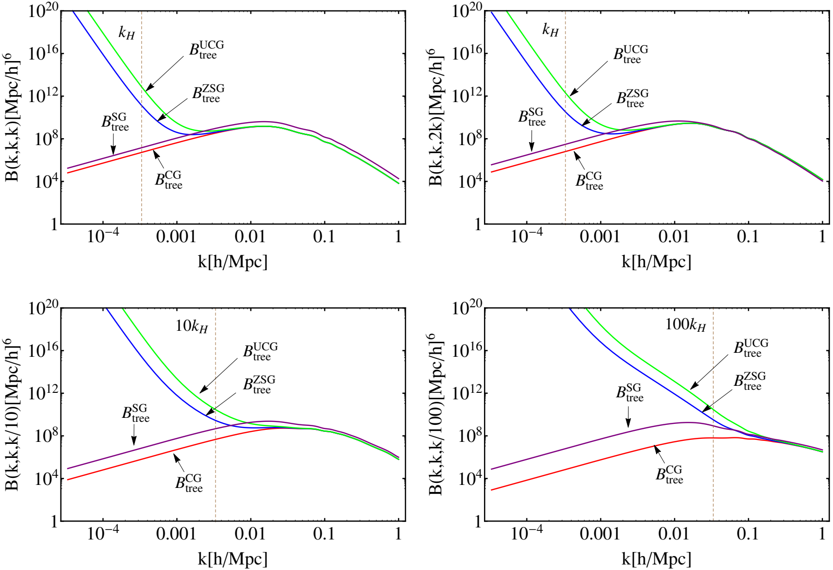
Power spectrum with leading corrections in CG



Bispectrum with leading corrections in CG



Leading bispectrum in various gauges



Synchronous gauge

We set the gauge condition as

$$g_{00} = -1 \quad \text{and} \quad g_{0i} = 0$$

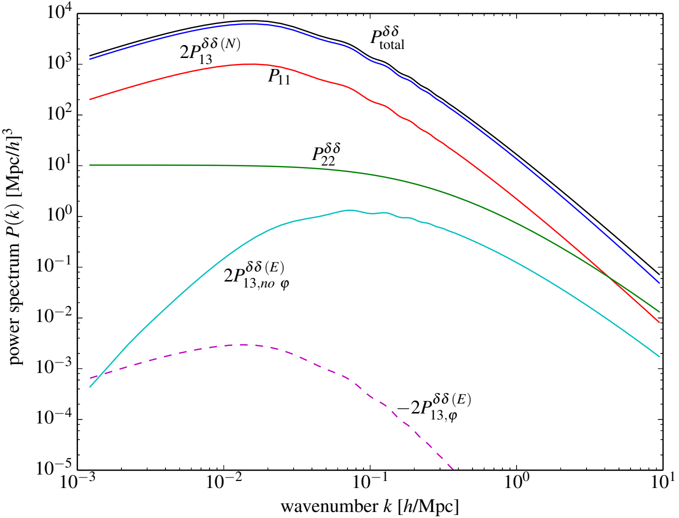
Kernels are found to be:

$$F_2 = \frac{5}{7} + \frac{2}{7}\frac{(\boldsymbol{q}_1 \cdot \boldsymbol{q}_2)^2}{q_1^2 q_2^2}$$

$$F_3 = F_{3\mathrm{N}} + F_{3\mathrm{GR},\varphi} + F_{3\mathrm{GR,no}\,\varphi}$$

- Newtonian kernels are *different* from standard ones
- Some GR contributions are not from φ but from non-linear coupling w/o k_H (thus time independent)

Power spectrum with leading corrections in SG



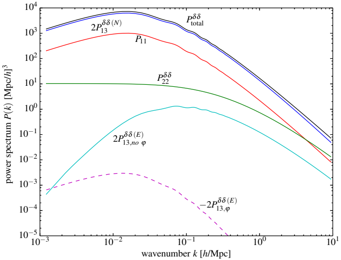
Newtonian interpretation of CG and SG

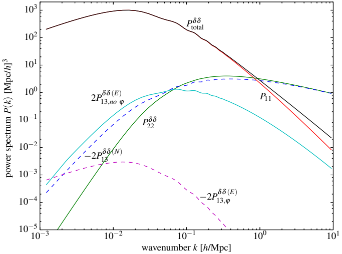
The problem lies in the Newtonian contributions

$$\dot{\delta} + \frac{1}{a}(1+\delta)\nabla\cdot\boldsymbol{u} = 0\,, \quad \dot{\theta} + 2H\theta + 4\pi G\rho\delta + \frac{1}{a^2}u^{i,j}u_{j,i} = (\text{NL terms})$$

$$\Downarrow \qquad \frac{d}{dt} \to \frac{d}{dt} + \frac{1}{a}\boldsymbol{u}\cdot\nabla$$

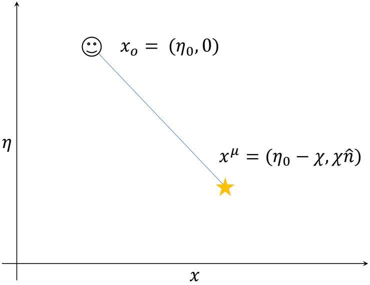
$$\dot{\delta} + \frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{u}] = 0\,, \quad \dot{\theta} + 2H\theta + 4\pi g\rho\delta + \frac{1}{a^2}\nabla\cdot[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}] = (\text{NL terms})$$





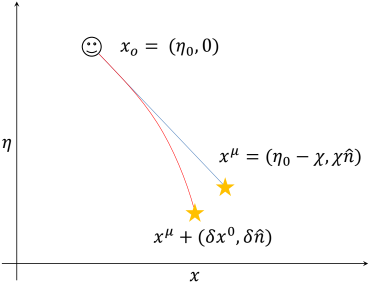
Galaxy power spectrum directly

We observe as if photons come to us along a straight, unperturbed geodesic...



Galaxy power spectrum directly

We observe as if photons come to us along a straight, unperturbed geodesic... but in fact the path is distorted due to perturbations at the locations of the observer and the source, and in between



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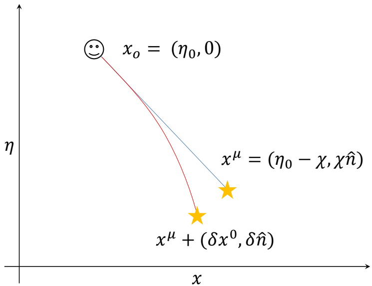

Observed number of galaxies N contained in vol $\widetilde{V}$

$$N = \int_{\widetilde{V}} \sqrt{-g} n_g \varepsilon_{\mu\nu\rho\sigma} u^\mu \frac{\partial x^\nu}{\partial \tilde{x}^1} \frac{\partial x^\rho}{\partial \tilde{x}^2} \frac{\partial x^\sigma}{\partial \tilde{x}^3} d^3\tilde{x} \quad \rightarrow \quad \text{Galaxy field } \delta_g = (\cdots)$$

Conclusions

- As galaxy surveys are deeper and deeper, fully GR description is relevant
- Gauge dependence at non-linear order:
  - In CG the standard perturbation theory is reproduced
  - Pure GR corrections are heavily suppressed in almost all cases
  - Naively using SG leads to pathologies
  - Transformation by hands cures the problem
- Gauge invariant description based on observations should help