Galaxy clustering in the large deviations regime

Sandrine Codis (CITA)

in collaboration with F. Bernardeau, C. Pichon (IAP) and C. Uhlemann (Utrecht)
How is the cosmic web woven? How do structures grow in the Universe?

Gaussian primordial fluctuations

Gravity

Expansion

Effect of non-linearities on the PDF

Gaussian PDF

(PDF: probability for this random variable $x$ to take a given value)

$G(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$

$\xi_3(\vec{r}_1, \vec{r}_2) = \xi_3(r_1, r_2, \theta)$

$\frac{\chi^2}{\chi^2_{LN}} = 1.28$

Clerkin’16 (DES)
How is the cosmic web woven?
How do structures grow in the Universe?

Gaussian primordial fluctuations

Can we predict the non-linear density PDF?

\[ G(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \]

Effect of non-linearities on the PDF

Clerkin’16 (DES)

Gaussian PDF

(PDF: probability for this random variable x to take a given value)
How is the cosmic web woven?
How do structures grow in the Universe?

Gaussian primordial fluctuations

Can we predict the non-linear density PDF?
Yes! In the large-deviations regime.

\[ G(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}} \]

Effect of non-linearities on the PDF

Clerkin’16 (DES)
Our goal: predict multi-scale densities PDF for $\sigma \sim 1$

Upshot: Large deviation principle: an unlikely fluctuation is brought about by the least unlikely of all unlikely paths.

Spherical collapse consistent with symmetry of measurement
Outline

1. Large deviation principle (LDP)
2. One-point density PDF
3. A new cosmological probe?
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2. One-point density PDF
3. A new cosmological probe?
Large-deviation Theory
what is the most likely initial configuration
a final density originates from?

This most likely path can be found for very specific configurations with sufficient degree of symmetry e.g. density in concentric spheres. In that case:

Different initial configurations can lead to the same final state! What is the most likely one?

Spherical symmetry enforces this most likely path to be the so-called Spherical Collapse dynamics:

\[ \tau \rightarrow \rho = \zeta_{SC}(\tau) \]
\[ r_0 \rightarrow r = r_0 \rho^{-1/3} \]
Large-deviation Theory

what is the most likely initial configuration
a final density originates from?

This most likely path can be found for very specific configurations with sufficient degree of symmetry e.g. density in concentric spheres. In that case:

This is not equivalent to a simple change of variable: we mix scales!

Different initial configurations can lead to the same final state! What is the most likely one?

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Large-deviation Theory

in a nutshell

LDP tells us how to compute the **cumulant generating function** from the initial conditions using the spherical collapse as the « mean dynamics »:

\[
\varphi(\{\lambda_k\}) = \sup_{\rho_i}(\lambda_i \rho_i - I(\rho_i))
\]

Varadhan’s theorem
Large-deviation Theory
in a nutshell

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\]

**Varadhan’s theorem**

<table>
<thead>
<tr>
<th>Why?</th>
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</table>
| \[
\varphi(\lambda_k) = \left\langle \exp(\sum_i \lambda_i \rho_i) \right\rangle = \int_0^\infty \Pi d\rho_i P(\{\rho_k\}) \exp \left( \sum_i \lambda_i \rho_i \right)
\]
| \[
\simeq \lambda_i \langle \rho_i \rangle + \lambda_i \lambda_j \langle \rho_i \rho_j \rangle + \ldots
\] |

**contraction principle**

\[
| = \int \mathcal{D}[\tau(\vec{x})] \mathcal{P}[\tau(\vec{x})] \exp(\lambda_i \rho_i [\tau(\vec{x})])
\]

**known Gaussian PDF** \( \mathcal{P}(\tau) \propto e^{-I(\tau)} \)

\[
= \int d\tau_i \exp(\lambda_i \zeta_{SC}(\tau_i) - I(\tau_i))
\]
Large-deviation Theory
in a nutshell

LDP tells us how to compute the **cumulant generating function** from the initial conditions using the spherical collapse as the « mean dynamics »:

\[ \varphi(\{\lambda_k\}) = \sup(\lambda_i \rho_i - I(\rho_i)) \]

*Varadhan’s theorem*

The density **PDF** is then obtained via an inverse Laplace transform of the CGF

\[ \exp \varphi(\lambda) = \int P(\rho) \exp(\lambda \rho) \leftrightarrow P(\rho) = \int_{-i\infty}^{i\infty} \frac{d\lambda}{2\pi} \exp(\lambda \rho - \varphi(\lambda)) \]

**Parameter-free** theory which depends on cosmology through : the spherical collapse dynamics and the linear power spectrum.

Predictions are fully **analytical** if one applies the LDP to the log. *(Uhlemann, SC’ 16)*
Outline

1. Large deviation principle (LDP)
2. One-point density PDF
3. A new cosmological probe?
Horizon-Run: 3.1 h^{-1} Gpc
R = 10...15 h^{-1} Mpc

One-cell density PDF

\[ \rho = 1 + \delta \]

\[ \log P(\rho|z=0.7) \]

\[ \sigma_R = 0.51 \]

\[ \sigma_R = 0.36 \]

(Uhlemann, SC+ 16)
We have developed a fast and easy-to-use public code...

LSSFAST

A Mathematica package to compute cosmic density PDF in the large-deviation regime

Author: Sandrine Codis (CITA)
Last modified: 04/03/2016
Code:  LSSFast.tar.gz

This code is based on theoretical works in collaboration with Francis Bernardeau (IAP, CEA-Saclay), Christophe Pichon (IAP, KIAS) and Cora Uhlemann (Utrecht University)

The LSSFast code is a free software distributed under the terms of the GNU-General Public License 3. It can be redistributed and modified at your own risk. This program is made publicly available in the hope that it will be useful in scientific research but without any warranty.

The companion paper "Constraining the nature of dark energy via density PDF" by S. Codis, F. Bernardeau, C. Pichon, C. Uhlemann and S. Prunet illustrates the possible use of LSSFast for cosmological data analysis.

Any questions or remarks can be emailed to codis@cita.utoronto.ca

http://cita.utoronto.ca/~codis/LSSFast.html
Two-cell PDF

Bernardeau, SC, Pichon 15
Uhlemann, SC+16

measurements (HR4)

theory
Two-cell PDF: statistics of the slope

Higher density environments have more negative slopes (peaks!).

Bernardeau, SC, Pichon 15
Uhlemann, SC+ 16

$z = 0.97$

$\log P(s)$

$s =$ slope

$\rho > 1$

$\rho < 1$

measurements

theory
Error budget?

Maximum likelihood requires proper handling of correlations between spheres at finite separations. The large-deviation principle provides a framework to compute the expected 2-pt correlations in the (not so) large separation limit:

\[ P(\rho(x), \rho'(x + r_e)) = P(\rho)P(\rho')[1 + \xi(r_e)b(\rho)b(\rho')] \]

where the large-deviations bias is

\[ b(\rho) = \frac{\zeta_{SC}^{-1}(\rho)}{\sigma^2(R\rho^{1/3})} \]

This bias encodes the spherical collapse prediction of dark matter correlation, density bias, and directly relates to the density bias in Poisson shot noise.
Error budget?

Maximum likelihood requires proper handling of **correlations** between spheres at **finite** separations. The large-deviation principle provides a framework to compute the expected 2-pt correlations in the (not so) large separation limit.

\[
P(\rho(x), \rho'(x + r_e)) = P(\rho)P(\rho')[1 + \xi(r_e)b(\rho)b(\rho')]
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where the large-deviations bias is

\[
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\]

which in turn can be expressed in terms of the variables \(\{\xi, b\}\) which in turn can be expressed in terms of the variables \(\{\rho_1, \rho_2, \rho_3\}\).

Finally note that the domain for \(r_e \gg 1\) and \(\rho_1, \rho_2, \rho_3\) to be computed as a density bias.

**Spherical collapse**

\[
\text{encodes } P_{\text{lin}}(k)
\]

**Dark matter correlation**

**Density bias**

**Measurements**

**Theory**

\[\text{decorrelation@ } \rho = 1\]
Maximum likelihood requires proper handling of correlations between spheres at finite separations. The large-deviation principle provides a framework to compute the expected 2-pt correlations in the (not so) large separation limit

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\[ b(\rho) = \frac{\xi^{-1}_{SC}(\rho)}{\sigma^2(R\rho^{1/3})} \]

The typical cosmic variance on the density PDF is then:

\[ \langle \hat{P}^2(\rho) \rangle - \langle \hat{P}(\rho) \rangle^2 = \frac{P(\rho)}{N\Delta\rho} + \xi b^2(\rho)P^2(\rho) \]
Outline

1. Large deviation principle (LDP)
2. One-point density PDF
3. A new cosmological probe?
Where is the cosmology dependence?

To get one-cell PDF, one has to:

1) know the rate function of the initial conditions e.g (Gaussian):

\[ I(\tau(R_0)) = \sigma^2(R_p) \times \frac{1}{2\tau(R_0)^2 / \sigma^2(R_0)} \]

where the initial variance is a function of the \textbf{linear power spectrum}

\[ \sigma^2(R) = \frac{1}{(2\pi)^3} \int d^3k \, P_{\text{lin}}(k) W_{\text{TH}}^2(kR) \]

2) deduce the rate function of the final densities from the Contraction Principle

\[ I(\rho) = I(\tau = \zeta^{-1}(\rho)) \]

3) compute CGF and then PDF

\[ P(\rho|\nu, P_{\text{lin}}, \sigma_{\text{NL}}(R, z)) \]

spherical collapse dynamics
ML estimator for the variance

The full knowledge of the PDF can be used to estimate the redshift evolution of the density variance $\sigma$ and therefore the DE e.o.s through $D(z)$.

Maximum Likelihood estimator: $\hat{\sigma}_{\text{ML}}^2 = \arg\max_{\tilde{\sigma}^2} \prod_{i=1}^{N} P(\rho_i | \tilde{\sigma}^2)$

Sample variance: $\hat{\sigma}_A^2 = \frac{1}{N} \sum_{i=1}^{N} (\rho_i - 1)^2$

When the PDF becomes non-Gaussian (high $\sigma$), the sample variance is sub-optimal compared to the ML estimator.
ML estimator for $\xi$

The full knowledge of the PDF can be used to estimate the DM 2-point correlation function:

Maximum Likelihood estimator: $\hat{\xi}_{\text{ML}} = \arg \max_{\xi} \prod_{i,j} P(\rho_i, \rho_j | \xi)$

Sample estimator: $\hat{\xi}_A = \frac{1}{N} \sum_{i,j} \rho_i \rho_j - 1$

ML estimate from Horizon-Run at 1 and 3 sigma
Use density PDF for cosmology

15,000 square degrees
R = 10 h\(^{-1}\) Mpc
0.1<z<1
One-cell velocity divergence PDF

Hahn+15, velocity stat obtained by tessellation of the DM sheet
One-cell velocity divergence PDF

(Uhlemann, SC+, in prep.)

Hahn+15, velocity stat obtained by tessellation of the DM sheet
Use **velocity** PDF for cosmology

Here the rest of the cosmology is fixed...

\[ \theta_{SC} = f(\Omega_m)\nu(1 - \rho^{1/\nu}) \]

15,000 square degrees
\[ R = 10 \, h^{-1} \, \text{Mpc} \]
\[ 0.1 < z < 1 \]
Use *velocity* PDF for cosmology

The PDFs also depend on: linear $P_k$, SC dynamics...
Use PDF for cosmology

Halo bias? Redshift space distortions? ...
PRELIMINARY RESULTS!!

15,000 square degrees
R = 10 h^{-1} Mpc
0.1<z<1
How to deal with redshift space distortions?

Projected densities?
Same formalism applies with cylindrical collapse

\[ \zeta_{CC}(\tau_{2D}) = \left(1 - \frac{\tau_{2D}}{\nu}\right)^{-\nu} \]

\[ \nu \approx 1.3 \]
How to deal with biased tracers?

Halo bias can be accounted for and marginalised over for cosmological experiments...

See Martin’s talk!
Conclusion:

Multi-scale densities PDF can be predicted in the mildly non-linear regime with surprising accuracy e.g. $<1\%$ on $P(\rho)$ for $\sigma=O(1)$ even in the rare event tails.

Predictions are fully analytical and explicitly cosmology-dependent!

Cosmic variance due to spatial correlation between spheres is predicted by the theory.

We can have a model for biased tracers of the density, velocities, projected densities and (in progress) cosmic shear maps.

Future prospects are numerous: PNG, correction to the spherical collapse dynamics, application to mocks and real data, etc.

Large deviation principle:

an unlikely fluctuation is brought about by the least unlikely of all unlikely paths.
A. One-cell density PDF

1st step: compute the cumulant generating function \( \varphi(\lambda) = \sup_{\lambda} (\lambda \rho - I(\rho)) \)

or equivalently \( \varphi(\lambda) = \lambda \rho - I(\rho) \) with stationary condition \( \lambda = I'(\rho) \)

⚠ inverting the stationary condition is not possible for all \( \lambda \)!
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This cumulant generating function gives directly access to **ALL** the density cumulants at tree order through:

\[
\varphi(\lambda) = 1 + \lambda + \sigma_1^2 \frac{\lambda^2}{2} + S_3 \sigma_1^4 \frac{\lambda^3}{3!} + \ldots \quad \text{where} \quad S_p = \frac{\langle \delta_1^p \rangle_c}{\sigma_1^{2(p-1)}}
\]
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\]

This is shown in the graphs for \( \sigma_8 = 1 \) and \( \sigma_8 = 1.25 \) respectively. The graphs demonstrate the cumulants all at once for various orders.

\( S_3 = \frac{34}{7} + \gamma_1, \)

\( S_4 = \frac{60712}{1323} + \frac{62}{3} \gamma_1 + \frac{7}{3} \gamma_1^2 + \frac{2}{3} \gamma_2, \)

\( S_5 = \frac{200575880}{305613} + \frac{1847200}{3969} \gamma_1 + \frac{6940}{63} \gamma_1^2 + \frac{235}{27} \gamma_1^3 + \frac{1490}{63} \gamma_2 + \frac{50 \gamma_1 \gamma_2}{9} + \frac{10 \gamma_3}{27}, \)

\( S_6 = \frac{12650 + 12330}{305613} \gamma_1 + \frac{4512}{305613} \gamma_1^2 + \frac{734.0}{305613} \gamma_1^3 + \frac{44.81}{305613} \gamma_1^4 + \frac{775.8}{305613} \gamma_2 + \frac{375.9}{305613} \gamma_1 \gamma_2 + \frac{45.56}{305613} \gamma_1^2 \gamma_2 + \frac{3.889}{305613} \gamma_2^2 + \frac{20.05}{305613} \gamma_3 + \frac{4.815}{305613} \gamma_1 \gamma_3 + \frac{0.1852}{305613} \gamma_4, \)

\( S_7 = \frac{307810 + 383000}{305613} \gamma_1 + \frac{190700}{305613} \gamma_1^2 + \frac{47460}{305613} \gamma_1^3 + \frac{5914}{305613} \gamma_1^4 + \frac{294.8}{305613} \gamma_1^5 + \frac{27340}{305613} \gamma_2 + \frac{20300}{305613} \gamma_1 \gamma_2 + \frac{5026}{305613} \gamma_1^2 \gamma_2 + \frac{414.8}{305613} \gamma_1^3 \gamma_2 + \frac{358.1}{305613} \gamma_1^2 \gamma_2^2 + \frac{88.15}{305613} \gamma_1 \gamma_2^2 + \frac{902.6}{305613} \gamma_3 + \frac{443.3}{305613} \gamma_1 \gamma_3 + \frac{54.44}{305613} \gamma_1^2 \gamma_3 + \frac{7.778}{305613} \gamma_2 \gamma_3 + \frac{14.20}{305613} \gamma_4 + \frac{3.457}{305613} \gamma_1 \gamma_4 + \frac{0.0864}{305613} \gamma_5, \)

\ldots

\[ \text{Bernardeau' 94} \]
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\]

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\]

\[
S_5 = \frac{200575880}{305613} + 1847200 \gamma_1 + \frac{6940 \gamma_1^2}{63} + \frac{235 \gamma_1^3}{27} + 1490 \gamma_2 + \frac{50 \gamma_1 \gamma_2}{9} + \frac{10 \gamma_3}{27},
\]

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S_6 = 12650 + 12330 \gamma_1 + 4512 \gamma_1^2 + 734.0 \gamma_1^3 + 44.81 \gamma_1^4 + 775.8 \gamma_2 + 375.9 \gamma_1 \gamma_2 + 45.56 \gamma_1^2 \gamma_2 + 3.889 \gamma_2^2 + 20.05 \gamma_3 + 4.815 \gamma_1 \gamma_3 + 0.1852 \gamma_4,
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S_7 = 307810 + 383000 \gamma_1 + 190700 \gamma_1^2 + 47460 \gamma_1^3 + 5915 \gamma_1^4 + 294.8 \gamma_1^5 + 27340 \gamma_2 + 20300 \gamma_2 \gamma_1 + 5026 \gamma_1^2 \gamma_2 + 414.8 \gamma_1^3 \gamma_2 + 358.1 \gamma_1^2 \gamma_2^2 + 88.15 \gamma_1 \gamma_2^2 + 902.6 \gamma_3 + 443.3 \gamma_1 \gamma_3 + 54.44 \gamma_1^2 \gamma_3 + 7.778 \gamma_2 \gamma_3 + 14.20 \gamma_4 + 3.457 \gamma_1 \gamma_4 + 0.08642 \gamma_5,
\]

\[
S_8 = \ldots
\]

instead of complex PT calculations, e.g

\[
S_3 = \frac{1}{\sigma_1^4} \int \frac{d^3k_1}{(2\pi)^3} \frac{d^3k_2}{(2\pi)^3} P(k_1) P(k_2) F_2(k_1, k_2) W(R k_1) W(R k_2) W(R |k_1 + k_2|)
\]

All 1-cell density cumulants at tree order at once!
A. One-cell density PDF

1st step: compute the cumulant generating function
\[ \varphi(\lambda) = \sup_{\lambda} (\lambda \rho - I(\rho)) \]
or equivalently \[ \varphi(\lambda) = \lambda \rho - I(\rho) \] with stationary condition \[ \lambda = I'(\rho) \]

2nd step: compute the PDF
The inverse Laplace transform requires integration into the complex plane:

\[ P(\rho) = \int_{-i\infty}^{+i\infty} \frac{d\lambda}{2\pi i} \exp(-\lambda \rho + \varphi(\lambda)) \]

Numerical integration AND **analytical** approximations at low and large densities:

\[ P(\rho) = \sqrt{\frac{I''(\rho)}{2\pi}} \exp(-I(\rho)) \quad \text{at low density} \]

\[ P(\rho) = \frac{3a_{3/2}^3}{4\sqrt{\pi}} \exp(\varphi_c - \lambda_c \rho) \frac{1}{(\rho + \ldots)^{5/2}} \]

functions of the cosmology via the power spectrum + the spherical collapse

branch cut provides asymptote
A. One-cell density PDF

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The inverse Laplace transform requires integration into the complex plane:

\[
P(\rho) = \int_{-i\infty}^{+i\infty} \frac{d\lambda}{2\pi i} \exp(-\lambda \rho + \varphi(\lambda))
\]

Numerical integration technically done by choosing the path of zero imaginary part

---

Bernardeau, Pichon, Codis, 14
500 h⁻¹ Mpc
R = 10 h⁻¹ Mpc

Agreement even in the mildly non-linear regime and in the rare event tails of the PDF! 1% precision until σ of order unity!

Bernardeau, Pichon, Codis, 14
Introduce slope = possible proxy for peaks & voids

\[ P(\rho_1, \rho_2) d\rho_1 d\rho_2 \rightarrow P(\rho, s) d\rho ds \]

\[ s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1} \]

density slope

1st step: compute the cumulant generating function

\[ \varphi(\lambda, \mu) = \sup_{\lambda,\mu} (\lambda \rho + \mu s - I(\rho, s)) \]

or equivalently \[ \varphi(\lambda, \mu) = \lambda \rho + \mu s - I(\rho, s) \]

with stationary condition

\[ \lambda = \frac{\partial I(\rho, s)}{\partial \rho} \]
\[ \mu = \frac{\partial I(\rho, s)}{\partial s} \]

⚠️ There is a critical line where the stationary condition is singular.

B. Two-cell PDF

Same formalism can be used to compute the statistics of cosmic densities in N>1 concentric cells

Introduce slope = possible proxy for peaks & voids

Bernardeau, Pichon, Codis, 14
B. Two-cell PDF

Same formalism can be used to compute the statistics of cosmic densities in \( N > 1 \) concentric cells. Introduce slope = possible proxy for peaks & voids

\[
\frac{\rho = \rho_1}{s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1}} \quad \text{density slope}
\]

1st step: compute the cumulant generating function

\[
\varphi(\lambda, \mu) = \sup_{\lambda, \mu} (\lambda \rho + \mu s - I(\rho, s))
\]

or equivalently

\[
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\]

with stationary condition

\[
\left\{
\begin{array}{l}
\lambda = \frac{\partial I(\rho, s)}{\partial \rho} \\
\mu = \frac{\partial I(\rho, s)}{\partial s}
\end{array}
\right.
\]

2nd step: compute the PDF via 2D Inverse Laplace Transform

\[
P(\rho, s) = \int_{-i\infty}^{i\infty} d\lambda \int_{-i\infty}^{i\infty} d\mu \exp(-\lambda \rho - \mu s + \varphi(\lambda, \mu))
\]

This is difficult because we need to choose a 2D path in 4D space with lots of the oscillations and analytical approximations have a poor range of validity.
B. Two-cell PDF

Same formalism can be used to compute the statistics of cosmic densities in $N>1$ concentric cells

Introduce slope = possible proxy for peaks & voids

$$P(\rho_1, \rho_2) d\rho_1 d\rho_2 \quad \rho = \rho_1$$

$$s = R_1 \frac{\rho_2 - \rho_1}{R_2 - R_1} \text{ density slope}$$

1st step: compute the cumulant generating function $\varphi(\lambda, \mu) = \sup_{\lambda,\mu}(\lambda \rho + \mu s - I(\rho, s))$

or equivalently $\varphi(\lambda, \mu) = \lambda \rho + \mu s - I(\rho, s)$ with stationary condition

$$\begin{cases} 
\lambda = \frac{\partial I(\rho, s)}{\partial \rho} \\
\mu = \frac{\partial I(\rho, s)}{\partial s}
\end{cases}$$

2nd step: compute the PDF via 2D Inverse Laplace Transform

$$P(\rho, s) = \int_{-i\infty}^{i\infty} d\lambda \int_{-i\infty}^{i\infty} d\mu \exp(-\lambda \rho - \mu s + \varphi(\lambda, \mu))$$

This is difficult because we need to choose a 2D path in 4D space with lots of the oscillations and analytical approximations have a poor range of validity.

Apply the large-deviation principle to the log of the density!
This is a simple change of variable but it removes the singularities and provides very accurate analytical approximations (almost indistinguishable from the numerical integration)!
B. Two-cell PDF: density profiles

![Graphs showing expected density as a function of radius given a constraint at a given scale: expectation+scatter.](image)

Expected density as a function of radius given a constraint at a given scale: expectation+scatter.

The cosmic scatter is reduced in low-density regions motivating the study of void profiles...
Fiducial cosmological experiment

- Power-law power spectrum with index $n_s$ (-2.5)
- PDF characterized by $n_s$ and $\nu$ (which parametrizes the spherical collapse, 3/2 here)
- n=2,000 and 11,000 measurements corresponding to a survey volume of $(200 \, h^{-1} \, \text{Mpc})^3$ and $(360 \, h^{-1} \, \text{Mpc})^3$

**Prediction for full joint PDF densities in concentric cells:**

$$P(\rho(R_1), \rho(R_2)) \, d\rho(R_1) \, d\rho(R_2)$$

which is gravity and cosmology-dependent through the linear power spectrum and the dynamics of the spherical collapse.
Fiducial cosmological experiment

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Log-likelihood contours of the data at 1, 3 and 5 sigmas.
Fiducial cosmological experiment

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Log-likelihood contours of the data at 1, 3 and 5 sigmas.

Error budget for finite volume surveys?

[encodes modifications of gravity]
Fiducial cosmological experiment: Error budget?

Maximum likelihood requires proper handling of correlations between spheres at finite separations. The large-deviation principle provides a framework to compute the expected 2-pt correlations in the (not so) large separation limit.

\[
P(\rho(x), \rho'(x + r_e)) = P(\rho)P(\rho')[1 + \xi(r_e)b(\rho)b(\rho')]\]

where the large-deviations bias is

\[
b(\rho) = \frac{-\zeta_{SC}^{-1}(\rho)}{\sigma^2(R\rho^{1/3})}\]

The typical cosmic variance on the density PDF is then:

\[
\langle \hat{P}^2(\rho) \rangle - \langle \hat{P}(\rho) \rangle^2 = \frac{P(\rho)}{N\Delta \rho} + \xi b^2(\rho)P^2(\rho)
\]
Fiducial cosmological experiment: Error budget?

Maximum likelihood requires proper handling of correlations between spheres at finite separations. The large-deviation principle provides a framework to compute the expected 2-pt correlations in the (not so) large separation limit

\[ P(s(x), s'(x + r_e)) = P(s)P(s')[1 + \xi(r_e)b(s)b(s')] \]

where the large-deviations bias is

\[ b(s) = \mathcal{F}(\zeta_{SC}, P_{lin}) \]

The typical cosmic variance on the slope PDF is then:

\[ \langle \hat{P}^2(s) \rangle - \langle \hat{P}(s) \rangle^2 = \frac{P(s)}{N\Delta s} + \xi b^2(s)P^2(s) \]
Thank you