On 5d descriptions of 6d SCFTs

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Based on the collaboration with

* Sung-Soo Kim, Kimyeong Lee, Masato Taki, Futoshi Yagi
  [arXiv:1512.08239]

Current Topics in String Theory: Conformal Field Theories
December 2016 at KIAS,
1. Introduction
• 6d SCFTs are mysterious quantum field theories which are superconformal field theories in the highest dimensions.

Nahm 78
• 6d SCFTs are mysterious quantum field theories which are superconformal field theories in the highest dimensions.

• A surprising feature:

\[
\begin{align*}
6d \, \mathcal{N} = (1,0) \text{ SCFT on } S^1 &\quad \leftrightarrow \quad 5d \, \mathcal{N} = 1 \text{ gauge theory} \\
\text{KK modes} &\quad \leftrightarrow \quad \text{instantons} \\
1/R_{S^1} \sim 1/g_{YM}^2
\end{align*}
\]
• A famous example:

\[ 6d \mathcal{N} = (1,0) \text{ E-string} \quad \rightarrow \quad 5d \mathcal{N} = 1 \text{ SU(2) gauge theory with 8 flavors} \]
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\[
6d \mathcal{N} = (1,0) \text{ E-string} \quad \rightarrow \quad 5d \mathcal{N} = 1 \text{ SU(2) gauge theory with 8 flavors}
\]

• Recently, the generalization of the 5d - 6d correspondence has been proposed.

\[
6d \ (D_{N+4}, D_{N+4}) \text{ minimal conformal matter theory} \quad \rightarrow \quad 5d \text{ SU(N+2) gauge theory with } 2N+8 \text{ flavors}
\]

HH, Kim, Lee, Taki, Yagi 15
Yonekura 15
In fact, its 5d description is not unique but there are other 5d descriptions.

6d \((D_{N+4}, D_{N+4})\) minimal conformal matter theory

**“UV dualities”**

- SU\((N+2)\) gauge theory with 2N+8 flavors
- Sp\((N+1)\) gauge theory with 2N+8 flavors

[4] – SU(2) - ... - SU(2) - [4]

See also S-S. Kim’s Talk
• The argument is based on brane construction of the 5d and 6d theories. In particular, various techniques for 5-brane webs have developed.

• In this talk, I will focus on further generalization.

• It is actually possible to further extend the 5d–6d correspondence and dualities by introducing so-called an ON$^0$-plane in the brane construction.

→ it can yield a D-type quiver or SO-Sp gauge groups
1. Introduction

2. 5-brane webs with an $\text{ON}^0$-plane

3. From 6d to 5d with an $\text{ON}^0$-plane

4. Conclusion
2. 5-brane webs with an ON$^0$-plane
• An ON$^0$-plane is S-dual to a combination of a D5-brane and an O5$^-$-plane.

• Therefore, in the original frame, an ON$^0$-plane might consist of an NS5-brane and an ON$^-$-plane which is S-dual to an O5$^-$-plane.
• An easy example with an O5⁻ or ON⁻-plane.

roots of SO(4)

D5₁

D5₂

e₁-e₂

e₁+e₂

O5⁻

S-dual

ON⁻

NS5₂

NS5₁

D₁
• Performing T-duality yields a 5-brane web with an ON-plane.

• In this case, we need to care about the slope of the 5-branes.
• Performing T-duality yields a 5-brane web with an ON-plane.

• In this case, we need to care about the slope of the 5-branes.

(1, 0) 5-brane (D5) (1, 1) 5-brane (0, 1) 5-brane (NS5) “5-brane web”

Aharony, Hanany 97, Aharony, Hanany, Kol 97
• A 5-brane web configuration in a ten-dimensional spacetime.

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\hline
D5-brane & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
NS5-brane & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
(p, q) 5-brane & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
\hline
\end{array}
\]

slope is q/p

5-brane web

Aharony, Hanany 97, Aharony, Hanany, Kol 97, DeWolfe, Iqbal, Hanany, Katz 99
In order to be consistent with the slopes, we propose a 5-brane web diagram with an ON⁻-plane as follows.
• This gives a microscopic description of an ON⁰-plane.
An important point of introducing an ON$^0$-plane is that the 5-brane web realizes a quiver gauge theory of D-type.

\[ \text{ON}^+ \quad 2k \quad 2k \quad 2k \]

\[ \text{SU}(k) \quad \text{SU}(2k) - ... - \text{SU}(2k) - [2k] \]

\[ \text{SU}(k) \quad \text{N-2} \]

Sen 98
Kapustin 98
Hanany, Zaffaroni 99
• We can also discuss a S-dual picture of a 5-brane with an ON\textsuperscript{−}-plane.
• In fact, we can further transform the diagram into an equivalent 5-brane web with an O5$^+$-plane.
• We then enlarge the Coulomb branch moduli.
• 5-branes eventually reach the $O5^-$-plane and create an $O5^+$-plane.
We will make use of this transition in the later manipulation.
• An implication of the transition is that it yields an interesting duality.
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• Let us consider the following case.
• This transition also implies a duality.

```
"SU(1)" \ /
   /   /
SU(2) – ... – SU(2) - [2]  ↔  Sp(N-2) – [2N]
   \   /
"SU(1)"
```

N-2
• This can be also shown by a chain of known dualities.

\[ \text{[4]} \rightarrow \text{SU(2)} \rightarrow \ldots \rightarrow \text{SU(2)} \rightarrow \text{[2]} \leftrightarrow \text{Sp(N-2)} \rightarrow \text{[2N]} \]
• This can be also shown by a chain of known dualities.
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$[4] \rightarrow SU(2) \rightarrow \ldots \rightarrow SU(2) \rightarrow [2] \leftrightarrow Sp(N-2) \rightarrow [2N]

N-2

S-dual

SU(N-1)_{\pm 1} \rightarrow [2N]

SU-Sp duality

Gaiotto, Kim 15
HH, Kim, Lee, Yagi 15
• This can be also shown by a chain of known dualities.

\[ [4] \rightarrow SU(2) \rightarrow \ldots \rightarrow SU(2) \rightarrow [2] \rightarrow Sp(N-2) \rightarrow [2N] \]

N-2
S-dual
SU(N-1) \pm 1 \rightarrow [2N]

SU-Sp duality

• This gives support for the transition.

Gaiotto, Kim 15
HH, Kim, Lee, Yagi 15
• Higher rank generalization yields a new duality

\[
\begin{align*}
&SU(k) \nonumber \\
\rightarrow &SU(2k) – ... – SU(2k) - [2k] \\
\rightarrow &SU(k)
\end{align*}
\]
3. From 6d to 5d with an ON\(^0\) – plane
• We move on to a brane configuration in type IIA string theory, which yields a 6d theory.

• The brane configuration involves D6-branes and NS5-branes (and also D8-branes).

Hanany, Zaffaroni 97
Brunner, Karch 97
• We move on to a brane configuration in type IIA string theory, which yields a 6d theory.

• The brane configuration involves D6-branes and NS5-branes (and also D8-branes).

• Then we also have a choice of introducing orientifolds:
  (i) O8-plane
  (ii) O6-plane
  (iii) O8-plane and O6-plane

Hanany, Zaffaroni 97
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Hanany, Zaffaroni 97
Brunner, Karch 97
We will only consider cases with an O8^-plane. Then we have two choices for the intersection.

(1). [O8^- - ON^0 - O6^-]         (2). [O8^- - ON^0 - O6^+]
• We will only consider cases with an O8^-plane. Then we have two choices for the intersection.

(1). \([\text{O8}^- - \text{ON}^0 - \text{O6}^-]\) 

(2). \([\text{O8}^- - \text{ON}^0 - \text{O6}^+]\)
• We will only consider cases with an O8⁻-plane. Then we have two choices for the intersection.

(1) \([\text{O8}^- \text{ON}^0 \text{O6}^-]\)  
(2) \([\text{O8}^- \text{ON}^0 \text{O6}^+]\)  

• The brane configuration in ten-dimensional spacetime.

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• The brane configuration for the case (1).

\[ \begin{align*}
\text{ON}^0 & \quad 4N \text{ D6} \quad 4N-8 \text{ D6} \quad 4N \text{ D6} \quad 4N-8 \text{ D6} \quad 4N \text{ D6} \\
\text{O8}^- + 8\text{D8} & \quad \text{NS5} \quad \text{O6}^- \quad \text{O6}^+ \quad \text{O6}^- \\
& \quad \text{O6}^- \quad \text{O6}^+ \quad \text{O6}^- \quad \text{O6}^+ \quad \text{O6}^- \\
& \quad \text{2k} \quad \text{x}_7,8,9 \\
& \quad \text{x}_5
\end{align*} \]
• The brane configuration for the case (1).

The mirror image of \((O8^- + 8D8)\) appears and hence they are actually fractional.

Hanany, Zaffaroni 99
• The brane configuration for the case (1).

\[ \text{ON}^0 \quad 4N \text{ D6} \quad 4N-8 \text{ D6} \quad 4N \text{ D6} \quad \ldots \quad 4N-8 \text{ D6} \quad 4N \text{ D6} \]

\[ \text{O}8^- + 8\text{D}8 \]

\[ [8] - \text{SU}(2N) - \text{Sp}(2N-4) - \text{SO}(4N) - \ldots - \text{Sp}(2N-4) - [\text{SO}(4N)] \]

Hanany, Zaffaroni 99

(i). # of 6d tensor multiplets = 1 + k + (k − 1) = 2k

(ii). # of 6d vector multiplets in the Cartan subalgebra

\[ = (2N - 1) + (2N - 4)k + 2N (k - 1) = 4Nk - 4k - 1 \]

(iii). Global symmetry = SO(4N) x SU(8) x U(1)
• We then compactify the $x_6$-direction on $S^1$ and perform the **T-duality** along the direction to go to a 5d description.

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5-brane web
• Then we obtain a 5-brane web with O5-plane, O7-plane and ON-plane.
• Then we obtain a 5-brane web with O5-plane, O7-plane and ON-plane.

\[
\begin{align*}
\text{O7}^0 &= \text{O7}^- + 4\text{D7} \\
\text{O5}^0 &= \text{O5}^- + \text{D5}
\end{align*}
\]
• At this level, the theory is not a 5d theory due to a periodic direction in the vertical direction.

• We here propose a novel way to resolve the O7⁻-plane in this case.
• Note that we have a half of \( O7^0 = [O7^-\text{-plane and 4 D7-branes ("A-brane")}] \) at each intersection.

• \( O7^-\text{-plane itself consists of a [1, -1] 7-brane ("B-brane") and a [1, 1] 7-brane ("C-brane")}. \)

Sen 96
• Note that we have a half of $O7^0 = [O7^{-}\text{-plane and 4 D7-branes ("A-brane")}]$ at each intersection.

• $O7^{-}\text{-plane itself consists of a } [1, -1] 7\text{-brane ("B-brane") and a } [1, 1] 7\text{-brane ("C-brane")}.$
In fact, the monodromy transformation for the combination of $ANA$ exactly gives the S-duality transformation.

$$M_{ANA} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
• This is consistent with the 5-brane configuration.
• This is consistent with the 5-brane configuration.
$\text{ON}^0 \rightarrow \text{ON}^-$

$\text{O5}^0 \rightarrow \text{O5}^-$

$\text{NS5}$

$\text{D5}$
• We will make use of this transition for the 5-brane web obtained by T-duality from the 6d brane picture.
(I) Let us resolve the two O7-planes.
• At the same time, we perform the transition which we obtained before.
Therefore, the two transitions yield the following 5-brane web.
• The situation is different according to the relation between \( N \) and \( k \).

1. (i). \( N \geq 2k \)

1. (ii). \( N < 2k \)
$I - (i) \ [ k = 1 \ ]$
$1 - (i) \ [ k = 1 ]$
\[ I - (i) \quad [k = 1] \]

\[ [4] - SU(N+1) \]

\[ SU(2N-2) - [2N-4] \]

\[ [4] - SU(N+1) \]
1. (i). $N \geq 2k$

In general, we obtain the following quiver gauge theory of D-type.

[4] \rightarrow SU(N+2k-1) \rightarrow SU(2N+4k-6) \rightarrow ... \rightarrow SU(2N-4k+2) \rightarrow [2N-4k]

\[4] \rightarrow SU(N+2k-1) \rightarrow 2k-1
Check 1:

# of 5d vector multiplets in the Cartan subalgebra

\[
= (N + 2k - 2) \times 2 + \sum_{i=1}^{2k-1} (2N + 4k - 7 - 4(i - 1))
\]

\[
= 4Nk - 2k - 1
\]

\[
= 2k + (4Nk - 4k - 1)
\]

\[
= \text{# of 6d tensor multiplets} + \text{# of 6d vector multiplets in the Cartan subalgebra}
\]
Check 2:

The analysis of one instanton operators implies the correct global symmetry.

→ affine $SO(4N)$ symmetry!
1 – (ii). $N < 2k$

In this case, the B, C-branes do not move to the rightmost column but meet in the middle of the 5-brane web.
I – (ii). $N < 2k$

In this case, the B, C-branes do not move to the rightmost column but meet in the middle of the 5-brane web.

Then, a pair of B and C-branes can turn into an O7\(^-\)-plane.
$1 - (ii). \ N < 2k$

The quiver gauge theory involves an Sp gauge group at the right end.

\[ [4] - SU(2k+N-1) \]
\[ \downarrow \]
\[ SU(4k+2N-6) - \ldots - SU(4k-2N+6) - Sp(2k-N+1) \]

\[ [4] - SU(2k+N-1) \]

N-2
So far we have resolved two O7-planes. We may also resolve one of them. The analysis is essentially the same as before.
So far we have resolved two O7-planes. We may also resolve one of them. The analysis is essentially the same as before.

(II). Resolution of one O7-plane. \((2N \geq 2k + 1)\)

\[ [8] - SU(2N+2k-1) - Sp(2N+2k-5) - SO(4N+4k-10) - \ldots \]

\[ \ldots - SO(4N-4k+6) - Sp(2N-2k-1) - [2N-2k] \]
• Therefore, we again obtain different 5d gauge theory descriptions from a 6d SCFT.

\[8\] – SU(2N) – Sp(2N-4) – SO(4N) – ... – Sp(2N-4) – [SO(4N)]

Resolution of two O7-planes
5d quiver of D-type

Resolution of one O7-plane
5d SO – Sp linear quiver
4. Conclusion
We proposed novel resolution of an O7⁻-plane with 4 D7-branes which appear at the intersection between an ON-plane and an O5-plane.

The resolution yields new 5d gauge theory descriptions of 6d SCFTs.

The 5d gauge theory can be a quiver of D-type or can involve SO and Sp gauge groups and it further enlarge the landscape of UV complete 5d theories.