Equivalence and Classifications of 5d rank 2 theories

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Current Topics in String Theory: Conformal Field Theories

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Hirotaka Hayashi, Kimyeong Lee, Masato Taki, Futoshi Yagi
Introduction

Talks by Hirotaka Hayashi, Masato Taki, Futoshi Yagi on 1st day

Various N=(1,0) SCFTs → 5d descriptions: IIB (p,q) web or quiver

New perspectives on 6d and 5d SCFTs
UV duality, 5d dualities

[Bergman, Hayashi, SSK, Kimyeong Lee, Taki, Yagi, Yonekura, Zafrir]
Introduction

• 5d SU(2) $N_f=8$ $\rightarrow$ 6d E-string theory on $S^1$

  Tao diagram provides with
  (i) Infinite spirals (KK spectrum)
  (ii) constant period (compactification circle)
  (iii) computational tool: $Z_{\text{Nek}}$

5d SU($N+2$), $N_f=2N+8$
Talk by Hirotaka Hayashi

[Del Zotto - Heckman - Tomasiello - Vafa ’14]

• 6d \( (D_{N+4}, D_{N+4}) \) conformal matter
  \[ \rightarrow \quad 5d \ SU(N+2), \ N_f=2N+8 \]
Introduction

Talk by Hirotaka Hayashi
[Del Zotto - Heckman - Tomasiello - Vafa ’14]

- 6d \((D_{N+4}, D_{N+4})\) conformal matter
  \[\rightarrow\quad 5d \text{ SU}(N+2), N_f = 2N+8\]
  Not unique!

5d \([4]\) - SU(2) - SU(2) - … - SU(2) - [4]  \(S\)-duality

5d Sp\((N+1)\), \(N_f = 2N+8\)  Gaiotto-Kim

Various different looking 5d descriptions have the same UV theory in 6d  “UV duality”
UV duality check: equivalence

6d Sp(N) theory
$N_f = 2N + 8$, Tensor

5d SU(N+2) theory
$N_f = 2N + 8$

5d Sp(N+1) theory
$N_f = 2N + 8$
6d Sp(1) theory
$N_f=10$, Tensor

5d SU(3) theory
$N_f=10$

5d Sp(2) theory
$N_f=10$
Contents

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Equivalence: Brane configurations

Idea:

6d (1,0) SCFTs

Tensor branch

IIA brane configurations

[Hanany, Zaffaroni '97, Brunner, Karch '97]

on $S^1$ and T-dual

IIB (p,q) brane web diagrams

: 5d descriptions
Equivalence: Brane configurations

Idea:

6d (1,0) SCFTs: Conformal matter

Tensor branch

one O8- IIA brane configurations

[Hanany, Zaffaroni '97, Brunner, Karch '97]

on S¹ and T-dual

two O7- IIB (p,q) brane web diagrams

Resolve O7s: two O7s T-dual of 6d

O7- -plane 5d Sp description
= [1,1] 7-brane

one O7- 5d SU description

[Sen '96]

+ [1,-1] 7-brane

no O7-
M5-brane probing $D_5$ singularity

6d $Sp(1)$ gauge theory with $N_f=10$ and Tensor multiplet

[ Hanany, Zaffaroni '97, Brunner, Karch '97]
M5-brane probing $D_5$ singularity

Tensor branch

6d $Sp(1)$ gauge theory with $N_f=10$ and Tensor multiplet

T-duality
Resolving two O7-:

6d $Sp(1) \ N_f = 10$

5d $SU(3)_0 \ N_f = 10$

[Hayashi-SSK-Lee-Taki-Yagi '15]
[Yonekura '15]
5d Sp description

Resolving only one O7−:

6d $Sp(1) \ N_f = 10$

5d $Sp(2) \ N_f = 10$

[Hayashi-SSK-Lee-Yagi ’15]
Equivalence:
two different 5d Brane configurations, in fact, come from the same 6d brane setup!

5d SU(3)$_0$ theory
$N_f = 10$

5d Sp(2) theory
$N_f = 10$

both O7s resolved
only one O7 resolved

Origin of SU-Sp duality

[Gaiotto-Kim ’15]
Equivalence: explicit checks

Partition functions

6d \textbf{Sp}(1) \text{ theory} \\
N_f=10, \text{Tensor} \\

Elliptic genus \\
[’15 Joonho Kim, Seok Kim, Kimyeong Lee]

Wilson lines

5d \textbf{SU}(3) \text{ theory} \\
N_f=10 \\
[’16 Hayashi-SSK-Lee-Yagi]

Topological vertex \\
via Tao diagram

Map

5d \textbf{Sp}(2) \text{ theory} \\
N_f=10 \\
[Nekrasov, Shadchin ’04] \\
[’16 Hayashi-SSK-Lee-Yagi] \\
[’16 Yun]

Wilson lines

The next talk by \textbf{Youngbin Yun}
Wilson lines for 6d $\text{Sp}(1)$ and 5d $\text{Sp}(2)$

6d $\text{Sp}(1)$  5d $\text{Sp}(2)$

$\tilde{q} = q'^2$  \hspace{1cm} Instanton = KK mode  \\

\tilde{y}_i = y'_i, \hspace{0.5cm} (i = 1, \ldots 9)$

$\tilde{y}_{10} = y'_{10}q'^{-2}$

$\tilde{\varphi} = A'_1 q' y'_{10}^{-1}$  \hspace{1cm} Gauge

$\tilde{A} = A'_2$

\[
Z_{6d \text{ Sp}(1)}(\tilde{A}, \tilde{q}, \tilde{y}, \tilde{\varphi}) = Z_{\text{Sp}(2)}(A', q', y')
\]
Map between $\text{Sp}(2)$ and $\text{SU}(3)$

[$'15$ Gaiotto-Kim] [$'16$ Hayashi-SSK-Lee-Yagi]

\[
q' = q
\]

**Instanton**

**Flavors**

\[
y_i' = \lambda y_i, \quad (i = 1, \ldots, 5)
\]

\[
y_i'^{-1} = \lambda y_i, \quad (i = 6, \ldots, 10)
\]

**Gauge**

\[
A_i'^{-1} = \lambda A_i, \quad (i = 1, 2)
\]

\[
\lambda = q^{\frac{1}{2}} \prod_{i=1}^{10} y_i^{-\frac{1}{4}}
\]

\[
Z_{\text{Sp}(2)}(A', q', y') = Z_{\text{SU}(3)}(A, q, y)
\]
Equivalence: explicit checks

Global symmetry

6d $\text{Sp}(1)$ theory
$N_f=10$, Tensor

$\text{SO}(20)$

checked!

5d $\text{SU}(3)$ theory
$N_f=10$

$\text{SO}(20) \supset \text{U}(10)$

checked!

5d $\text{Sp}(2)$ theory
$N_f=10$

$\text{SO}(20)'$

checked!
Classifications

Work in progress
How many 5d rank 1 theories?

classified! all global symmetries, brane configurations

For example, 16 inequivalent dual toric diagrams

*Figure 2*: All the $GL(2,\mathbb{Z})$-inequivalent convex lattice polygons with single internal point and their dual web diagrams.

Also, toric-like diagrams (Nf=5, 6, 7)
How many 5d **rank 2** theories exit?

For 5d rank 2 theories, do we know all possible brane configurations (Lagrangian or non-Lagrangian), and global symmetries?

Or, can we classify dual toric(-like) diagrams? (convex, two interior points)

-> Mathematicians studied for the toric case. Some examples are

![Diagram](image-url)
Non-Lagrangian: i), s)

“Partial”-Lagrangian: e), m), n), g), j), r)
Toric-like diagrams: Nf > 6 or large CS

Class S theories

[’09 Benini-Benvenuti-Tachikawa]
[’15 Zafrir]
Summary

1. UV duality from brane configurations

2. UV duality is explicitly checked for non-trivial 5d gauge theories (5d SU(3) Nf=10 and 5d Sp(2) Nf=10)!

3. Classification of 5d rank 2 theories is work in progress