Abstract

Cluster Algebras in Combinatorics and Topology

Name	Karin Baur
Title	Dimer algebras and cluster categories with Grassmannian structure
Abstract	We use dimer models with boundary as quivers with potentials to introduce dimer algebras. This provides in particular a combinatorial model for cluster categories with Grassmannian structure.

Name	Arkady Berenstein
Title	Noncommutative marked surfaces.
Abstract	The goal of my talk (based on a recent joint work with Vladimir Retakh) is to attach a noncommutative cluster-like structure to each marked surface. This is a noncommutative algebra generated by "noncommutative geodesics" between marked points subject to certain triangular relations and noncommutative analogues of Ptolemy-Plucker relations. It turns out that the algebra exhibits a noncommutative Laurent Phenomenon with respect to any triangulation of the surface, which confirms it "cluster nature". As a surprising byproduct, we obtain a new topological invariant of the surface, which is a free or a 1-relator group easily computable in terms of any triangulation of the surface. Another application is the proof of Laurentness and positivity of certain discrete noncommutative integrable systems.

Name	Ilke Canakci
Title	Snake graphs, skein relations and their application in Jacobian algebras
Abstract	This talk will focus on abstract snake graphs which were inspired by snake graphs associated to surface triangulations. This work is joint work with Ralf Schiffler. Snake graphs are constructed from the crossing pattern of arcs in triangulated surfaces whereas the notion of abstract snake graphs is untied from the geometry of surfaces and is given in an elementary way. Moreover, the skein relations, which give a formula for the product of cluster variables, can be interpreted in this abstract setting. Furthermore, this talk will cast an

	application where the snake graph calculus is the main ingredient. In this work, joint work with Sibylle Schroll, we give a dimension formula for the extension space of string modules associated to Jacobian algebras arising from (unpunctured) surface triangulations.

Name	Frédéric Chapoton
Title	Cluster varieties for tree-shaped quivers, and their cohomology
Abstract	Understanding the cohomology of varieties associated with cluster algebras would be interesting to gain geometric understanding. I will present some partial results on the case of quivers whose underlying graph is a tree. This study involves a surprising canonical 3-coloring of the vertices of all trees, related to maximal matchings.

Name	Laurent Demonet
Title	Ice quivers with potential associated with triangulations and Cohen–Macaulay modules over orders (case An and Dn)) (joint with Xueyu Luo)
Abstract	In this talk, we attach an ice quiver with potential (Q, W, F) to each triangulation s of a polygon (resp. a polygon with one puncture) where the set F of frozen vertices correspond to the sides of the polygon. This quiver extends the one introduced by Labardini–Fragoso and Cerulli Irelli. Thus, we consider the (non-completed) frozen Jacobian algebra G = P(Q, W, F). One of the main result is that G as the structure of an order over K[x], that L = e G e is a Gorenstein order independent of s (e is the sum of idempotents at the frozen vertices of s). Moreover, the cluster tilting objects in CM(L) are the modules e G for all triangulations s. More precisely, its stable category CM(L) is equivalent to a cluster category of type A (resp. type D).

Name	Anna Felikson
Title	CW-complexes from Coxeter groups and their mutations
Abstract	We construct a geometric interpretation of the results Barot and Marsh and their generalizations: for any quiver or a diagram of a certain mutation class (in particular, this applies to finite and affine types, and unpunctured surfaces/orbifolds) we provide a CW-complex which is a quotient of the Davis complex of the corresponding Coxeter group with a proper Coxeter group action. We define an operation of mutation of these complexes agreeing with

	mutations of quivers. We also investigate some partial cases when these complexes are geometric manifolds. This is a joint work with Pavel Tumarkin.

Name	Michael Gekhtman
Title	Cluster Structures on simple Lie groups and their doubles
Abstract	We will discuss results and conjectures on natural cluster structures in the rings of regular functions on simple complex Lie groups and Poisson–Lie structures compatible with these cluster structures. Much of this talk is based on an ongoing collaboration with M. Shapiro and A. Vainshtein.

Name	Kiyoshi Igusa
Title	Categories of noncrossing partitions
Abstract	In joint work with Gordana Todorov, we constructed a category whose morphisms are given by clusters. We call it the ``cluster morphism category''. This has applications to the cohomology of groups and to maximal green sequences. In a separate paper, I gave a combinatorial version of a special case of this category. Objects of this category are noncrossing partitions in the traditional sense. Hubery and Krause have also constructed a category of noncrossing partitions in which the objects are generalized noncrossing partitions. In this talk I will use a variation of the cluster morphism category to compare the two categories of noncrossing partitions. Roughly speaking, there is a projection functor from this new cluster morphism category to the Hubery-Krause category of noncrossing partitions and the fiber category is my category of noncrossing partitions. I will illustrate this in several special cases.

Name	Philipp Lampe
Title	Almost periodic sequences attached to non-crystallographic root systems and cluster theory
Abstract	We study Fomin-Zelevinsky's mutation rule in the context of non-crystallographic root systems. In particular, we construct almost periodic sequences of real numbers for the non- crystallographic root systems of rank 2 by adjusting the exchange relation for cluster algebras. Moreover, we describe a matrix mutation class in rank 3.

Name	Kyu-Hwan Lee
Title	Combinatorial approach to root multiplicities of rank two hyperbolic Kac-Moody algebras
Abstract	Root multiplicities are fundamental data in the structure of a Kac-Moody algebra. For finite and affine Kac-Moody algebras, root multiplicities are completely known. However, root multiplicities of hyperbolic Kac-Moody algebras are still mysterious. In this talk, we will consider a new combinatorial approach to root multiplicities of rank two hyperbolic Kac-Moody algebras.

Name	Li Li
Title	Bases of Cluster algebras
Abstract	Cluster algebras were introduced by Fomin and Zelevinsky in trying to understand canonical bases in algebraic groups. A lot of activity in the theory of cluster algebras has been directed towards various constructions of natural bases in them. In this talk, I will discuss some recent advances in the study of cluster algebras, in particular, various natural bases of the cluster algebras and their connection to two classes of special varieties, namely the quiver Grassmannians and Nakajima's graded quiver varieties.

Name	Travis Mandel
Title	Tropical curve counts and a Frobenius structure on cluster algebras
Abstract	I will describe a Frobenius trace on "middle" cluster algebras (which often equal the upper cluster algebra containing them). With respect to the canonical bases defined by Gross, Hacking, Keel, and Kontsevich, this trace can be expressed in terms of certain weighted counts of tropical curves. In particular, this gives a different perspective on the GHKK construction of these bases. Analogous statements hold for rings of global regular functions on cluster X-varieties.

Name	Greg Muller
Title	Twists for positroid cells
Abstract	A Grassmannian admits a stratification into positroid cells, analogous to the double Bruhat stratification of a semisimple Lie group. Each positroid variety has a (conjectural) cluster structure, for which some of the cluster tori are indexed by certain bipartite graphs in the disc. Curiously, the same graphs

	each determine a different algebraic torus inside the same positroid variety, via Postnikov's `boundary measurement map'. Reconciling these two sets of tori has been an open problem since Postnikov's original manuscript featured a missing chapter on the subject. Joint with David Speyer, we introduce a simple `twist' automorphism of each positroid cell, which takes one type of torus to the other. This provides simple proofs of several open questions about these tori, as well as a generalization of the `Chamber Ansatz' of Berenstein-Fomin-Zelevinsky.

Name	Gregg Musiker
Title	Combinatorics of the Del-Pezzo 3 Quiver: Aztec Dragons, Castles, and Beyond
Abstract	In this talk, I will focus on combinatorial aspects of connections between cluster algebras and string theory. In particular, I will focus on a specific six-vertex quiver, the dP3 quiver, and toric mutation sequences of it, i.e. mutations which only occur at a vertex with two in-coming arrows and two out-going arrows. Work with REU students I. Jeong and S. Zhang, and later with REU students M. Leoni, S. Neel, and P. Turner, led to combinatorial interpretations for cluster variables arising from a two-dimensional subspace of toric mutations. I will also discuss recent work in progress with T. Lai towards extending this combinatorial interpretation to a three-dimensional subspace. I will not assume prior background on string theory and the talk will include a partial dictionary between cluster algebras and string theory.

Name	Nathan Reading
Title	Models for cluster algebras of affine type
Abstract	Cluster algebras of finite type constitute the best-understood class of cluster algebras, aside from rank-2 cluster algebras. One of the first results in the structural theory of cluster algebras was the connection between cluster algebras of finite type and finite root systems. Recent work of Felikson, Shapiro, Thomas, and Tumarkin reveals that the notion of "affine type" is also a cluster-theoretic notion, rather than simply a root-system-theoretic notion: Outside of rank two, "affine type" means "linear growth." Much of what is known in finite type is proved using combinatorial models. I will describe work (completed, in progress, and conjectural) on similar models in affine type. Specifically, I will discuss joint work with Speyer realizing the g-vector

	fan in affine type as a "doubled Cambrian fan" and joint work in progress with Stella realizing the d-vector fan in affine type in terms of compatibility of "almost positive Schur roots." I will also discuss universal coefficients for cluster algebras of affine type. It seems possible that cluster algebras of affine type will soon be just as well-understood combinatorially as cluster algebras of finite type.

Name	Dylan Rupel
Title	Greedy bases for rank 2 generalized cluster algebras
Abstract	A remarkable basis for rank 2 cluster algebras known as the "greedy basis" was introduced by Lee, Li, and Zelevinsky. In this talk I will describe how all of the nice properties of this basis including the combinatorial construction can be extended to provide a similar basis in rank 2 generalized cluster algebras, i.e. where the standard binomial exchange relations are replaced by essentially arbitrary polynomial exchange relations. If there is time I will also discuss the existence of a quantum lift of this greedy basis in the spirit of a joint work with Lee, Li, and Zelevinsky for the binomial case.

Name	Ahmet Seven
Title	Invariants for quivers of finite mutation type
Abstract	In this talk, we will discuss some algebraic and numerical invariants for quiver mutations. Main examples will be the groups of transvections associated with quivers and their orbits. We will establish combinatorial properties of these invariants for quivers from surfaces and discuss applications to c-vectors and maximal green sequences.

Name	Vasilisa Shramchenko
Title	Unistructurality and automorphisms of cluster algebras
Abstract	We introduce and study the notion of a cluster automorphism. In the case of cluster algebras arising from triangulations of surfaces, we relate the group of cluster automorphisms to the mapping class group of the corresponding surface. We also introduce and discuss the notion of unistructurality for cluster algebras. In particular, we prove that cluster algebras of Dynkin type are unistructural, which means that the set of their cluster variables cannot be obtained as a set of cluster variables of some other cluster algebra. This

	is a joint work with I. Assem and R. Schiffler.

Name	Michael Shapiro
Title	Exotic cluster structures on SL(N).
Abstract	We will construct cluster structure in the space of regular functions on $SL(N)$ compatible with the Cremmer-Gervais Poisson-Lie structure. We prove that on $SL(3)$ the cluster algebra does not coincide with the corresponding cluster algebra. This a joint work with M.Gekhtman and A.Vainshtein.

Name	Gordana Todorov
Title	Maximal green sequences, Picture groups and Spherical do-mains of semi-invariants
Abstract	Green mutations and maximal green sequences are studied by considering pictures L(Q) and picture groups G(Q) associated to valued quivers Q. Spherical domains of semi-invariants on presentation spaces are shown to dene such pictures and the normal orientation on these pictures is used in an essential way in order to determine green mutations. Using truncated pictures we have shown that the number of maximal green sequences is nite in the tame valued case and we are working on the general case. For nite type, we have shown that maximal green sequences are in bijec-tion with positive expressions of the Coxeter element in the picture group G(Q) associated to the picture L(Q) dened using domains of semi-invariants.

Name	Milen Yakimov
Title	Cluster structures or quantum Richardson varieties
Abstract	Richardson varieties play an important role in Schubert calculus and total positivity for flag varieties. Leclerc defined a cluster algebra inside the coordinate ring of each open Richardson variety for a symmetric Kac-Moody group, and Muller and Speyer studied these cluster algebras in the case of Richardson varieties in Grassmannnians. We will show how to realize the quantized coordinate ring of each open Richardson variety as a normal localization of a prime factor of a quantum Schubert cell algebra. This can be used to produce families of chains of subalgebras whose sequences of normal elements produce toric frames. They can be used to extend Leclerc's

	result to the quantum situation and to all symmetrizable Kac-Moody algebras, and to control the size of the cluster algebra from below. This is a joint work with Tom Lenagan (Univ Edinburgh).

Name	Yu Zhou
Title	Intersection-dimension formulae from decorated marked surfaces
Abstract	For each triangulation T of a decorated marked surface S, there is an associated differential graded algebra whose finite dimensional derived category D is a 3-Calabi-Yau triangulated category. In this talk, we give a correspondence between spherical twists on spherical objects in D and braid twists on closed curves in S and show that it is independent of the choice of the triangulation T. We prove formulae about intersection numbers of curves in S and dimensions of graded morphism spaces in D. This is a joint work with Yu Qiu.