## Workshop on Algebraic Surfaces and Moduli

April 11-15, 2016

Rm 1501, Natural Science B/D E6-1,

KAIST, Daejeon, Korea

Organizers

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Inquiries

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### Monday April 11 (Room 1501)

09:40 - 10:40	Fabrizio Catanese (Univ. of Bayreuth) Canonical surfaces of high degree for $p_g=4$ , 5, 6.
11:00 - 12:00	Wenfei Liu (Xiamen Univ.) Surfaces of general type with q=2 are rigidified
12:00 - 13:30	Lunch
13:30 - 14:30	Lidia Stoppino (Univ. of Poitiers) Eventual properties of continuous linear systems on irregular varieties
14:50 - 15:50	DongSeon Hwang (Ajou Univ.) <i>Cascades of singular del Pezzo surfaces</i>

#### **Tuesday April 12 (Room 1501)**

09:40 - 10:40	Gian Pietro Pirola (Univ. of Pavia) On rational dominant maps from surfaces of general type
11:00 – 12:00	Miguel Angel Barja (Polytechnic Univ. of Catalonia) <i>Continuous rank and higher dimensional Castelnuovo and Clifford-</i> <i>Severi inequalities on varieties of maximal Albanese dimension.</i>
12:30 - 13:30	Lunch
13:30 – 14:30	Dongsoo Shin (Chungnam National Univ.) Deformations of weighted homogeneous surface singularities admitting rational homology disk smoothings
15:00 – 15:30	Contributed Talk: Yonghwa Cho (KAIST) <i>Line bundles on surfaces associated with singular degenerations</i>
15:45 – 16:15	Contributed Talk: Gebhard Martin (Tech. U. of Munich) Enriques surfaces with finite automorphism groups in positive characteristic
16:30 - 17:00	Contributed Talk: Sangho Byun (Seoul National University) Stability of pencils of quadrics and nets of quadrics
18:00 -	Banquet

## Wednesday April 13 (Room 1501)

09:40 - 10:40	Francesco Polizzi (Univ. of Calabria) On triple planes with $p_g=q=0$
11:00 - 12:00	Giancarlo Urzua (Pontificia Univ. of Catolica de Chile) The Craighero-Gattazzo surface is simply-connected
12:00 - 12:50	Lunch
12:50 - 20:30	Excursion

#### Thursday April 14 (Room 1501)

09:40 - 10:40	Sheng-Li Tan (East China Normal Univ.) Effective behavior of the pluricanonical systems of an ODE on an algebraic surface
11:00 - 12:00	Xavier Roulleau (Univ. of Poitiers) <i>An explicit construction of Schoen surfaces</i>
12:00 - 13:30	Lunch
13:30 - 14:30	Joonyeong Won (KAIST) <i>Cylinders in rational surfaces</i>
14:50 - 15:20	Contributed Talk: Yeongrak Kim (KAIST) <i>Chow form and Ulrich bundles on surfaces</i>
15:30 - 16:00	Contributed Talk: In-Kyun Kim (Seoul National University) <i>Log canonical thresholds of birationally rigid Fano threefolds</i>
16:15 – 17:15	Department Colloquium: Fabrizio Catanese (Univ. of Bayreuth) Configurations of lines and interesting algebraic surfaces

#### Friday April 15 (Room 1501)

09:40 - 10:40	Ingrid Bauer (Univ. of Bayreuth) <i>Line configurations and new rigid surfaces</i>
11:00 - 12:00	Valery Alexeev (Univ. of Georgia) <i>Geometric compactifications of moduli of K3 surfaces</i>

#### Valery Alexeev

#### Univ. of Georgia

#### Geometric compactifications of moduli of K3 surfaces

I will talk about concrete compactifications of the moduli spaces of K3 surfaces obtained by adding stable pairs on the boundary, concentrating on the case of degree 2.

#### Miguel Angel Barja

#### Polytechnic Univ. of Catalonia

## Continuous rank and higher dimensional Castelnuovo and Clifford-Severi inequalities on varieties of maximal Albanese dimension.

I will present new Clifford-Severi inequalities relating the volume and the continuous rank of line bundles on varieties of maximal Albanese dimension. I also will give an extension for such varieties of the classical results of Castelnuovo for the rank of the multiplication map on global sections for curves. Even in the case of irregular surfaces, we obtain new geographical bounds. These results rely on two main techniques for the study of continuous linear series on irregular varieties, which are of independent theoretical interest. The first one is the eventual behaviour of such continuous systems, explained in the talk given by L. Stoppino. The second ingredient is the extension of the continuous rank of a line bundle to a real function with good regularity properties. I will show how these two ingredients provide short and conceptual proofs of the above mentioned theorems and allow us to classify varieties and line bundles on the Severi lines. This is a joint work with R. Pardini and L. Stoppino.

#### **Ingrid Bauer**

#### Univ. of Bayreuth

#### Line configurations and new rigid surfaces

Rigid surfaces are surfaces which have no nontrivial deformations. Up to now not many rigid surfaces of general type are known. In fact, the known surfaces are all projective classifying spaces (i.e., their universal cover is contractible). They are uniformised by the ball or the bidisk, or are the examples of Mostow and Siu, or the Kodaira fibrations of Catanese-Rollenske.

Together with F. Catanese we showed that for all natural numbers  $n \ge 4$  the Kummer covering of the plane branched over a complete quadrangle of exponent  $n \ge 1$  has no infinitesimal deformations, in particular is rigid. Instead this is not true for n = 3.

I will explain the main ingredients of the proof and how to extend these results to obtain similar results for Abelian covers of \$\mathbb P^2\$ branched in other line configurations, as e.g. the Hesse configuration which is a joint project with R. Pignatelli.

#### Fabrizio Catanese

#### Univ. of Bayreuth

#### Canonical surfaces of high degree for $p_g=4,5,6$ .

A canonical surface Y is the image of a minimal surface S under a birational canonical map f\_1. For this to happen, we must have  $p_g \ge 4$ , and for  $p_g = 4,5$  there are only 3 cases where f\_1 is a biregular embedding. But one expects many such examples for  $p_g = 6$ .

By BMY the canonical volume K\_S<sup>2</sup> is bounded by 9 ( $p_g + 1$ ), and we know that the degree d of Y is bounded by the canonical volume.

For  $p_g=4$  the question posed by Enriques was whether the canonical volume would be strictly smaller than 25 (as a consequence of the Enriques-Kuranishi inequality for the number of moduli).

Enriques' question raises interest about the highest possible canonical volume and degree of canonical surfaces with small  $p_g=4,5,6$ .

For  $p_g = 4$  together with Ingrid Bauer we found examples (ball quotients) with the highest volume K<sup>2</sup>= 45, but it is not yet clear which is the highest possible value for d= d(Y) (using the method of singular bidouble covers, I found values reaching until 28).

For  $p_g=5$  the problem is less explored, whereas for  $p_g=6$  one asks for canonically embedded surfaces S.

If one further looks at the equations of canonical surfaces, there are very interesting connections with homological algebra, and one can deal with these when the canonical degree is small.

For instance, for  $p_g=6$ , by a theorem of Walter, there is an alternating form on a vector bundle E on projective 5-space such that S is the Pfaffian locus.

This vector bundle can be described (cf.my Santa Cruz article) in the range  $11 \le d = K^2 \le 17$ , but for higher K<sup>2</sup> it becomes very hard to describe it. The Kapustzka brothers succeeded for d=18, and they asked whether 18 would be the maximum possible degree.

In a quite recent paper I exhibited a family of surfaces S with  $K^2_S = 24$ ,  $p_g(S) = 6$ , q(S)=0, and which are canonically embedded with degree d =24.

I shall finish the talk with other examples and questions.

#### **DongSeon Hwang**

#### Ajou Univ.

#### Cascades of singular del Pezzo surfaces

The classification of smooth del Pezzo surfaces is classical. They are either the projective plane, the quadric surface, and blow-ups of the projective plane in at most \$8\$ general points. Similar classification was obtained by Demazure, Hidaka and Watanabe for rational Gorenstein del Pezzo surfaces. The key idea is to consider points on the projective plane in almost general position as a blowup locus to get minimal resolutions of those surfaces.

I would like to talk about its generalization for del Pezzo surfaces with worse singularities. Special emphasis will be put on log del Pezzo surfaces of Picard number one, which leads to the classification of those surfaces. The proof is based on the complete description of Miyanishi-Zhang's theory on log del Pezzo surfaces of rank one.

#### Wenfei Liu

#### Xiamen Univ.

#### Surfaces of general type with q=2 are rigidified

A compact complex manifold X is called rigidified if there is no nontrivial holomorphic automorphism of X that is isotopic to the identity. Catanese asked if compact complex manifolds of general type are rigidified. In this talk I will address this question for surfaces of general type with a special focus on those with q=2.

#### **Gian Pietro Pirola**

#### Univ. of Pavia

#### On rational dominant maps from surfaces of general type

Let X be a complex algebraic surface of general type. We would like to study the set L(X) of rational dominant maps  $X \dashrightarrow Y$ , up to birational equivalence, where Y is not rational surface:

 $L(X) = \{f : X \dashrightarrow Y; f \text{ dom.}; Y \text{ non rat.}\}/_{bir.equiv}$ 

I will present some results obtained, by combining Hodge theoretical and deformation methods, in joint works with Yongnam Lee. In particular we have proved:

**Theorem** Let l(X) = #L(X) be the cardinality of L(X).

- (1) If  $X \subset \mathbb{P}^3$  is the very general surface of degree  $d \ge 5$  then l(X) = 1.
- (2) If  $X = C \times D$  where C and D are the very general curve of genus respectively g(C) > 2 and g(D) > 6 then l(X) = 3.
- (3) If  $X = C_2$  is the 2- symmetric product of a very general curve of genus g > 10 then l(X) = 1.

Similar results hold for hyperplane (of high degree) of some Fano 3-folds.

#### Francesco Polizzi

#### Univ. of Calabria

#### *On triple planes with p\_g=q=0*

We will outline a classification of triple planes with  $p_g=q=0$ , and we will relate them to the geometry of some rank 2 Steiner bundles on  $\Lambda P^2$ . This is a joint work with D. Faenzi and J. Valles.

Xavier Roulleau

Univ. of Poitiers

#### An explicit construction of Schoen surfaces

Schoen surfaces are surfaces of general type ( $K^2=2c_2=16$ , q=4, pg=5) which cary many remarkable properties and have been constructed by smoothing some reducible surfaces. In this talk, we will explain how to construct such a surface as a double cover of an explicit 40 nodal surface, which is a complete intersection of the Igusa quartic threefold with a quadric in special position. One irreducible component of the moduli space of these surfaces of general type appears to be unirational and 4 dimensional.

This is a joint work (in progress) with Carlos Rito and Alessandra Sarti, plus a previous work with Ciro Ciliberto and Margarida Mendes Lopes.

#### **Dongsoo Shin**

#### Chungnam National Univ.

#### Deformations of weighted homogeneous surface singularities admitting rational homology disk smoothings

We investigate the relations between certain partial resolutions and the base space of a semiuniversal deformation space of a weighted homogeneous surface singularity admitting a rational homology disk smoothing. This is a joint work in progress with Heesang Park.

#### Lidia Stoppino

#### Univ. of Insubria

#### Eventual properties of continuous linear systems on irregular varieties

This is a joint work with M. A. Barja and R. Pardini. Consider a natural generalization of varieties of Albanese maximal dimension: a variety X admitting a finite map a on an abelian variety A. We study the linear system |L+P|, where P is a general element of Pic<sup>0</sup>(A) (the so-called continuous systems).

I want to discuss eventual properties of continuous linear systems, i.e. properties that are preserved or that "stabilize" taking a base change of X by the multiplication by d map of A, for d going to infinity. I will describe a structural factorization theorem for the map a, and its consequences on the eventual properties of line bundles. This will be the starting point for a proof of new geographical inequalities which will be described by M. A. Barja in his talk.

I will construct and discuss some examples, mainly for the case where the continuous system is the paracanonical one.

#### Sheng-Li Tan

#### East China Normal Univ.

# *Effective behavior of the pluricanonical systems of an ODE on an algebraic surface*

A holomorphic foliation on an algebraic surface is the set of holomorphic solutions of an ordinary differential equation. One can define its canonical and pluricanonical systems, Kodaira dimension and Chern numbers. We will talk about some effective bounds for the k-very ampleness of the pluricanonical systems.

#### Giancarlo Urzua

#### Pontificia Univ. Católica de Chile

#### The Craighero-Gattazzo surface is simply-connected

This is a joint work with Julie Rana (UMinnesota) and Jenia Tevelev (Umass Amherst).

We show that the Craighero-Gattazzo surface, the minimal resolution of an explicit complex quintic surface with four elliptic singularities, is simply-connected. This was conjectured by Dolgachev and Werner, who proved that its fundamental group has a trivial profinite completion. The Craighero-Gattazzo surface is the only explicit example of a smooth simply-connected complex surface of geometric genus zero with ample canonical class. We hope that our method will find other applications: to prove a topological fact about a complex surface we use an algebraic reduction mod p technique and deformation theory.

#### Joonyeong Won

#### KAIST

#### Cylinders in rational surfaces

For a projective variety X and an ample divisor H on it, an H-polar cylinder in X is an open ruled affine subset whose complement is a support of an effective Q-divisor Q-rationally equivalent to H. This notion links together affine, birational and Kaehler geometries. We investigate rational surfaces (in particular del Pezzo surfaces) to find effective ample Qdivisors such that the complements are cylinders. This is joint work with Ivan Cheltsov and Jihun Park.

#### [Contributed Talk]

#### Sangho Byun

#### Seoul National University

#### Stability of pencils of quadrics and nets of quadrics

In this talk, we will introduce results for the GIT stability of pencils of quadrics in P^n. Then we will analyze GIT stability of nets of quadrics in P^5 up to projective equivalence and discuss some connections between a net of quadrics and the associated discriminant sextic curve.

#### Yonghwa Cho

#### KAIST

#### Line bundles on surfaces associated with singular degenerations

In this talk, we discuss how to understand line bundles on a general fiber of a deformation in terms of information coming from the central fiber. If the deformation is a \$\mathbb Q\$-Gorenstein smoothing of a singular surface, then line bundles on the general fibers are usually associated with coherent sheaves on the central fiber. We apply Hacking's exceptional vector bundle construction to compare the coherent sheaves on the central fiber and the line bundles on the general fiber. As an application, we present a semiorthogonal decomposition of a derived category of a surface with Kodaira dimension one in terms of an exceptional collection of line bundles of length 12 and a phantom category. This is a joint work with Yongnam Lee.

#### In-Kyun Kim

#### Seoul National University

#### Log canonical thresholds of birationally rigid Fano threefolds

We calculate the global log canonical thresholds of birationally rigid orbifold Fano threefolds embedded in weighted projective spaces as codimension two or three. As an important application we show that most of them admit Kahler-Einstein metrics. This is a joint work with Takuzo Okada and Joonyeong Won.

#### Yeongrak Kim

#### KAIST

#### Chow form and Ulrich bundles on surfaces

In 1860, Cayley introduced a geometric invariant by considering lines intersect with a given space curve. Such lines sweep a 3-dimensional family in the Grassmannian G(2,4). Indeed, it becomes a hypersurface. Later, Chow and van der Waerden generalized this notion for arbitrary projective varieties. It allows a viewpoint that regarding a given projective variety as a divisor of a Grassmannian whose defining equation is called the Chow form. However, quite less is known about the Chow form, and even the calculation is difficult. On the other hand, Grothendieck extended Cayley's idea, and defined so called the "Chow complex". Eisenbud and Schreyer gave a concrete construction, and observed that the complex becomes extremely simple when we are handling "Ulrich bundles". In this talk, I will briefly review the notion of the Chow form, Ulrich bundles, and a computing method using an Ulrich bundle. Then I will introduce how we can construct Ulrich bundles on some nice surfaces.

#### **Gebhard Martin**

#### Tech. U. of Munich

#### Enriques surfaces with finite automorphism groups in positive characteristic

Enriques surfaces with finite automorphism groups over the complex numbers have been classified explicitely by S. Kondo (1986) and in terms of their root invariants by V.V. Nikulin (1983). Using an approach similar to that of S. Kondo, we show that his explicit classification still holds if the characteristic of the base field is big enough. If time permits, recent progress in small characteristics will also be discussed.