2016 Combinatorics Workshop
2016 조합론 학술대회

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Organized by
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KIAS Center for Mathematical Challenges (KIAS CMC).
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	July 21	July 22	July 23
09:00 - 10:00		Kyungyong Lee	Mihyun Kang
10:00 - 11:00			
11:00 - 12:00		Seongmin Ok Younjin Kim	Dong Han Kim Hanbaek Lyu
12:00 - 13:00		Lunch (E5, 1st floor)	Lunch (E5, 1st floor)
13:00 - 14:00			
14:00 - 15:00	Andreas Holmsen	Seung Jin Lee	
15:00 - 16:00			
16:00 - 17:00	Ringi Kim Zhicong Lin	Seog-Jin Kim	
17:00 - 18:00		Ilkyoo Choi	
18:00 - 19:00	Dinner (TBA)	Banquet (Faculty Club)	

Lecture Hall:

- Room 1501 (공동강의실), Building E6-1 (자연과학동 수리과학과)

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Abstracts

Homogeneous selections and regularity in oriented matroids

ANDREAS HOLMSEN

KAIST

In recent years several researchers have noticed that the combinatorial structure regarding collections of basic geometric objects can be encoded as hypergraphs defined by *semi algebraic relations*. This has led to several deep results witnessing a drop in the combinatorial complexity of such hypergraphs, such as reduced Ramsey numbers and $\{0, 1\}$-regularity partitions. In this talk we will survey some of the basic results in this area and also show that the same drop in complexity is witnessed by hypergraphs defined by an oriented matroid which generalizes several of the previous results.

Domination in tournaments

RINGI KIM (김린기)

(joint work with Maria Chudnovsky, Chun-Hung Liu, Paul Seymour and Stéphan Thomassé)

Princeton University

For a tournament T, the *domination number* of T is the size of a minimum set $S \subseteq V(T)$ such that every vertex $v \in V(T) \setminus S$ has an in-neighbor in S. Hehui Wu conjectured that for every tournament T, the class of T-free tournaments has bounded domination number. We show that the conjecture is false in general, but true when T is 2-colorable. Moreover, we give a non-2-colorable tournament that satisfies the conjecture.

On 1212-avoiding restricted growth functions

ZHICONG LIN

(joint work with Shishuo Fu)

NIMS

Restricted growth functions (RGFs) avoiding the pattern 1212 are in natural bijection with noncrossing partitions. Motivated by recent work of Campbell et al, we study five classical statistics $\mathrm{bk}, \mathrm{ls}, \mathrm{lb}, \mathrm{rs}$ and rb on 1212-avoiding RGFs. We show the equidistribution of $(\mathrm{ls}, \mathrm{rb}, \mathrm{lb}, \mathrm{bk})$ and $(\mathrm{rb}, \mathrm{ls}, \mathrm{lb}, \mathrm{bk})$ on 1212-avoiding RGFs by constructing a simple involution. To our surprise, this result was already proved by Simion 22 years ago via an involution on noncrossing partitions. Our involution, though turns out essentially the same as Simion's, is defined quite differently and has the advantage that makes the discussion more transparent. Consequently, a multiset-valued extension of Simion's result is discovered.

Through two bijections to Motzkin paths, we also prove that $(\mathrm{exc}+1, \mathrm{den}, \mathrm{inv}-\mathrm{exc})$ on 321-avoiding permutations is equidistributed with $(\mathrm{bk}, \mathrm{rb}, \mathrm{rs})$ on 1212-avoiding RGFs, which generalizes another result of Simion. In the course, an interesting q-analog of the γ-positivity of Narayana polynomials is found. This talk is based on joint work with Shishuo Fu.

Homological connectivity of random hypergraphs

Mihyun Kang (강미현)
(joint work with Oliver Cooley, Penny Haxell, and Philipp Sprüssel)
Graz University of Technology

We consider simplicial complexes that are generated from the binomial random 3-uniform hypergraph by taking the downward-closure. We determine when this simplicial complex is homologically connected, meaning that its zero-th and first homology groups with coefficients in $\mathbb{F}_2$ vanish. Although this is not intrinsically a monotone property, we show that it nevertheless has a single sharp threshold, and indeed prove a hitting time result relating the connectedness to the disappearance of the last minimal obstruction.

Real roots of the Tutte polynomial

Seongmin Ok (옥성민)
(joint work with T. Perrett)
KIAS

A. Sokal proved that the complex zeros of the Tutte polynomial are dense in the whole complex plane. On the other hand, B. Jackson proved that the chromatic polynomial has no real zero between 1 and $32/27$. Jackson and Sokal later identified some regions in the real plane where the Tutte polynomial never becomes zero. They conjectured that outside their region the real roots are dense, and we prove that the conjecture holds for most of the real plane.

On the fast enumeration algorithm on the lattice reduced basis

Younjin Kim (김연진)
(joint work with Hyang-Sook Lee)
Ewha Womans University

In a lattice based cryptography, finding a shortest nonzero vector is known as the Shortest Vector Problem (SVP), is NP-hard under randomized reductions. The goal of lattice reduction is to find a good basis from an arbitrary basis of a given lattice. Enumeration algorithms are an useful tool to find the shortest vector in a lattice. In this talk, we give a fast enumeration algorithm on the given lattice reduced basis.

Introduction to the affine Schubert polynomials.

Seung Jin Lee (이승진)
KIAS

The Schubert polynomials are generalizations of Schur polynomials that represent cohomology classes of the Schubert varieties in flag varieties, defined by Lascoux and Schűtzenberger in 1982. Since then, the Schubert polynomials are studied in various point of views in combinatorics, algebraic geometry and representation theory.

In this talk, we discuss the affine version of the Schubert polynomials. The affine Schubert polynomials represent cohomology classes of the Schubert varieties in the affine flag variety. It turns out that the affine Schubert polynomials generalize both the Schubert polynomials and affine Stanley symmetric functions. I will present combinatorial properties of the affine Schubert polynomials.

Nine Dragon Tree Conjecture

Seog-Jin Kim (김석진)
Konkuk University

For a loopless multigraph G, the *fractional arboricity* $Arb(G)$ is the maximum of $\frac{|E(H)|}{|V(H)|-1}$ over all subgraphs H with at least two vertices. Generalizing the Nash-Williams Arboricity Theorem, the Nine Dragon Tree Conjecture asserts that if $Arb(G) \leq k + \frac{d}{k+d+1}$, then G decomposes into $k+1$ forests with one having maximum degree at most d. In this talk, we introduce Nine Dragon Tree Conjecture and present recent results.

Characterization of cycle obstructions for improper coloring planar graphs with two parts

Ilkyoo Choi (최일규)
(joint work with Chun-Hung Liu and Sang-il Oum)
KAIST

Given a graph property $\mathcal{P}$, a set of integers X is a cycle obstruction for $\mathcal{P}$ if and only if a graph with no cycle length in X has property $\mathcal{P}$. A graph is (d_1, d_2)-colorable if its vertex set can be partitioned into two parts V_1 and V_2 where V_i has maximum degree at most d_i for each $i \in \{1, 2\}$. A graph is balanced partitionable and unbalanced partitionable if there exists an integer D such that it can be (D, D)-colorable and $(0, D)$-colorable, respectively. We characterize the cycle obstructions for planar graphs to be balanced partitionable and unbalanced partitionable. Namely, we prove that a planar graph is balanced partitionable if and only if it does not contain a 4-cycle, and a planar graph is unbalanced partitionable if and only if it does not contain 3-, 4-, 6-cycles. This is joint work with Chun-hung Liu and Sang-il Oum.

Maximal green sequences for quivers of finite mutation type

Kyungyong Lee (이경용)
University of Nebraska–Lincoln

A quiver is a directed graph, and a quiver mutation is an operation which transforms a quiver into another quiver. A maximal green sequence is a particular sequence of mutations, which arises in string theory, hyperbolic geometry, and tropical geometry. I will talk about a result of my student Matthew Mills, which shows that a maximal green sequence exists for almost all quivers of finite mutation type.

Sturmian colorings on regular trees

Dong Han Kim (김동한)
(joint work with Seonhee Lim)
Dongguk University

We introduce Sturmian colorings of regular trees, which are colorings of the minimal unbounded factor complexity. Then, we classify Sturmian colorings into two families, namely cyclic and acyclic ones. We characterize acyclic Sturmian colorings in a way analogous to the continued faction algorithm of Sturmian words.

Synchronization of finite-state pulse-coupled oscillators and an application to distributed algorithm

HANBAEK LYU (류한백)
(joint work with David Sivakoff and Janko Gravner)
The Ohio State University

A generalized cellular automaton (GCA) is a discrete model for complex dynamical systems, which is defined by a triple (G, X_0, τ), where $G = (V, E)$ a fixed simple graph, $X_0 : V \to \mathbb{Z}_\kappa$ a κ-coloring for some fixed integer $\kappa \geq 2$, and τ a locally defined deterministic transition map that maps a given κ-coloring X_t to the next X_{t+1}. The iteration of τ then generates a discrete-time deterministic dynamical system, and the limiting behavior of the orbit $(X_t)_{t\geq 0}$ is of interest.

Cyclic cellular automaton (CCA) and Greenberg-Hastings Model (GHM) are two particular GCA models for excitable media such as chemical reaction or nerve cells that has been studied extensively from 90's. Most of earlier works consider limiting behavior of the orbit $(X_t)_{t\geq 0}$ on the integer lattice $\mathbb{Z}$ starting from a uniformly chosen random coloring. By considering their embedded particle system structure, one relates the limiting behavior with survival of random walks with i.i.d. increments. Along this line, it is known that CCA for $\kappa = 3$ and GHM for any κ, the the probability of seeing synchronized colors on any finite fixed interval at time t behaves as $1 - \Theta(t^{-1/2+o(1)})$.

However, limiting behaviors of these models on general graphs are largely unexplored. In this work, we characterize limiting behavior of 3-color CCA and GHM on arbitrary graphs, by constructing a monotone comparison process on the universal covering space of G. In particular, for finite G, we show that $(X_t)_{t\geq 0}$ synchronizes iff an induced 1-form dX_0 is "irrotational". We find an interesting connection to the literature of tree-indexed random walks when $G = \Gamma$ is an infinite tree. In this case, starting from a random 3-coloring X_0, we show that every vertex is fluctuated by its neighbors in the dynamic in a common linear average rate, where proportionality dictated by certain notions of dimension of Γ and large deviations rate of an associated random walk. [4]

In addition to the two GCA models mentioned, we propose another GCA model for pulse-coupled inhibitory oscillators, which we call the *firefly cellular automaton* (FCA) [1]. Its embedded particle system on $\mathbb{Z}$ has an additional queuing structure and the associated walk has local dependence, in contrast to the other two models. We apply traditional methods as well as our new techniques and show that FCA on $\mathbb{Z}$ belongs to the same universality class; that is, for any $\kappa \geq 3$, the random κ-color FCA trajectory on $\mathbb{Z}$ locally clusters with probability $1 - \Theta(t^{-1/2+o(1)})$ as $t \to \infty$ [2].

In two or more dimensions, both CCA and GHM for any κ show spontaneous emergence of self-organizing spirals in resemblance of a Belousov-Zhabotinsky reaction. These spirals divide the lattice into Voronoi-like cells, so the correlation length is finite and no local clustering occurs like in one dimension. Indeed, FCA for $\kappa \neq 4$ shows the similar behavior on $\mathbb{Z}^2$ [3]. However, this universal phenomenon of spiral formation is somehow inhibited in the four color FCA, and we do observe clustering on $\mathbb{Z}^2$. Even more surprisingly, this critical behavior does not seem to depend on the dimension d of the lattice.

As an application for clock synchronization problem in distributed networks, we generalize the four color FCA into a continuous model and show that on any finite tree with diameter d, arbitrary initial configuraiton $\Lambda_0 : V \to \mathbb{R}/\mathbb{Z}$ synchronizes in $O(d)$ times. Combining this model with a distributed spanning tree algorithm, we obtain a distributed clock synchronization algorithm in a system of processors with distinct label and identical local clocks on any connected graph, which runs in $O(|V|)$ times [5].

References

[1] H. Lyu, *Synchronization of finite-state pulse-coupled oscillators*. Physica D: Nonlinear Phenomena 303 (2015): 28-38.

[2] H. Lyu and D. Sivakoff, *Synchronization of finite-state pulse-coupled oscillators on Z*. In preperation.

[3] H. Lyu, D. Sivakoff, and J. Gravner *Non-critical self-organization of firefly cellular automaton in two dimension*. In preperation.

[4] H. Lyu, D. Sivakoff, and J. Gravner *Limiting behavior of 3-color excitable media on arbitrary graphs*. In preperation.

[5] H. Lyu, *Phase-synchronization of pulse-coupled excitable clocks*. arXiv preprint arXiv:1604.08381 (2016).

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