2016 Syzygies, exterior algebras, coherent sheaf cohomology and applications workshop

Y-Resort, Jeju, South Korea

August 24 – 27, 2016

Invited Speakers: Hirotachi Abo (University of Idaho)

Christian Bopp (Universität des Saarlandes)

Gunnar Fløystad (University of Bergen)

Suk Moon Huh (Sungkyunkwan University)

Yeongrak Kim (IMAR)

Eui-Sung Park (Korea University)

Jinhyung Park (KIAS)

Hal Schenck (University of Illinois Urbana-Champaign)

Frank-Olaf Schreyer (Universität des Saarlandes)

Ngo Viet Trung (Vietnam Academy of Science and Technology)

Organizer: Sijong Kwak (KAIST)

Frank-Olaf Schreyer (Universität des Saarlandes)

KIAS Center for Mathematical Challenges (CMC).

Dept. of Mathematical Sciences, KAIST.

Title & Abstract

Hirotachi Abo

University of Idaho

Title*: The discriminant of a global section of a vector bundle I+II*

Abstract: The main purpose of the talks is to discuss the variety formed by global sections of a vector bundle on projective space whose zero scheme is singular. We call such a variety the discriminant variety of a global section of the vector bundle.

The discriminant variety of a global section of a vector bundle can be very naturally considered as a generalization of the classical discriminant of a polynomial in one variable, i.e., a polynomial in the coefficients which vanishes at the polynomial whenever it has a multiple root. It is very natural to ask "What is the dimension of the discriminant of a global section of a vector bundle? What about the degree?"

The discriminant variety of a global section of a line bundle is also naturally identified with the dual variety of the Veronese variety, and hence its dimension and degree are already known. The focus of this talk is therefore on the discriminant variety for a vector bundle of higher rank.

In the first talk, I define the discriminant variety of a global section of a vector bundle and discuss a lower bound for the dimension of such a discriminant variety. There are examples of discriminant varieties whose dimension is strictly bigger than the above-mentioned lower bound. If time permits, I will discuss such "exceptional" cases. In the second talk, I plan to give an outline of the proof that the discriminant variety of a global section of the so-called the Schwarzenberger type bundle (STB) is an irreducible hypersurface. I also plan to show that the geometry of a non-singular curve associated with STB helps us find the degree of such a hypersurface.

Christian Bopp

Universität des Saarlandes

K3 surfaces and the Hurwitz space of degree 6 covers of P^1 by genus 9 curves

Abstract: Let C be a general canonically embedded curve of genus 9 together with a pencil of degree 6 and let X be the rational normal scroll swept out by this pencil. We will show that C lies on a one dimensional family of K3 surfaces of degree 14, which is contained in the scroll X and parametrized by a P^1. This yields an unirationality proof of the moduli space of h-polarized K3 surfaces, where h is the Picard lattice of a general element in this family.

Gunnar Fløystad

University of Bergen

The Chow Form of the Essential Variety in Computer Vision

Abstract:

We first give a general introduction to Chow forms of projective varieties and Ulrich sheaves, and show how the Chow form of a variety may be computed from an Ulrich sheaf on this variety.

We apply this to the essential variety in computer vision. This is a certain variety of real 3×3-matrices arising in two-camera theory in computer vision. Our objective is to compute the Chow form of this variety.

The invariants of the essential variety are the same as for the variety of 3×5-matrices of rank 2, and for the variety of symmetic 4×4-matrices of rank 2. We show that the essential variety identifies as the variety of traceless symmetric 4×4 matrices of rank 2. We then find an Ulrich sheaf on this variety. From this we may compute the Chow form of the essential variety.

In the end we discuss several problems and questions related to these determinantal varieties and to the exterior algebra.

Sukmoon Huh

Sungkyunkwan University

ACM sheaves on the double plane

Abstract: A coherent sheaf on a projective variety is called arithmetically Cohen-Macaulay(aCM), if it has no intermediate cohomology. Ever since Horrocks proved that direct sums of line bundles are the only aCM bundles, there have been several attempts to classify aCM bundles on specific varieties. In this talk we report our recent work on the classification of aCM sheaves on the double plane. We first show that every aCM bundle is a direct sum of line bundles. Then we classify aCM sheaves up to rank 3/2.

This is a joint work with E. Ballico, F. Malaspina and J. Pons-Llopis.

Yeongrak Kim

IMAR

Special Ulrich bundles on surfaces

Abstract: In a pioneering work by Eisenbud and Schreyer, they introduced the notion of Ulrich bundles which discovers many fruitful connections between commutative algebra and projective algebraic geometry. For instance, the existence of Ulrich bundles on a given projective variety implies that the cone of cohomology tables has the same structure as the one of the projective space of the same dimension. Unfortunately, the existence problem seems to be difficult even for smooth surfaces. It is also interesting to consider special Ulrich bundles, which are Ulrich bundles of rank 2 with determinant $\omega_S(3)$, since it provides a Pfaffian representation of the Chow form and has technical merits. In this talk, I will review a few known constructions of special Ulrich bundles using the Brill-Noether theory of curves by Beauville and Aprodu-Farkas-Ortega. And I will introduce a relation between special Ulrich bundles on a blown-up and the original surface.

Eui-Sung Park

Korea University

Multisecant Line and Regularity

Abstract. For a projective subscheme $X \subset \mathbb{P}^r$ of dimension n and codimension c, we consider the integers $\text{reg}(X)$, $m(X)$ and $\ell(X)$ where $m(X)$ is the maximal degree of a minimal generator of the homogeneous ideal of X and $\ell(X)$ is the largest integer ℓ such that X admits a proper ℓ-secant line. Therefore it holds always that

$$\text{reg}(X) \leq m(X) \leq \ell(X).$$

I will speak about some interesting cases where the equality $\text{reg}(X) = \ell(X)$ is attained. This is a report on joint works with Markus Brodmann, Kiryong Chung, Wanseok Lee and Peter Schenzel.

Jinhyung Park

KIAS

Classification and syzygies of algebraic varieties with 2-regular structure sheaves

Abstract: It is well known that the regularity of the structure sheaf of a smooth projective variety is greater than or equal to 1, and the equality holds if and only if the variety is obtained by an isomorphic projection from a variety of minimal degree. In this talk, I show a geometric characterization of smooth projective varieties with 2-regular structure sheaves using adjunction theory. Then I study syzygies of section rings of projected varieties with 2-regular structure sheaves, and give a sharp bound for Castelnuovo-Mumford regularity of such varieties. This is joint work in progress with Sijong Kwak.

Hal Schenk

University of Illinois Urbana-Champaign

Syzygies, Exterior Algebra, and Hyperplane Arrangements.

Abstract: In 1980, Orlik and Solomon published their pioneering paper describing the cohomology ring of the complement X of a complex hyperplane arrangement $\mathcal{A}$. The cohomology ring $A=H^*(X,\mathbb{Z})$ is a quotient of an exterior algebra E with a generator for each hyperplane, and relations determined by the combinatorics of $\mathcal{A}$. While A is somewhat analogous to a Stanley-Reisner ring, it is much more subtle, and much remains to be understood. A key tool in the analysis of A is the Bernstein-Gelfand-Gelfand correspondence. In the first talk, I will discuss the basics of BGG and examples, as well as discussing the basics of Hyperplane Arrangements; this will be accessible to non-experts. In the second talk, I will discuss two landmark results on syzygies of A and $Hom(A,E)$. First, we'll prove the Eisenbud-Popsecu-Yuzvinsky theorem that $Hom(A,E)$ has a linear free resolution; the proof brings in flat deformations and Alexander duality. Second, we'll discuss the proof of the Chen rank conjecture, which gives a geometric description, in terms of cohomology jump loci, of the linear syzygies of A. (the latter is joint work Cohen and Suciu).

Frank-Olaf Schreyer

Universitat des Saarlandes

Tate resolutions and direct image complexes I + II

Abstract: The direct image complex $Rf_*F \in D(Y)$ along a projective morphism $f\colon X \to Y$ of a coherent sheaf F on X is the fundamental tool to study jump phenomena.

In the fi talk I will report how syzygies over the exterior algebra lead to a computation of direct image complex in the local case $Y = Spec A$, and illustrate these in series of example.

In the second talk, I report on the attempt to tackle the global case, Y projective, which builds upon the much more involved theory of Tate resolution on products of projective spaces.

Ngo Viet Trung

Vietnam Academy of Science and Technology

Symbolic powers of sums of ideals

Abstract: Let I and J be nonzero proper ideals in two different polynomial rings A and B. We study algebraic invariants and properties of symbolic powers of the sum (I+J) in the tensor product of A and B. Our main technical result is the binomial expansion (I+J)^{(n)} = \sum_{i+j = n} I^{(i)} J^{(j)}. This binomial expansion allows us to to give formulas for the depth and the regularity of (I+J)^{(n)} in terms of those of I and J if char(k) = 0. The formulas are upper bounds if char(k) > 0. This is a report on a joint work with H.T. Ha (New Orleans), N.D. Hop (Genova), T.N. Trung (Hanoi).