Bayesian Naturalness

of the CMSSM and CNMSSM

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Naturalness Problem

- Fine-tuning problem of Higgs mass:
  Defined by a tension between gravity and weak interaction.

- In supersymmetric models, the big hierarchy problem is translated into the little hierarchy problem between the EWSB & SUSY breaking scale.
Naturalness Problem

• Fine-tuning problem of Higgs mass:
  Defined by a tension between gravity and weak interaction.

• In supersymmetric models, the big hierarchy problem is translated into the little hierarchy problem between the EWSB & SUSY breaking scale.

• How do we define the Fine-tuning problem?
Fine-tuned Higgs Mass & SUSY

• In the MSSM, it is hard to find a natural solution to the Higgs mass

\[ m_h^{2\text{Tree}} \leq M_Z^2 \]

• $\mu$ Problem

\[ \frac{M_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \]

• Next-to-MSSM ameliorates the situation introducing additional scalar S
  : Lifts up $m_h^2$ more at tree level
  : All mass scales are introduced at the SUSY breaking scale
NMSSM better than MSSM

• It is generally believed that the additional F-term helps to relax the tension between $M_z$ and $M_h$.

• For CMSSM, extensive studies have suggested the problem to get a realistic Higgs mass with low fine-tuning.

• If the singlet vev helps to increase the $M_h$, then it will reduce the fine-tuning. Then will the CNSSM also be better than the CMSSM?
Definitions of Fine-tuning

- \( \frac{\delta m_h}{m_h} \), or \( \frac{\delta m_Z}{m_Z} \): Compare the size of quantum fluctuation of \( m_h/Z \) relative to its tree mass.


- \( \Delta_{BG} \equiv \max \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i^2} \right| \): Sensitivity of EW observable to model parameters.

G. F. Giudice, [arXiv:1307.7879]

- \( \Delta_J = \left| \frac{\partial \ln \sigma_j^2}{\partial \ln p_i^2} \right| \): Counts the correlations bet. the observables.

Definitions of Fine-tuning

- $\Delta_{EW}$: Hierarch Based
  Focus on Radiative Stability & Cancellation

$$\frac{M_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

Each terms: $C_i$

e.g. $C_{H_u} = -m_{H_u}^2 / (\tan^2 \beta - 1) / (M_Z^2 / 2)$

$\Delta_{EW} \equiv \max(C_i)$

Definitions of Fine-tuning

- $\Delta_{BG}$: Focus on the Stable adjustment of parameters to fit data
  Usually $M_Z$, Single variable

\[
\Delta_{BG} \equiv \max \left| \frac{\partial \ln M_Z^2}{\partial \ln p_i^2} \right|
\]

$M_Z$  $p_i = \mu, B, m_0, m_{1/2}, A_0$

G. F. Giudice, [arXiv:1307.7879]
Definitions of Fine-tuning \[\text{arXiv:1312.4150}\]

- In Bayesian Analysis

\[p(\mathcal{M}|\mathcal{D}) = \frac{p(\mathcal{D}|\mathcal{M})}{p(\mathcal{D})}p(\mathcal{M}) = \frac{1}{p(\mathcal{D})} \int p(\mathcal{D}|p_i)p(p_i)dp_i\]

- For CMSSM

\[\int \mathcal{L}p(\mu, B, y)d\mu dBdy = \int \mathcal{L}|J_{\mathcal{T}_1}|p(M_Z, y, m_t)dM_Zdm_tdt\]

\[\mathcal{T}_1: \{\mu, B, y\} \rightarrow \{M_z, t, m_t\}\]

Definitions of Fine-tuning [arXiv:1312.4150]

- $\Delta_J$ : Same as $\Delta_{BG}$ but deals with a SET of low energy variables (all vevs)

& the correlation among them

$$
\Delta_J = \left| \frac{\partial \ln \mathcal{O}_j^2}{\partial \ln p_i^2} \right| \rightarrow \frac{\delta V_{\mathcal{O}}}{\delta V_p}
$$
Definitions of Fine-tuning [arXiv:1312.4150]

- For CMSSM

\[ \Delta J = \left| \frac{\partial \ln(M_Z^2, \tan^2 \beta, m_t^2)}{\partial \ln(\mu^2, B^2, y_t^2)} \right| \]

- For CNMSSM

\[ \Delta J = \left| \frac{\partial \ln(M_Z^2, \tan^2 \beta, s^2, m_t^2)}{\partial \ln(\lambda^2, \kappa^2, m_s^2, y_t^2)} \right| \]
Definitions of Fine-tuning [arXiv:1312.4150]

• For CMSSM

\[ \Delta J^{-1}_{\text{CMSSM}} = \frac{M_Z^2}{2\mu^2} \frac{B}{B_0} \frac{t^2 - 1}{t^2 + 1} \partial \ln y^2 \]

• For CNMSSM

\[ \Delta J^{-1}_{\text{CNMSSM}} = \begin{vmatrix} b_1 & e_1 & f_1 \\ b_2 & e_2 & f_2 \\ b_3 & e_3 & f_3 \end{vmatrix} \]

• Convergence of NMSSM to MSSM

\[
\begin{align*}
\lambda, \kappa & \to 0 \\
\frac{m_S^2}{A_\kappa^2} & \to 0 \\
\Delta J|_{\text{CNMSSM}} & \to \Delta J|_{\text{CMSSM}}
\end{align*}
\]
\[ d\lambda = -\frac{\lambda}{s} ds + \frac{1}{2\lambda s^2} \frac{2t}{(t^2 - 1)^2} (m_{Hu}^2 - m_{Hd}^2) dt - \frac{1}{4\lambda s^2} dM_z^2 + \frac{1}{2\lambda s^2} \frac{dM_{Hu}^2}{t^2 - 1} \]

\[ + \frac{1}{2\lambda s^2} \partial \mu^2 \partial y_i^2 \]

\[ \equiv b_1 ds + b_2 dt + b_3 dM_z^2 + b_5 dy_i^2 + b_7 dm_{Hu}^2 + b_8 dm_{Hd}^2, \]

\[ 0 = \left[ \frac{A_\lambda + \kappa s}{\lambda s} - 2 \sin 2\beta \left( 1 + \frac{M_z^2}{\bar{g}^2 s^2} \right) \right] d\lambda - \frac{2\lambda s \sin 2\beta - (A_\lambda + 2\kappa s)}{s^2} ds \]

\[ - \frac{1 - t^2}{(1 + t^2)^2} \frac{m_{Hu}^2 + m_{Hd}^2 + 2\mu^2 \left( 1 + \frac{M_z^2}{\bar{g}^2 s^2} \right)}{\lambda s^2} dt - \frac{\lambda \sin 2\beta}{\bar{g}^2 s^2} dM_z^2 + d\kappa \]

\[ + \frac{1}{\lambda s^2} \frac{t}{1 + t^2} \frac{2\lambda^2 M_z^2}{\bar{g}^2 s^2} d\bar{g}^2 - \frac{1}{\lambda s^2} \frac{t}{1 + t^2} (dm_{Hu}^2 + dm_{Hd}^2) + \frac{1}{s} dA_\lambda - \frac{1}{\lambda s^2} \frac{\partial B_\mu}{\partial y_i^2} dy_i^2 \]

\[ \equiv e_0 d\lambda + e_1 ds + e_2 dt + e_3 dM_z^2 + e_4 d\kappa + e_5 dy_i^2 + e_6 d\bar{g}^2 + e_7,8 dm_{Hu,d}^2 + e_9 dA_\lambda, \]

\[ dm_S^2 = -\frac{4M_z^2}{\bar{g}^2 s} \left[ \lambda s - \frac{t}{1 + t^2} (A_\lambda/2 + \kappa s) \right] d\lambda - \left( 4\kappa^2 s + \kappa A_\kappa + \frac{2M_z^2}{\bar{g}^2 s^2} \frac{t}{1 + t^2} \lambda A_\lambda \right) ds \]

\[ + \frac{4M_z^2}{\bar{g}^2 s^2} \frac{1 - t^2}{(1 + t^2)^2} \lambda s (A_\lambda/2 + \kappa s) dt - \left[ \frac{2\lambda^2}{\bar{g}^2 s^2} - \frac{4\lambda}{\bar{g}^2 s} \frac{t}{1 + t^2} (A_\lambda/2 + \kappa s) \right] dM_z^2 \]

\[ - \left[ 4\kappa s^2 + s A_\kappa - \frac{4M_z^2}{\bar{g}^2 s^2} \lambda s \frac{t}{1 + t^2} \right] d\kappa + \frac{M_z^2}{\bar{g}^2} \left[ \frac{2\lambda^2}{\bar{g}^2 s^2} - \frac{4\lambda}{\bar{g}^2 s} \frac{t}{1 + t^2} (A_\lambda/2 + \kappa s) \right] d\bar{g}^2 \]

\[ + \frac{2M_z^2}{\bar{g}^2 s} \frac{t}{1 + t^2} \lambda dA_\lambda - \kappa s dA_\kappa + \frac{\partial m_S^2}{\partial y_i^2} dy_i^2 \]

\[ \equiv f_0 d\lambda + f_1 ds + f_2 dt + f_3 dM_z^2 + f_4 d\kappa + f_5 dy_i^2 + f_6 d\bar{g}^2 + f_9 dA_\lambda + f_{10} dA_\kappa. \]
MSSM/NMSSM Scalar Potential

\begin{align*}
V_{\text{Higgs}} &= (|\mu|^2 + m_{H_u}^2) \left(|H_u^0|^2 + |H_u^-|^2\right) + (|\mu|^2 + m_{H_d}^2) \left(|H_d^0|^2 + |H_d^-|^2\right) \\
&\quad + \left[B\mu \left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \text{c.c.}\right] \\
&\quad + \frac{g^2 + g'^2}{8} \left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right) \\
&\quad + \frac{1}{2} g^2 \left|H_u^0 H_d^* + H_u^* H_d^0\right|^2 ,
\end{align*}

\begin{align*}
m_{H_u}^2 &= -|\mu|^2 + B\mu \cot \beta + \left(M_z^2/2\right) \cos 2\beta , \\
m_{H_d}^2 &= -|\mu|^2 + B\mu \tan \beta - \left(M_z^2/2\right) \cos 2\beta .
\end{align*}

\begin{align*}
V_{\text{Za NMSSM Scalar}} &= |\lambda \left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \kappa S|^2 \\
&\quad + (|\lambda S|^2 + m_{H_u}^2) \left(|H_u^0|^2 + |H_u^+|^2\right) + (|\lambda S|^2 + m_{H_d}^2) \left(|H_d^0|^2 + |H_d^-|^2\right) \\
&\quad + \frac{g^2 + g'^2}{8} \left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right) \\
&\quad + \frac{1}{2} g^2 \left|H_u^0 H_d^* + H_u^* H_d^0\right|^2 \\
&\quad + m_f^2 |S|^2 + \left(\lambda A \lambda \left(H_u^+ H_d^- - H_u^0 H_d^0\right) S + \frac{1}{3} \kappa A \kappa S^3 + \text{h.c.}\right) , \tag{2.49}
\end{align*}

\begin{align*}
m_{H_u}^2 &= -\lambda^2 s^2 + \lambda s(A \lambda + \kappa s) \cot \beta + \left(M_z^2/2\right) \cos 2\beta - 2\lambda^2 \frac{M_z^2}{g^2} \cos^2 \beta , \tag{2.50}
\end{align*}

\begin{align*}
m_{H_d}^2 &= -\lambda^2 s^2 + \lambda s(A \lambda + \kappa s) \tan \beta - \left(M_z^2/2\right) \cos 2\beta - 2\lambda^2 \frac{M_z^2}{g^2} \sin^2 \beta , \tag{2.51}
\end{align*}

\begin{align*}
m_f^2 &= -2\kappa^2 s^2 + 2\lambda(A \lambda/2 + \kappa s) \frac{M_z^2}{g^2} \sin 2\beta - \kappa A \kappa s - 2\lambda^2 \frac{M_z^2}{g^2} . \tag{2.52}
\end{align*}
Numerical Results
CMSSM w/o Yukawa

$A_0 = -1$ TeV, $\tan \beta = 10$
CMSSM w/ Yukawa

\[ A_0 = -1 \text{ TeV}, \tan \beta = 10 \]
CNMSSM w/o Yukawa

\[ A_0 = -1 \text{ TeV}, \tan \beta = 10 \]
CNMSSM w/ Yukawa

\[ A_0 = -1 \text{ TeV}, \tan \beta = 10 \]
CMSSM w/ Yukawa

$A_0 = -2.5 \text{ TeV, } \tan \beta = 10$
CNMSSM w/ Yukawa

\[ A_0 = -2.5 \text{ TeV}, \tan \beta = 10 \]
Higgs in CMSSM and CNMSSM

$A_0 = -1 \text{ TeV}, \tan \beta = 10$

- CMSSM
- CNMSSM
Higgs for CMSSM and CNMSSM

\[ A_0 = -1 \text{ TeV}, \tan \beta = 10 \]

- It is not a Good idea to simply extend CMSSM to NMSSM!
Higgs for CMSSM and CNMSSM

- CMSSM

\[ A_0 = -2.5 \text{ TeV}, \quad \tan \beta = 3 \]

- CNMSSM

\[ A_0 = -2.5 \text{ TeV}, \quad \tan \beta = 10 \]
CNMSSM with other experimental data

- $A_0 = -2.5$ TeV, $\tan \beta = 10$
- $A_\lambda, A_\kappa$ are set released

<table>
<thead>
<tr>
<th>Observable</th>
<th>Experimental value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Omega_{DM} h^2$</td>
<td>$0.1187 \pm 0.0017$</td>
</tr>
<tr>
<td>$m_h$</td>
<td>$125.9 \pm 0.4$ GeV</td>
</tr>
<tr>
<td>BR ($B_s \to \mu^+ \mu^-$)</td>
<td>$(2.9 \pm 1.1) \times 10^{-9}$</td>
</tr>
<tr>
<td>BR ($b \to s\gamma$)</td>
<td>$(343 \pm 21 \pm 7) \times 10^{-6}$</td>
</tr>
<tr>
<td>BR ($B \to \tau\nu$)</td>
<td>$(114 \pm 22) \times 10^{-6}$</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_1^0}$</td>
<td>$&gt; 46$ GeV</td>
</tr>
<tr>
<td>$m_{\tilde{\chi}_1^\pm}$</td>
<td>$&gt; 94$ GeV if $m_{\tilde{\chi}<em>1^\pm} - m</em>{\tilde{\chi}_1^0} &gt; 3$ GeV</td>
</tr>
<tr>
<td>$m_{\tilde{q}}$</td>
<td>$&gt; 1.43$ TeV</td>
</tr>
<tr>
<td>$m_{\tilde{g}}$</td>
<td>$&gt; 1.36$ TeV</td>
</tr>
</tbody>
</table>
Summary

• Sensitive to EW scale cancellation
  Cares for the sensitivity of param. to observables
  Generic in the Bayesian analysis

• The simplest initial condition of the NMSSM must not just a simple extension of CMSSM. A-term constraints should be released in order for a flexible EWSB compatible with the Higgs mass. Then what must be the reasonable starting point for the NMSSM?

• This is a fine-tuning map for given models. For a systematic comparison of models, we need to compare Bayesian evidence.

• This is a fixed \((A_0, \tan \beta)\) slice scanning
  -> Complete scanning to appear soon
Higgs for CMSSM and CNMSSM

$A_0 = -1 \text{ TeV, } \tan \beta = 10$