Perturbative unitarity and identification of Higgs coupling constants in the Minimal Composite Higgs model

Naoki Machida (Univ. of Toyama)

In collaboration with:

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arXiv:1410.xxxx

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Contents

Introduction

- The Higgs boson has been discovered.

  m_h = 126GeV, spin/parity = 0^+,

  Coupling constants are consistent with SM predictions

However, a big question still remains.

Is the Higgs boson an *Elementary scalar* or a *Composite state*?

Elementary scalar   Simple structure

SUSY? GUT over the grand desert?

Composite state   Rich structure

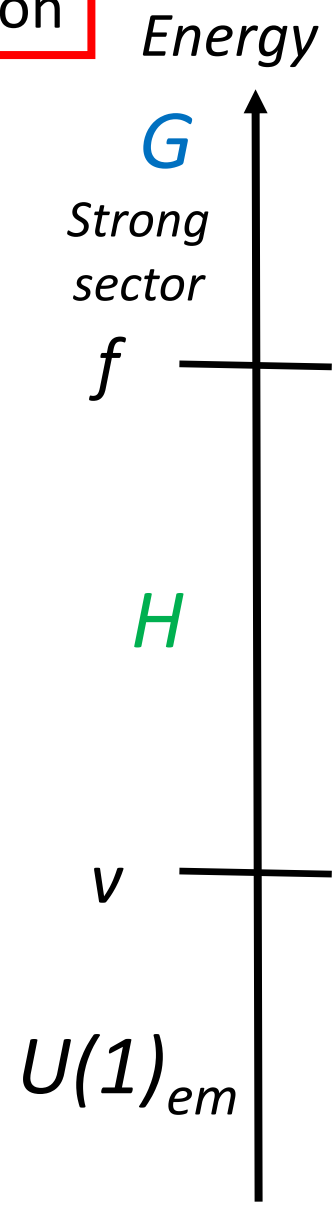
Technicolor

Little Higgs

(Minimal) Composite Higgs Model

Composite Higgs Models

The Higgs boson = pseudo Nambu-Goldstone boson



A global symmetry G is broken down to a subgroup H.

- The subgroup H contains the SM gauge group, $SU(2)_L \times U(1)_Y$.
- No. of NGB = dim(G) – dim(H).
- The breaking scale f (the analog of f_π) is higher than 246 GeV.
- V_{Higgs} is generated by the Coleman-Weinberg mechanism.
- The loop contributions from 3rd generation quarks and $SU(2)_L$ gauge bosons are dominant.
- The Higgs boson mass is naturally light.

Composite Higgs Models

G/*H*

G	*H*	pNGB	Ref.
SO(5)	*SO(4)*	4	Agashe et al., NPB 719
SO(6)	*SO(5)*	4+1	Gripaios et al., JHEP0904
SO(6)	*SO(4)xSO(2)*	4+4	Mrazek et al., NPB 853
SO(7)	*SO(6)*	4+1+1	Mrazek et al., NPB 853
…	…	…	…

- *SO(5)/SO(4)* : Minimal, one doublet
- *SO(6)/SO(5)* : Next to minimal, one doublet and one singlet (dark matter)
- *SO(6)/SO(4)xSO(2)* : two doublets

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Composite Higgs Models

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Minimal Composite Higgs Model

Agashe, Contino, Pamarol, Nucl.Phys.B719

Contino, arXiv.1005.4269

- **Minimal setup** : *G*/*H* = *SO(5)*/*SO(4)* (No. of NGB = 10 – 6 = **4**)

Higgs fields : $\Sigma = \Sigma_0 \exp\left(-iT^{\hat{a}} h^{\hat{a}}/f\right), \quad \Sigma_0 = (0,0,0,0,1)$

$T^{\hat{a}}$: broken generators, $h^{\hat{a}}$: Higgs fields, f : decay constant

$\hat{a} = 1\text{-}4$

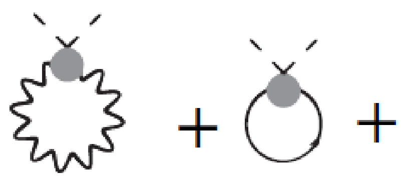
- Matter fields of the MCHM_4

"4" : 4-dimensional rep. of *SO(5)*.

$$\Psi_q^{(4)} = \begin{pmatrix} q_L \\ Q_L \end{pmatrix}, \quad \Psi_u^{(4)} = \begin{pmatrix} q_R^u \\ \begin{pmatrix} u_R \\ d'_R \end{pmatrix} \end{pmatrix}, \quad \Psi_d^{(4)} = \begin{pmatrix} q_R^d \\ \begin{pmatrix} u'_R \\ d_R \end{pmatrix} \end{pmatrix}.$$

q_L, u_R, d_R : SM quarks
Others : non-SM quarks

- Higgs potential : the Coleman-Weinberg mechanism

$V_{\mathrm{eff}}^{\mathrm{Higgs}} =$  $+ \cdots$

Contributions of 3rd generation quarks and $SU(2)_L$ gauge boson loop are dominant.

Agashe, Contino, Pamarol, Nucl.Phys.B719

Minimal Composite Higgs Model

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Higgs fields : $\Sigma = \frac{\sin(h/f)}{h}(h^1, h^2, h^3, h^4, h\cot(h/f)) \quad h = \sqrt{h^{\hat{a}}h^{\hat{a}}}$

$T^{\hat{a}}$: broken generators, $h^{\hat{a}}$: Higgs fields, f : decay constant

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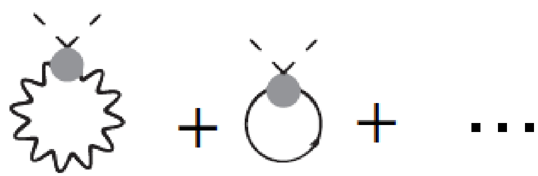
$V_{\text{eff}}^{\text{Higgs}} =$ 

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$A^\mu : SO(5)$ gauge, $X^\mu : U(1)_X$ gauge, Π_0^X, Π_0, Π_1 : Form factors

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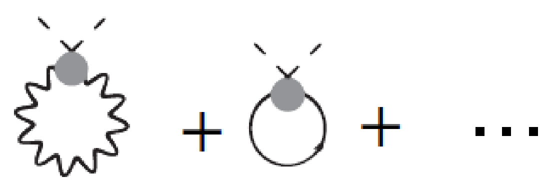

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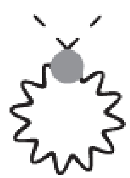
$V_{\rm eff}^{\rm Higgs} =$  $+$  $+ \cdots$

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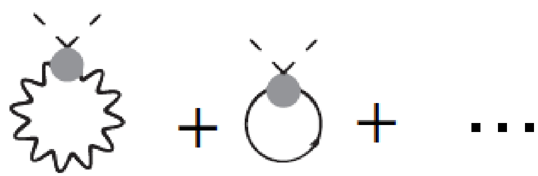
$V_{\text{eff}}^{\text{Higgs}} =$ 

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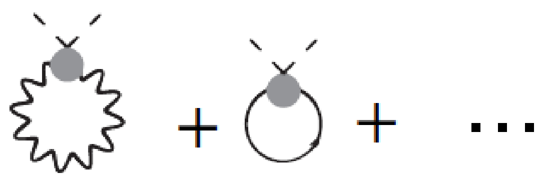
$V_{\rm eff}^{\rm Higgs} =$ 

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$$V_{\rm eff}^{\rm Higgs} = \frac{9}{2} \int \frac{d^4p}{(2\pi)^4} \ln \Pi_W - 2N_C \int \frac{d^4p}{(2\pi)^4} \left[\ln \not p \Pi_{b_L} + \ln(p^2 \Pi_{t_R} \Pi_{t_L} - |\Pi_{t_L t_R}|^2) \right]$$

Gauge bosons

$$\Pi_W = \Pi_0 + \frac{\Pi_1}{4} \sin^2 \frac{h}{f}, \quad \Pi_{b_L} = \Pi_{t_L} = \Pi_0^q + \Pi_1^q \cos \frac{h}{f},$$

$$\Pi_{t_R} = \Pi_0^u - \Pi_1^u \cos \frac{h}{f}, \quad \Pi_{t_L t_R} = M_1^u \sin \frac{h}{f}.$$

Quarks

Agashe, Contino, Pamarol, Nucl.Phys.B719

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The electroweak symmetry is broken.

Minimal Composite Higgs Model

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  Compositeness parameter : $\xi = v^2/f^2$ (< 1, $\xi = 0$ is the SM limit)

  - Higgs-gauge couplings

$$\mathcal{L}_{\rm eff}^{\rm gauge} = \frac{g_V^2 v^2}{4} V_\mu V^\mu + \frac{g_V^2 v}{2}\sqrt{1-\xi}\,\hat{h} V_\mu V^\mu + \frac{g_V^2}{4}(1-2\xi)\hat{h}^2 V_\mu V^\mu$$

$$V = W^\pm, Z,\ g_W = g,\ g_Z = \sqrt{g^2+g'^2},\ \hat{h} : \text{physical Higgs field}$$

These deviations don't depend on matter representations.

- Matter fields of the MCHM$_4$

  "4" : 4-dimensional rep. of *SO(5)*.

  - Higgs-fermion couplings

$$m^t(h) \simeq m^t\left(1+\sqrt{1-\xi}\,\frac{\hat{h}}{v} - \frac{1}{2}\xi\frac{\hat{h}^2}{v^2}\right)\bar{t}t$$

m^t : top quark mass

These deviations depend on matter representations.

- Higgs potential : the Coleman-Weinberg mechanism

  - Higgs-self couplings

$$\left\langle \frac{\partial V_h}{\partial h}\right\rangle = 0\,,\ \left\langle \frac{\partial^2 V_h}{\partial h^2}\right\rangle = m_h^2 > 0\,,\ \left\langle \frac{\partial^3 V_h}{\partial h^3}\right\rangle = \frac{3m_h^2}{v}\sqrt{1-\xi} \equiv \lambda_{hhh}$$

These deviations depend on matter representations.

Agashe, Contino, Pamarol, Nucl.Phys.B719

Minimal Composite Higgs Model

Contino, arXiv.1005.4269

$G/H = SO(5)/SO(4)$

- MCHM_4 : 4-dimensional rep.
- MCHM_5 : 5-dimensional rep.
- MCHM_{10} : 10-dimensional rep.
- MCHM_{14} : 14-dimensional rep.
- ・・・ total : 14 models

By using same procedure, we can derive Higgs boson coupling deviations. They will be changed.

➢ Matter fields of the MCHM_4

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$$V = W^\pm, Z,\ g_W = g,\ g_Z = \sqrt{g^2 + g'^2},\ \hat{h} : \text{physical Higgs field}$$

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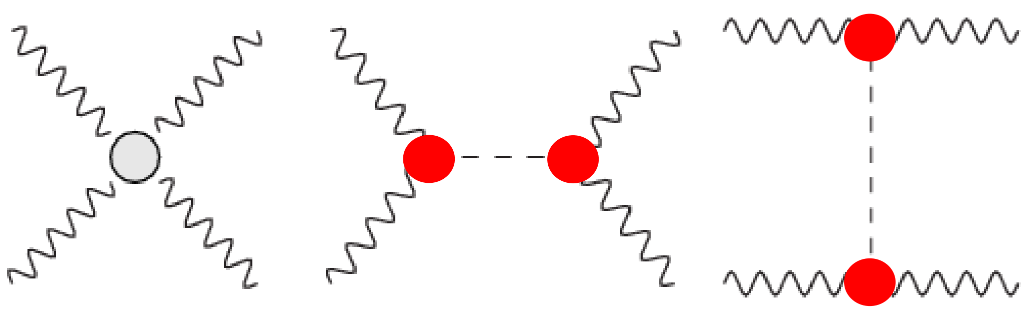
The hVV coupling deviation, $\sqrt{1-\xi}$, violates perturvative unitarity.

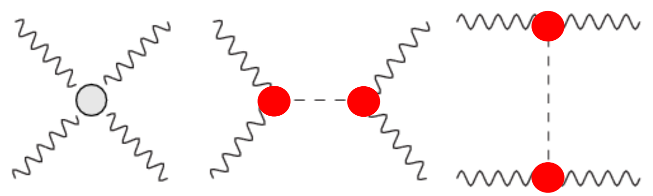
$$W_L W_L \to W_L W_L$$



Perturbative unitarity

Lee, Quigg, Thacker, PRD16

WW scattering $g_{hVV} = g_{hVV}^{\rm SM}\sqrt{1-\xi}$



In the SM, energy dependence vanished.

In the MCHM, energy dependence remains.

$$\left|{\rm Re}\left[a_{\ell=0}^{W_LW_L\to W_LW_L}\right]\right| = \left|\frac{G_F\xi S}{16\sqrt{2}\pi} + \frac{G_F(m_h^2-m_W^2)(1-\xi)}{4\sqrt{2}\pi}\right| \le \frac{1}{2}$$

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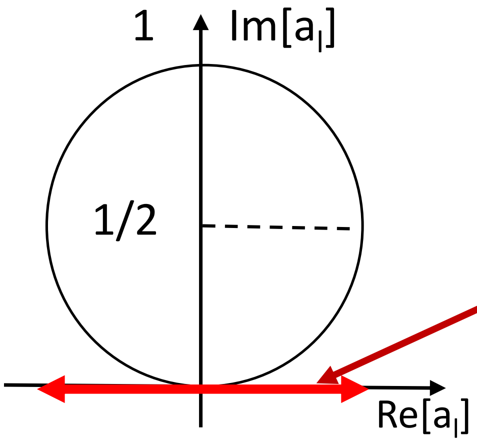
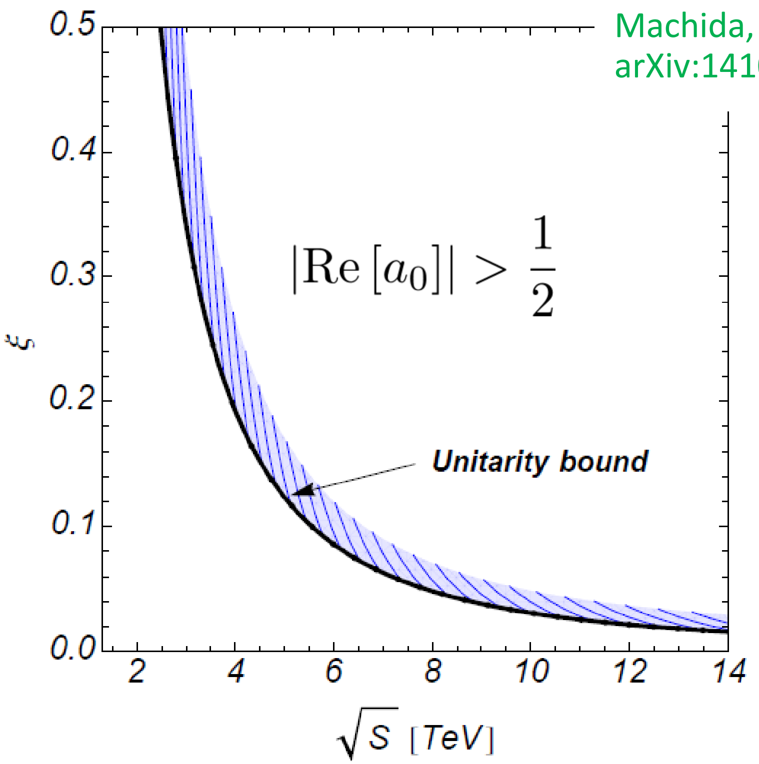


Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx



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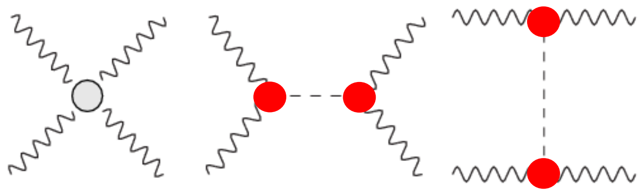

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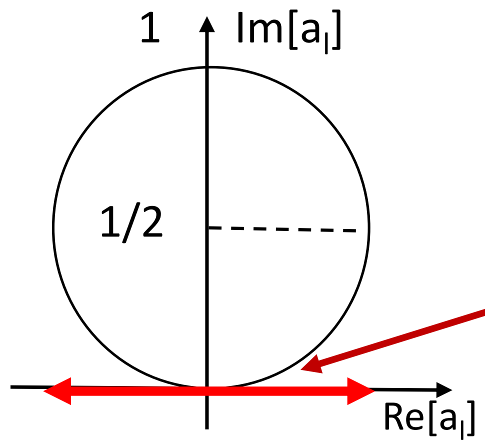
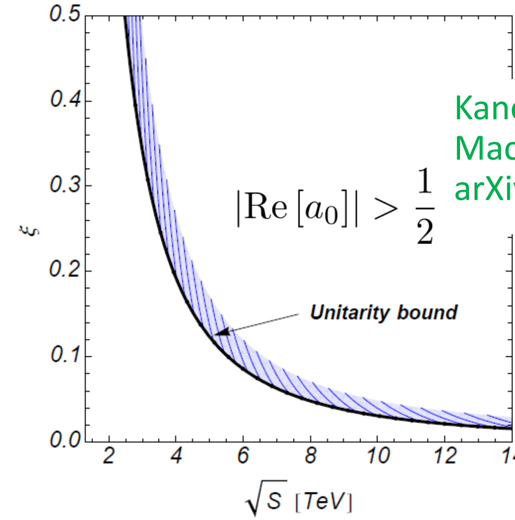
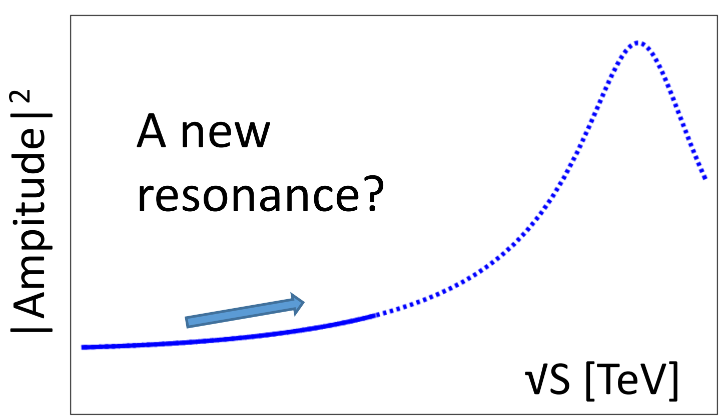
$$\left|G_F\xi S/(16\sqrt{2}\pi) + G_F(m_h^2 - M_W^2)(1-\xi)/(4\sqrt{2}\pi)\right| \le 1/2$$





Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx

What does the unitarity violation indicate?



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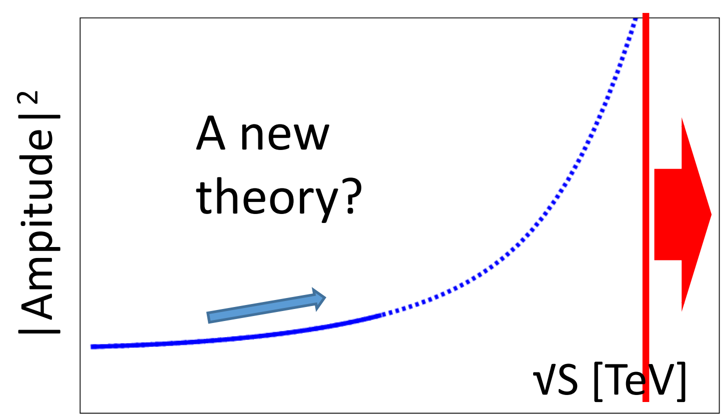

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Lee, Quigg, Thacker, PRD16

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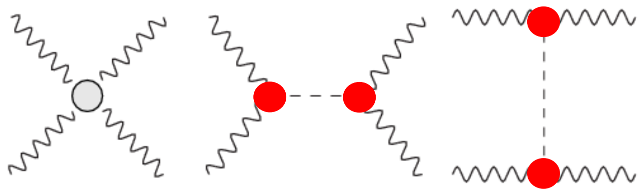

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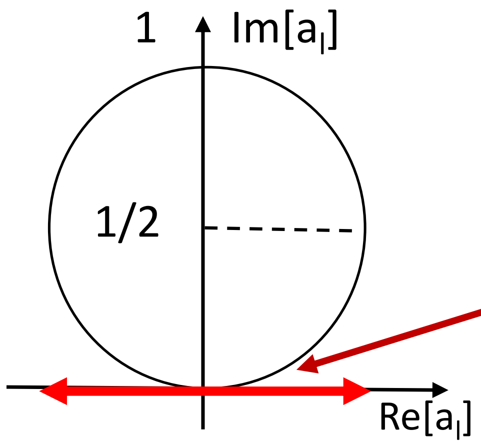
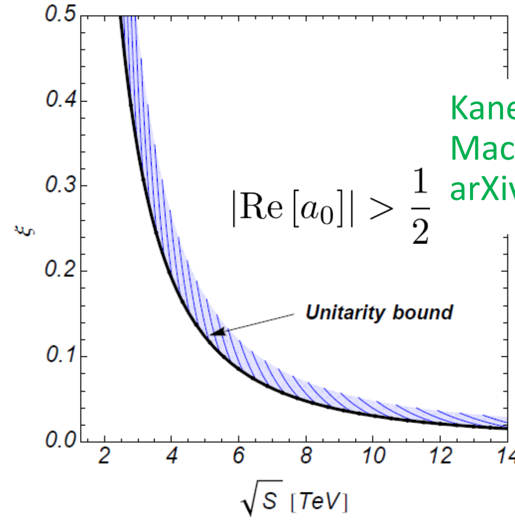
In the MCHM, energy dependence remains.

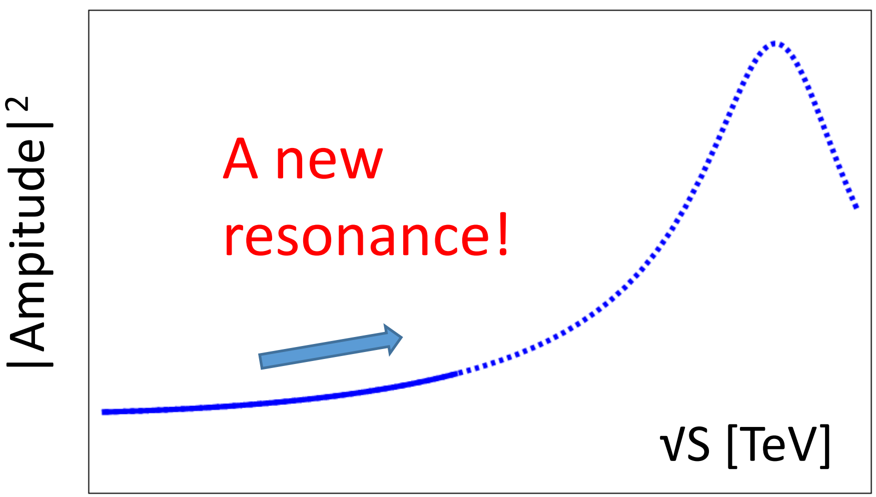
$$\left|G_F\xi S/(16\sqrt{2}\pi) + G_F(m_h^2 - M_W^2)(1-\xi)/(4\sqrt{2}\pi)\right| \leq 1/2$$





Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx

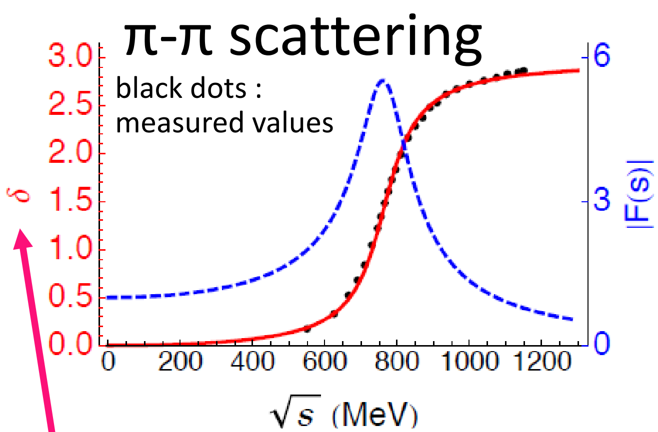
What does the unitarity violation indicate?



We know the existence of such a pseudo scalar, *pion*.
By using the analogy of pion physics, we can estimate **a new resonance scale**.

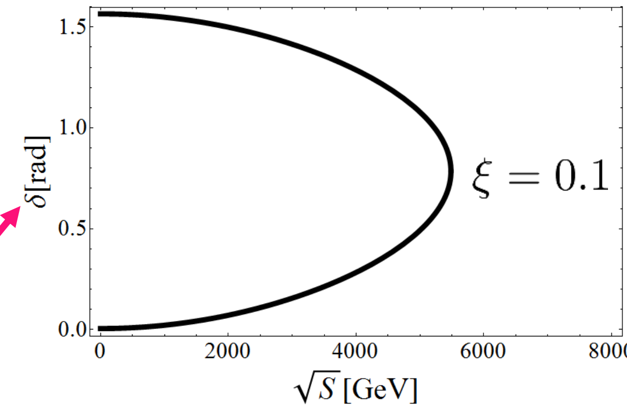
New resonance scale

Murayama, et al. arXiv :1401. 3761



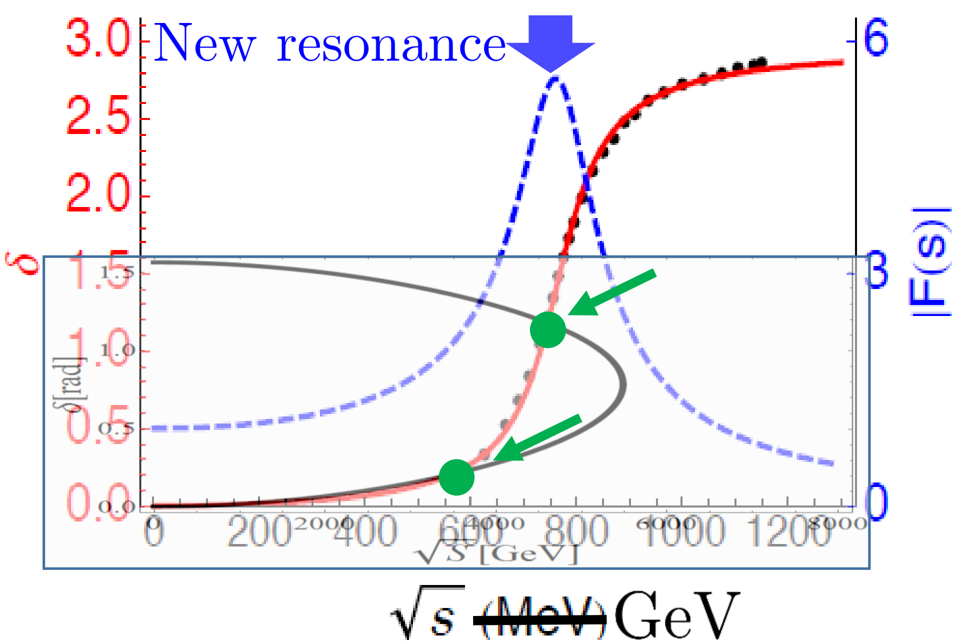
It is well known that the phase shift occurs in π-π scattering. Then, we take phase into account for the WW scattering amplitude of the MCHM.

δ of $W_L W_L$ -> $W_L W_L$ in the MCHM

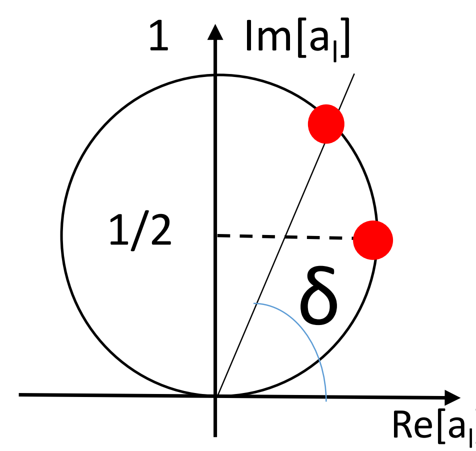


Assumption

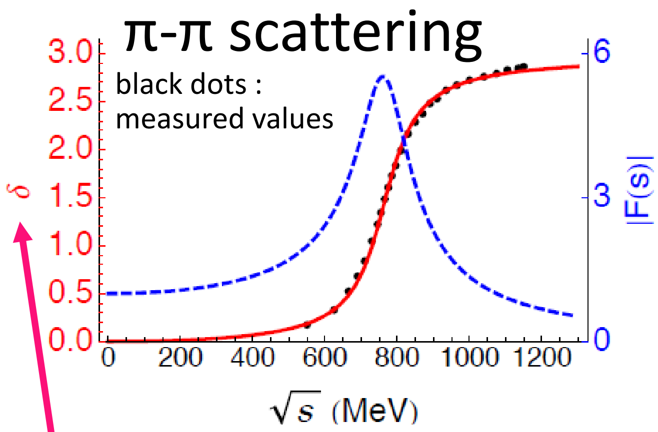
$$\delta = \begin{cases} \tan^{-1}[(\Gamma/m)S/(m^2+\Gamma^2-S)] & \text{for} \quad \sqrt{S} < \sqrt{m^2+\Gamma^2} \\ \tan^{-1}[(\Gamma/m)S/(m^2+\Gamma^2-S)]+\pi & \text{for} \quad \sqrt{S} \geq \sqrt{m^2+\Gamma^2} \end{cases} = \delta = \tan^{-1}\left[1/(2\mathrm{Re}[a_0]) \pm \sqrt{1/(4\mathrm{Re}[a_0]^2)-1}\right]$$



$$a_0 = \frac{G_F \xi S}{16\sqrt{2}\pi} + \frac{G_F(m_h^2 - m_W^2)(1-\xi)}{4\sqrt{2}\pi}$$

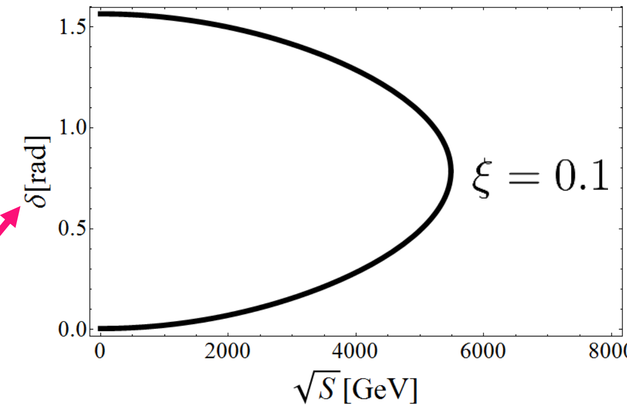


New resonance scale



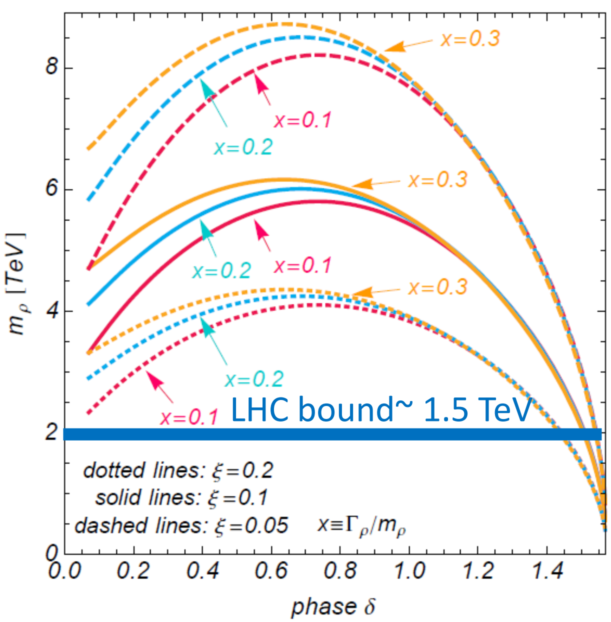
It is well known that the phase shift occurs in π-π scattering. Then, we take phase into account for the WW scattering amplitude of the MCHM.

δ of $W_L W_L \to W_L W_L$ in the MCHM



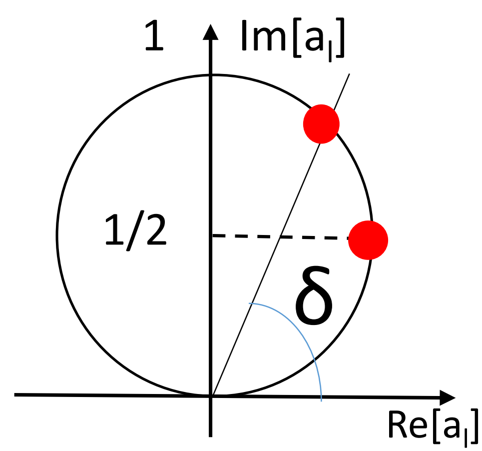
Assumption

$$\delta = \begin{cases} \tan^{-1}[(\Gamma/m)S/(m^2+\Gamma^2-S)] & \text{for} \quad \sqrt{S} < \sqrt{m^2+\Gamma^2} \\ \tan^{-1}[(\Gamma/m)S/(m^2+\Gamma^2-S)] + \pi & \text{for} \quad \sqrt{S} \geq \sqrt{m^2+\Gamma^2} \end{cases} = \delta = \tan^{-1}\left[1/(2\mathrm{Re}[a_0]) \pm \sqrt{1/(4\mathrm{Re}[a_0]^2) - 1}\right]$$



$$a_0 = \frac{G_F \xi S}{16\sqrt{2}\pi} + \frac{G_F(m_h^2 - m_W^2)(1-\xi)}{4\sqrt{2}\pi}$$

How to measure the phase δ ?



Measurement at LHC and Higher-energy Colliders

Murayama et al
arXiv: 1401. 3761

$$u\bar{d} \to W^+ Z \to \ell^+ \nu \ell^+ \ell^-$$

λ_W and λ_Z are W and Z helicity eigenvalues.
0 is longitudinal mode.

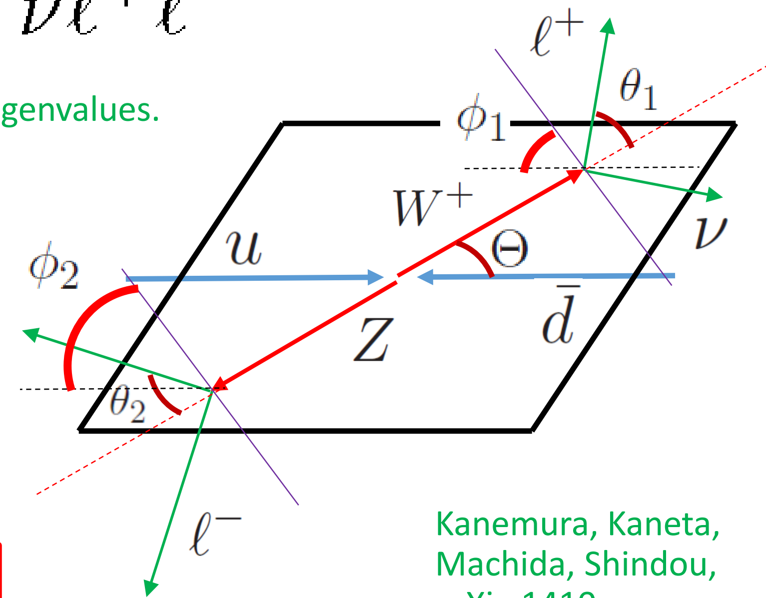
$$\mathcal{M}_{\rm Prod}(\Theta, \lambda_W, \lambda_Z)$$

$$\mathcal{M}_{\rm Prod}(\Theta, 0, 0) \to \mathcal{M}_{\rm Prod}(\Theta, 0, 0) e^{i\delta}$$

The phase δ is extracted by

Azimuthal angle of the charged lepton from W

$$|\text{amplitude}|^2 \supset \sin(\lambda_W \phi_1 - \lambda_Z \phi_2) \sin\delta$$



Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx

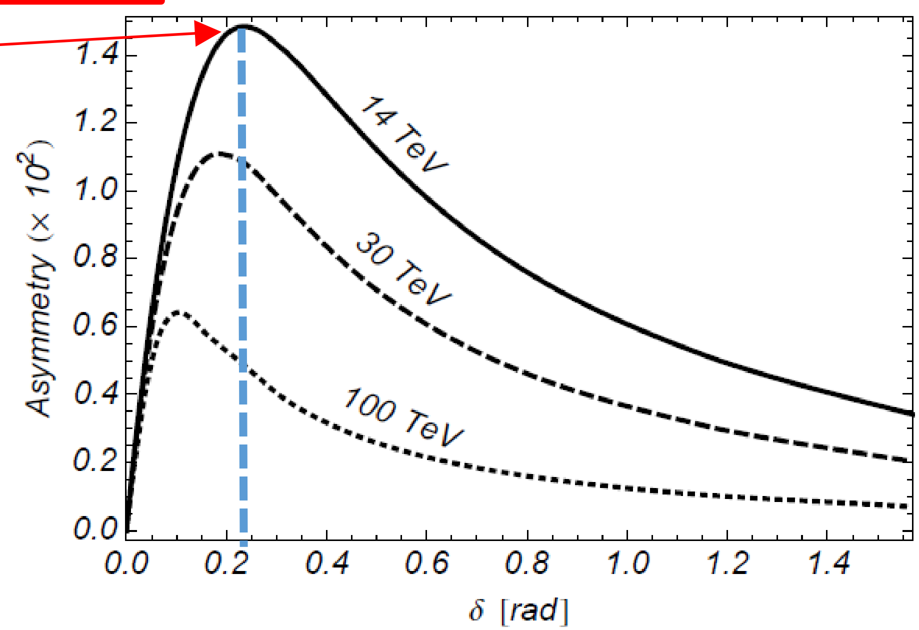
Asymmetry

+ : Above production plane, -- : Below production plane

$$A_\pm \equiv |\sigma_+ - \sigma_-|/(\sigma_+ + \sigma_-), \quad \sigma_\pm \equiv \sigma(\sin\phi_1 \gtrless 0)$$

The asymmetry with 14 TeV becomes maximum at δ=0.24 .

$$A_\pm \simeq 0.015$$
$$\sigma \simeq 0.037 \text{ pb}$$



This asymmetry can be observed if we accumulate 3000 fb^{-1} with HL-LHC.

Matter sector of the MCHM

Many variations depending on matter representations

1, 4, 5, 10, 14-dimensional representations

ex) MCHM_4

$$\Psi_q^{(4)} = \begin{pmatrix} q_L \\ Q_L \end{pmatrix}, \quad \Psi_u^{(4)} = \begin{pmatrix} q_R^u \\ \begin{pmatrix} u_R \\ d'_R \end{pmatrix} \end{pmatrix}, \quad \Psi_d^{(4)} = \begin{pmatrix} q_R^d \\ \begin{pmatrix} u'_R \\ d_R \end{pmatrix} \end{pmatrix}.$$

q_L, u_R, d_R : SM quarks
Others : non-SM quarks

$$\mathcal{L}_{\text{eff}}^{\text{matter}} = \sum_{r=q,u,d} \overline{\Psi}_r^{(4)} \not{p} \left[\Pi_0^r(p) + \Pi_1^r(p)\Gamma^i\Sigma_i\right] \Psi_r^{(4)} + \sum_{r=u,d} \overline{\Psi}_q^{(4)} \left[M_0^r(p) + M_1^r(p)\Gamma^i\Sigma_i\right] \Psi_r^{(4)}$$

MCHM variations (Introducing q_L, u_R, d_R)

MCHM_5 (q_L, u_R, d_R)=5 rep.

$\mathrm{MCHM}_{14\text{-}5\text{-}10}$ (q_L, u_R, d_R)=(14,5,10) rep.

We investigate 14 variation models of MCHMs.

Coupling deviations from the SM $\kappa_{h\phi\phi} = g^{\rm MCHM}_{h\phi\phi} / g^{\rm SM}_{h\phi\phi}$

Carena et al. JHEP 1406 (2014)

Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx

Higgs-gauge couplings are ***universal***. They do not depend on matter representations.

The Higgs-fermion and Higgs-self couplings are ***not universal***. They depend on matter representations. We can distinguish models by **correlations among several coupling deviations**.

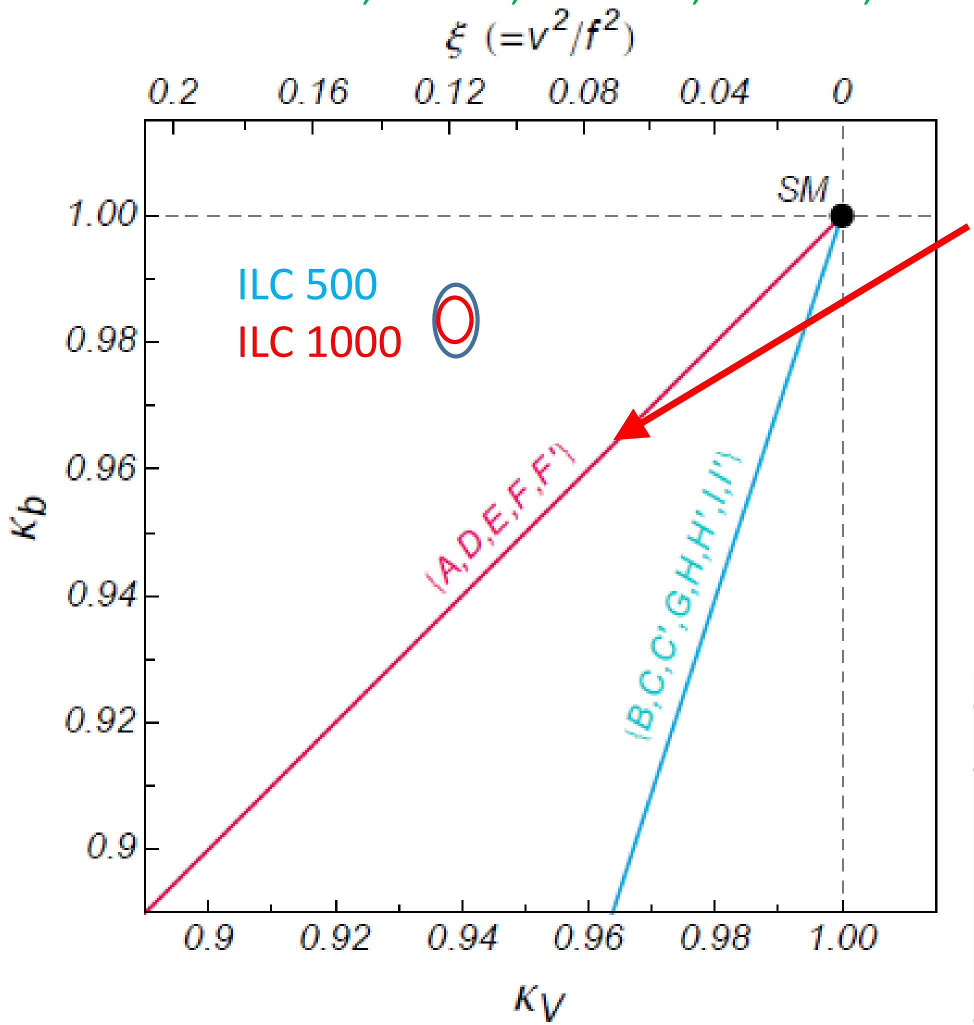
Depends on *G/H*

Depends on Matter representation

Label	Model	κ_V	c_{hhVV}	κ_{hhh}	c_{hhhh}	κ_t	κ_b	c_{hhtt}	c_{hhbb}
A	MCHM_4	$\sqrt{1-\xi}$	$1-2\xi$	$\sqrt{1-\xi}$	$1-\frac{7}{3}\xi$	$\sqrt{1-\xi}$	$\sqrt{1-\xi}$	$-\xi$	$-\xi$
B	MCHM_5	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
B	MCHM_{10}	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
C, C'	MCHM_{14}	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_3	$\frac{1-2\xi}{\sqrt{1-\xi}}$	F_6	-4ξ
D	$\text{MCHM}_{5\text{-}5\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\sqrt{1-\xi}$	-4ξ	$-\xi$
E	$\text{MCHM}_{5\text{-}10\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\sqrt{1-\xi}$	$\sqrt{1-\xi}$	$-\xi$	$-\xi$
F, F'	$\text{MCHM}_{5\text{-}14\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_5	$\sqrt{1-\xi}$	F_8	$-\xi$
G	$\text{MCHM}_{10\text{-}5\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\sqrt{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$-\xi$	-4ξ
B	$\text{MCHM}_{10\text{-}14\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
B	$\text{MCHM}_{14\text{-}1\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
H, H'	$\text{MCHM}_{14\text{-}5\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_4	$\frac{1-2\xi}{\sqrt{1-\xi}}$	F_7	-4ξ
B	$\text{MCHM}_{14\text{-}10\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
I, I'	$\text{MCHM}_{14\text{-}14\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_3	$\frac{1-2\xi}{\sqrt{1-\xi}}$	F_6	-4ξ

Fingerprint identification of MCHMs : κ_V vs. κ_b

Kanemura, Kaneta, Machida, Shindou, arXiv:x1410.xxxx



If measured value is on this line, (A, D, E, F, F') are still degenerate.

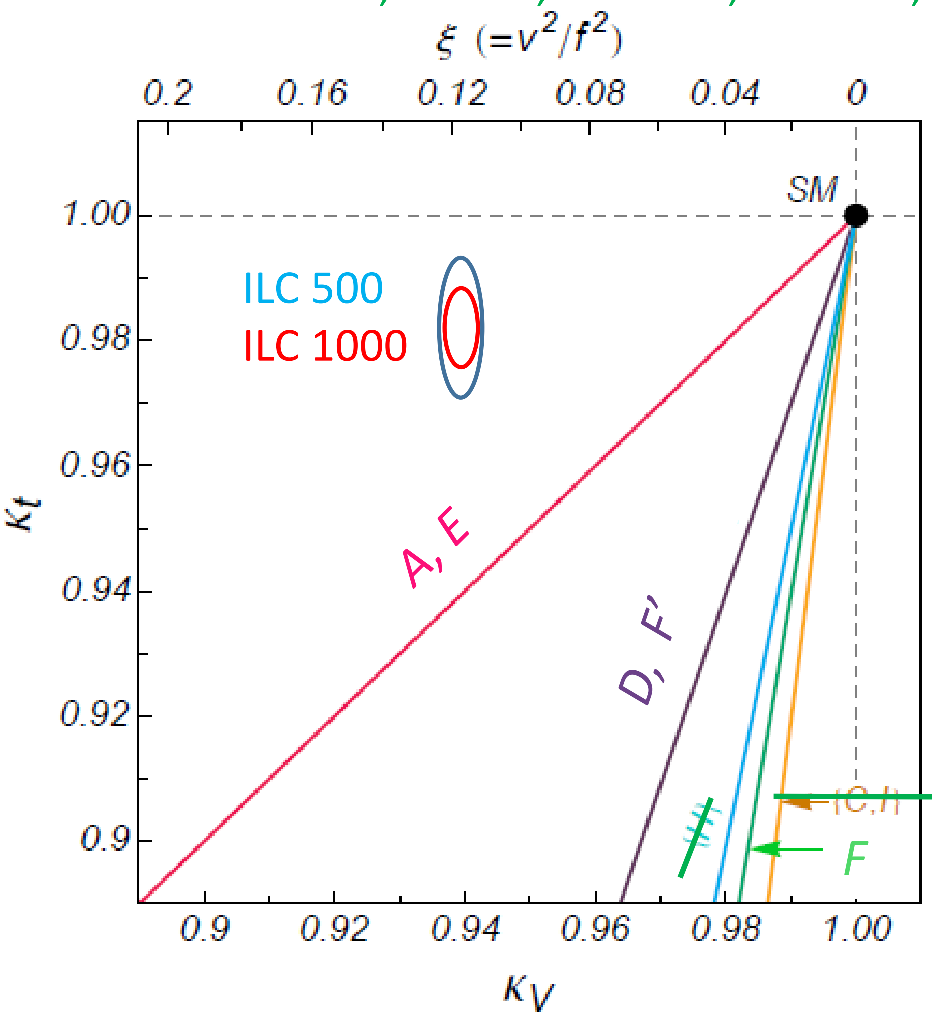
	ILC(500)	ILC(1000)
$\sqrt{s}$ (GeV)	250+500	250+500+1000
L (fb^{-1})	250+500	250+500+1000
$\gamma\gamma$	8.3 %	3.8 %
gg	2.0 %	1.1 %
WW	0.4 %	0.3 %
ZZ	0.5 %	0.5 %
$t\bar{t}$	2.5 %	1.3 %
$b\bar{b}$	1.0 %	0.6 %
$\tau^+\tau^-$	1.9 %	1.3 %
$\Gamma_T(h)$	1.7 %	1.1 %
$\mu^+\mu^-$	91 %	16 %
hhh	83 %	21 %
BR(invis.)	< 0.9 %	< 0.9 %
$c\bar{c}$	2.8 %	1.8 %

Label	Model
A	$MCHM_4$
B	$MCHM_5$
B	$MCHM_{10}$
C, C'	$MCHM_{14}$
D	$MCHM_{5\text{-}5\text{-}10}$
E	$MCHM_{5\text{-}10\text{-}10}$
F, F'	$MCHM_{5\text{-}14\text{-}10}$
G	$MCHM_{10\text{-}5\text{-}10}$
B	$MCHM_{10\text{-}14\text{-}10}$
B	$MCHM_{14\text{-}1\text{-}10}$
H, H'	$MCHM_{14\text{-}5\text{-}10}$
B	$MCHM_{14\text{-}10\text{-}10}$
I, I'	$MCHM_{14\text{-}14\text{-}10}$

ILC Higgs White paper

Fingerprint identification of MCHMs : κ_V vs. κ_t

Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx



(A, E)

or

(D, F')

or

F

	ILC(500)	ILC(1000)
$\sqrt{s}$ (GeV)	250+500	250+500+1000
L (fb^{-1})	250+500	250+500+1000
$\gamma\gamma$	8.3 %	3.8 %
gg	2.0 %	1.1 %
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$t\bar{t}$	2.5 %	1.3 %
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hhh	83 %	21 %
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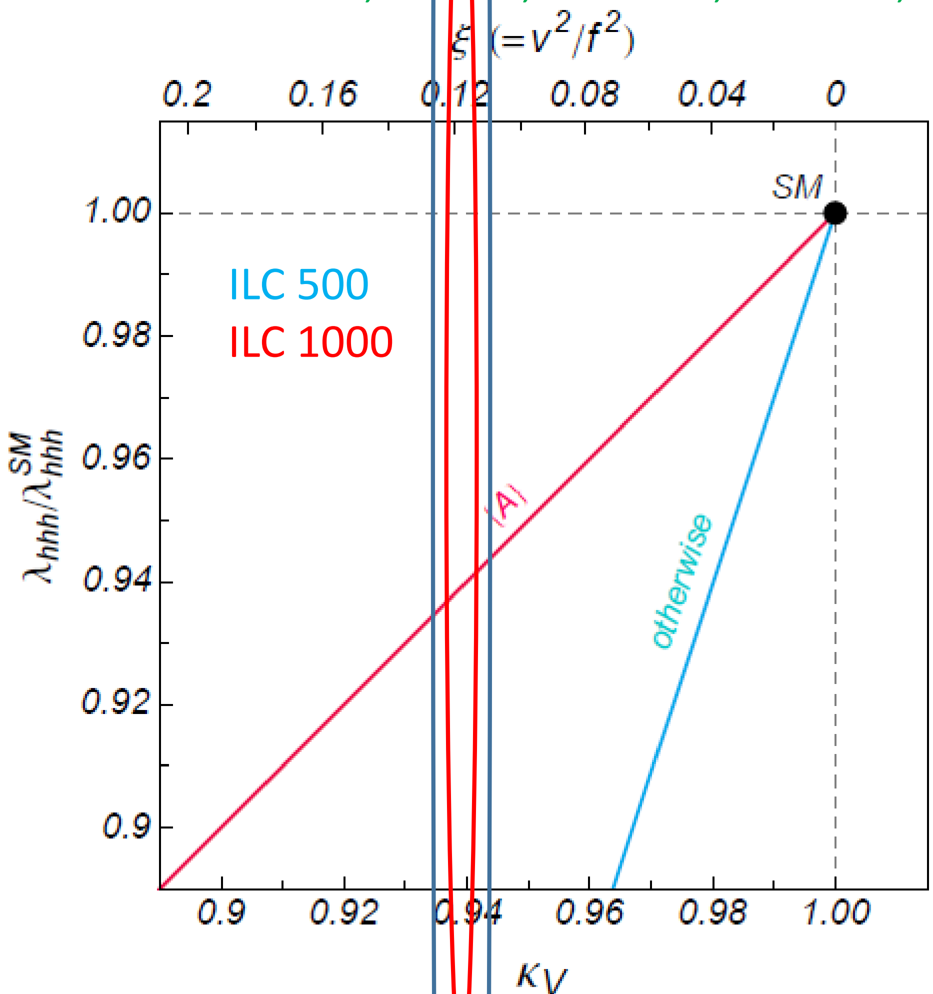
$c\bar{c}$	2.8 %	1.8 %

Label	Model
A	$MCHM_4$
~~B~~	~~$MCHM_5$~~
~~B~~	~~$MCHM_{10}$~~
~~C, C'~~	~~$MCHM_{14}$~~
D	$MCHM_{5\text{-}5\text{-}10}$
E	$MCHM_{5\text{-}10\text{-}10}$
F, F'	$MCHM_{5\text{-}14\text{-}10}$
~~G~~	~~$MCHM_{10\text{-}5\text{-}10}$~~
~~B~~	~~$MCHM_{10\text{-}14\text{-}10}$~~
~~B~~	~~$MCHM_{14\text{-}1\text{-}10}$~~
~~H, H'~~	~~$MCHM_{14\text{-}5\text{-}10}$~~
~~B~~	~~$MCHM_{14\text{-}10\text{-}10}$~~
~~I, I'~~	~~$MCHM_{14\text{-}14\text{-}10}$~~

ILC Higgs White paper

Fingerprint identification of MCHMs : κ_V vs. $\lambda_{hhh}/\lambda^{SM}_{hhh}$

Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx



We may distinguish A, E, (D, F') and F.

We may distinguish A and E.

	ILC(500)	ILC(1000)
$\sqrt{s}$ (GeV)	250+500	250+500+1000
L (fb^{-1})	250+500	250+500+1000
$\gamma\gamma$	8.3 %	3.8 %
gg	2.0 %	1.1 %
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ZZ	0.5 %	0.5 %
$t\bar{t}$	2.5 %	1.3 %
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$\tau^+\tau^-$	1.9 %	1.3 %
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$c\bar{c}$	2.8 %	1.8 %

Label	Model
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~~B~~	~~$MCHM_5$~~
~~B~~	~~$MCHM_{10}$~~
~~C, C'~~	~~$MCHM_{14}$~~
D	$MCHM_{5\text{-}5\text{-}10}$
E	$MCHM_{5\text{-}10\text{-}10}$
F, F'	$MCHM_{5\text{-}14\text{-}10}$
~~G~~	~~$MCHM_{10\text{-}5\text{-}10}$~~
~~B~~	~~$MCHM_{10\text{-}14\text{-}10}$~~
~~B~~	~~$MCHM_{14\text{-}1\text{-}10}$~~
~~H, H'~~	~~$MCHM_{14\text{-}5\text{-}10}$~~
~~B~~	~~$MCHM_{14\text{-}10\text{-}10}$~~
~~I, I'~~	~~$MCHM_{14\text{-}14\text{-}10}$~~

ILC Higgs White paper

Summary

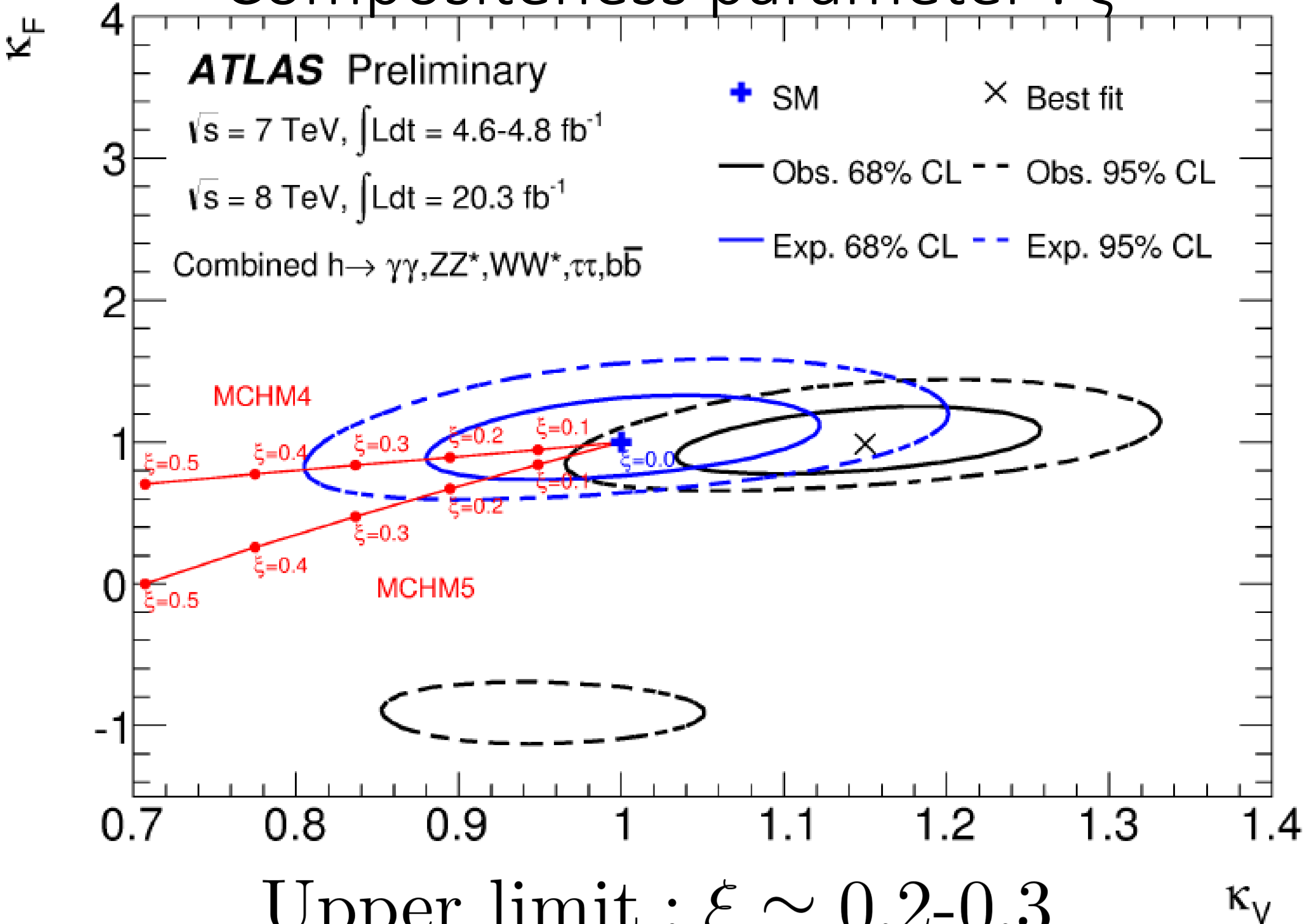
The Composite Higgs model is one of the promising candidates of the essence of the Higgs sector.

By using phase shift information, we can explore new resonance scale above the LHC direct reach .

We have discussed how to distinguish various models of MCHM by using the precise measurement of Higgs boson couplings.

Back up slides

Compositeness parameter : ξ



Upper limit : $\xi \simeq 0.2\text{-}0.3$

Coupling deviation from the SM

$\kappa_{h\phi\phi} = g_{h\phi\phi}^{\mathrm{MCHM}} / g_{h\phi\phi}^{\mathrm{SM}}$

Kanemura, Kaneta, Machida, Shindou, arXiv:1410.xxxx

Label	Model	κ_V	κ_{hhVV}	κ_{hhh}	κ_{hhhh}	κ_t	κ_b	κ_{hhtt}	κ_{hhbb}
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B	MCHM_5	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
B	MCHM_{10}	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
C, C'	MCHM_{14}	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_3	$\frac{1-2\xi}{\sqrt{1-\xi}}$	F_6	-4ξ
D	$\mathrm{MCHM}_{5\text{-}5\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\sqrt{1-\xi}$	-4ξ	$-\xi$
E	$\mathrm{MCHM}_{5\text{-}10\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\sqrt{1-\xi}$	$\sqrt{1-\xi}$	$-\xi$	$-\xi$
F, F'	$\mathrm{MCHM}_{5\text{-}14\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_5	$\sqrt{1-\xi}$	F_8	$-\xi$
G	$\mathrm{MCHM}_{10\text{-}5\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\sqrt{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$-\xi$	-4ξ
B	$\mathrm{MCHM}_{10\text{-}14\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
B	$\mathrm{MCHM}_{14\text{-}1\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-28\xi/3+28\xi^2/3}{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
H, H'	$\mathrm{MCHM}_{14\text{-}5\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_4	$\frac{1-2\xi}{\sqrt{1-\xi}}$	F_7	-4ξ
B	$\mathrm{MCHM}_{14\text{-}10\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	$\frac{1-2\xi}{\sqrt{1-\xi}}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$	-4ξ	-4ξ
I, I'	$\mathrm{MCHM}_{14\text{-}14\text{-}10}$	$\sqrt{1-\xi}$	$1-2\xi$	H_1	H_2	F_3	$\frac{1-2\xi}{\sqrt{1-\xi}}$	F_6	-4ξ

Coupling deviation from the SM

$$\kappa_{h\phi\phi} = g_{h\phi\phi}^{\text{MCHM}} / g_{h\phi\phi}^{\text{SM}}$$

$$\begin{aligned}
F_3 &= \frac{1}{\sqrt{1-\xi}} \frac{3(1-2\xi)M_1^t + 2(4-23\xi+20\xi^2)M_2^t}{3M_1^t + 2(4-5\xi)M_2^t}, \\
F_4 &= \sqrt{1-\xi}\frac{M_1^t + 2(1-3\xi)M_2^t}{M_1^t + 2(1-\xi)M_2^t}, \quad F_5 = \sqrt{1-\xi}\frac{M_1^t - (4-15\xi)M_2^t}{M_1^t - (4-5\xi)M_2^t}, \\
F_6 &= -4\xi\frac{3M_1^t + (23-40\xi)M_2^t}{3M_1^t + 2(4-5\xi)M_2^t}, \quad F_7 = -\xi\frac{M_1^t + 2(7-9\xi)M_2^t}{M_1^t + 2(1-\xi)M_2^t}, \\
F_8 &= -\xi\frac{M_1^t - (34-45\xi)M_2^t}{M_1^t - (4-5\xi)M_2^t}, \\
H_1 &= 1 - \frac{3\xi}{2} - \frac{5\xi^2}{8} + \frac{\xi^3}{3m_h^2}\left[-\frac{21m_h^2}{16} + \frac{48\gamma}{v^2}\right], \\
H_2 &= 1 - \frac{25\xi}{2} + \xi^2 + \frac{\xi^3}{3m_h^2}\left[3m_h^2 + \frac{288\gamma}{v^2}\right],
\end{aligned}$$

Matter sector of MCHM

Many variations depending on matter representations

1, 4, 5, 10, 14 representations

ex) MCHM_4

$$\Psi_q^{(4)} = \begin{pmatrix} q_L \\ Q_L \end{pmatrix}, \quad \Psi_u^{(4)} = \begin{pmatrix} q_R^u \\ \begin{pmatrix} u_R \\ d'_R \end{pmatrix} \end{pmatrix}, \quad \Psi_d^{(4)} = \begin{pmatrix} q_R^d \\ \begin{pmatrix} u'_R \\ d_R \end{pmatrix} \end{pmatrix}$$

SM quarks
(other fields are non-dynamical)

$$\mathcal{L}_{\text{eff}}^{\text{matter}} = \sum_{r=q,u,d} \overline{\Psi}_r^{(4)} \not{p} \left[\Pi_0^r(p) + \Pi_1^r(p)\Gamma^i\Sigma_i\right] \Psi_r^{(4)} + \sum_{r=u,d} \overline{\Psi}_q^{(4)} \left[M_0^r(p) + M_1^r(p)\Gamma^i\Sigma_i\right] \Psi_r^{(4)}$$

MCHM variations (Introducing q_L, u_R, d_R)

MCHM_5 (q_L, u_R, d_R)=5 rep.

$\text{MCHM}_{14\text{-}5\text{-}10}$ (q_L, u_R, d_R)=(14,5,10) rep.

We discuss 14 variation models of MCHMs.

Matter sector

- MCHM$_4$

$$\mathcal{L}_{\rm eff}^{\rm matter} = \sum_{r=q,u,d} \overline{\Psi}_r^{(4)} \not{p} \left[\Pi_0^r(p) + \Pi_1^r(p)\Gamma^i\Sigma_i\right] \Psi_r^{(4)} + \sum_{r=u,d} \overline{\Psi}_q^{(4)} \left[M_0^r(p) + M_1^r(p)\Gamma^i\Sigma_i\right] \Psi_r^{(4)} \; ,$$

- MCHM$_5$

$$\mathcal{L}_{\rm eff}^{\rm matter} = \sum_{r=t_L,t_R,b_L,b_R} \overline{\Psi}_r^{(5)} \left[\not{p}\Pi_0^r + \Sigma^\dagger \not{p}\Pi_1^r \Sigma\right] \Psi_r^{(5)}$$
$$+ \overline{\Psi}_{t_L}^{(5)} \left[M_0^t + \Sigma^\dagger M_1^t \Sigma\right] \Psi_{t_R}^{(5)} + \overline{\Psi}_{b_L}^{(5)} \left[M_0^b + \Sigma^\dagger M_1^b \Sigma\right] \Psi_{b_R}^{(5)} + \text{h.c.} \; .$$

Matter sector

- MCHM_{10}

$$\mathcal{L}_{\text{eff}}^{\text{matter}} = \sum_{r=q_L,t_R,b_R} \left[\overline{\Psi}_r^{(10)} \not{p} \Pi_0^r \Psi_r^{(10)} + (\Sigma \overline{\Psi}_r^{(10)}) \not{p} \Pi_1^r (\Psi_r^{(10)} \Sigma^\dagger) \right]$$
$$+ \overline{\Psi}_{q_L}^{(10)} M_0^t \Psi_{t_R}^{(10)} + (\Sigma \overline{\Psi}_{q_L}^{(10)}) M_1^t (\Psi_{t_R}^{(10)} \Sigma^\dagger)$$
$$+ \overline{\Psi}_{q_L}^{(10)} M_0^b \Psi_{b_R}^{(10)} + (\Sigma \overline{\Psi}_{q_L}^{(10)}) M_1^b (\Psi_{b_R}^{(10)} \Sigma^\dagger) + \text{h.c.} \, .$$

- MCHM_{14}

$$\mathcal{L}_{\text{eff}}^{\text{matter}} = \sum_{r=q_L,t_R,b_R} \left[\overline{\Psi}_r^{(14)} \not{p} \Pi_0^r \Psi_r^{(14)} + (\Sigma \overline{\Psi}_r^{(14)}) \not{p} \Pi_1^r (\Psi_r^{(14)} \Sigma^\dagger) + (\Sigma \overline{\Psi}_r^{(14)} \Sigma^\dagger) \not{p} \Pi_2^r (\Sigma \Psi_r^{(14)} \Sigma^\dagger) \right]$$
$$+ \overline{\Psi}_{q_L}^{(14)} M_0^t \Psi_{t_R}^{(14)} + (\Sigma \overline{\Psi}_{q_L}^{(14)}) M_1^t (\Psi_{t_R}^{(14)} \Sigma^\dagger) + (\Sigma \overline{\Psi}_{q_L}^{(14)} \Sigma^\dagger) M_2^t (\Sigma \Psi_{t_R}^{(14)} \Sigma^\dagger)$$
$$+ \overline{\Psi}_{q_L}^{(14)} M_0^b \Psi_{b_R}^{(14)} + (\Sigma \overline{\Psi}_{q_L}^{(14)}) M_1^b (\Psi_{b_R}^{(14)} \Sigma^\dagger) + (\Sigma \overline{\Psi}_{q_L}^{(14)} \Sigma^\dagger) M_2^b (\Sigma \Psi_{b_R}^{(14)} \Sigma^\dagger) + \text{h.c.} \, .$$

SO(5) generators & eigenvectors

- **5**-representation

$$(T^{a\,L,R})_{ij} = -\frac{i}{2}\left[\frac{1}{2}\epsilon^{abc}\left(\delta_i^b\delta_j^c - \delta_j^b\delta_i^c\right) \pm \left(\delta_i^a\delta_j^4 - \delta_j^a\delta_i^4\right)\right]$$

$$T_{ij}^{\hat{a}} = -\frac{i}{\sqrt{2}}\left(\delta_i^{\hat{a}}\delta_j^5 - \delta_j^{\hat{a}}\delta_i^5\right).$$

$$v_{(-,-)} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},\ v_{(-,+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ i \\ 1 \\ 0 \end{pmatrix},$$

$$v_{(+,-)} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ -i \\ 1 \\ 0 \end{pmatrix},\ v_{(+,+)} = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix},\ v_{(0,0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

SO(5) generators & eigenvectors

- 10-representation

$$
\begin{aligned}
(\mathbf{3},\mathbf{1}) \ &: \ v_{(\pm1,0)} = \frac{1}{\sqrt{2}}(T_L^1 \pm iT_L^2), \qquad v_{(0,0)} = T_L^3, \\
(\mathbf{1},\mathbf{3}) \ &: \ v_{(0,\pm1)} = \frac{1}{\sqrt{2}}(T_R^1 \pm iT_R^2), \qquad v_{(0,0)} = T_R^3, \\
(\mathbf{2},\mathbf{2}) \ &: \ v_{(-1/2,-1/2)} = \frac{1}{\sqrt{2}}(T^{\hat{1}} - iT^{\hat{2}}), \quad v_{(+1/2,+1/2)} = \frac{1}{\sqrt{2}}(T^{\hat{1}} + iT^{\hat{2}}), \\
& \quad\ v_{(-1/2,+1/2)} = \frac{1}{\sqrt{2}}(T^{\hat{3}} - iT^{\hat{4}}), \quad v_{(+1/2,-1/2)} = \frac{1}{\sqrt{2}}(T^{\hat{3}} + iT^{\hat{4}}).
\end{aligned}
$$

$$
v_{(0,0)} = \frac{1}{2}\begin{pmatrix} & -i & & & 0 \\ +i & & & & 0 \\ & & & -i & 0 \\ & & +i & & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
$$

Anti-symmetric

$$
\Sigma^T \bar{\Psi} \Sigma \to 0
$$

SO(5) generators & eigenvectors

- 14-representation

$$(\mathbf{3},\bar{\mathbf{3}}) \ : \ T^{ab}_{ij} = \frac{1}{\sqrt{2}}(\delta^a_i\delta^b_j + \delta^a_j\delta^b_i), \qquad a<b,\ a,\ b=1,\cdots,4$$

$$T^{aa}_{ij} = \frac{1}{\sqrt{2}}(\delta^a_i\delta^a_j - \delta^{a+1}_i\delta^{a+1}_j), \qquad a=1,2,3$$

$$(\mathbf{2},\bar{\mathbf{2}}) \ : \ T^{\hat{a}}_{ij} = \frac{1}{\sqrt{2}}(\delta^a_i\delta^5_j + \delta^a_j\delta^5_i), \qquad a=1,\cdots,4$$

$$(\mathbf{1},\bar{\mathbf{1}}) \ : \ T^{0}_{ij} = \frac{1}{2\sqrt{5}}\mathrm{diag}(1,1,1,1,-4).$$

$$v_{(+1,+1)} = \frac{1}{4}\begin{pmatrix} 1 & 2i & & & 0 \\ 2i & -2 & & & 0 \\ & & 1 & & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Symmetric

$$\Sigma^T \bar{\Psi} \Sigma \neq 0$$