

[Talk 22] A Numerical Study of the Disorder-Order Transition in a Two-Dimensional Incompressible Flocking Model

Wanming Qi, Beijing Computational Science Research Center

Active matter, which is driven out of equilibrium by the energy injection at the single-particle level as in many biological systems, displays fascinating collective behavior. One such example is the emergence of collective motion (or “flocking”) in large groups of self-propelled particles. The flocking models generally show phase transition from random to collective motion. At the transition, both the physics of nonlinear advection as in the Navier-Stokes fluid and the physics of ferromagnetic interaction as in the XY model come into play. Despite much theoretical, numerical and experimental efforts, we still lack a fundamental understanding of the flocking transition.

Recently the incompressible Toner-Tu model has attracted some attention partly due to its minimality. The Toner-Tu model is a stochastic continuum theory that captures the flocking behavior of polar active particles with short-ranged polar alignment interaction. The density-polarization coupling makes the phase transition discontinuous in the compressible Toner-Tu model that have been studied, but this mechanism does not work in the incompressible case. A perturbative dynamical renormalization group analysis in $4-\epsilon$ dimensions found a continuous phase transition indeed, and further found the critical exponents of a new universality class [1]. We numerically simulate the two-dimensional incompressible Toner-Tu equation on the square torus to study the phase transition. Our numerical results confirm a continuous transition. We further find that the statistics of the global velocity behave differently for weak and strong noises, and the Binder cumulant crossing values show strange non-convergence at available system sizes. We will also discuss the nature of self-similarity at phase transition from the flow patterns of vortices and anisotropic jets.

[1] L. Chen, J. Toner, and C.F. Lee, *New Journal of Physics* 17, 042002 (2015).