The current status of NRQCD descriptions of $J/\psi$ and $\Upsilon$ system

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HaPhy2012-11: APCTP-KIAS miniworkshop for Physics on Heavy Quarkonia
November 23 (Fri), 2012, KIAS, Seoul, Korea
1 Introduction

2 J/psi production at the B factories
   - double charmonium production
   - Inclusive $J/\psi$ production

3 $J/\psi$ production at the Tevatron and LHC
   - QCD Correction to color-singlet $J/\psi$ production
   - QCD Correction to color-octet $J/\psi$ production
   - Other important Progress

4 the other
   - $c \rightarrow J/\psi$ fragmentation function
   - $J/\psi$ production in $Z$ decay
   - $J/\psi$ production from $\Upsilon$ Decay

5 The $\alpha_s \nu^2$ correction or higher-order correction

6 Summary
Introduction

- Perturbative and non-perturbative QCD, hadronization, factorization
- Color-singlet and Color-octet mechanism was proposed based on NRQCD for heavy quarkonium
- Why so serious to on the test: Clear signal to detect $J/\psi$, very limited number of nonperturbative parameters, double perturbative expansions on $\alpha_s$ and $v$ (the velocity of heavy quark in quarkonium) are better since b and c-quark is heavy.
- $J/\psi$ production at the B factories
- $J/\psi$ photoproduction at HERA
- $J/\psi$ production and polarization at the Tevatron and LHC
- LO theoretical predication were given before more than 15 years
- NLO theoretical predcations were given within last 5 years.
- The QCD NLO calculations can adequately describe the experimental data?
- There are still many difficulties.
FIG. 4 (color online). Prompt polarizations as functions of $p_T$: (a) $J/\psi$ and (b) $\psi(2S)$. The band (line) is the prediction from NRQCD [4] (the $k_T$-factorization model [9]).
\[ e^+ e^- \rightarrow J/\psi + \eta_c \]

**Experimental Data**

**BELLE:** \[ \sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (25.6 \pm 2.8 \pm 3.4) \text{ fb} \]

**BARAR:** \[ \sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 2] = (17.6 \pm 2.8^{+1.5}_{-2.1}) \text{ fb} \]

[?, ?, ?]

**LO NRQCD Predictions**

2.3 \sim 5.5 \text{ fb}

[?, ?, ?]
\[ e^+ e^- \rightarrow J/\psi + \eta_c \]

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**LO NRQCD Predictions**

\( 2.3 \sim 5.5 \text{ fb} \)

**NLO QCD corrections**

\( K \equiv \frac{\sigma^{NLO}}{\sigma^{LO}} \sim 2 \)

Confirmed by the analytic result in PRD77, (2008), B. Gong and J. X. Wang

**Relativistic corrections**

\( K \sim 2 \)

PRD67, (2007) E. Braaten and J. Lee
PRD75, (2007), Z. G. He, Y. Fan and K. T. Chao
PRD77,(2008),G.T. Bodwin, J. Lee and C. Yu
$e^+ e^- \rightarrow J/\psi + J/\psi$

**Problem**

LO NRQCD prediction indicates that the cross section of this process is larger than that of $J/\psi + \eta_c$ production by a factor of 1.8, but no evidence for this process was found at the B factories.

PRL90, (2003) G. T. Bodwin, E. Braaten and J. Lee
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PRL90, (2003) G. T. Bodwin, E. Braaten and J. Lee

**NLO QCD corrections**

- Greatly decreased, with a K factor ranging from \(-0.31 \sim 0.25\) depending on the renormalization scale.
- Might explain the situation.

LO NRQCD Predictions:

<table>
<thead>
<tr>
<th>Process</th>
<th>Predicted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^- \to J/\psi + c\bar{c}$</td>
<td>$0.07 \sim 0.20 \text{pb}$</td>
</tr>
<tr>
<td>$e^+e^- \to J/\psi + gg$</td>
<td>$0.15 \sim 0.3 \text{pb}$</td>
</tr>
<tr>
<td>$e^+e^- \to J/\psi(3P^8_J, 1S^0_0) + g$</td>
<td>$0.3 \sim 0.8 \text{pb}$</td>
</tr>
</tbody>
</table>

Experimental Data:

- **BARAR**
  \[ \sigma[e^+e^- \to J/\psi + X] = (2.54 \pm 0.21 \pm 0.21) \text{ pb} \]

- **CLEO**
  \[ \sigma[e^+e^- \to J/\psi + X] = (1.9 \pm 0.20) \text{ pb} \]

- **BELLE**
  \[ \sigma[e^+e^- \to J/\psi + c\bar{c} + X] = (0.87^{+0.21}_{-0.19} \pm 0.17) \text{ pb} \]

New BELLE Data

- \[ \sigma[e^+e^- \to J/\psi + X] = (1.17 \pm 0.02 \pm 0.07) \text{ pb} \]
- \[ \sigma[e^+e^- \to J/\psi + c\bar{c}] = (0.74 \pm 0.08^{+0.09}_{-0.08}) \text{ pb} \]
- \[ \sigma[e^+e^- \to J/\psi + X_{\text{non}-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09) \text{ pb} \]
Cross section at NLO for $e^+e^- \rightarrow J/\psi + gg$

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln \left( \frac{\mu}{2m_c} \right) \right] \right\}$$

<table>
<thead>
<tr>
<th>$m_c$(GeV)</th>
<th>$\alpha_s(\mu)$</th>
<th>$\sigma^{(0)}$(pb)</th>
<th>$a(\hat{s})$</th>
<th>$\sigma^{(1)}$(pb)</th>
<th>$\sigma^{(1)}/\sigma^{(0)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.267</td>
<td>0.341</td>
<td>2.35</td>
<td>0.409</td>
<td>1.20</td>
</tr>
<tr>
<td>1.5</td>
<td>0.259</td>
<td>0.308</td>
<td>2.57</td>
<td>0.373</td>
<td>1.21</td>
</tr>
<tr>
<td>1.6</td>
<td>0.252</td>
<td>0.279</td>
<td>2.89</td>
<td>0.344</td>
<td>1.23</td>
</tr>
</tbody>
</table>

Consistent results from two group:
PRL102, (2009) Y. Q. Ma, Y. J. Zhang and K. T. Chao

Relativistic Correction enhance results about a factor 1.3 from two group:
PRD82, (2010) Y. Jia
$e^+ e^- \rightarrow J/\psi + c\bar{c}$

$$\sigma^{(1)} = \sigma^{(0)} \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[ a(\hat{s}) + \beta_0 \ln \left( \frac{\mu}{2m_c} \right) \right] \right\}$$

<table>
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<tr>
<th>$m_c$ (GeV)</th>
<th>$\alpha_s(\mu)$</th>
<th>$\sigma^{(0)}$ (pb)</th>
<th>$a(\hat{s})$</th>
<th>$\sigma^{(1)}$ (pb)</th>
<th>$\sigma^{(1)}/\sigma^{(0)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.267</td>
<td>0.224</td>
<td>8.19</td>
<td>0.380</td>
<td>1.70</td>
</tr>
<tr>
<td>1.5</td>
<td>0.259</td>
<td>0.171</td>
<td>8.94</td>
<td>0.298</td>
<td>1.74</td>
</tr>
<tr>
<td>1.6</td>
<td>0.252</td>
<td>0.129</td>
<td>9.74</td>
<td>0.230</td>
<td>1.78</td>
</tr>
</tbody>
</table>

Cross sections with different charm quark mass $m_c$ with the renormalization scale $\mu = 2m_c$ and $\sqrt{s} = 10.6$ GeV. The former result given by PRL98, (2007) Y. J. Zhang and K. T. Chao confirmed by PRD80, (2009) B. Gong and J. X. Wang.
Use Brodsky, Lepage and Mackenzie (BLM) scale setting

$$\sigma^{(1)} = \sigma^{(0)}(\mu^*)[1 + \frac{\alpha_s(\mu^*)}{\pi} b(\hat{s})].$$

<table>
<thead>
<tr>
<th>$m_c$(GeV)</th>
<th>$\alpha_s(\mu^*)$</th>
<th>$\sigma^{(0)}$(pb)</th>
<th>$b(\hat{s})$</th>
<th>$\sigma^{(1)}$(pb)</th>
<th>$\sigma^{(1)}/\sigma^{(0)}$</th>
<th>$\mu^*$(GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>0.348</td>
<td>0.381</td>
<td>3.77</td>
<td>0.540</td>
<td>1.42</td>
<td>1.65</td>
</tr>
<tr>
<td>1.5</td>
<td>0.339</td>
<td>0.293</td>
<td>4.31</td>
<td>0.429</td>
<td>1.47</td>
<td>1.72</td>
</tr>
<tr>
<td>1.6</td>
<td>0.332</td>
<td>0.222</td>
<td>4.90</td>
<td>0.337</td>
<td>1.52</td>
<td>1.79</td>
</tr>
</tbody>
</table>

Cross sections with different charm quark mass $m_c$. The renormalization scale $\mu = \mu^* \sim m_c$. 
Momentum distribution of inclusive $J/\psi$ production with $\mu = \mu^*$ and $m_c = 1.4$ GeV is taken for the $J/\psi cc$ channel. The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.
Momentum and angular distributions of inclusive $J/\psi$ production.

The contribution from the feed-down of $\psi'$ has been added to all curves by multiplying a factor of 1.29.
Polarization parameter $\alpha$ and angular distribution parameter $A$ of $J/\psi$ as functions of $p$. 
Constraint for color-octect matrix element of $c\bar{c}(^{1}S_{0}^{8},^{3}P_{j}^{8})$

\[ \sigma[e^+e^- \rightarrow J/\psi + X_{\text{non}-c\bar{c}}] = (0.43 \pm 0.09 \pm 0.09) \text{ pb} \]

\[ \sigma[e^+e^- \rightarrow J/\psi + X_{\text{non}-c\bar{c}}]^{\text{color-singleTh}} > (0.43) \text{ pb} \]

\[ \sigma[e^+e^- \rightarrow J/\psi + X_{\text{non}-c\bar{c}}]^{\text{color-octetTh}} > (0.6) \text{ pb} \]

From the contribution of $e^+e^- \rightarrow J/\psi(^{1}S_{0}^{8},^{3}P_{j}^{8}) + g$ at NLO

The cross section of $h$ hadroproduction is

$$\sigma[pp \rightarrow hx] = \sum \int \, d\mathbf{x}_1 d\mathbf{x}_2 \, G_p^i G_p^j \hat{\sigma}[ij \rightarrow (c\bar{c})_n x] \langle O_n^h \rangle,$$  \hspace{1cm} (1)

where $p$ is either a proton or anti-proton, the indices $i, j$ run over all the partonic species and $n$ represents the $c\bar{c}$ intermediate states $(^3S_1, ^3S_1^8, ^1S_0^8, ^3P_1^8)$ for $J/\psi$ and $\psi'$, and $(^3P_1^1, ^3S_1^8)$ for $\chi_{cJ}$.

- double expansions in $\alpha_s$ and the heavy-quark velocity $v$.
- predication can be systematically improved with these two perturbative expansions.
- limited number of universal long-distance matrix elements to be extracted from experiment.
In last five years, there were a few very important progresses in the next-to-leading Order (NLO) QCD correction calculation:

- QCD Correction to color-singlet $J/\psi$ production
- QCD Correction to color-singlet $J/\psi$ polarization
- QCD Correction to color-octet $J/\psi(1S_0^8, 3S_1^8)$ production and polarization
- QCD Correction to color-octet $J/\psi(1S_0^8, 3S_1^8, 3P_J^8)$ production
- QCD Correction to color-octet $J/\psi(1S_0^8, 3S_1^8, 3P_J^8)$ polarization
- QCD Correction to $\chi_{cJ}(3S_1^8, 3P_J^1)$ production

Until now, there are:

- $p_t$ distribution of $J/\psi$ yield for prompt $J/\psi$ hadroproduction at QCD NLO
- $p_t$ distribution of $J/\psi$ polarization for direct $J/\psi$ hadroproduction at QCD NLO
- Feeddown of $\chi_{cJ}$ about 20 – 30% to prompt $J/\psi$ production and very important.

Prompt: included the $J/\psi$ feeddown from excited charmonium state than direct production.

We need:

- $p_t$ distribution of $J/\psi$ polarization for prompt $J/\psi$ hadroproduction at QCD NLO

We have finished this work and presented in a recent paper ArXiv:1205.6682, Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang.
The polarisation is defined from the dilepton decay of inclusively produced vector mesons and the general observable distribution is

$$W(\cos \vartheta, \varphi) \propto \frac{1}{(3 + \lambda \vartheta)} \left(1 + \lambda \cos^2 \vartheta + \lambda \varphi \sin^2 \vartheta \cos 2\varphi + \lambda \vartheta \varphi \sin 2\vartheta \cos \varphi\right),$$

where $\vartheta$ and $\varphi$ are the (polar and azimuthal) angles of the positive lepton with, respectively, the polarisation axis $z$ and the production plane $xz$ (with the colliding particles and the decaying meson in it).

The polarization factor $\lambda$ is

$$\lambda = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$

(2)

where $\lambda = +1$ completely transverse polarization $\lambda = 0$ unpolarization $\lambda = -1$ completely longitudinal polarization

In the helicity frame the polar axis coincides with the flight direction of the meson in the centre-of-mass frame of the colliding hadrons.

In the Collins-Soper (CS) frame, the polar axis reflects, the direction of the relative velocity of the colliding partons.
QCD Correction to color-singlet $J/\psi$ production

$P_t$ distribution of $J/\psi$ production at QCD NLO was calculated in \textit{PRL98,252002 (2007)}, J. Campbell, F. Maltoni F. Tramontano

Some technique problems must be solved to calculate $J/\psi$ polarization

$P_t$ distribution of $J/\psi$ polarization at QCD NLO was calculated in \textit{PRL100,232001 (2008)}, B. Gong and J. X. Wang
QCD Correction to color-singlet $\Upsilon$ production

$\Upsilon$ polarization drastically changes from transverse polarization dominant at LO into longitudinal polarization dominant at NLO

$P_t$ distribution of $\Upsilon$ polarization at QCD NLO was calculated with detail in PRD78 074011 (2008), B. Gong and J. X. Wang

Partly NNLO calculation for $\Upsilon$ production calculated by PRL101, 152001(2008), P. Artoisenet, John M. Campbell, J.P. Lansberg, F. Maltoni, F. Tramontano
NLO QCD corrections to $J/\psi$ production via S-wave color octet states

3 tree processes at LO

\[ g(p_1) + g(p_2) \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right](p_3) + g(p_4), \quad (267, 413) \]
\[ g(p_1) + q(p_2) \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right](p_3) + q(p_4), \quad (49, 111) \]
\[ q(p_1) + \bar{q}(p_2) \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right](p_3) + g(p_4). \quad (49, 111) \]

Real Correction (8 processes at NLO)

\[ gg \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]gg, \quad gg \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]q\bar{q}, \]
\[ gq \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]gq, \quad q\bar{q} \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]gg, \]
\[ q\bar{q} \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]q\bar{q}, \quad q\bar{q} \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]q'\bar{q}', \]
\[ qq \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]qq, \quad qq' \rightarrow J/\psi\left[ ^1S_0^{(8)}, ^3S_1^{(8)} \right]qq', \]
QCD Correction to $J/\psi(3S_1, 1S_0, 3S_1)$ production

To fit the Tevatron $P_t$ distribution give more $\langle O_8^{\psi}(S_0) \rangle = 0.075$ GeV$^3$ and less $\langle O_8^{\psi}(S_1) \rangle = 0.0021$ GeV$^3$ than they are at LO fitting. The experimental data with $p_t < 6$ GeV have to abandon.


Correction to $J/\psi(3S_1, 1S_0, 3S_1, 3P_1)$ production was done without calculation of polarization, by PRL 106, 042002,2011, Yan-Qing Ma, Kai Wang, Kuang-Ta Chao

PRL 106, 022003,2011, Mathias Butenschoen, Bernd A. Kniehl
QCD Correction to prompt $J/\psi(1S_0^8, 3S_1^8, 3P_J^8)$ production

PRL 106, 042002, 2011, Yan-Qing Ma, Kai Wang, Kuang-Ta Chao
QCD Correction to $J/\psi(3S_{1}^{1}, 1S_{0}^{8}, 3S_{1}^{8}, 3P_{j}^{8})$ production

PRL 106, 022003, 2011, Mathias Butenschoen, Bernd A. Kniehl
QCD Correction to polarization of $J/\psi(1S_0^8, 3S_1^8, 3P_J^8)$ direct production

PRL 108, 248004, 2012 Kuang-Ta Chao, Yan-Qing Ma, Hua-Sheng Shao, Kai Wang, Yu-Jie Zhang
QCD Correction to polarization of $J/\psi(3S_1^1, 1S_0^8, 3S_1^8, 3P_1^8)$ direct production

ALICE data

Collins-Soper frame

- CS, LO
- CS, NLO
- CS+CO, LO
- CS+CO, NLO

$\sqrt{s} = 7$ TeV

$pp \rightarrow J/\psi + X$

Helicity frame

- CS, LO
- CS, NLO
- CS+CO, LO
- CS+CO, NLO

CDF data: Run I / II

- CS, LO
- CS, NLO
- CS+CO, LO
- CS+CO, NLO

$|y| < 0.6$

$pp \rightarrow J/\psi + X$

PRL 108, 172002, 2012, Mathias Butenschoen, Bernd A. Kniehl
Relativistic Correction to color-octet $J/\psi(3S_1^1, 1S_0^8, 3S_1^8, 3P_1^8)$ production

![Graphs showing various plots for different energy conditions and angular momenta](image)

ArXiv:1203.0207, gng-Zhi Xu, Yi-Jie Li, Kui-Yong Liu, and Yu-Jie Zhang
QCD Correction to prompt $J/\psi(3S^1_1, 1S^8_0, 3S^8_1, 3P^8_j)$ production and polarization

In addition to all the calculation for $J/\psi$, we need to calculate the polarization of $\chi_{cJ}(3P^1_j, 3S^8_1)$. At LO, there are three partonic processes:

$$g(p_1) + g(p_2) \rightarrow J/\psi \left[ 3P^1_j \right] (p_3) + g(p_4),$$
$$g(p_1) + q(p_2) \rightarrow J/\psi \left[ 3P^1_j \right] (p_3) + q(p_4),$$
$$q(p_1) + \bar{q}(p_2) \rightarrow J/\psi \left[ 3P^1_j \right] (p_3) + g(p_4).$$

There are eight processes involved in the real corrections:

$$gg \rightarrow J/\psi \left[ 3P^1_j \right] gg, \quad gg \rightarrow J/\psi \left[ 3P^1_j \right] q\bar{q},$$
$$gq \rightarrow J/\psi \left[ 3P^1_j \right] gq, \quad q\bar{q} \rightarrow J/\psi \left[ 3P^1_j \right] gg,$$
$$q\bar{q} \rightarrow J/\psi \left[ 3P^1_j \right] q\bar{q}, \quad q\bar{q} \rightarrow J/\psi \left[ 3P^1_j \right] q'\bar{q'},$$
$$qq \rightarrow J/\psi \left[ 3P^1_j \right] qq, \quad qq' \rightarrow J/\psi \left[ 3P^1_j \right] qq'.$$
The present status and problem

It is well known that the uncertainties in $p_t$ distribution of charmonium hadroproduction yield from charm-quark mass $m_c$, NRQCD scale $\mu_\Lambda$, renormalization scale $\mu_r$ and factorization scale $\mu_f$ are large at small $p_t$ range. And recent work on relativistic correction to $J/\psi$ hadroproduction shows that the correction is negative and large at small $p_t$ range ($p_t < 10\text{GeV}$). For large $p_t$ range, the large logarithm term $\ln(p_t/m_c)$ will appear even with the default choice of all the scales $\mu_r = \mu_f = p_t$, $\mu_\Lambda = m_c$ and it may ruin the result without proper treatment of this term. In the other side, it very clearly show in the previous fitting and also this fitting that the experimental data for $p_t < 7$ and $p_t > 15$ can not be represented very well simultaneously.
QCD Correction to prompt $J/\psi(3S_1^1, 1S_0^8, 3S_1^8, 3P_j^8)$ production

**Figure:** $p_t$ distribution of prompt $J/\psi$ and $\psi'$ hadroproduction. The CDF and LHCb data are taken in the fitting.

*ArXiv:1205.6682, Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang*
QCD Correction to $\psi'(3S_1^1, 1S_0^8, 3S_1^8, 3P_8^8)$ polarization

Figure: Polarization parameter $\lambda$ of $J/\psi'$ in helicity(left) and CS(right) frames.

ArXiv:1205.6682, Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang
QCD Correction to $\chi c J(3P^1_J, 3S^8_1) \rightarrow J/\psi$ polarization

Figure: Polarization parameter $\lambda$ of $J/\psi$ in helicity(left) and CS(right) frames.

ArXiv:1205.6682, Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang
QCD Correction to prompt $J/\psi(3S^1_1, 1S^8_0, 3S^8_1, 3P^8_j)$ polarization

Figure: Polarization parameter $\lambda$ of prompt $J/\psi$ hadroproduction in helicity(left) and CS(right) frames.

ArXiv:1205.6682, Bin Gong, Lu-Ping Wan, Jian-Xiong Wang and Hong-Fei Zhang
QCD Correction to color-octet $\Upsilon(3S_1^1, 1S_0^8, 3S_1^8)$ production

QCD Correction to color-octet $\Upsilon(3S_1^{1}, 1S_0^{8}, 3S_1^{8}, 3P_0^{8})$ production

ArXiv:1201.6012 Kai Wang, Yan-Qing Ma, Kuang-Ta Chao
A new factorization scheme for $J/\psi$ hadron production

The main point is to extend the fragmentation factorization from: one-parton fragment into hadron to: two-parton fragment into hadron. There will be more fragmentation function needed in this scheme.

PRL 108 (2012) 102002, Zhong-Bo Kang, Jian-Wei Qiu and George Sterman
Testing Charmonium Production Mechanism via Polarized $J/\psi$ Pair Production at the LHC

J.Phys. G37 (2010) 075019, Cong-Feng Qiao, Li-Ping Sun, Peng Sun

Inclusive double-quarkonium production at the Large Hadron Collider

JHEP 1101 (2011) 070, P. Ko, Chaehyun Yu, Jungil Lee

$p + p \rightarrow J/\psi + \Upsilon + X$ as a clean probe to the quarkonium production mechanism


$J/\psi$ Pair Production at the Tevatron with $\sqrt{s} = 1.96$ TeV

arXiv:1204.1700, Cong-Feng Qiao, Li-Ping Sun
The fragmentation function of $c \rightarrow J/\psi$ at QCD NLO

The limit without $\sqrt{s}$ dependence is seen.

$\sqrt{s}$ dependence is seen.

ArXiv:1102.0118, B. Gong, R. Li and J. X. Wang
Experimental and Leading-order Theoretical Results.[Acciarri:1998]

\[ Br(Z \to J/\psi_{prompt} + X) = (2.1^{+1.4}_{-1.2}) \times 10^{-4} \]

Dominant process: \( Z \to J/\psi + c\bar{c} + X \), and the total decay width is presented as

\[ \Gamma^{NLO}(\mu) = \Gamma^{LO}(\mu)[1 + \frac{\alpha_s(\mu)}{\pi}(A + \beta_0 \ln \frac{\mu}{2m_Q} + Bn_f)]. \]  

\[ Br^{total} = (7.3 \sim 10) \times 10^{-5} \]

The situation for $J/\psi$ production in $\Upsilon$ decay

### LO NRQCD Predictions:

- $Br(\Upsilon \rightarrow J/\psi(^3S_1^0) + gg) = 6.2 \times 10^{-4}$, M. Napsuciale, Phys. Rev. D 57, 5711 (1998)
- $Br(\Upsilon \rightarrow J/\psi + c\bar{c}g) = 5.9 \times 10^{-4}$, S. Y. Li, Q. B. Xie and Q. Wang, Phys. Lett. B 482, 65 (2000)
- $Br(\Upsilon \rightarrow J/\psi + gg) = order at \times 10^{-4}$, ???

### Experimental Data for $Br(\Upsilon \rightarrow J/\psi + X)$:

- CLEO $(11 \pm 4 \pm 2) \times 10^{-4}$ Phys. Lett. B 224, 445
- CLEO $(6.4 \pm 0.4 \pm 0.6) \times 10^{-4}$ Phys. Rev. D70, 072001(2004)

The situation is quite strange ???

The correct leading order prediction is

$$B_{Direct}(\Upsilon \rightarrow J/\psi + c\bar{c}g) = 3.9 \times 10^{-5}.$$  

Part of NLO prediction from $\Upsilon \rightarrow J/\psi + gg$ is

$$B_{Direct}(\Upsilon \rightarrow J/\psi + gg) = 3.1 \times 10^{-5}.$$  

The full QCD correction for the inclusive $J/\psi$ production in $\Upsilon$ decay would be a very interesting and challenge work for explaining the experimental data.
### The $\alpha_s$ correction

QCD corrections to $e^+e^- \rightarrow J/\psi(\psi(2S)) + \chi_{cJ}(J = 0, 1, 2)$ at B Factories

<table>
<thead>
<tr>
<th></th>
<th>Belle $\sigma \times B_{&gt;2(0)}$</th>
<th>BaBar $\sigma \times B_{&gt;2}$</th>
<th>LO result ($\mu = 2m_c$)</th>
<th>NLO result ($\mu = 2m_c$)</th>
<th>LO result ($\mu = \sqrt{s}/2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(J/\psi + \chi_{c0})$</td>
<td>$6.4 \pm 1.7 \pm 1.0$</td>
<td>$10.3 \pm 2.5^{+1.4}_{-1.8}$</td>
<td>$6.0 \pm 0.9$</td>
<td>$9.5 \pm 1.2$</td>
<td>$4.0 \pm 0.8$</td>
</tr>
<tr>
<td>$\sigma(J/\psi + \chi_{c1})$</td>
<td>-</td>
<td>-</td>
<td>$1.02^{+0.08}_{-0.10}$</td>
<td>$0.93^{+0.04}_{-0.07}$</td>
<td>$0.68^{+0.09}_{-0.10}$</td>
</tr>
<tr>
<td>$\sigma(J/\psi + \chi_{c2})$</td>
<td>-</td>
<td>-</td>
<td>$1.47^{+0.01}_{-0.05}$</td>
<td>$1.15^{+0.05}_{-0.08}$</td>
<td>$0.97^{+0.07}_{-0.08}$</td>
</tr>
<tr>
<td>$\sigma(J/\psi + \chi_{c1}) + \sigma(J/\psi + \chi_{c2})$</td>
<td>$&lt;5.3$ at 90% C.L.</td>
<td>-</td>
<td>$2.49^{+0.09}_{-0.16}$</td>
<td>$2.08^{+0.08}_{-0.14}$</td>
<td>$1.65^{+0.16}_{-0.19}$</td>
</tr>
<tr>
<td>$\sigma(\psi(2S) + \chi_{c0})$</td>
<td>$12.5 \pm 3.8 \pm 3.1$</td>
<td>-</td>
<td>$2.6 \pm 0.4$</td>
<td>$4.1 \pm 0.5$</td>
<td>$1.7^{+0.4}_{-0.3}$</td>
</tr>
<tr>
<td>$\sigma(\psi(2S) + \chi_{c1})$</td>
<td>-</td>
<td>-</td>
<td>$0.44^{+0.03}_{-0.05}$</td>
<td>$0.40^{+0.01}_{-0.03}$</td>
<td>$0.29 \pm 0.04$</td>
</tr>
<tr>
<td>$\sigma(\psi(2S) + \chi_{c2})$</td>
<td>-</td>
<td>-</td>
<td>$0.63^{+0.01}_{-0.02}$</td>
<td>$0.49^{+0.02}_{-0.03}$</td>
<td>$0.42^{+0.02}_{-0.04}$</td>
</tr>
<tr>
<td>$\sigma(\psi(2S) + \chi_{c1}) + \sigma(\psi(2S) + \chi_{c2})$</td>
<td>$&lt;8.6$ at 90% C.L.</td>
<td>-</td>
<td>$1.06^{+0.05}_{-0.06}$</td>
<td>$0.89^{+0.04}_{-0.06}$</td>
<td>$0.71^{+0.06}_{-0.08}$</td>
</tr>
</tbody>
</table>

Phys.Rev. D84 (2011) 034022, Kai Wang, Yan-Qing Ma, Kuang-Ta Chao

QCD corrections to $J/\psi(\psi(2S)) + \chi_{cJ}$ production at B Factories

JHEP 1110 (2011) 141, Hai-Rong Dong, Feng Feng, Yu Jia
The $\alpha_s v^2$ correction

$\alpha_s v^2$ correction to Hadronic and Electromagnetic Decays of $1S0$ Heavy Quarkonium

Phys. Rev. D83 (2011) 114038, Huai-Ke Guo, Yan-Qing Ma, Kuang-Ta Chao

The $\alpha_s v^2$ correction to pseudoscalar quarkonium decay to two photons

JHEP 1106 (2011) 097, Yu Jia, Xiu-Ting Yang, Wen-Long Sang
The $\alpha_s v^2$ correction

The $\alpha_s v^2$ correction to $J/\psi \rightarrow 3\gamma$. Very huge $\alpha_s v^2$ correction

$O(\alpha_s v^2)$ correction to $e^+ e^- \rightarrow J/\psi + \eta_c$ at $B$ factories

Table: Individual contributions to the predicted $\sigma[e^+ e^- \rightarrow J/\psi + \eta_c]$ at $\sqrt{s} = 10.58$ GeV, labeled by powers of $\alpha_s$ and $v$. The cross sections are in units of fb.

<table>
<thead>
<tr>
<th>$\alpha_s(\mu_R)$</th>
<th>$\sigma_0^{(0)}$</th>
<th>$\sigma_0^{(1)}$</th>
<th>$\sigma_2^{(0)}$</th>
<th>$\sigma_2^{(1)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_s(\sqrt{s}) = 0.211$</td>
<td>4.40</td>
<td>5.22</td>
<td>1.72</td>
<td>0.73</td>
</tr>
<tr>
<td>$\alpha_s(2m_c) = 0.267$</td>
<td>7.00</td>
<td>7.34</td>
<td>2.73</td>
<td>0.24</td>
</tr>
</tbody>
</table>


The two exclusive processes, $e^+ e^- \rightarrow \eta_c + \gamma$ and $e^+ e^- \rightarrow J/\psi + J/\psi$, at the center-of-momentum energy $\sqrt{s} = 10.58$ GeV within the framework of the nonrelativistic QCD factorization approach. A class of relativistic corrections is resummed to all orders in the heavy-quark velocity $v$ and the corrections are large negative.

arXiv:1211.4111, Ying Fan, Jungil Lee, Chaehyun Yu
For B-factories: NRQCD at NLO of $\alpha_s$ and $\nu$ can well described $J/\psi$ production data. strong constraint to the values of color-octect matrix element of $c\bar{c}(1S_0^8, 3P_j^8)$ to almost zero. The dominant part $c\bar{c}(3S_1^8)$ for hadronproduction is still there.

The polarization problem for prompt $J/\psi$ hadroproduction is still there even when the QCD NLO fitting and prediction are archived.

The more precision experimental measurements at LHC are needed to clarify the situation.

The experimental measurements at BESIII on $J/\psi$ polarization where $J/\psi$ feeddown from $\chi_{cJ}$ or $\psi(2s)$ will be very helpful to clarify the situation.

More theoretical Progresses are needed on relativistic coorection, .... to solve the polarization puzzle.
Thank you!