Lectures on extra dimensions

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Overview

Lecture #1

References

- Csáki TASI04 (hep-ph/0404096) and TASI(hep-ph/0510275)
- Sundrum TASI04 (hep-ph/0508134)
- Cheng TASI09 (arXiv:1003.1162)
- Gherghetta TASI09 (arXiv:1008.2570)
- Ponton TASI11(arXiv:1207.3827)
- Tait TASI13 (..)
- Park, A Review (arXiv:1203.4683)

More references

- Open KIAS school
  http://workshop.kias.re.kr/KWS2013/?Program
- PDG
  http://pdg.lbl.gov/

A fundamental question

Q. What do you know about spacetime?

What do we know about spacetime?

- Q1. How does it look like ?
- Q2. How did it begin?
- Q3. what's the fate? fate of universe =fate of ST?
- Q4. How many dimensions?
- ⋮
- Q137. What's the role of spacetime in collider physics?

The modern view of spacetime (1/3)

- In the SM, spacetime is assumed to be $D = 1 + 3$ with one temporal and three spatial dimensions
- Lorentz metric: $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu = dt^2 - d\vec{x}^2$
- Lorentz symmetric : $V^\mu \to V^{\mu'} = L^{\mu'}_\nu V^\nu$ with $\eta_{\mu'\nu'} = L^\mu_{\mu'} L^\nu_{\nu'} \eta_{\mu\nu}$
- Gravity neglected since it effect is suppressed as $\sim (G_N E^2) \sim \left(\frac{E}{M_P}\right)^2$

The modern view of spacetime (2/3)

- With gravity, in GR, $\eta_{\mu\nu} \to g_{\mu\nu}(x^\mu)$ thus non-trivial curvature $R \sim \partial^2 g - (\partial g)^2$ following the Einstein equations $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$
- Spacetime is dynamical!
- It can expand as we know the universe has been expanding for $13.7b$ years. The universe expanded even exponentially during inflation
- It can shrink as a big energy density makes the spacetime even singular in a black hole

The modern view of spacetime (3/3)

- In pure gravity, D is free parameter. In string/M theory, $D = 10$ or 11.
- It could happen (at least theoretically possible) that only our $1+3$ dimensions remains large (or expanded a lot) but other dimensions stay small .. There could be extra dimensions!
- From the beginning of 21st century, many models with extra dimensions have been suggested to address phonomenological problems of particle physics: hierarchy problem (m_h^2), flavor structure, Dark matter, grand unification..
- Model building to avoid difficulties : large flavor violation ($K^0 - \overline{K^0}, \mu \to e\gamma$), proton decay ($\not{B}$), precision electroweak measurements, compactification..

(Ex-1) A real scalar field in curved spacetime

- $S = \int d^4x \frac{1}{2} \left(\eta^{ab}\partial_a\phi\partial_b\phi - m^2\phi^2\right)$
- When curved, $S = \int d^4x \sqrt{-g} \frac{1}{2} \left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right)$ with $g = det(g^{\mu\nu})$
- $S_D = \int d^Dx \sqrt{-g} \frac{1}{2} \left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right)$, $[\phi] = M^{-1+D/2}$

(Note) $d^4x\sqrt{-g}$ is covariant volume element.

(Ex-2) A Dirac field in curved spacetime

- $S = \int d^4x\ \bar{\psi}(i\gamma^a\partial_a - m)\psi$, $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$
- When curved, with vierbeins, $\gamma^\mu(x) = E^\mu_a \gamma^a$ where a is index for flat-coordinate then $\{\gamma^\mu, \gamma^\nu\} = 2E^\mu_a E^\nu_b \eta^{ab} = 2g^{\mu\nu}$
- $S = \int d^4x E \bar{\psi}(i E^\mu_a \gamma^a \partial_\mu - m)\psi$ since $\sqrt{-g} = \sqrt{det(E^\mu_a E^\nu_b \eta^{ab})} = E$
- $S_D = \int d^Dx E \bar{\psi}(i E^\mu_a \gamma^a \partial_\mu - m)\psi$, $[\psi] = M^{(D-1)/2}$
- $[Fermion] = M^{(D-1)/2}$ Q.for $s = 3/2$?

(Ex-3) Maxwell field in curved spacetime

- $S = \int d^4x \; -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- $S = \int d^4x \sqrt{g} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- $S_D = \int d^Dx \sqrt{g} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, $[A_\mu] = M^{(D-2)/2}$
- $[Boson] = M^{(D-2)/2}$ why?

How does XD look like?

A quantum particle in $D = 2$ box

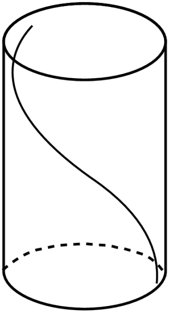
- Find the energy spectrum for a massive particle in 2D potential box $: V(x,y) = \begin{cases} 0 & \text{if } 0 \leq x \leq L_1, 0 \leq y \leq L_2 \\ \infty & \text{otherwise} \end{cases}$
- Inside the box: $-\frac{\hbar^2}{2m}(\partial_x^2 + \partial_y^2)\psi(x,y) = E\psi(x,y)$
- The solution is $\psi(x,y) \sim \sin(\frac{n_1 \pi x}{L_1})\sin(\frac{n_2 \pi y}{L_2})$ satisfying Dirichlet B.C.s $\psi(0,y) = 0 = \psi(L_1,y)$ and $\psi(x,0) = 0 = \psi(x,L_2)$.
- $E_{n_1,n_2} = \frac{\hbar^2\pi^2}{2m}(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2})$
- If $L_1 \gg L_2$, there are many excitations satisfying $E_{n,1} < E_{1,2}$
- In a sense, the presence of y-direction is hardly seen below $E_{1,2}$ thus effectively we find $D = 1$ theory with $E_n \equiv E_{n,1}$ well below a cutoff scale $\Lambda \sim E_{1,2}$.

Quiz: A particle on the surface of thin cylinder

1. Find the energy spectrum of a particle on a infinite cylinder with a radius r by solving the Schrödinger equation:

$$-\frac{\hbar^2}{2m}(\partial_z^2 + \frac{\partial^2}{r^2\partial\phi^2})\psi(z,\phi) = E\psi(z,\phi)$$

2. If $r \ll 1/E$, what's the effective description of the theory?

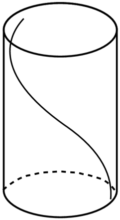


Sol: A particle on the surface of thin cylinder

1. Ansatz: $\psi(z,\phi) \sim e^{ik_z z} e^{ik_\phi r\phi}$ gives

$$-\frac{\hbar^2}{2m}(\partial_z^2 + \frac{\partial^2}{r^2\partial\phi^2})\psi(z,\phi) = \frac{\hbar^2}{2m}(k_z^2 + k_\phi^2)\psi$$

$$\Rightarrow E = \frac{\hbar^2}{2m}(k_z^2 + k_\phi^2) \quad (2)$$



2. Periodic B.C.
$\psi(\phi) = \psi(\phi + 2\pi) \Rightarrow k_\phi = \frac{n\pi}{r}$ with
$n = 0, \pm 1, \pm 2, \cdots$.
3. k_z is continuous Q. Why?
4. If $1/r \gg E$, $E \approx \frac{\hbar^2 k_z^2}{2m}$ and $\psi \sim e^{ik_z z}$.
Excitation along ϕ calls for extremely high energy so that at low energy, irrelevant.

A toy model: A complex scalar in 5D (1/5)

- $\Phi(x^\mu) \to \Phi(x^\mu, x^5)$
  - $x^\mu = (t, x^1, x^2, x^3)$: "ordinary dimensions"
  - $x^5 \equiv y$: extra coordinate.
  - "Φ can go into the extra dimensions", "Φ can propagate through the bulk" (it is really not shocking that an extra dimension can capture other theories with internal dynamics such as compositeness!)
- Assume that y is a circle compactified. How?
  - $0 \leq y \leq 2\pi R$ and $\Phi(x^\mu, y) = \Phi(x^\mu, y + 2\pi R)$.
- Gravity neglected as $E \ll M_p$.

A toy model: A complex scalar in 5D(2/5)

$Z[J] = \int \mathcal{D}\Phi\mathcal{D}\Phi^* Exp(iS_5)$

- $S_5 = \int d^5x\mathcal{L}_5$
- $\mathcal{L}_5 = \partial_M\Phi^*\partial^M\Phi + \cdots + J\cdot\Phi$
- $\int d^5x = \int d^4x \int dy$
- $M = 0, 1, 2, 3, 5$
- $\eta^{MN} = \mathrm{diag}(1, -1, -1, -1, -1)$
- $\Phi(x^\mu, y) = \sum_n \phi_n(x^\mu) f_n(y)$
  - f_n: wave functions
  - $\phi_n(x^\mu)$: Kaluza-Klein (KK) modes

A toy model: A complex scalar in 5D(3/5)

$$
\begin{aligned}
S_5 &= \int d^5x \partial_M \Phi^* \partial^M \Phi \\
&= \int d^4x \int_0^{2\pi R} dy \left[\partial_\mu \Phi^* \partial^\mu \Phi - \partial_5 \Phi^* \partial_5 \Phi\right] \\
&= \sum_n \sum_\ell \int d^4x dy \left[\partial_\mu \phi_n \partial^\mu \phi_\ell f_n^*(y) f_\ell(y) - \phi_n^* \phi_\ell f^{*\prime}_n f_\ell^\prime\right]
\end{aligned}
$$

- $\delta_{n\ell} = \int dy f_n^* f_\ell$: orthonormal choice of wave functions
- $\int dy (\partial_y f_n^*)(\partial_y f_\ell) = m_n^2 \delta_{n\ell}$: diagonal mass matrix
- $S_5 = \int d^4x \sum_n \left(\partial_\mu \phi_n^* \partial^\mu \phi_n - m_n^2 \phi_n^* \phi_n\right) = \int dy \sum_n \mathcal{L}_4^{(n)}$
- $\mathcal{D}\Phi^* \mathcal{D}\Phi = \Pi_n \mathcal{D}\phi_n^* \mathcal{D}\phi_n$.

A toy model: A complex scalar in 5D(4/5)

$$\int dy(\partial_y f_n^*)(\partial_y f_\ell) = m_n^2 \int dy f_n^* f_\ell$$

- $L.H.S. = \int dy \partial_y (f_n^* \partial_y f_n) - f_n^* \partial_y^2 f_n = - \int dy f_n^* \partial_y^2 f_n$
- $R.H.S. - L.H.S. = 0 = \int dy f_n (\partial_y^2 + m_n^2) f_n$
  or $(\partial_y^2 + m_n^2) f_n = 0$.
- Sol: $f_n(y) = N_n e^{i m_n y}$ Q. Show $N_n = 1/\sqrt{2\pi R}$
- $f_n(y) = f_n(y + 2\pi R)$ implies $m_n = \frac{n}{R}$ with $n = 0, \pm 1, \pm 2, \ldots$

A toy model: A different approach(5/5)

Euler-Lagrange equation reads:

$$\partial_M \frac{\partial \mathcal{L}}{\partial \partial_M \Phi} = \partial \mathcal{L} / \partial \Phi = 0$$

- $\partial_M^2 \Phi = \partial_\mu^2 \Phi - \partial_5^2 \Phi = 0$
- $\sum_n [(\partial_\mu^2 \phi_n) f_n(y) - \phi_n(x^\mu) \partial_5^2 f_n(y)] = 0$
- Use $\partial_\mu^2 \phi_n = -m_n^2 \phi_n$ then $\sum_n (\partial_5^2 + m_n^2) f_n = 0$ which coincide the previous result
- One can arrive the same results by $P_M P^M = p_\mu^2 - p_5^2 = 0$ for massless scalar in 5D. As $p_5 = n/R$ (quantized), $p_\mu^2 = m_n^2 = p_5^2 = n^2/R^2$ or $m_n = n/R$ with $n = 0, \pm 1, \pm 2, ...$ (left-mover and right-mover)
- Below $1/R$, only zero mode is relevant..we are made of zero mode particles!

Quiz-2: A massive complex scalar field in 5D

1. Find the Kaluza-Klein spectrum of the action

$$S_5 = \int d^5x \left[\partial_M \Phi^* \partial^M \Phi - M^2 \Phi^* \Phi\right]$$

2. Add quartic coupling term $\Delta\mathcal{L} = -\frac{\lambda_5}{4}(\Phi^*\Phi)^2$ then answer the following questions:
   - Find the mass dimensions of Φ, $\mathcal{L}_5$, λ_5 and M
   - Show that the Feynman rules for the quartic couplings among KK-states are given by $\frac{1}{4}\lambda_{nmpq} = \frac{1}{4}\frac{\lambda_5}{2\pi R}\delta_{n+m,p+q}$

Sol-2: A massive complex scalar field in 5D

1. Sol: By varying $\delta\Phi^*$, one gets

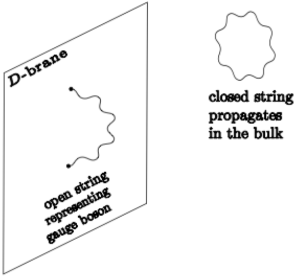
$$\partial_M^2\Phi = -M^2\Phi$$
$$\partial_\mu^2\Phi - \partial_5^2\Phi + M^2\Phi = 0$$
$$\Rightarrow \sum_n \phi_n \left[-m_n^2 - \partial_5^2 + M^2\right] f_n = 0$$
$$\Rightarrow m_n^2 = M^2 + n^2/R^2, \quad n = 0, \pm 1, \pm 2, \ldots$$

2.
   - $[\Phi] = M^{3/2}, [\mathcal{L}_5] = M^5, [\lambda_5] = M^{-1}, [M] = M^1$
   - $\mathcal{L} \to \sum_{nmpq} -\frac{\lambda_5}{4}\phi_n^*\phi_m\phi_p^*\phi_q \int dy f_n^* f_m f_p^* f_q$

     Put $f_n(y) = \frac{1}{\sqrt{2\pi R}} e^{iny/R}$ and use $\delta_{nm} = \int_0^{2\pi R} dy \frac{1}{2\pi R} e^{i(n-m)y/R}$.

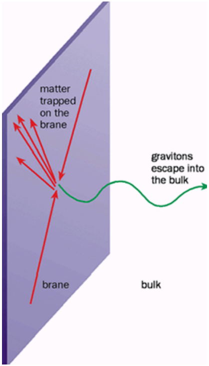
Large Extra Dimensions: ADD model (1/6)

- The first potentially realistic model of extra dimension by Arkani-Hamed, Dimopoulos and Dvali (ADD in short)
- The entire SM sector is confined to a 3-brane, a hyper surface in higher dimensional volume Q. How to confine?
- Only graviton can go into extra dimensions Q. Why not graviton?



Large Extra Dimensions: ADD model (2/6)

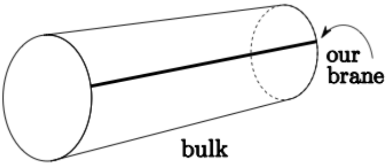
- Pure gravity theory of Einstein in higher dimensions $D = 4 + n$
- $S_4 = \int d^4x \sqrt{g} \frac{M_4^2}{2} R$
  $\Rightarrow S_D = -\int d^Dx \sqrt{g} \frac{M_D^{2+n}}{2} R$
- If the extra dimension is flat (i.e. n-torus) Q. if not?,
  $S_D = -\int d^4 \sqrt{g_4} \int dy \frac{M_D^{2+n}}{2} R + \cdots$
  $\Rightarrow M_4^2 = M_D^{2+n} V_n$ where the volume of n-torus is $V_n = (2\pi R)^n$
- $M_4^2 = M_D^2 (2\pi R M_D)^n$ or the hierarchy M_4/M_D is given by the volume in unit of M_D.



Large Extra Dimensions: ADD model (3/6)

- ADD provides a cute way of understanding why gravity is so weak!
- Assuming that $M_D \sim m_W$, there is no hierarchy between the gravity scale and the week scale
- ADD= a low scale strong gravity theory = TeV gravity theory = low scale string model (?)...
- When $r < R$, gravitational potential looks $V(r) = \frac{G_4 m}{r} \to \frac{G_D m}{r^{1+n}}$
- The LHC may be able to see gravitational interactions $\sim 1/M_D$ instead of $\sim 1/M_P$

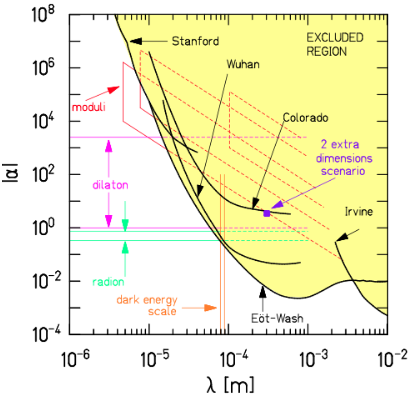
$$S = -\int d^D x \frac{M_D^{2+n}}{2} \sqrt{g} R + \delta(y - y_0)\mathcal{L}_{SM}$$



Large Extra Dimensions: ADD model (4/6)

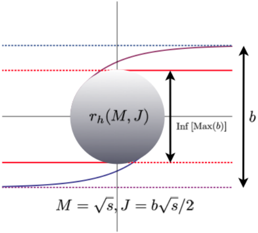
- How to test this model?
- KK-mass spacing $\Delta m \sim 1/R$ which is given by $M_4^2/M_D^2 = (2\pi R M_D)^n$ or $(10^{19}\mathrm{GeV}/10^3\mathrm{GeV})^{2/n} = 2\pi R_{TeV}$ or $1/R_{TeV} \sim 2\pi \times 10^{-32/n}$
- $n = 1, 2, \cdots, 6 \to R \sim 9\mathrm{km}, 0.5\mathrm{mm}, \cdots 0.1\mathrm{MeV}^{-1}$ Note: $\hbar c \approx 200\mathrm{MeV}\cdot\mathrm{fm}$
- A joke(?) by J. Hewett, if $n \sim 32$, $1/R \sim m_W \sim M_D$ ... no hierarchy at all! [Phys.Rev.Lett. 95 (2005) 261603]

$$V(r) = -\frac{G_N m_1 m_2}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$



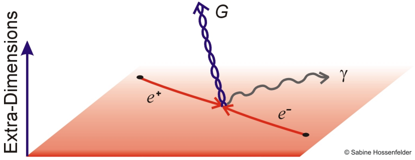
Large Extra Dimensions: ADD model (5/6)

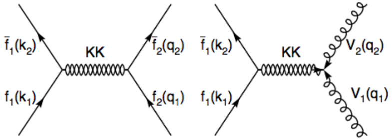
- If the LHC energy jumps to energies past M_D, we can produce microscopic black holes. Park[arXiv:1203.4683]
- If two partons are close enough ($b < b_{max} = f(D) r_s(E)$) , they feel strong gravity and eventually forms a black hole (Hoop conjecture)
- $\sigma \simeq \pi r_s(E)^2$
- Once produced, bh decays through Hawking process nearly thermal with large multiplicity
- BlackMax, Charybdis used and a new MC is under development



Large Extra Dimensions: ADD model (6/6)

- $\gamma + G_{KK}$, $jet + G_{KK}$ is interpreted as photon(j)+MET ($M_D \geq 2.7\,TeV$)
- $q\bar{q} \to G_{KK} \to \mu^+\mu^-$..
  ($M_D \geq 4\,TeV$)





Today's summary

Extra dimensions

$d^4x \to d^{4+n}x$, $\Phi(x^\mu) \to \Phi(x^\mu, y)$

KK-picture

$\Phi(x,y) = \sum_n \phi_n(x) f_n(y)$:A higher dimensional field = many $4D$ fields

Large Extra Dimension

$M_4^2 = M_D^{2+n} V_n$, strong gravity at $M_D \sim \mathrm{TeV}$, rich-phenomenology (KK-graviton, mini blackhole)

Lecture #2

Quiz

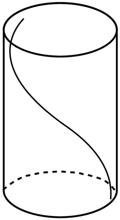
1. Why a small dimension invisible at low energy?
2. How does the presence of large extra dimension explain the weakness of gravity?

A particle on the surface of thin cylinder

1. Ansatz: $\psi(z,\phi) \sim e^{ik_z z} e^{ik_\phi r\phi}$ gives

$$-\frac{\hbar^2}{2m}(\partial_z^2 + \frac{\partial^2}{r^2\partial\phi^2})\psi(z,\phi) = \frac{\hbar^2}{2m}(k_z^2 + k_\phi^2)\psi$$

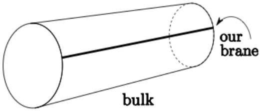
$$\Rightarrow E = \frac{\hbar^2}{2m}(k_z^2 + k_\phi^2) \quad (3)$$



2. Periodic B.C.
   $\psi(\phi) = \psi(\phi + 2\pi) \Rightarrow k_\phi = \frac{n\pi}{r}$ with
   $n = 0, \pm 1, \pm 2, \cdots$.
3. k_z is continuous Q. Why?
4. If $1/r \gg E$, $E \approx \frac{\hbar^2 k_z^2}{2m}$ and $\psi \sim e^{ik_z z}$.
   Excitation along ϕ calls for extremely high energy so that at low energy, irrelevant.

Why gravity looks weak?

$$S = -\int d^D x \frac{M_D^{2+n}}{2} \sqrt{g} R + \delta(y - y_0)\mathcal{L}_{SM}$$



1. Only gravity can see XD! (thus XD can be large mm to MeV^{-1})
   $M_4^2 = M_D^{2+n} R^n \Rightarrow R_{\mathrm{TeV}} \sim (M_4/\mathrm{TeV})^{2/n}$
2. Phenomena of TeV gravity (KK graviton, blackhole) are expected (thus get tested at the LHC)

Q. What's the mass of KK gravitons? Where are they?

Starter of Lecture #2: Questions

- ADD looks cute ..
- ..but is not general enough
- What happens if others also can go into XD?
- Flavor? Neutrino mass? Dark matter?
- Any other problem?
- New models (UED, RS..)!

Starter of Lecture #2: Models

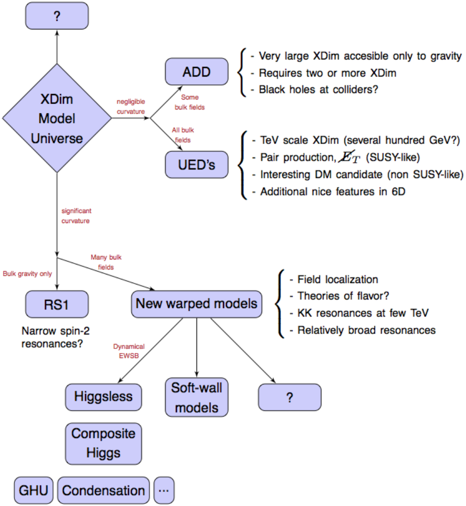


Figure : Ponton's TASI lecture

RS (1) Warp factor

- The most general higher dimensional geometry containing 4D Lorentz symmetry: $ds^2 = A(y)\eta_{\mu\nu}dx^\mu dx^\nu - B(y)dy^2$
- $dz^2 = B(y)dy^2$ leads $ds^2 = e^{-2C(z)}\eta_{\mu\nu}dx^\mu dx^\nu - dz^2$
  - $e^{-C(z)} = W(z)$ is called "warp factor"
  - Flat: $C(z) = 0$ without CC for ADD
  - RS(=warped XD): $C(z) = kz$ a slice of AdS_5 with CC

  Randall+Sundrum, [Phys. Rev. Lett. 83, 3370 (1999)], [Phys. Rev. Lett. 83, 4690 (1999)]

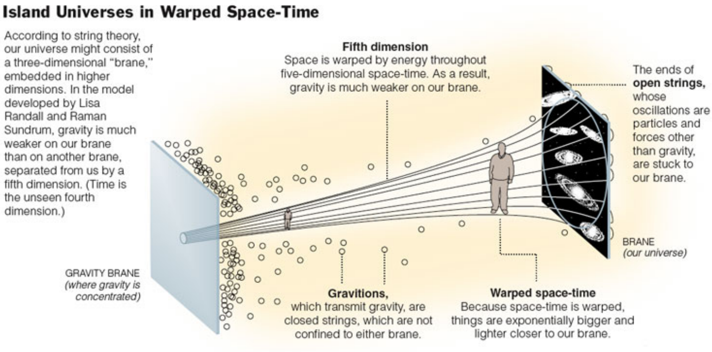
RS (2) Hierarchy

- Due to the "warp factor" $W(z) = e^{-kz}$, the standard measure of energy varies with respect to z as $M(z) = M_0 e^{-kz}$
- $S_{\text{Higgs}} = \int d^5x \sqrt{g}\delta(z - z_0)\left[|D_\mu H|^2 - \lambda(|H|^2 - v_0^2)^2\right]$
  - $\sqrt{g} = \sqrt{Det(g_{MN})} = e^{-4kz}\sqrt{g_4}$
  - $g^{\mu\nu}(z)(D_\mu H)^\dagger D_\nu H = e^{2kz}\eta^{\mu\nu}(D_\mu H)^\dagger D_\nu H$
  - $H \to \hat{H} = e^{kz}H$ is canonically normalized
  - $V(H) = e^{-4kz}\lambda\left(|H|^2 - v_0^2\right)^2 = \lambda\left(|\hat{H}|^2 - (v_0 e^{-kz})^2\right)^2$
  - The physical VEV is red-shifted $\hat{v} = e^{-kz}v_0 \ll v_0$.
  - As a result, we can understand why the scale of Higgs is much smaller than fundamental scale $M_5 \sim v_0$. (The hierarchy problem solved)

RS (3) Gravity

- $S_G = \int d^5x \sqrt{g} \quad \frac{M_5^3}{2} R$
  - $R = e^{2kz} \eta^{\mu\nu} R^{(4)}_{\mu\nu} = e^{2kz} R^{(4)}$
  - $\sqrt{g} = e^{-4kz} \sqrt{g_4}$
  - $M_4^2 = M_5^3 \int_{-L}^{L} dz \; e^{-2kz} = \frac{M_5^3}{k}(1 - e^{-2kL}) \sim \frac{M_5^3}{k}$
  - $M_4 \sim M_5 \sim k$ but still solves hierarchy problem!

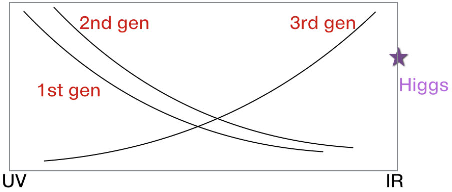
RS (4) Pictorial



Q. Where do we live in extra dimensions?

RS (5) Flavor

- The location of the Higgs is at IR thus $\hat{v} = e^{-kL} v \sim \mathrm{TeV}$
- If a fermion's wave function is localized toward the IR, it can efficiently interact with the Higgs..leading a large Yukawa coupling
- It is *attempting* to consider the RS as a theory of flavor physics!
- See e.g. Randall [0710.1869], Neubert et al [0807.4937], [0912.1625]



$$y_{ij}^{\mathrm{eff}} = y_5 \int dy f_i^{(0)}(z) f_j^{(0)}(z) \delta(z - z_0)$$

RS (6) Fermions in 5D

$$S_\Psi = \int d^5x \sqrt{g}\left(i\overline{\Psi}E_a^M\Gamma^a\overleftrightarrow{\partial_M}\Psi - M\overline{\Psi}\Psi\right)$$

- $a\overleftrightarrow{\partial}b = \frac{1}{2}(a\partial b - (\partial a)b)$
- $\Gamma^M = (\gamma^\mu, i\gamma^5)$ satisfying $\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}$ (cf) $(\gamma^\mu, -i\gamma^5)$
- $E_a^\mu = e^{C(z)}\delta_a^\mu$, $E_5^\mu = 1$ and $\sqrt{g} = e^{-4C}$
  (Check! $E_a^M E_b^N \eta^{ab} = g^{MN}$)
- $S_\Psi = \int d^5x\ e^{-4C}\overline{\Psi}(ie^C\gamma^\mu\partial_\mu - \gamma^5(\partial_5 - \frac{1}{2}C') - m)\Psi$
  $\delta\overline{\Psi} \Rightarrow (ie^C\gamma^\mu\partial_\mu - \gamma^5(\partial_5 - \frac{1}{2}C') - m)\Psi = 0$
- BC: $\delta S|_{bdy} = \delta\overline{\Psi_L}\Psi_R - \delta\overline{\Psi_R}\Psi_L|_{z=0}^{z=L} = 0$
  $\Rightarrow \Psi_L| = 0$ or $\Psi_R| = 0$.
- Four choices of BC's $(+,+), (+,-), (-,+), (-,-)$

RS (7) KK decomposition Fermions in 5D

$$\Psi_{L/R}(x^\mu, y) = \sum_n \frac{e^{3C(z)/2}}{\sqrt{L}} \psi^{(n)}_{L/R}(x^\mu) f^{(n)}_{L/R}(y)$$

- $\left(\partial_z \pm M - \frac{1}{2}C'\right) f^{(n)}_{L/R} = \pm m_n e^C f^{(n)}_{R/L}$
- $\frac{1}{L}\int_0^L dz\ f^{(n)}_{L/R} f^{(m)}_{L/R} = \delta_{nm}$
- $M(y) \approx \pm c C'(z) \Rightarrow f^{(0)}_{L/R} = N_0 e^{-(c-1/2)C(z)}$
  .. L/R for $(+,+)$ and $(-,-)$.
- KK modes are Bessel functions for $C = kz$ ..localized toward IR
- Similarly for gauge bosons

RS (7) KK states

- Differently from the zero modes, KK excitations are all localized toward the IR
- Naturally, KK mass gap $\delta m \sim ke^{-kL} \sim \mathrm{TeV}$ thus subject to get tested by the LHC
- RS-GIM mechanism

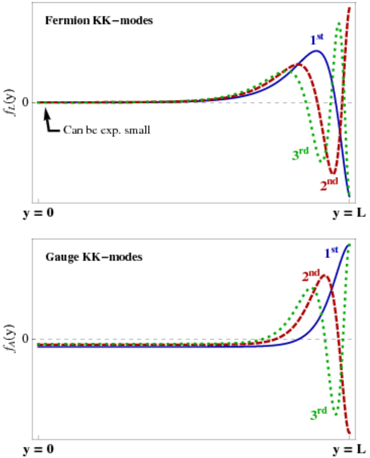


Figure : Ponton's TASI lecture note

RS (8) LHC phenomenology

- $W_{KK}, Z_{KK}, g_{KK}, G_{KK}$ in resonances all in TeV range
- Only third generation fermions (and Higgs) would couple rather strongly to these KK gauge bosons and KK graviton
- Gauge boson's have flat profile thus less suppressed couplings with KK bosons

$$
\begin{aligned}
\frac{g_{\rm RS}^{q\bar{q},l\bar{l}A^{(1)}}}{g_{\rm SM}} &\simeq -\xi^{-1} \approx -\frac{1}{5} \\
\frac{g_{\rm RS}^{Q^3\bar{Q}^3A^{(1)}}}{g_{\rm SM}}, \frac{g_{\rm RS}^{t_R\bar{t}_RA^{(1)}}}{g_{\rm SM}} &\simeq 1 \text{ to } \xi \ (\approx 5) \\
\frac{g_{\rm RS}^{HHA^{(1)}}}{g_{\rm SM}} &\simeq \xi \approx 5 \quad (H = h, W_L, Z_L) \\
\frac{g_{\rm RS}^{A^{(0)}A^{(0)}A^{(1)}}}{g_{\rm SM}} &\sim 0 \\
g_{\rm RS}^{q\bar{q},l\bar{l}G^{(1)}} &\sim \frac{E}{\bar{M}_P e^{-k\pi r_c}} \times 4D \text{ Yukawa} \\
g_{\rm RS}^{A^{(0)}A^{(0)}G^{(1)}} &\sim \frac{1}{k\pi r_c} \frac{E^2}{\bar{M}_P e^{-k\pi r_c}} \\
g_{\rm RS}^{Q^3\bar{Q}^3A^{(1)}}, g_{\rm RS}^{t_R\bar{t}_RG^{(1)}} &\sim \left(\frac{1}{k\pi r_c} \text{ to } 1\right) \frac{E}{\bar{M}_P e^{-k\pi r_c}} \\
g_{\rm RS}^{HHG^{(1)}} &\sim \frac{E^2}{\bar{M}_P e^{-k\pi r_c}}
\end{aligned}
$$

Snowmass 2013 benchmark [1309.7847]

RS (9) LHC phenomenology-2

KK particle	total σ (fb)	(SM) final states	references
graviton	~ 0.1 for $k \sim M_{\rm Pl}$	$t\bar{t}$, $b\bar{b}$, WW, hh, ZZ	hep-ph/0701186
gluon	~ 100	$t\bar{t}$, $b\bar{b}$	0706.3960
Z	a few	$t\bar{t}$, $b\bar{b}$, Zh, WW	0709.0007
W	10	WZ, Wh, $t\bar{b}$	0810.1497

Snowmass 2013 benchmark [1309.7847]

Universal Extra Dimensions (1)

$$\text{UED} = \text{Effective theory of RS}^{KK-parity}$$

Universal Extra Dimensions(2)

$$\text{UED} = \text{Effective theory of RS}^{KK-parity}$$

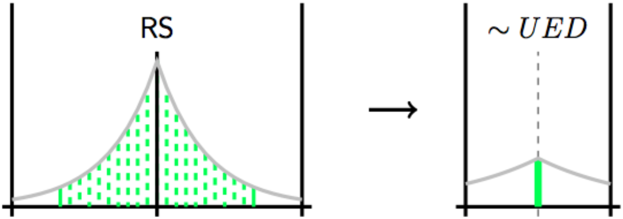


Figure : Csaki, Heinonen, Hubisz, SCP, Shu [JHEP 1101 (2011) 089]

Universal Extra Dimensions(3)

- Nearly flat geometry
- KK-parity = Inversion about the middle point of XD
- $n = even$ modes are KK-even. The SM particles are KK-even (why?)
- $n = odd$ modes are KK-odd. The 1st excited KK modes are odd
- An KK-odd particle cannot decay into KK-even particles
- The lightest KK-odd particle (LKP) is stable
- A perfect DM candidate if neutral!

  Servant, Tait [Nucl.Phys. B650 (2003) 391-419]

  Kong, Matchev [JHEP 0601 (2006) 038]

Universal Extra Dimensions(4)

In minimal realization of UED

- $\mathcal{M}^5 = \mathcal{R}^{3,1} \times S^1/\mathcal{Z}_2$
  - $\{Q, u, d, L, e\} \Rightarrow \Psi(x^\mu, y)$
  - $\{g, W, Z\} \Rightarrow V^M(x^\mu, y)$
  - all bulk $=0$
  - $f_n(y) \sim$ sin or cos
- γ_1 turned out to be LKP

Appelquist, Cheng, Dobrescu [Phys.Rev. D64 (2001) 035002]

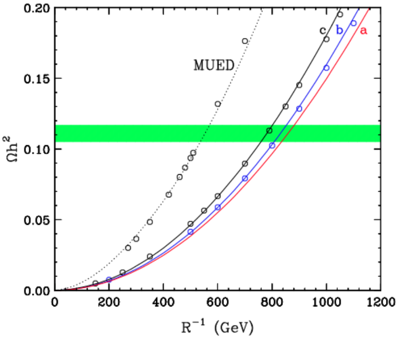


Figure : Backovíc, Kong, McCaskey [1308.4955] MadDM1.0

Universal Extra Dimensions(5)

In non-minimal UED

- $\mathcal{M}^5 = \mathcal{R}^{3,1} \times S^1/\mathcal{Z}_2$
  - $\{Q, u, d, L, e\} \Rightarrow \Psi(x^\mu, y)$
  - $\{g, W, Z\} \Rightarrow V^M(x^\mu, y)$
  - with bulk masses and boundary localized terms

    SCP,Shu '09, Kong, SCP, Rizzo '10, Csaki, Heinonen, Hubisz, SCP, Shu '11, Flacke, Kong, SCP 13'
  - $f_n(y) \sim \sin'$ or $\cos'$
- γ_1, Z_1, ν_1 can be LKP .. rich phenomenology

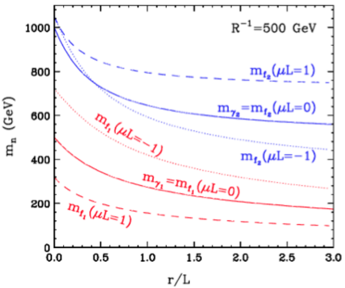


Figure : Flacke, Kong, SCP [JHEP 1305 (2013) 111]

Universal Extra Dimensions(6)

Collider phenomenology of UED

- Single KK-odd production is forbidden due to KK-parity
- Resonance e.g. $pp \to \gamma_2, Z_2 \to \ell^+\ell^-$
- W' search $pp \to W_2 \to \ell\nu_\ell$
- $pp \to f_1\bar{f}_1, V_1V_1$ then cascade decay down to $LKP + SMs$

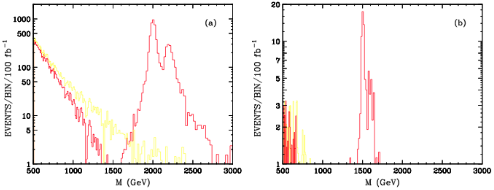


Figure 6: *Invariant mass distributions at the LHC for (a) $R^{-1} = 1$ TeV, $\sqrt{s} = 14$ TeV and $\mathcal{L} = 100$ fb^{-1} and (b) $R^{-1} = 0.75$ TeV, $\sqrt{s} = 10$ TeV and $\mathcal{L} = 1$ fb^{-1}. The yellow histogram is the SM background while the red histogram includes both signal and backgrounds.*

Figure : Kong, SCP, Rizzo [JHEP 1004 (2010) 081]

AdS/CFT(1) Introduction

- So far we have seen XD models in terms of KK decomposition
- ..which is good, as KK-states are mass eigenstates which are subject to get measured at the LHC
- ...however, which is not the only way to study XD
- According to AdS/CFT, XD can be understood by a dual 4D theory
- ..and also new intuition
- ..provides new computing methods

AdS/CFT(2)

AdS/CFT dictionary

memorize!

AdS 5D	CFT 4D
Type IIB String on $\mathrm{AdS}_5 \times S^5$	4D $\mathcal{N} = 4$ SU(N) super YM
UV localization	mostly elementary
IR localization	mostly composite
flat profile	partially composite and elementary
5D gauge symmetry	(weakly gauged) Global symmetry

Table : see e.g. T. Gherghetta TASI '10

AdS/CFT (3)

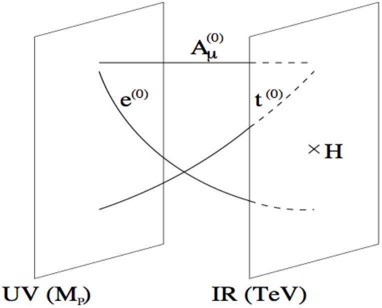


Figure : T. Gherghetta's TASI lecture '10

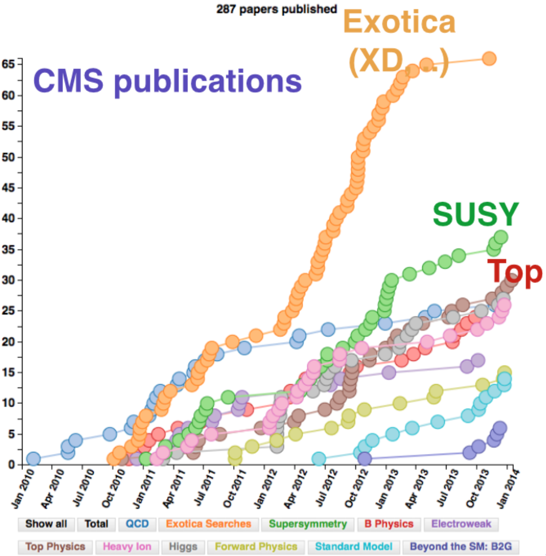
1. Composite : 3rd gen fermions, Higgs, KK-states
2. Elementary : 1,2nd gen fermions
3. Mixed : gauge boson

AdS/CFT (4)

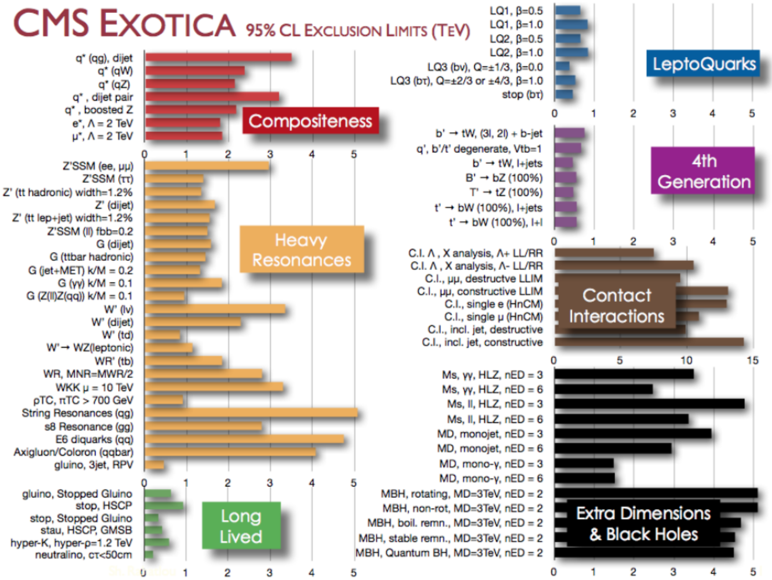
Holographic interpretation of 5D physics

1. Higgs is a CFT composite state. Its mass is generated at CFT breaking scale $\sim ke^{-kL} \sim \mathrm{TeV}$
2. Gauge bosons are admixture of elementary and composite states. Gets corrections from strong interaction at IR
3. Top is mostly composite and gets large mass from the strong dynamics
4. KK modes are 'excitations' of strong dynamics
5. Custodial symmetry can be realized by imposing $[SU(2)_L \times SU(2)_R \times U(1)_X]_{\mathrm{local}}$ in the bulk $\Rightarrow [SU(2)_L \times SU(2)_R \times U(1)_X]_{\mathrm{global}} \supset [SU(2)_L \times U(1)_Y]_{\mathrm{local}}$ by BC

Conclusion (1)



Conclusion(2)



Conclusion(3)

- XD opens new era of model building
- á la theoretically deep and phenomenologically interesting
- Within relatively short time ($\sim$ 10+years), XD have provided a powerful framework to address long-lasting hard problems in particle physics
- Several realistic models are already under the LHC test especially the models with TeV scale KK-particles for hierarchy problem
- Flavor problems (why top is heavy!) may be best understood by XD so far ...
- ...but we should not think they are the last models!