Project on MSSM

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- Higgs potential: Including the radiative corrections in the MSSM or in the general 2HDM, the Higgs potential is given by

$$
\begin{aligned}
\mathrm{V} \;=\; & -\mu_1^2(\Phi_1^\dagger\Phi_1) - \mu_2^2(\Phi_2^\dagger\Phi_2) - m_{12}^2(\Phi_1^\dagger\Phi_2) - m_{12}^{*2}(\Phi_2^\dagger\Phi_1) \\
& +\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\
& +\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \frac{\lambda_5^*}{2}(\Phi_2^\dagger\Phi_1)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1) \\
& +\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\,. \qquad (1)
\end{aligned}
$$

- Tadpole conditions: With the parameterization

$$
\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}\,(v_1 + \phi_1^0 + ia_1) \end{pmatrix}\,; \quad \Phi_2 = \mathrm{e}^{i\xi} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}\,(v_2 + \phi_2^0 + ia_2) \end{pmatrix} \qquad (2)
$$

and denoting $v_1 = v\cos\beta = vc_\beta$ and $v_2 = v\sin\beta = vs_\beta$, derive the following three tadpole conditions:

$$
\begin{aligned}
\mu_1^2 &= v^2\left[\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_3 s_\beta^2 + c_\beta s_\beta \Re\mathrm{e}(\lambda_6 \mathrm{e}^{i\xi})\right] - s_\beta^2 M_{H^\pm}^2 \qquad (3) \\
\mu_2^2 &= v^2\left[\lambda_2 s_\beta^2 + \frac{1}{2}\lambda_3 c_\beta^2 + c_\beta s_\beta \Re\mathrm{e}(\lambda_7 \mathrm{e}^{i\xi})\right] - c_\beta^2 M_{H^\pm}^2 \\
\Im\mathrm{m}(m_{12}^2\mathrm{e}^{i\xi}) &= \frac{v^2}{2}\left[c_\beta s_\beta \Im\mathrm{m}(\lambda_5 \mathrm{e}^{2i\xi}) + c_\beta^2 \Im\mathrm{m}(\lambda_6 \mathrm{e}^{i\xi}) + s_\beta^2 \Im\mathrm{m}(\lambda_7 \mathrm{e}^{i\xi})\right]
\end{aligned}
$$

with

$$
M_{H^\pm}^2 = \frac{\Re\mathrm{e}(m_{12}^2\mathrm{e}^{i\xi})}{c_\beta s_\beta} - \frac{v^2}{2c_\beta s_\beta}\left[\lambda_4 c_\beta s_\beta + c_\beta s_\beta \Re\mathrm{e}(\lambda_5 \mathrm{e}^{2i\xi}) + c_\beta^2 \Re\mathrm{e}(\lambda_6 \mathrm{e}^{i\xi}) + s_\beta^2 \Re\mathrm{e}(\lambda_7 \mathrm{e}^{i\xi})\right] \qquad (4)
$$

- Mass matrix: The Higgs potential includes the mass terms which can be cast into the form

$$V_{\rm mass} = (G^+\, H^+) \left(\begin{array}{cc} 0 & 0 \\ 0 & M_{H^\pm}^2 \end{array}\right) \left(\begin{array}{c} G^- \\ H^- \end{array}\right) + \frac{1}{2}(\phi_1^0\, \phi_2^0\, a)\, \mathcal{M}_0^2 \left(\begin{array}{c} \phi_1^0 \\ \phi_2^0 \\ a \end{array}\right) \qquad (5)$$

where

$$\left(\begin{array}{c} \phi_1^- \\ \phi_2^- \end{array}\right) = \left(\begin{array}{cc} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{array}\right) \left(\begin{array}{c} G^- \\ H^- \end{array}\right); \quad \left(\begin{array}{c} a_1 \\ a_2 \end{array}\right) = \left(\begin{array}{cc} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{array}\right) \left(\begin{array}{c} G^0 \\ a \end{array}\right). \qquad (6)$$

Show that the 3×3 mass matrix of the neutral Higgs bosons is given by

$$\mathcal{M}_0^2 = M_A^2 \left(\begin{array}{ccc} s_\beta^2 & -s_\beta c_\beta & 0 \\ -s_\beta c_\beta & c_\beta^2 & 0 \\ 0 & 0 & 1 \end{array}\right) + \mathcal{M}_\lambda^2 \qquad (7)$$

with

$$M_A^2 = M_{H^\pm}^2 + \frac{1}{2}\lambda_4 v^2 - \frac{1}{2}\Re{\rm e}(\lambda_5 {\rm e}^{2i\xi}) v^2 \qquad (8)$$

and

$$\frac{\mathcal{M}_\lambda^2}{v^2} = \left(\begin{array}{ccc} \begin{array}{l} 2\lambda_1 c_\beta^2 + \Re{\rm e}(\lambda_5 {\rm e}^{2i\xi}) s_\beta^2 \\ +2\Re{\rm e}(\lambda_6 {\rm e}^{i\xi}) s_\beta c_\beta \end{array} & \begin{array}{l} \lambda_{34} c_\beta s_\beta + \Re{\rm e}(\lambda_6 {\rm e}^{i\xi}) c_\beta^2 \\ +\Re{\rm e}(\lambda_7 {\rm e}^{i\xi}) s_\beta^2 \end{array} & \begin{array}{l} -\frac{1}{2}\Im{\rm m}(\lambda_5 {\rm e}^{2i\xi}) s_\beta \\ -\Im{\rm m}(\lambda_6 {\rm e}^{i\xi}) c_\beta \end{array} \\ & & \\ \begin{array}{l} \lambda_{34} c_\beta s_\beta + \Re{\rm e}(\lambda_6 {\rm e}^{i\xi}) c_\beta^2 \\ +\Re{\rm e}(\lambda_7 {\rm e}^{i\xi}) s_\beta^2 \end{array} & \begin{array}{l} 2\lambda_2 s_\beta^2 + \Re{\rm e}(\lambda_5 {\rm e}^{2i\xi}) c_\beta^2 \\ +2\Re{\rm e}(\lambda_7 {\rm e}^{i\xi}) s_\beta c_\beta \end{array} & \begin{array}{l} -\frac{1}{2}\Im{\rm m}(\lambda_5 {\rm e}^{2i\xi}) c_\beta \\ -\Im{\rm m}(\lambda_7 {\rm e}^{i\xi}) s_\beta \end{array} \\ & & \\ \begin{array}{l} -\frac{1}{2}\Im{\rm m}(\lambda_5 {\rm e}^{2i\xi}) s_\beta \\ -\Im{\rm m}(\lambda_6 {\rm e}^{i\xi}) c_\beta \end{array} & \begin{array}{l} -\frac{1}{2}\Im{\rm m}(\lambda_5 {\rm e}^{2i\xi}) c_\beta \\ -\Im{\rm m}(\lambda_6 {\rm e}^{i\xi}) s_\beta \end{array} & 0 \end{array}\right) \qquad (9)$$

where $\lambda_{34} = \lambda_3 + \lambda_4$.

- Inputs: With $m_{12}^2 = |m_{12}|^2 e^{i\phi_{12}}$ and $\lambda_{5,6,7} = |\lambda_{5,6,7}| e^{i\phi_{5,6,7}}$,

$$\begin{array}{l} v\,,\, t_\beta\,,\, |m_{12}|\,; \\ \lambda_1\,,\, \lambda_2\,,\, \lambda_3\,,\, \lambda_4\,,\, |\lambda_5|\,,\, |\lambda_6|\,,\, |\lambda_7|\,; \\ \phi_5 + 2\xi\,,\, \phi_6 + \xi\,,\, \phi_7 + \xi\,,\, {\rm sign}[\cos(\phi_{12} + \xi)]\,. \end{array} \qquad (10)$$

When the CP-odd phases $\phi_5+2\xi$, $\phi_6+\xi$ and $\phi_7+\xi$ are given, $\sin(\phi_{12}+\xi)$ is fixed by the CP-odd tadpole condition and, accordingly, $\cos(\phi_{12}+\xi)$ is determined up to the two-fold ambiguity. One may take the convention with $\xi = 0$ without loss of generality.