The 6d (1,0) and (2,0) SCFTs

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High1-2014, KIAS-NCTS Joint Workshop
February 2014

Hee-Cheol Kim, Seok Kim, Sung-Soo Kim, KM [arXiv:1307.7660] The general M5-brane superconformal Index
Hee-Cheol Kim, KM [arXiv:1210.0853] M5 brane theories on R x CP2
Hee-Cheol Kim, Seok Kim, Eunkyung Ko, KM [arXiv:1110.2175] On instantons as KK modes of M5 branes
Stefano Bolognesi, KM [arXiv:1105.5073] 1/4 BPS string junctions and N3 problem in 6-dim conformal field theories
String/M Theory

* Fundamental Formulation
* From 11-dimension to 4-dimension
* AdS-CFT Correspondence

* D3 branes, 4d N=4 SYM, AdS$_5 \times S^5$
* M2 branes, 3d N=8 ABJM, AdS$_4 \times S^7$
* M5 branes, 6d (2,0) Theory, AdS$_7 \times S^4$
6d (2,0) Superconformal Theories

* A, D, E type: type IIB on $R^{1+5} \times C^2/\Gamma_{ADE}$
* $A_{N-1}, D_N$ type: N M5 branes, N M5 (+OM5)
* superconformal symmetry: $OSp(2,6|2) \supset O(2,8) \times USp(4)_R$
* fields: $B, \Phi_I$ ($I=1,2,3,4,5$), $\Psi$
  * selfdual strength $H=dB=*H$, purely quantum $\hbar=1$

* We do not know how to write down the theory for nonabelian case.
  * Sorokin, Chu, Lambert, Papageorgakis, ....
* covariant derivative?
* $N^3$ degrees of freedom

* Calculate something exact on (2,0) theories?
Many types of Theories:

(1) 6d E8 (1,0) Superconformal Field Theory ;
- M5 branes near one of E8 Walls of M theory on $\mathbb{R}^{1+9} \times S^1/\mathbb{Z}_2$
- Tensor multiplet+ hypermultiplet......

(2) Mixed with tensor multiplets, gauge multiplets, matter hypermultiplets
- hypermultiplets coupled to vectors
- hypermultiplets coupled to tensors

Nonabelian tensor theories, $N^3$ degrees of freedom

* Witten, Ganor, Hanany, Seiberg, Duff, Lu, Pop, Morrison, Aspinwall, Berkooz, Leigh, Schwarz, Brunner, Karch, Zaffaroni, How, Sezgin, West, Pasti, Sorokin, Tonin, Bandos,
5-dim Approach to 6d (2,0) theories

* Instantons = KK modes
  * threshold bound states of k instantons for kk momentum k.

* Douglas, Lambert, Papageorgakis

* 5d N=2 super Yang-Mills theories could be complete when all non perturbative effects are included


* Perturbative approach: problem in higher order (6-loop)
* incomplete? (Need higher order operators to complete the theory?)
* strong coupling limit = 6d theory

* BPS object counting
  * dyonic instanton counting = S-dual of elliptic genus of selfdual strings in Coulomb phase of (2,0) theories (Nekrasov-partition function..)
  * DLCQ of the (2,0) theory = (gauge invariant operators..)

* monopole string junctions
  * selfdual string junctions in the Coulomb phase of 6d (2,0) theories

Berkooz, Rozali, Seiberg 1997, Berkooz, Douglas 1996
More Lessons from 5d SYM

* 1/4 BPS selfdual string junction in the Coulomb phase of 6d (2,0) theory
  * possible solution for $N^3$ degrees of freedom.
  * The rough entropy calculation in the Coulomb phase seems to work.

KL, Ho-UngYee:0606150
Stefano Bolognesi, KL:1105.5073
BPS Junction Math for A(DE)

- dimension of $A_{N-1}$: $d = N^2 - 1$
- rank of $A_{N-1}$: $r = N$
- Coxeter number = number of roots/rank: $h = N$
  - Coxeter = Dual Coxeter for simple laced groups
- Anomaly coefficient: $c = dh/3 = N(N^2 - 1)/3$
- Relation:
  $$c = N(N^2 - 1)/3 = N^2 - N + N(N-1)(N-2)/3$$
- # of roots = ij selfdual strings = # of roots: $N(N-1)$
- # of SU(3) imbedding = ijk of BPS (anti)junctions: $N(N-1)(N-2)/3$
- True for ADE algebras
High Temperature in Coulomb Phase

- Micro-canonical
- Massless on N M5 branes: $O(N)$
- Loops of self-dual strings excitations: $O(N^2)$
- Beyond the Hagedorn temperature
- Webs of junctions and anti-junctions: $O(N^3)$
- Excitations of webs of tensionless strings with junctions act as atoms.
- $N^3$ degrees of freedom
Nonzero Temperature in Symmetric Phase
5-dim Approach to 6d (1,0) Theories

* 6d $E_8$ SCFT compactified on $R^{1+4} \times S^1$
  * 5-dim non-Lagrangian theory + KK modes

* With Wilson-loop along the flavor direction which breaks $E_8$ to $SO(16)$

* 5d $N=1$ susy $USp(N)$ gauge theory with $N_f=8$ fundamental hyper +
  antisymmetric hyper

  * strongly coupled limit = 6d theory (‘KK modes’ should be captured by instanton )

* 5d $N=1$ susy $USp(N)$ theory with $N_f \leq 7$ fundamental hyper+ antisymmetric hyper

  * strongly coupled limit = 5d supersymmetric theory with $E_{N_f+1}$ global symmetry

  * the index calculation on $S^1 \times S^4$ has shown this recently by us and others.

* It is not known whether the instantons of the 5d version of 6d SCFT ($H^2 + d\Phi^2 + \Phi F^2 + BFF + ..$) captures KK modes 6d theory
Difficulties with nonabelianization of the B field and its strength H=dB

- after the torus compactification to 4d, instantons are magnetically charged KK modes.
- its electric dual is the adjoint fields of the 5d theory kk modes from 5d to 4d
- A explicit construction of a local field theory including the nonabelian 2-form field would leads a 4d local field theory with nonabelian electric and magnetically charged fields. (Besides abelian case, it is not known yet.)

Generalize ABJM to M5 brane theory

- Mode out by $\mathbb{R}^8/\mathbb{Z}_k$
- Weak coupling limit
- No fixed point

$\text{AdS}_7 \times S^4/\mathbb{Z}_k$ (Tomasiello): 6d theory with D6 and $\text{D6}^*$: still 6d theory with $H=\star H$
6d (2,0) Theory on R x S^5

* Radial quantization

* S^5 = a circle fibration over CP^2
  * \( ds^2_{S^5} = ds^2_{CP^2} + (dy + V)^2 \), \( dV = 2J, J = *J, y \sim y + 2\pi \)

* S^5/Z_K (y \sim y + 2\pi/K) has no fixed point

* Hamiltonian = the conformal dimension operator H

* Superconformal index

\[
Q_{j_1,j_2,j_3}^{R_1,R_2} \Rightarrow Q = Q^{\frac{1}{2},\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}}, S = Q^\dagger
\]
Index Function on $S^1 \times S^5$

* Supercharge: $Q^{R_1, R_2}_{j_1, j_2, j_3} \Rightarrow Q = Q_{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}}, S = Q^\dagger$

* BPS bound: $E = j_1 + j_2 + j_3 + 2(R_1 + R_2)$

* Index function:

$$I = \text{Tr} \left[ (-1)^F e^{-\beta' \{Q,S\}} e^{-\beta \left( E - \frac{R_1 + R_2}{2} - m(R_1 - R_2) + a j_1 + bj_2 + cj_3 \right)} \right], \ a + b + c = 0$$

* Euclidean Path Integral of (2,0) Theory on $S^1 \times S^5$:

* $S^5 = S^1$ fiber over $\mathbb{CP}^2$: $-i \partial_y = \text{KK modes}$

$$k \equiv j_1 + j_2 + j_3$$

* $Z_k$ modding keeps only $k/K = \text{integer modes}$
6d Abelian Theory (Fermion+ Scalar)

* on $\mathbb{R} \times S^5$, …… (Could Include $H=dB$)

\[- \frac{i}{2} \lambda \Gamma^M \hat{\nabla}_M \lambda - \frac{1}{2} \partial_M \phi_I \partial^M \phi_I - \frac{2}{r^2} \phi_I \phi_I \]

* gamma matrices $\Gamma^M$, $\rho^a$

* Symplectic Majorana $\lambda = -BC \lambda^*$, $\epsilon = BC \epsilon^*$

* Weyl: $\Gamma^7 \lambda = \lambda$, $\Gamma^7 \epsilon = -\epsilon$

* 32 supersymmetry

\[
\delta \phi_I = -\bar{\lambda} \rho_I \epsilon = +\bar{\epsilon} \rho_I \lambda, \\
\delta \lambda = +\frac{i}{6} H_{MNP} \Gamma^{MNP} \epsilon + i \partial_M \phi_I \Gamma^M \rho_I \epsilon - 2 \phi_I \rho_I \bar{\epsilon}, \\
\delta \bar{\lambda} = -\frac{i}{6} H_{MNP} \bar{\epsilon} \Gamma^{MNP} + i \partial_M \phi_I \bar{\epsilon} \Gamma^M \rho_I - 2 \bar{\epsilon} \rho_I \phi_I. 
\]

* additional condition on Killing spinor:

\[
\hat{\nabla}_M \epsilon = \frac{i}{2r} \Gamma_M \bar{\epsilon}, \quad \Gamma^M \hat{\nabla}_M \bar{\epsilon} = 2i \epsilon, \quad \bar{\epsilon} = \pm \Gamma_0 \epsilon.
\]
**Dimensional Reduction to CP$^2$**

* define new variables so some of Killing spinors are $y$-independent

* New variables with twisting

$$
\epsilon_{\text{old}} = e^{-\frac{y}{4}M_{IJ}p_{IJ}} \epsilon_{\text{new}}, \\
\lambda_{\text{old}} = e^{-\frac{y}{4}M_{IJ}p_{IJ}} \lambda_{\text{new}}, \\
(\phi_1 + i\phi_2)_{\text{old}} = e^{+(3+p)i\gamma/2(\phi_1 + \phi_2)}_{\text{new}} \\
(\phi_4 + i\phi_5)_{\text{old}} = e^{+(3-p)i\gamma/2(\phi_4 + i\phi_5)}_{\text{new}}.
$$

* $y$-independent supersymmetry: $Q = Q^{++}, S = Q^{--}$

* singlet $\epsilon_+, \epsilon_- : 2$ surviving supersymmetries

* kk mode $= -i\partial_y$

$$
k \equiv j_1 + j_2 + j_3 + \frac{3}{2}(R_1 + R_2) + \frac{p}{2}(R_1 - R_2), \ p = \text{odd integer}
$$
5d Lagrangian

\[ Q = Q_{-+-}, S = Q_{++-} \]

* Lagrangian on \( R \times \text{CP}^2 \) with 2 supersymmetries for any \( p \):

\[
S = \frac{K}{4\pi^2} \int_{R \times \text{CP}^2} d^5 x \sqrt{|g|} \text{tr} \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\sqrt{|g|}} \epsilon^{\mu\nu\rho\sigma\eta} J_{\mu\nu} \left( A_\rho \partial_\sigma A_\eta - \frac{2i}{3} A_\rho A_\sigma A_\eta \right) - \frac{1}{2} D_\mu \phi_I D^\mu \phi_I + \frac{1}{4} [\phi_I, \phi_J]^2 - 2\phi_I^2 - \frac{1}{2} (M_{IJ} \phi_J)^2 - i(3-p)[\phi_1, \phi_2] \phi_3 - i(3+p)[\phi_4, \phi_5] \phi_3 - \frac{i}{2} \bar{\lambda} \gamma^\mu D_\mu \lambda - \frac{i}{2} \bar{\lambda} \rho_I [\phi_I, \lambda] - \frac{1}{8} \bar{\lambda} \gamma^{mn} \lambda J_{mn} + \frac{1}{8} \bar{\lambda} M_{IJ} \rho_{IJ} \lambda \right],
\]

(2.27)

* Supersymmetry Transformation

\[
\delta A_\mu = + i \bar{\lambda} \gamma_\mu \epsilon = - i \bar{\epsilon} \gamma_\mu \lambda, \quad \delta \phi_I = - \bar{\lambda} \rho I \epsilon = \bar{\epsilon} \rho_I \lambda, \\
\delta \lambda = + \frac{1}{2} F_{\mu\nu} \gamma^{\mu\nu} \epsilon + i D_\mu \phi_I \rho_I \gamma^\mu \epsilon - \frac{i}{2} [\phi_I, \phi_J] \rho_{IJ} \epsilon - 2 \phi_I \rho_I \bar{\epsilon} - M_{IJ} \phi_I \rho_J \epsilon.
\]

* \( p/2 = -1/2 \) : \( k = j_1 + j_2 + j_3 + R_1 + 2R_2 \)

* additional supersymmetries: Total 8 supersymmetries

\[ Q^{+-+}, Q^{++-}, Q^{++-} \text{ conjugates} \]
Coupling Constant Quantization

* Instanton number on CP2

\[ \nu = \frac{1}{8\pi^2} \int_{\text{CP}^2} \text{Tr}(F \wedge F) = \frac{1}{16\pi^2} \int_{\text{CP}^2} d^4 x \sqrt{|g|} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} . \]

* Instantons represents the momentum K or energy K:

\[ \frac{1}{g_{YM}^2} = \frac{K}{4\pi^2 r} \]

* Another approach to quantization: F=2J: \( 2\pi \) flux on a cycle, 1/2 instanton for abelian theory

\[ \frac{K}{4\pi^2} \int_{D \times \text{CP}^2} d^5 x \frac{1}{2} \epsilon^{\mu\nu\rho\sigma\eta} J_{\mu\nu} \partial_\rho A_\sigma A_\eta \Rightarrow K \int dt A_0 \]

* 't Hooft coupling constant: \( \lambda = N/K \)

* Large K => Free Theory
Diluting degrees of freedom with $Z_K$ modding + Twisting

![Graph showing energy levels for different KK modes.](image)
Diluting degrees of freedom with $Z_K$ modding + Twisting

Energy

KK modes: $k$

-4 -3 -2 -1 0 1 2 3 4
Expected Enhanced Supersymmetries

* Killing spinors with $p/2=-1/2$, $k = j_1+j_2+j_3+R_1+2R_2$
  * $k=0$: 8 kinds
  * $k=\pm 1$: 14 kinds
  * $k=\pm 2$: 8 kinds
  * $k=\pm 3$: 2 kinds

* # of supersymmetries
  * $K \geq 4$: 8 supersymmetries
  * $K=3$: 10 supersymmetries
  * $K=2$: 16 supersymmetries
  * $K=1$: 32 supersymmetries
the index function on $S^1 \times S^5$

* 5d SYM on $S^5$  
  Hee-Cheol Kim, Seok Kim: 1206.6339; Hee-Cheol Kim, Joonho Kim, S.K. 1211.0144
  * S-dual version of the index
  * Vacuum energy:
    \[
    (\epsilon_0)_{\text{index}} = \lim_{\beta' \to 0} \text{Tr} \left[ (-1)^F \frac{E - R}{2} e^{-\beta'(E - R_1)} \right] \\
    = \frac{N(N^2 - 1)}{6} + \frac{N}{24}
    \]

* $S^1 \times \text{CP}^2$ path integral off-shell

* Stationary phase: $D^1=D^2=0$, $F = 2s \ J$, $\phi + D^3=4s$, $s = \text{diag}(s_1,s_2,\ldots,s_N)$

* Path Integral: Off-shell, localization

\[
\sum_{s_1,s_2,\ldots,s_N=-\infty}^{\infty} \frac{1}{|W_s|} \oint \frac{d\lambda_i}{2\pi} e^{\frac{\beta}{2} \sum_{i=1}^{N} s_i^2 - i \sum_{i} s_i \lambda_i} Z^{(1)}_{\text{pert}} Z^{(1)}_{\text{inst}} \cdot Z^{(2)}_{\text{pert}} Z^{(2)}_{\text{inst}} \cdot Z^{(3)}_{\text{pert}} Z^{(3)}_{\text{inst}}.
\]

* For $K=1$, well-confirmed for $k \leq N$ with $N=1,2,3$ with the AdS/CFT calculation
Strange Vacua

* K=1, F=2sJ background

\[ U(2) \ (1, -1) \]
\[ U(3) \ (2, 0, -2), (2, -1, -1), (1, 1, -2), (1, 0, -1) \]
\[ U(4) \ (3, 1, -1, -3), (3, 1, -2, -2), (2, 2, -1, -3), (3, 0, -1, -2), (2, 1, 0, -3), (2, 0, 0, -2), (2, 0, -1, -1), (1, 1, 0, -2), (1, 0, 0, -1) \]

* the Lowest one \( s_G = 2\rho \cdot \mathbf{H} \) with negative energy \(-2\rho^2\), where \( \rho \) = Weyl vector

* Ground State for Index: \( K \leq N \) (Strong 't Hooft coupling \( \lambda = N/K \))

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<th>( K )</th>
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Table 1: Vacuum energies divided by \( K \), at general \( \mathbb{Z}_K \) (and fluxes)
Check with AdS/CFT

- E.g. $k = N = 3$: (all results multiplied by vacuum energy factor & $e^{-3\beta}$)
  $$ y_i = e^{-\beta a_i}, \quad y = e^{\beta(m - \frac{1}{2})} $$

  $$ Z_{(2,0,-2)} = 3 \left[ y^2(y_1 + y_2 + y_3) + y(y_1^2 + y_2^2 + y_3^2) + y^{-1}(y_1 + y_2 + y_3) - (1 + \frac{y_1}{y_2} + \frac{y_2}{y_1} + \cdots) + y^3 \right] $$
  $$ + 6y \left[ y(y_1 + y_2 + y_3) - (y_1^{-1} + y_2^{-1} + y_3^{-1}) + y^{-1} + y^2 \right] + y^3 $$

  $$ Z_{(2,-1,-1)} + Z_{(1,1,-2)} = -2y \left[ y(y_1 + y_2 + y_3) - (y_1^{-1} + y_2^{-1} + y_3^{-1}) + y^{-1} + y^2 \right] $$
  $$ - 2y \left[ y(y_1 + y_2 + y_3) - (y_1^{-1} + y_2^{-1} + y_3^{-1}) + y^{-1} + y^2 \right] $$

  $$ Z_{(1,0,-1)} = y^3 + y^2(y_1 + y_2 + y_3) - y(y_1^{-1} + y_2^{-1} + y_3^{-1}) + 1 $$

  $$ Z_{SUGRA} = 3y^3 + 2y^2(y_1 + y_2 + y_3) + y \left( y_1^2 + y_2^2 + y_3^2 - \frac{1}{y_1} - \frac{1}{y_2} - \frac{1}{y_3} \right) - \left( \frac{y_1}{y_2} + \frac{y_2}{y_3} + \cdots \right) + y^{-1}(y_1 + y_2 + y_3) $$

- Non-zero flux states contributing to the index
  - $s = (N-1, N-3, \ldots, -(N-1)) = s_0 : SU(N)$ Weyl vector
  - Index vacuum energy: $E_0 = -\frac{N(N^2 - 1)}{6}$
SU(2) Case

- BPS Eq. for Homogeneous Configuration with instanton number $n^2$
  
  $$A = V \text{diag}(n, -n), \quad F = 2J \text{diag}(n, -n),$$

- homogeneous solutions possible only with $n=+1,-1$
- but gauss law is violated
- for one of the constant bps solutions, the homogeneous fermionic zero mode is possible.
- gauss law can be satisfied with fermionic contribution for $K=1$ but not for $K>1$.
- energetic is more complicated to due to zero-point contribution to the classical one,...
6d (1,0) Theories

- Similar approach on R x CP2 works for some theories
- It would be interesting to check some index functions

- For other (1,0) 6d theories with instanton strings, it is not clear whether the corresponding 5d theories capture KK modes via instantons
  - Again we need to understand the instanton physics better
Conclusion

* New 5d supersymmetric theories for M5 are found with discrete coupling constant
* Index Function of 6d $A_N (2,0)$ is partially obtained.
* Highly nontrivial vacuum structure in the strong coupled regime

* UV finite? How rigid is the theory with eight supersymmetries.
* Enhanced supersymmetry to $K=1,2,3$?
* Wilson-loop can be included.
* Near BPS objects? Perturbative approach?
* Formulation and Calculation of 6d N=1 SCFT Theories
* Relate our result to Rastelli’s result?
5d $\mathcal{N}=2$ SYM as the M5 brane theory

* compactification on $\mathbb{R}^{1+4} \times S^1$ with radius $r$
* the lowest KK modes $\Rightarrow$ 5d SYM
* coupling constant $1/g_{YM}^2 = 4\pi^2/r$
* instanton = quantum of KK modes of unit momentum
* drop KK modes and keep instantons
  * otherwise, it is over-counting
* there may quantum-gauge-invariance identifying two
* dyonic instanton index
* monopole string + momentum

5d SYM + instantons = ? 6d (2,0) theory

* 6-loop UV divergence in four-point function


Hee-Cheol Kim, Seok Ki, E. Koh, KL, Sungjay Lee 2011
Haghighat, Iqbal, Kozcaz, Lockhart, Vafa: M-strings