Lessons and challenges from PLANCK

Jinn-Ouk Gong

APCTP, Pohang 790-784, Korea

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Outline

Why inflation?

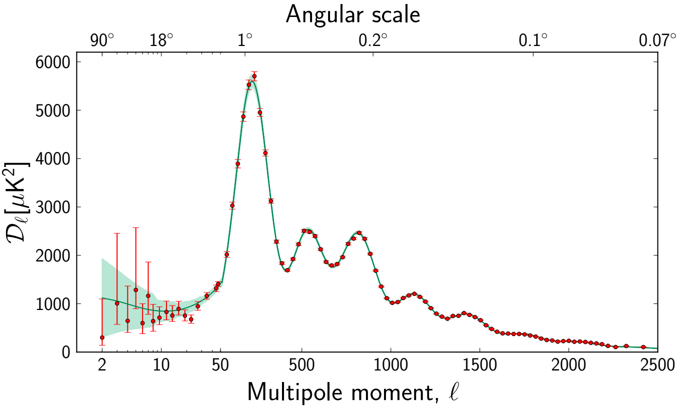
Hot big bang	Inflation
Horizon problem	Single causal patch
Flatness problem	Locally flat
Monopole problem	Diluted away
Initial perturbations	Quantum fluctuations

1. Initial conditions for hot big bang
2. A certain amount of expansion is required

$$\text{Number of } e\text{-folds}: N = \log\left(\frac{a_e}{a_i}\right) \sim 60 \text{ is necessary}$$

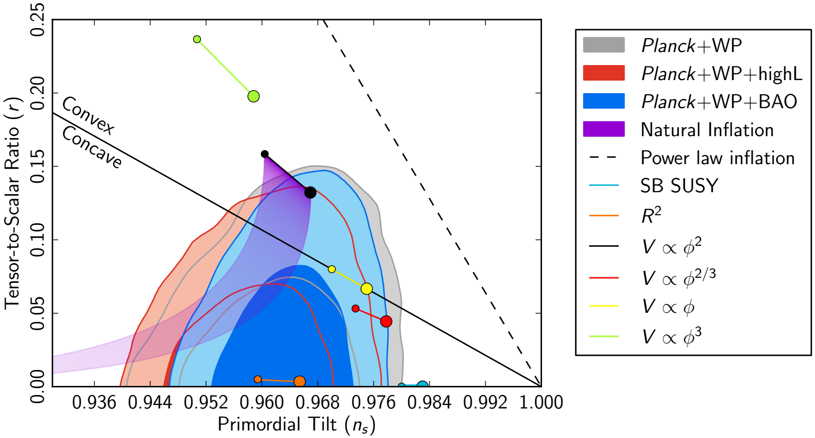
3. Consistent with most recent observations
4. Typically driven by inflaton with a specific potential $V(\phi)$

PLANCK 2013 and new physics



- PLANCK seems to suggest nothing new
- No way to probe physics beyond ΛCDM?
- If so, what is it and how can we probe?

Status of inflation models



Fiducial model: single field slow-roll inflation

Lesson 1: nearly scale invariant power spectrum

$$\text{Power spectrum}: \langle \mathcal{R}_k \mathcal{R}_q \rangle \equiv (2\pi)^3 \delta^{(3)}(\boldsymbol{k}+\boldsymbol{q}) P_{\mathcal{R}}(k) = (2\pi)^3 \delta^{(3)}(\boldsymbol{k}+\boldsymbol{q}) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k)$$

- Spectral index $n_{\mathcal{R}}$: $\mathcal{P}_{\mathcal{R}} \propto k^{n_{\mathcal{R}}-1}$
- Harrizon-Zeldovich spectrum: $n_{\mathcal{R}} = 1$ (const $\mathcal{P}_{\mathcal{R}}$ over all k)

For fiducial case,

- $n_{\mathcal{R}} = 1 - 6\epsilon + 2\eta$ with $\epsilon = \dfrac{m_{\rm Pl}^2}{2}\left(\dfrac{V'}{V}\right)^2$ and $\eta = m_{\rm Pl}^2 \dfrac{V''}{V}$
- If $V \sim \phi^n$, $n_{\mathcal{R}} \approx 1 - \dfrac{n/2+1}{N} \sim 0.95 - 0.97$

PLANCK found departure from HZ: $n_{\mathcal{R}} = 0.9603 \pm 0.0073$, 5.5σ away

Lesson 2: no gravity waves

Primordial gravitational waves

- Transverse ($h^i{}_{j,i} = 0$), traceless ($h^i{}_i = 0$) parts of spatial metric
- Directly related to the energy scale of inflation

$$\mathcal{P}_T = \frac{8}{m_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2 = \frac{2}{3\pi^2}\frac{\rho_{\rm inf}}{m_{\rm Pl}^4} \sim \frac{V}{m_{\rm Pl}^4}$$

Tensor-to-scalar ratio: $r \equiv \dfrac{\mathcal{P}_T}{\mathcal{P}_\mathcal{R}} = 16\epsilon$ for fiducial case

PLANCK found no gravity waves: $r_{0.002} < 0.11$ at $2\sigma \to \dfrac{V^{1/4}}{m_{\rm Pl}} < 0.008$

Lesson 3: almost perfect Gaussianity

Bispectrum : $\langle \mathscr{R}_{k_1}\mathscr{R}_{k_2}\mathscr{R}_{k_3}\rangle \equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_{\mathscr{R}}(k_1,k_2,k_3)$

- Expanding $\mathscr{R}$ locally as $\mathscr{R} = \mathscr{R}_g + \frac{3}{5}f_{\rm NL}\mathscr{R}_g^2 + \cdots$ gives

$$B_{\mathscr{R}}(k_1,k_2,k_3) \underset{k_3\to 0}{\longrightarrow} \frac{12}{5}f_{\rm NL}P_{\mathscr{R}}(k_1)P_{\mathscr{R}}(k_3)$$

- If $\mathscr{R}$ is completely described by S_2, $f_{\rm NL}=0$

For fiducial case, $f_{\rm NL} = \frac{5}{12}(1-n_{\mathscr{R}}) \ll 1$

PLANCK found no nG: $f_{\rm NL} = 2.7 \pm 5.8$ at $2\sigma \to \mathscr{R}$ is 99.99% Gaussian

Challenge 1: nearly scale invariant power spectrum

η problem: a flat potential is difficult to obtain

- Nearly scale invariance requires $\epsilon, |\eta| \ll 1$
- When building inflation models based on particle physics...
  1. In supergravity,

$$V_F = \underbrace{e^{K/m_{\rm Pl}^2}}_{K=|\phi|^2+\cdots} V_0 \approx \left(1+\frac{|\phi|^2}{m_{\rm Pl}^2}\right) V_0 \to \eta = 1 + m_{\rm Pl}^2 \frac{V_0''}{V_0} = \mathcal{O}(1)$$

  2. On general ground, a new scale $\Lambda(\lesssim m_{\rm Pl})$ gives

$$\Delta V = cV(\phi)\frac{\phi^2}{\Lambda^2} \to \Delta\eta = m_{\rm Pl}^2 \frac{\Delta V''}{V} \approx 2c\left(\frac{m_{\rm Pl}}{\Lambda}\right)^2 = \mathcal{O}(1)$$

- Difficult to keep flat potential against corrections

Challenge 2: no gravity waves

For $V \sim \phi^n$, $r = \frac{4n}{N} \gtrsim 0.1$

1. Power-law potential in a corner
2. Either hill-top inflation
   - Initially near a local maximum: how to start there?
   - Usually min $> m_{\rm Pl}$: Taylor expansion not trustable
3. ... or low-scale inflation
   - $V^{1/4}$ as low as TeV scale (N.B. $E_{\rm LHC} = 14$ TeV)
   - Possible signatures at the collider experiments?
4. ... or more perturbation from other sources
   - Curvaton, modulated reheating... multi-field effects
   - More complex

Challenge 3: almost perfect Gaussianity

$\langle\mathcal{RRR}\rangle$ requires cubic order action: using in-in formalism

$$\langle\hat{\mathcal{O}}(t)\rangle = \sum_{n=1}^{\infty} i^n \int_{t_{\rm in}}^{t} dt_n \int_{t_{\rm in}}^{t_n} dt_{n-1} \cdots \int_{t_{\rm in}}^{t_2} dt_1 \left\langle 0 \middle| \left[H_{\rm int}(t_1), \left[H_{\rm int}(t_2), \cdots \left[H_{\rm int}(t_n), \hat{\mathcal{O}}(t) \right] \cdots \right] \right] \middle| 0 \right\rangle$$

with $H = H_0 + H_{\rm int}$

quadratic (H_0) — cubic and higher ($H_{\rm int}$): $S_3 = -\int dt H_{\rm int}$

$\mathcal{R}$ = free field, thus for non-zero $\langle\mathcal{RRR}\rangle$ we need at least S_3

$$\langle\mathcal{RRR}(t)\rangle = i\int_{t_{\rm in}}^{t} dt' \left\langle 0 \middle| \left[\mathcal{RRR}(t'), \mathcal{RRR}(t) \right] \middle| 0 \right\rangle + \cdots$$

1. Observable [$f_{\rm NL} \gtrsim \mathcal{O}(1)$] nG when interactions are appreciable
2. New model discriminator
3. Null detection: how to probe inflationary dynamics?

Why correlating correlation functions?

Features may arise from such inflationary dynamics:

1. $V(\phi)$ is not smooth → departure from SR (Starobinsky 1992, Adams et al. 2001)
2. Curved trajectory → non-trivial c_s (Achucarro et al. 2011–)

These effects permeate through mode function solution

- Features in a corr fct = features in other corr fcts
- Distinctive signatures to look for
- Unique way to probe higher order corr fct

Q: How are features in correlation functions correlated?

Q′: How can we correlate correlation functions?

General slow-roll approximation

- $\mathcal{R}_k(\tau)$ = de Sitter piece + corrections away from dS
- No hierarchy between SR parameters: $\epsilon \sim \eta \sim \cdots$
- More general contexts

Mode equation for the comoving curvature perturbation $\mathcal{R}$

$$\ddot{\mathcal{R}} + \left[\frac{c_s^2}{a^3\epsilon}\frac{d}{dt}\left(\frac{a^2\epsilon}{c_s^2}\right) + H\right]\dot{\mathcal{R}} - \frac{\Delta}{a^2}\mathcal{R} = 0$$

$$\Downarrow \quad z^2 \equiv \frac{2a^2 m_{\rm Pl}^2\epsilon}{c_s}, \; y \equiv \sqrt{2k}z\mathcal{R}_k, \; dx \equiv -kc_s\frac{dt}{a}, \; f \equiv 2\pi\frac{xz}{k}$$

$$\underbrace{\frac{d^2y}{dx^2} + \left(1 - \frac{2}{x^2}\right)y}_{\text{de Sitter solution}} = \underbrace{\frac{1}{x^2}\underbrace{\frac{f'' - 3f'}{f}}_{\equiv g(\log x)}y}_{\text{departure from dS}} \qquad \left(f' \equiv \frac{df}{d\log x}\right) \quad \rightarrow \quad y_0(x) = \left(1 + \frac{i}{x}\right)e^{ix}$$

Power spectrum in GSR

Green's function solution (JG & Stewart 2001)

$$
\begin{aligned}
y(x) &= y_0(x) + \frac{i}{2}\int_x^\infty \frac{du}{u^2} g(\log u)\left[y_0^*(u)y_0(x) - y_0^*(x)y_0(u)\right]y(u) \\
&\equiv y_0(x) + L(x,u)y(u) \\
&= y_0(x) + L(x,u)y_0(u) + L(x,u)L(u,v)y_0(v) + \cdots
\end{aligned}
$$

Power spectrum (Stewart 2002, Choe, JG & Stewart 2004)

$$
\begin{aligned}
\mathcal{P}_{\mathcal{R}}(k) &= \lim_{x\to0}\left|\frac{xy}{f}\right|^2 \\
&= \lim_{x\to0}\frac{1}{f^2}\left[1 - \frac{2}{3}x^3\int_x^\infty\frac{du}{u^4}g(\log u) + \frac{2}{3}\int_x^\infty\frac{du}{u}g(\log u)W(u) + \cdots\right] \\
&= \frac{1}{f_\star^2}\left\{1 + \frac{2}{3}\frac{f'_\star}{f_\star} + \frac{2}{3}\int_0^\infty\frac{dx}{x}\left[W(x) - \theta(x_\star - x)\right]g(\log x) + \cdots\right\}
\end{aligned}
$$

($\star$: evaluation time)

Windows functions $W(x)$ and $X(x)$ (see later) given by y_0

Inverting power spectrum

By inverting $\mathscr{P}_\mathcal{R}$ as (Joy, Stewart, JG & Lee 2005, Joy & Stewart 2006)

$$\log\left(\frac{1}{f^2}\right) = \int_0^\infty \frac{dk}{k} m(k\tau) \log \mathscr{P}_\mathcal{R}(k)$$

$$m(x) = \frac{2}{\pi}\left[\frac{1}{x} - \frac{\cos(2x)}{x} - \sin(2x)\right]$$

f and g can be written i.t.o. $\mathscr{P}_\mathcal{R}$, $\dfrac{d\log\mathscr{P}_\mathcal{R}}{d\log k}$, $\dfrac{d^2\log\mathscr{P}_\mathcal{R}}{d\log k^2}$

$$\frac{f'}{f} = -\frac{1}{2}\frac{d}{d\log\tau}\left[\log\left(\frac{1}{f^2}\right)\right] = \frac{1}{2}\int_0^\infty \frac{dk}{k} m(k\tau)\frac{d\log\mathscr{P}_\mathcal{R}}{d\log k}$$

$$g = \frac{f'' - 3f'}{f}$$

$$= -\frac{1}{2}\int_0^\infty \frac{dk}{k} m(k\tau)\frac{d^2\log\mathscr{P}_\mathcal{R}}{d\log k^2} - \frac{3}{2}\int_0^\infty \frac{dk}{k} m(k\tau)\frac{d\log\mathscr{P}_\mathcal{R}}{d\log k} + \left[\frac{1}{2}\int_0^\infty \frac{dk}{k} m(k\tau)\frac{d\log\mathscr{P}_\mathcal{R}}{d\log k}\right]^2$$

N.B. These results are valid for cases without features

3rd order action reprocessed

$\dot{\mathscr{R}}^3$ and $\dot{\mathscr{R}}^2\mathscr{R}$: cumbersome to compute with many derivatives

$$\int \dot{\mathscr{R}}^3 \sim \int \left(\dot{y}_0 + \dot{L}y_0 + L\dot{y}_0 + \cdots\right)^3 \sim ☹$$

Using partial int and linear eq to reduce the # of derivatives

$$\left.\frac{\delta L}{\delta \mathscr{R}}\right|_1 \equiv \frac{a^3\epsilon}{c_s^2}\left\{\ddot{\mathscr{R}} + \underbrace{\left[\frac{c_s^2}{a^2\epsilon}\frac{d}{dt}\left(\frac{a^2\epsilon}{c_s^2}\right) + H\right]}_{\equiv C = H(3+\eta-2s)}\dot{\mathscr{R}} - \frac{\Delta}{a^2}\mathscr{R}\right\}$$

$$\int A\dot{\mathscr{R}}^3 = \int \frac{\ddot{A} - 3\dot{A}C - 2A\dot{C} + 2AC^2}{2}\frac{d}{dt}\frac{\left(\mathscr{R}^3\right)}{3} + \cdots + \left.\frac{\delta L}{\delta \mathscr{R}}\right|_1 \frac{c_s^2}{a^3\epsilon}\left(\frac{\dot{A} - 2AC}{2}\mathscr{R}^2 + \cdots\right)$$

$$\int B\dot{\mathscr{R}}^2\mathscr{R} = \int \frac{-\dot{B} + BC}{2}\frac{d}{dt}\frac{\left(\mathscr{R}^3\right)}{3} + \cdots + \left.\frac{\delta L}{\delta \mathscr{R}}\right|_1 \frac{c_s^2}{a^3\epsilon}\frac{B}{2}\mathscr{R}^2$$

Field redefinition with more terms involved

$$S_3 = \int d\tau d^3x \underbrace{\frac{m_{\rm Pl}^2}{3}\frac{a^2\epsilon}{c_s}\left[-c_s aH\left(3s + \frac{\epsilon\eta}{2} + \epsilon s + 9us - 2s^2\right) - \frac{1}{2}\frac{d}{d\tau}\left(\frac{\eta}{c_s^2}\right)\right]}_{\equiv \mathfrak{C}}\frac{d}{d\tau}\left(\mathscr{R}^3\right) + (\text{higher SR terms})$$

1st order bispectrum in GSR

"Source" for the bispectrum

$$g_B(\log\tau) = \frac{c_s}{a^2 m_{\rm Pl}^2 \epsilon} \frac{-\tau}{f} \mathfrak{C} = \frac{1}{f}\left[c_s a H\left(3s + \frac{\epsilon\eta}{2} + \epsilon s + 9us - 2s^2\right) + \frac{1}{2}\frac{d}{d\log\tau}\left(\frac{\eta}{s}\right)\right]$$

Bispectrum up to 1st correction [i.e. $\mathcal{O}(g)$] (c.f. Adshead et al. 2011)

$$\begin{aligned}
B_{\mathcal{R}}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = & \frac{(2\pi)^4}{4}\frac{\sqrt{\mathcal{P}_{\mathcal{R}}(k_1)}}{k_1^2}\frac{\sqrt{\mathcal{P}_{\mathcal{R}}(k_2)}}{k_2^2}\frac{\sqrt{\mathcal{P}_{\mathcal{R}}(k_3)}}{k_3^2}\int_0^\infty \frac{d\tau}{\tau} g_B(\log\tau) \\
& \times\left\{\left(d_\tau - 3\frac{f'}{f}\right)W_B + \frac{1}{3}d_\tau\left(X_B + X_{B3}\right)\int_0^\infty\frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)X(k_3\tilde\tau) + 2\text{ perm}\right. \\
& - \frac{1}{3}d_\tau W_{B3}\int_\tau^\infty \frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)W(k_3\tilde\tau) - \frac{1}{3}d_\tau X_{B3}\int_0^\tau\frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)X(k_3\tilde\tau) + 2\text{ perm} \\
& \left. -\frac{1}{2}d_\tau\left(X_B + X_{B3}\right)\int_\tau^\infty\frac{d\tilde\tau}{\tilde\tau}g(\log\tilde\tau)\left(\frac{1}{k_3\tilde\tau} + \frac{1}{k_3^3\tilde\tau^3}\right) + 2\text{ perm}\right\} \quad \left(d_\tau \equiv \frac{d}{d\log\tau} + 3\right)
\end{aligned}$$

Window functions W_B and X_B constructed from y_0

Effective theory: source term g_B

Effective action (Cheung et al. 2008)

$$S_\pi = -\int d^4x a^3 \left\{ \dot{H}\left[\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right] - 2M_2^4\dot{\pi}\left[\dot{\pi} + \dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right] + \frac{4}{3}M_3^4\dot{\pi}^3 + \cdots \right\}$$

In principle M_n^4 are independent couplings (Achucarro et al. 2012a)

$$\frac{M_3^4}{M_2^4} = \begin{cases} -\frac{3}{4}\left(c_s^{-2}-1\right) & \text{for integrating out 1 heavy mode} \\ 3\left(c_s^{-2}-1\right) & \text{for DBI} \end{cases}$$

(c.f. JG, Pi & Sasaki 2013)

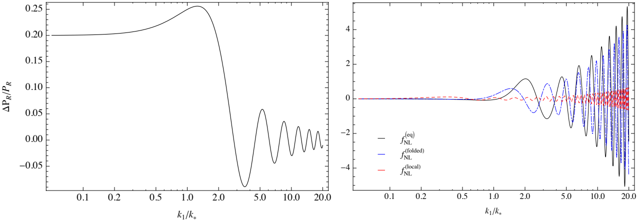
g_B can be explicitly connected to $\mathscr{P}_\mathscr{R}$ for specific cases (Achucarro et al. 2013)

Example: Starobinsky model

Linear $V(\phi)$ + sudden slope change (Starobinsky 1992)

$$V(\phi) = V_0 \times \begin{cases} \left[1 - A(\phi-\phi_0)\right] & \text{for} \quad \phi < \phi_0 \\ \left[1 - (A+\Delta A)(\phi-\phi_0)\right] & \text{for} \quad \phi > \phi_0 \end{cases}$$

de Sitter approx: $\dfrac{f'}{f} = -\dfrac{\ddot\phi}{H\dot\phi}$, $g = -3\dfrac{V''}{V}$, $g_B = \dfrac{1}{f}\dfrac{d}{d\log\tau}\left(\dfrac{\ddot\phi}{H\dot\phi}\right)$ (Choe, JG & Stewart 2004)



(cf. Arroja & Sasaki 2012)

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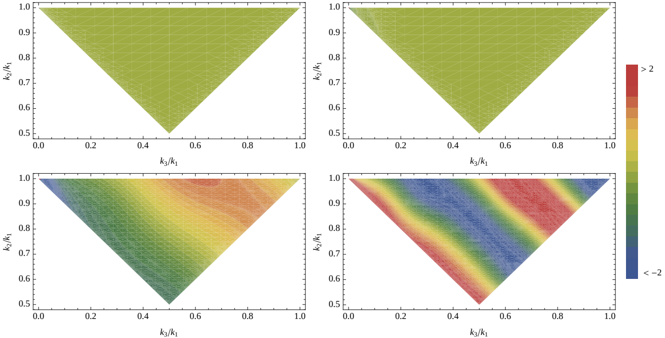


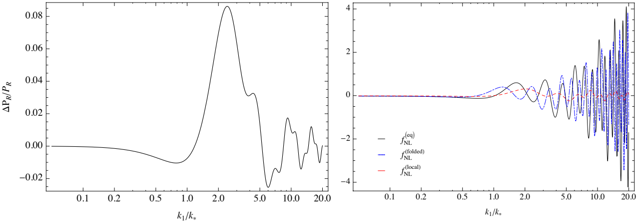
(cf. Arroja & Sasaki 2012)

Example: Top-hat change in c_s

c_s is changed for a limited interval by a small amount

$$\frac{1}{c_s^2} = 1 + \lambda\left[\theta(N-N_i) - \theta(N-N_f)\right] = \begin{cases} 1 & \text{for } N < N_i \\ 1+\lambda & \text{for } N_i < N < N_f \\ 1 & \text{for } N > N_f \end{cases}$$

f and g are given by: $\frac{f'}{f} = -\frac{s}{2}$, $g = \frac{3}{2}s$, $g_B = \frac{3}{f}s$



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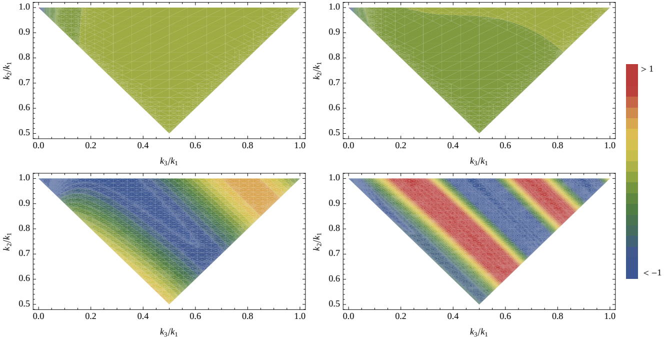

Summary

1. PLANCK suggests both lessons and challenges
   - Mostly single field slow-roll inflation
   - But not as simple as it seems
2. Correlated correlation functions
   - CMB outliers may arise from primordial features
   - GSR formalism: beyond standard SR including features
   - Correlation functions are correlated
   - Possible window to inflationary dynamics