QCD as a Hologram of a gravity: focused on viscosity and thermalization

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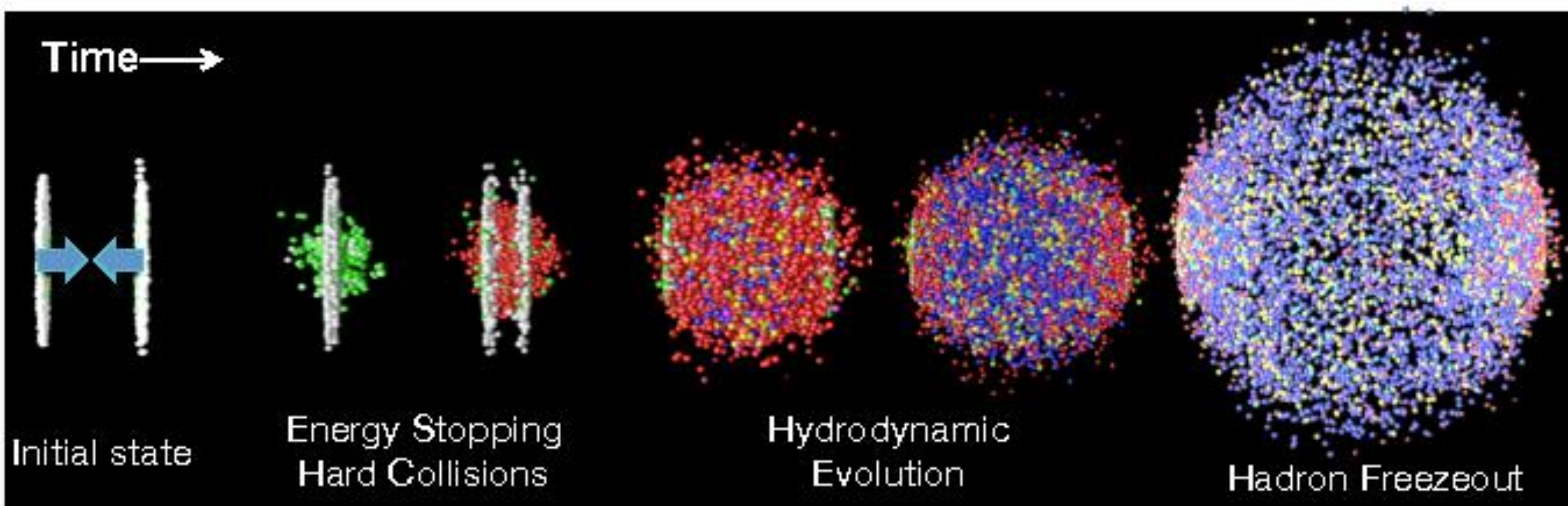
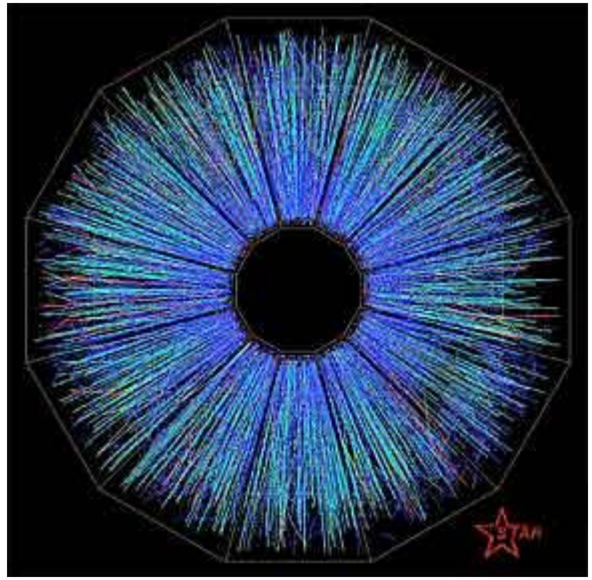
@kias, 2013.11.13

based on
arXiv:1302.1277 (PLB) with Eunseok Oh
and arXiv:1310.7179 by SJS)
And Presentations of D.T.Son, Romatschke

Puzzles in RHIC :

1. Low viscosity → Perfect liquid behavior
2. Early thermalization→Equilibrate in t ~ 0.2 fm/c





5000 particles strongly interacting!
→ Hydrodynamic description.

Hydrodynamics = Energy-Momentum Conservation

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = \eta \langle \nabla^\mu u^\nu \rangle,$$

Zeroth Order: Ideal Hydrodynamics (“Euler equation”)

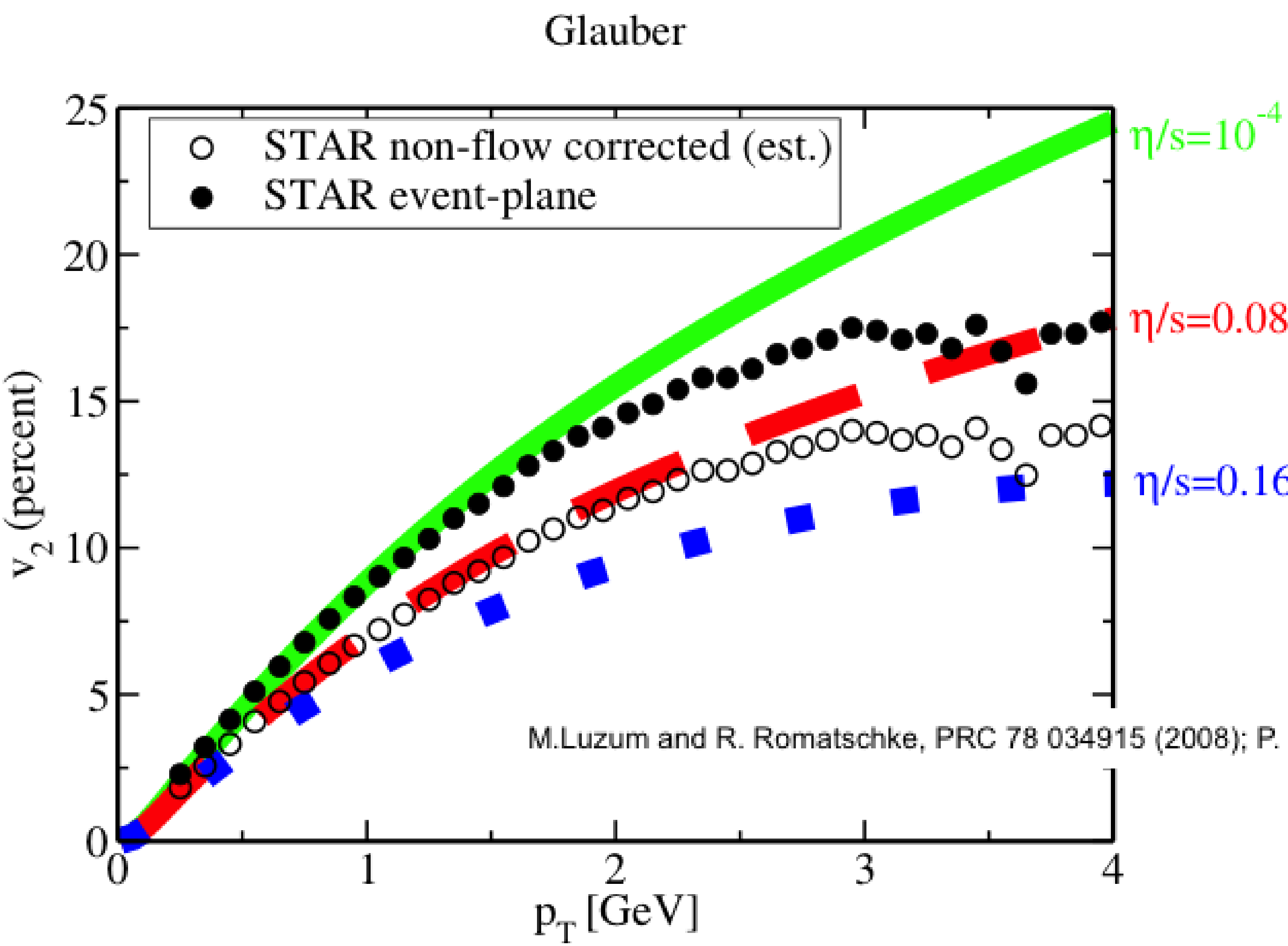
First-Order: Viscous Hydrodynamics (“Navier-Stokes equation”)

Ideal Hydro: $\Pi^{\mu\nu} = 0$, $Re = \infty$

$$Re \sim \frac{\epsilon + p}{|\Pi^{\mu\nu}|} \sim \frac{sT}{\eta / L}$$

RHIC fireball: almost ideal flow

Viscosity effect



M.Luzum and R. Romatschke, PRC 78 034915 (2008); P. Romatschke, arXiv:0902.3663.

$$\eta/s = 0.1 \pm 0.1(th.) \pm 0.08(exp)$$

Why Viscosity is so small?

- For pQCD:

$$\eta/s = \frac{C}{g^4 \ln 1/g}$$

which is too big to explain the rhic data.

→ Son et.al : Use string theory idea to solve it.

Duality

- Reformulate a theory with strong coupling in terms of weakly coupled theory.

- Best known: Kramers Wannier in 2d Ising.

$$\sinh 2K \cdot \sinh 2\tilde{K} = 1$$

For us, Use Gauge/gravity duality.

Gauge/gravity duality & RG

Describe all scale physics simultaneously ➔ Holography!
Einstein eq. encodes RG flow.

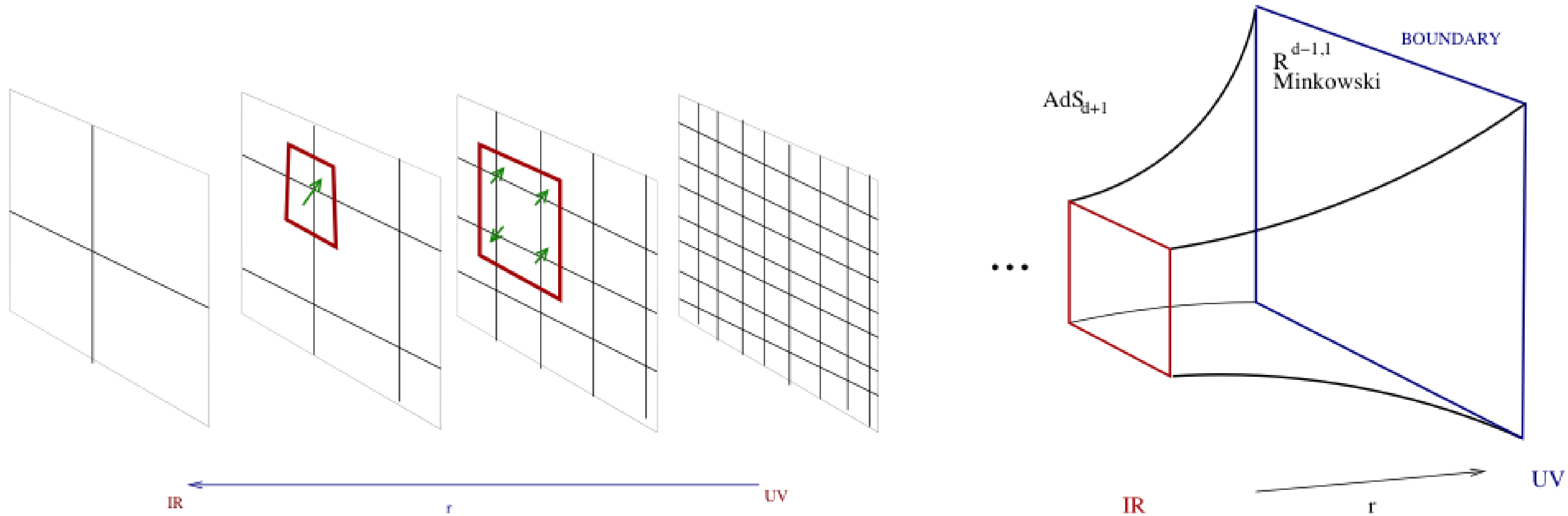


Figure 1. The left figure indicates a series of block spin transformations labelled by a parameter r. The right figure is a cartoon of AdS space, which organizes the field theory information in the same way. In this sense, the bulk picture is a hologram: excitations with different wavelengths get put in different places in the bulk image.

Figure from 1101.0597 by Faulkner et.al

Consequence of duality

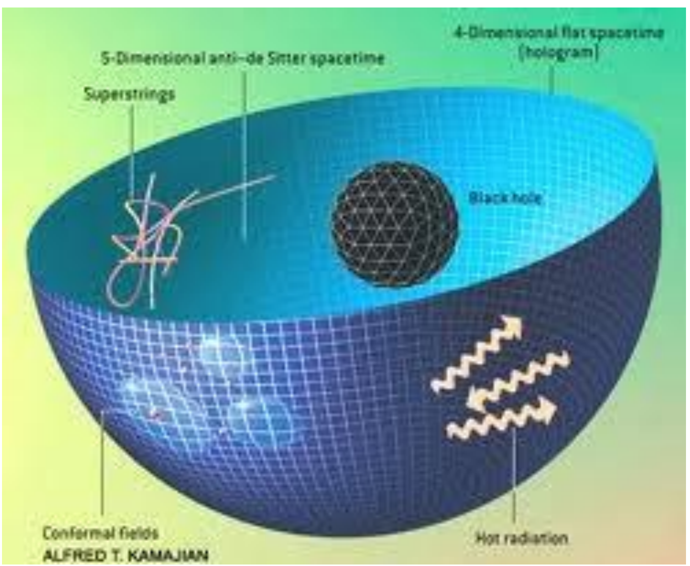
1. Gluon dynamics is replaced by ads gravity.

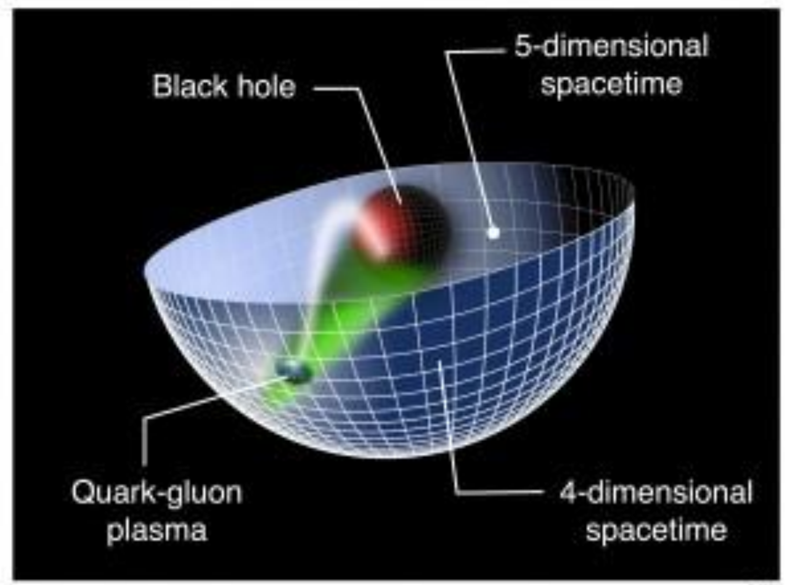
2. For large N, gravity is weakly coupled.

3. Calculability of Full Correlation function:

   by the classical dynamics at the 5 dim AdS.
   → holographic.

Dual of
Thermally Equilbriated state
is Black Hole in AdS.





Viscosity by Ads/cft

$$\eta = \lim_{\omega\to 0} \frac{1}{2\omega} \int dt\, d\vec{x}\, e^{i\omega t} \langle [T_{xy}(t,\vec{x}),\, T_{xy}(0,\mathbf{0})] \rangle \,.$$

Dual of graviton in AdS

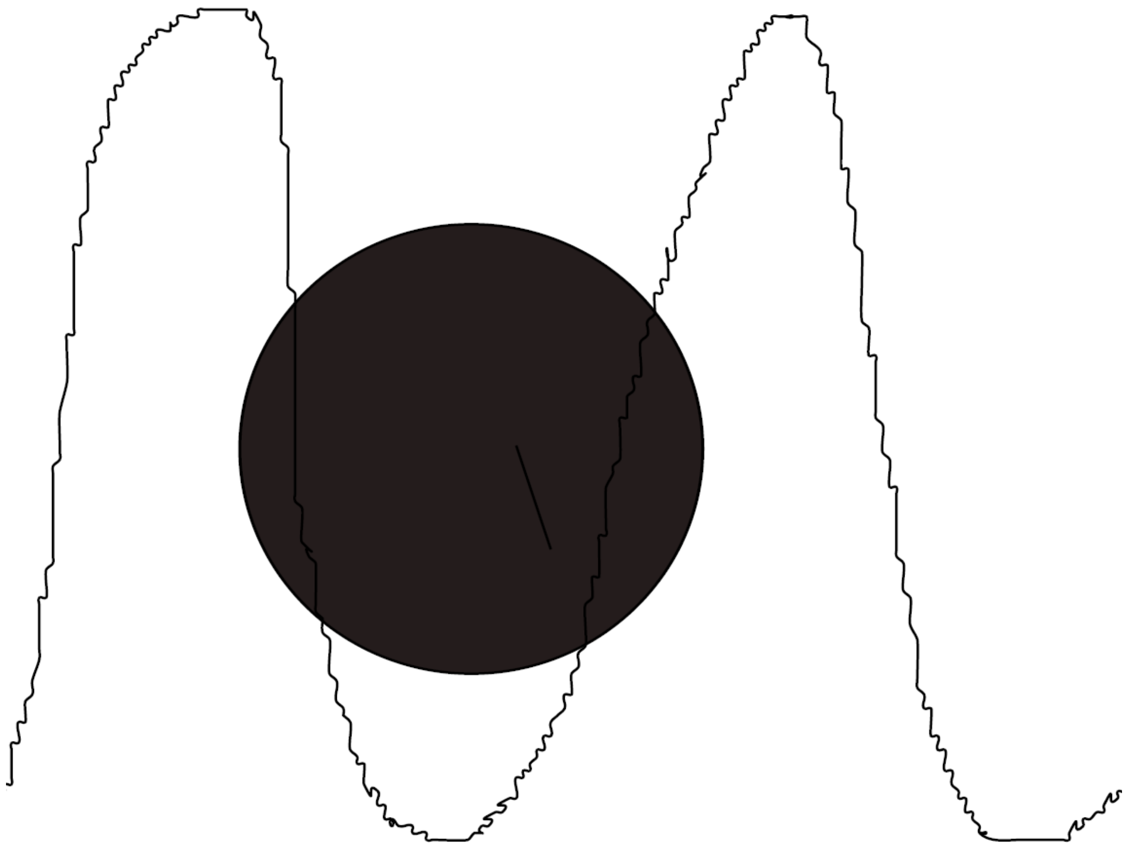
$$\begin{aligned} \sigma_{abs} &= -\frac{16\pi G}{\omega} \,\mathrm{Im}\, G^R(\omega) \\ &= \frac{8\pi G}{\omega} \int dt\, dx e^{i\omega t} \langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \rangle \end{aligned}$$

$$\eta = \frac{\sigma_{abs}(0)}{16\pi G} \qquad \sigma_{abs}(0) = A_H \qquad s = A_H/4G$$

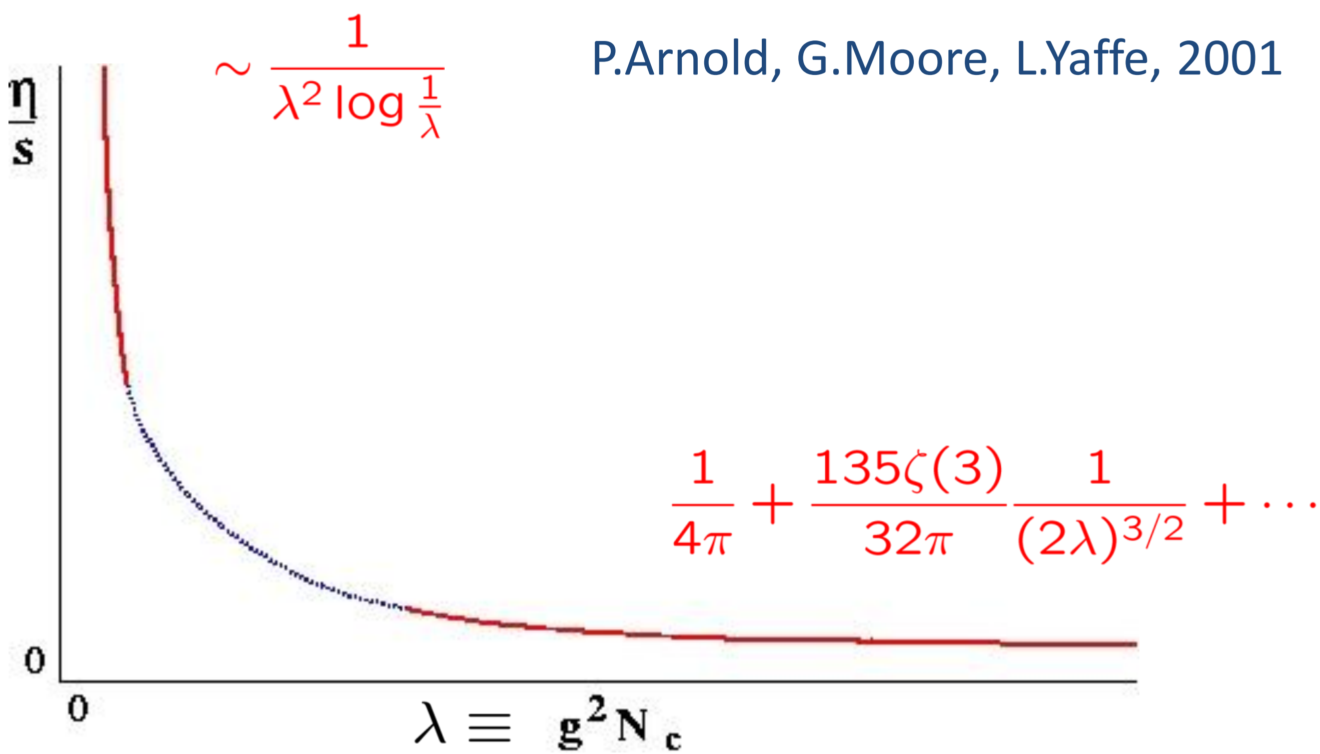
Bekenstein entropy

$$\eta = \frac{4G}{16\pi G} \frac{A_H}{4G} = \frac{1}{4\pi} s$$

Graviton scattering



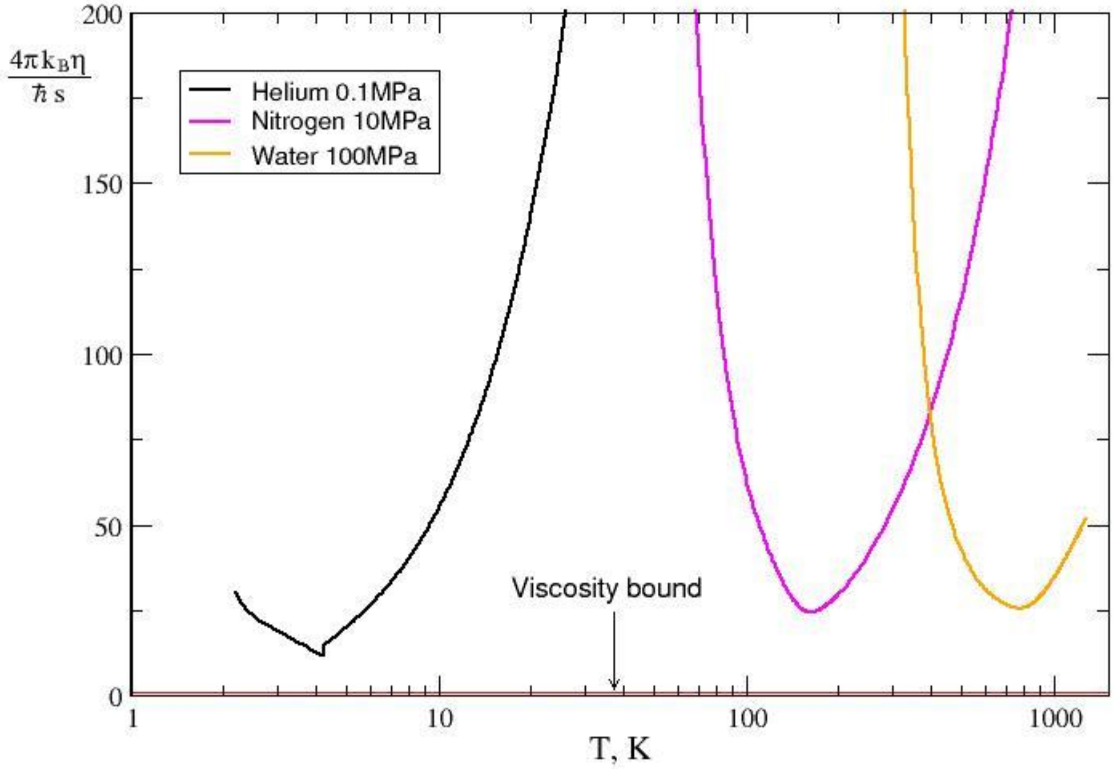
Shear viscosity in N=4 SYM



Correction to $1/4\pi$: A.Buchel, J.Liu, A.S., hep-th/0406264

conjecture : viscosity bound

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B} \approx 6.08 \cdot 10^{-13} K \cdot s$$

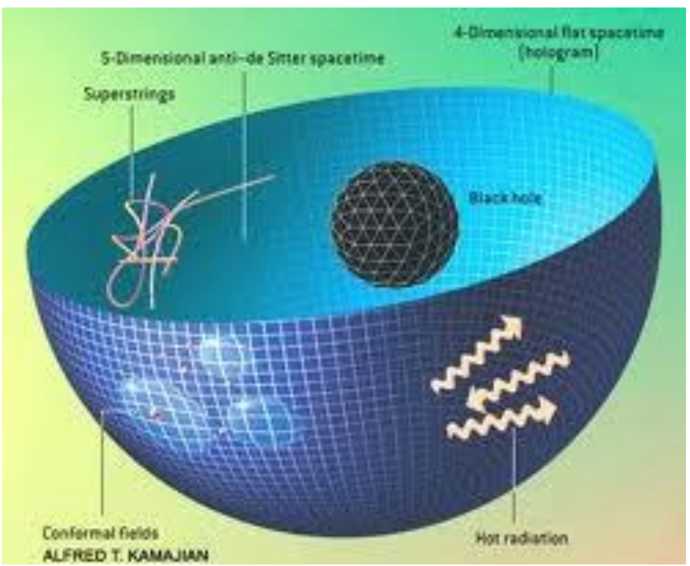


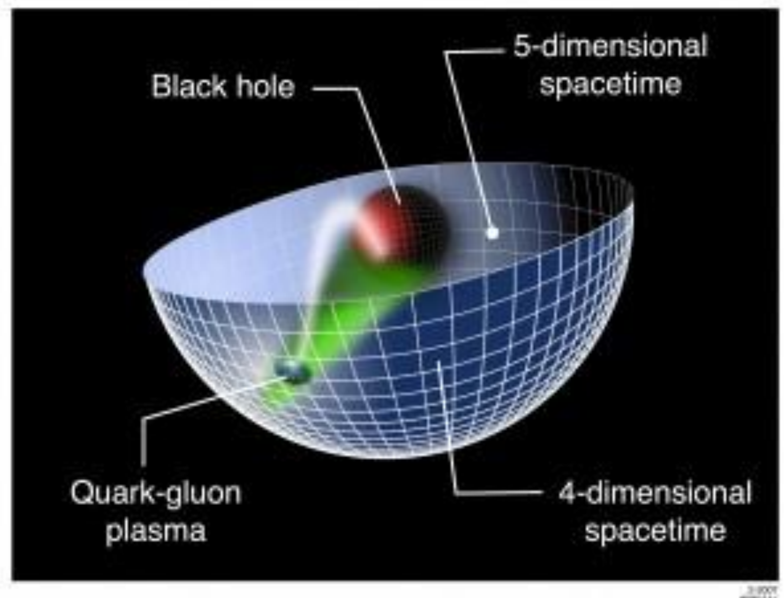
P.Kovtun, D.Son, A.Starinets, hep-th/0309213, hep-th/0405231

So far holography is applied to

- Viscosity
- Jet quenching
- Energy Loss
- Thermalization
- Elliptic flow
- EOS
- Non-fermi Liquid
- Hadron Spectrum,
- Heavy quarkonium
- Symmetry Energy
- Super conductivity

Dual of
Thermal Equilbriated state
is Black Hole in AdS.





Early thermalization = Easy BHformation

Why BH formation is easy in AdS ?

Huge list of works!

Aharony, Bizon, Chesler,…,Gubser, …, Horowitz, Liu, Minwalla,…. Yaffe, Zayas.

Spherical initial condition.

Experimentally, we can not fine tune the initial configurations and black hole forms without oscillation!

Thermalization: big subject. AdS instability

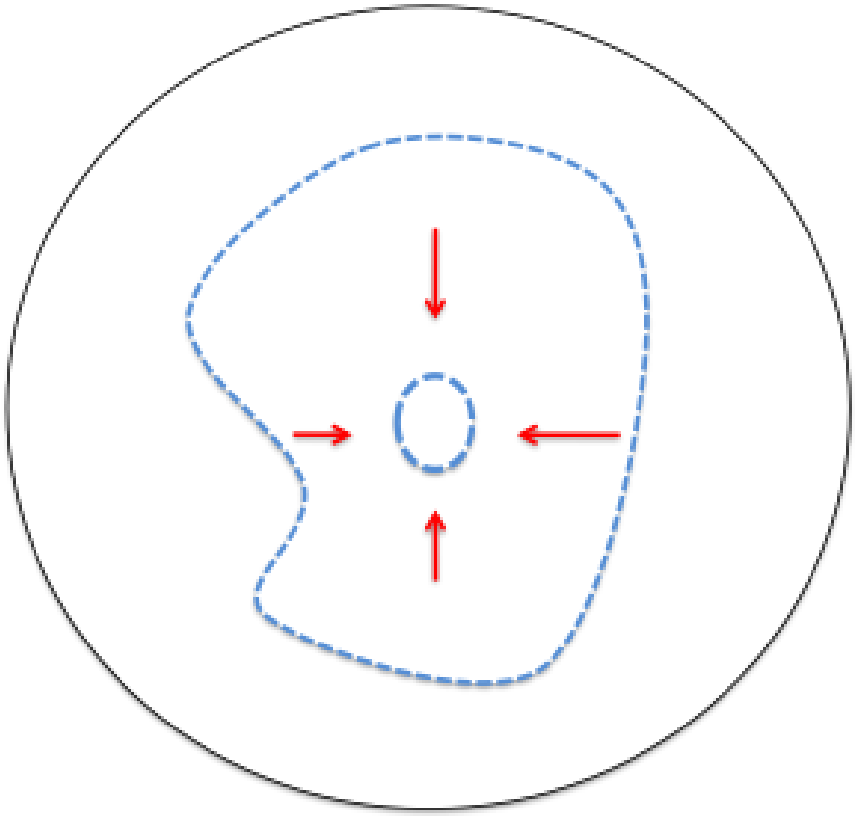
- 1. Unfortunately,
  All spherical collapse, it is fine tuned initial configuration →
  nothing to do with RHIC exp.

- 2. All used classical scalar field collapse to make GR solution doable, otherwise, difficult even in numerical level.

- 3. It turns out that even for spherical shape,
  it is not so easy to form a BH. It has to oscillate many times before BH formation, due to the wave property (Later).

- 4. Need to use particle picture, since
  classical field configuration is dual to the condensation and RHIC fireball is not.

- So the question is:
  Can a shell of generic shape in AdS collapse to form a BH in ONE dynamical time?



In AdS space

In flat space, this is impossible.

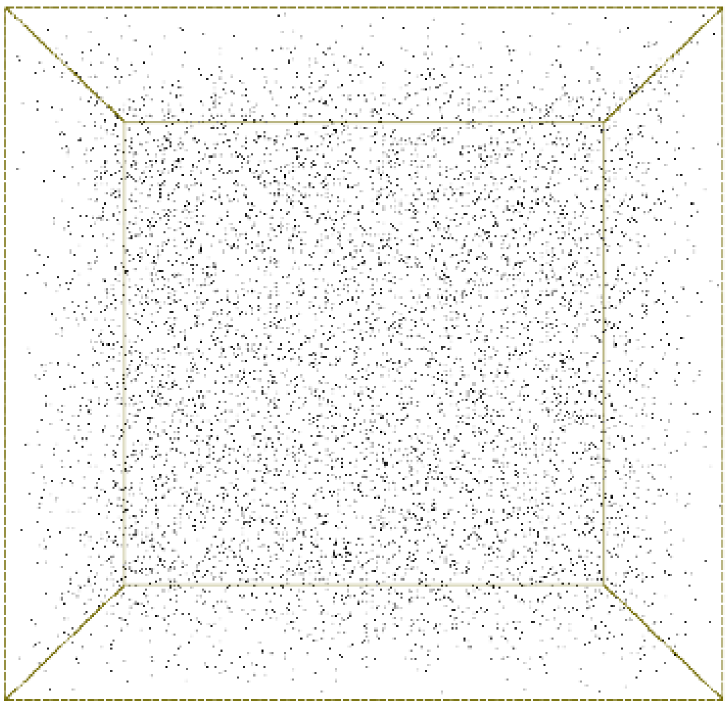


Figure 1: The system of particles after initialization.

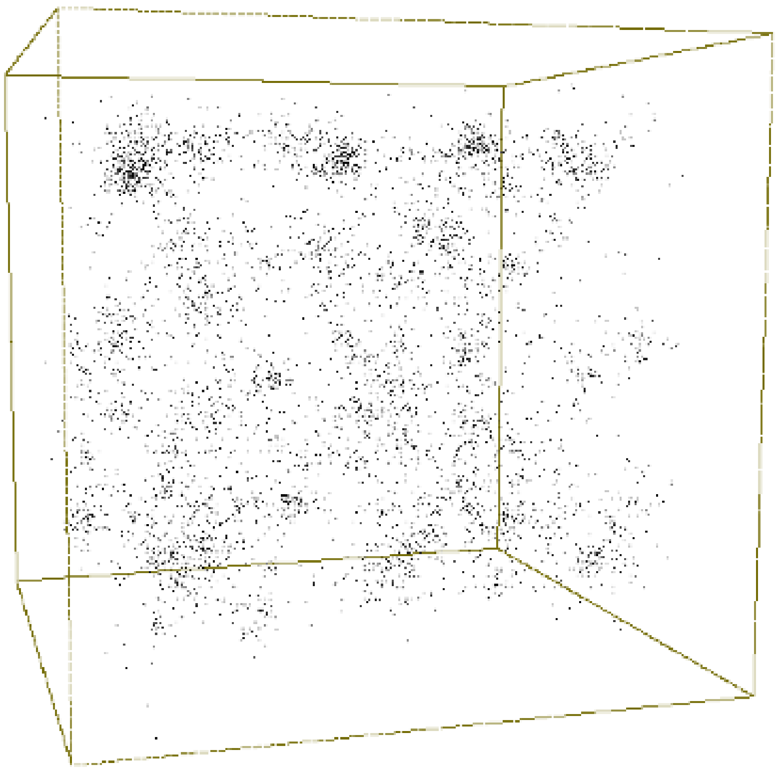


Figure 2: The system of particles after the simulation has run for 2.5 days nonstop.

Then Why possible in ads? Probe ads by test particles. 1302.1277

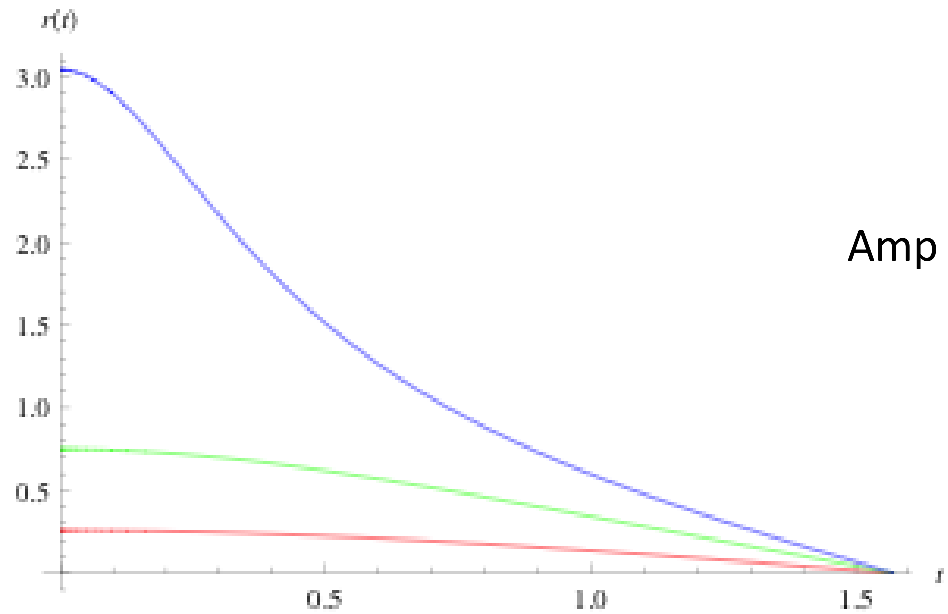
- Consider a free fall of a massive particle in ads.

$$ds^2 = -(1 + r^2/R^2)c^2dt^2 + r^2d\Omega^2 + \frac{dr^2}{1 + r^2/R^2}$$

$$S = -m\int\sqrt{-g_{\mu\nu}\dot{x^\mu}\dot{x^\nu}}dt, \text{ with } \dot{x} = \frac{dx}{dt}.$$

Eq. of M

$$\dot{r}^2 + (m/E)^2(1+r^2)^2(1+r^2-E^2/m^2) = 0$$



Amp independence of Period

Figure 1. Left: Free falling of a particle in AdS

Exact Solution

$$r = \frac{v_c \cos t}{\sqrt{1 - v_c^2 \cos^2 t}}.$$

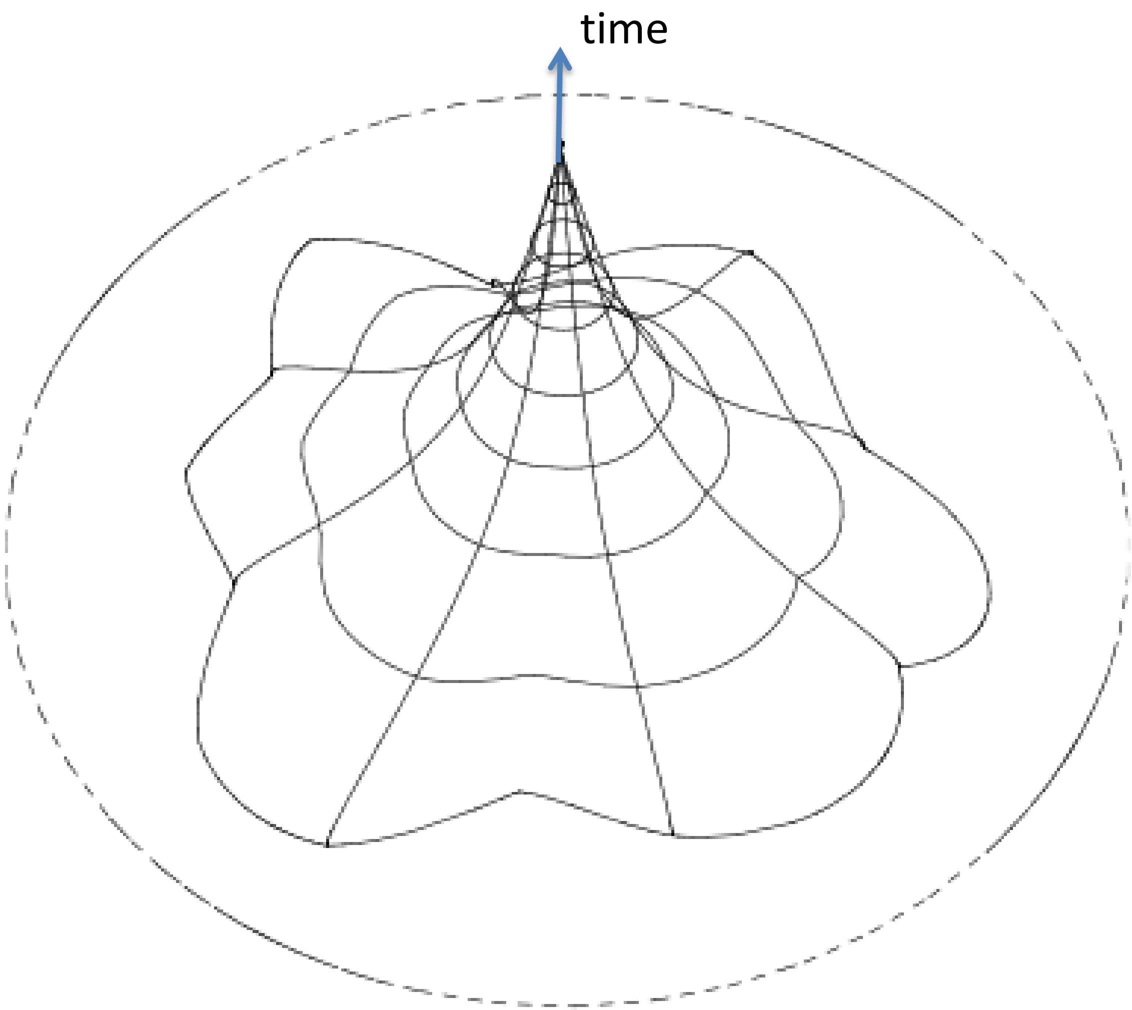
$$E = \frac{m}{\sqrt{1-v_c^2}}$$

r_0 is the initial radial position. v_c is the velocity at the center.

Falling time is the same independent of the initial condition of particle.

$$T_{fall} = \frac{\pi}{2}\frac{R}{c}.$$

Focusing effect in (Global) AdS



Two consequences:

1. Thick shell → thin shell (E. Cascade)
2. Beast → Beauty (Isotropization/Hydronization)

- Any shape of dust particle distribution will collapse to make a Black hole at the time when the shell pass its would-be-horizon.
  It is less than one Falling time.

- This is the mechanism of Early Thermalization.

Evidences for THE mechanism of early Thermalization? It can answer at least following questions.

- Q1. Why dust ?
- Q2. Why not wave? Explain dispersive nature.
- Q3. Why fall?
- Q4. Why non-interacting particle?
- Q5. Poincare patch?
- Q6. What is the initial velocity effect? Precise dictionary?
- Q7. What happen if we add interaction? Still yes.
- Q8. Any prediction? Two Time scales.
- Q9. Where is entropy generation/irreversibility?
- Q10. Is this mechanism universal? Yes
- Q11. What is wrong with Shock wave? Not encode the particle creation.
- Q12. Can we understand the eventual formation of BH wave collapse following the chaotic instability? Yes.

1. Why dust? BH at once.
2. What if wave? Cascade.

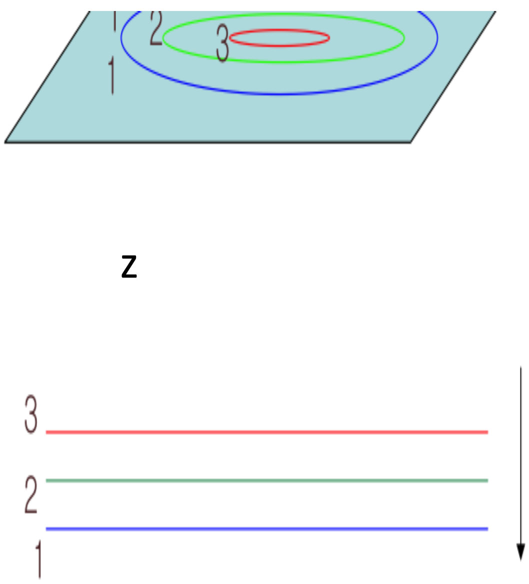
- Why the holographic image of the created particles is the dust in the bulk.

Dust$\rightarrow$ BH formation at once.
Wave$\rightarrow$ oscillation and cascade:
Difficult to localize the wave....
Uncertainty relation.
Pressure develop$\rightarrow$ mechanism of dispersion.
Thinnering$\rightarrow$ mechanism of cascade.

Experiment says, BH at once that is why it should be dust.

3. Why Fall ? It equilibrates after some Expansion

- Accodring to UV/IR relation,
  Expansion @bdry is dual to the Falling in bulk.



3. Why non-interacting particles ?

- Major component of the gluon mediated interactions are transformed away as background gravity

  → dusts are weakly interacting closed string in ads background. For BH formation, it is enough to consider the cm of the strings.

4. How about Poincare Patch?

- Motion not periodic. Falling time is infinite.

$$r = \frac{\epsilon R}{\sqrt{(\epsilon t/R)^2 + 1}} = R^2/t - (R/t)^3/2\epsilon^2 + \mathcal{O}(t^{-5}),$$

- Initial condition dependence rapidly disappear. → Synchronization effect is still there.

- Formation of trapped surface in “finite time”.

Precise dictionary?

- holographic image of a created   particle is at r> 0. Where?

$$\frac{m(1+r^2)}{\sqrt{1+r^2-\dot{r}^2/(1+r^2)}} = E.$$

If we want to attribute the energy due to a static particle in a bulk, we need to locate it at r_0 given bt

$$E = m\sqrt{1+r_0^2},$$

Dictionary in Poincare Patch

$$\frac{mr^2}{\sqrt{r^2(1-\dot{x}^2)-\dot{r}^2/r^2}} = E, \quad \frac{mr^2\dot{x}}{\sqrt{r^2(1-\dot{x}^2)-\dot{r}^2/r^2}} = p,$$

$$p = r_0 \frac{mv_x}{\sqrt{1-v_x^2}}$$

$$E = r_0 \frac{m}{\sqrt{1-v_x^2}}$$

$$p = r_0 \tilde{p}$$

$$\tilde{p} = p/r_0$$

This is nothing but the ansatz of Polchinski+Strassler (hep-th/0109174)

Notice that p, E are conserved one. v_x are language of Bulk.
The radial velocity goes to 0 unlike the global coordinate.
It is clear that r_0 is the parameter which sets the energy scale.

5. Effect of initial velocity

- Holographic Image at its creation moment does not have radial velocity.
- Need to work in Poincare patch.

$$\frac{mr^2}{\sqrt{r^2(1-\dot{x}^2)-\dot{r}^2/r^2}} = E, \frac{mr^2\dot{x}}{\sqrt{r^2(1-\dot{x}^2)-\dot{r}^2/r^2}} = p, \quad V = p/E.$$

$$r = \frac{\epsilon(1-V^2)}{\sqrt{1+(\epsilon(1-V^2)t)^2}}.$$

Remarkably the large time behavior of the radial position is independent of

all of the initial conditions m, E, p.

6. Residual Interaction effect

- Newtonian potential in AdS.

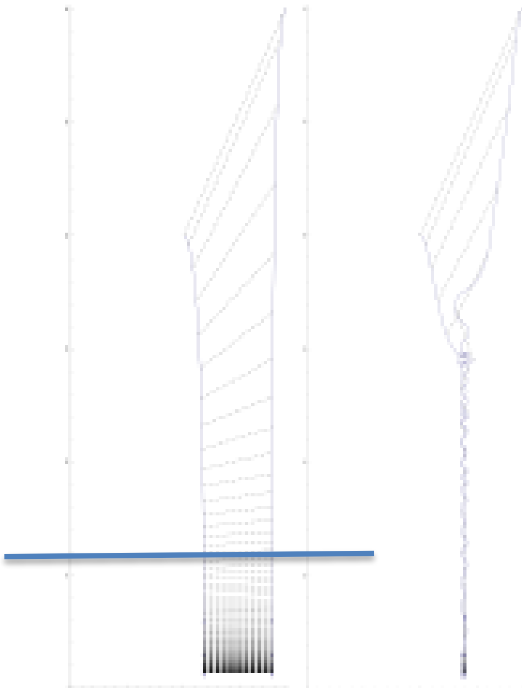
$$V(\{x_i, y_i\}) = \int\int d^5x d^5y J(x) G(x,y) J(y),$$
$$= G_N \sum_{i<j} \int dt \frac{(x_{i0} x_{j0})^{2\Delta}}{\left(|x_i(t) - x_j(t)|^2 + |x_{i0}(t) - x_{j0}(t)|^2\right)^{\Delta - 1/2}}$$

from $\quad G \sim \left(\frac{1}{u(2+u)}\right)^{\Delta}$ with $u = \frac{(x-y)^M (x-y)_M}{2x_0 y_0}$

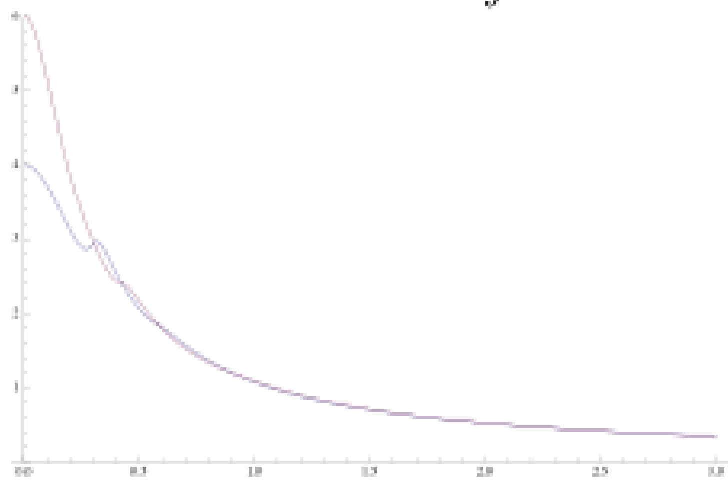
$$ds^2 = \frac{1}{x_0^2}(dx_0^2 + dx^\mu dx_\mu), \text{ with } x_0 = 1/r$$

6. Interaction effect I:

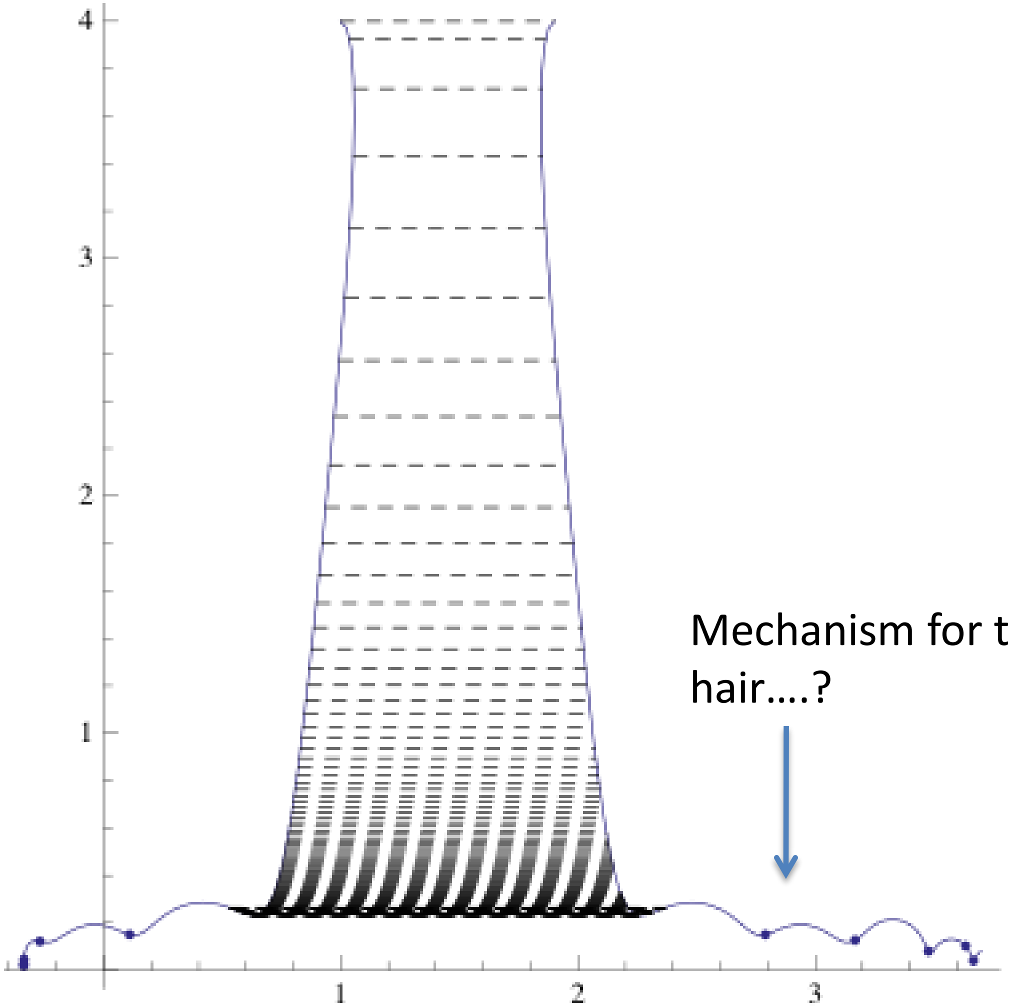
- Interaction is not so important if attractive.



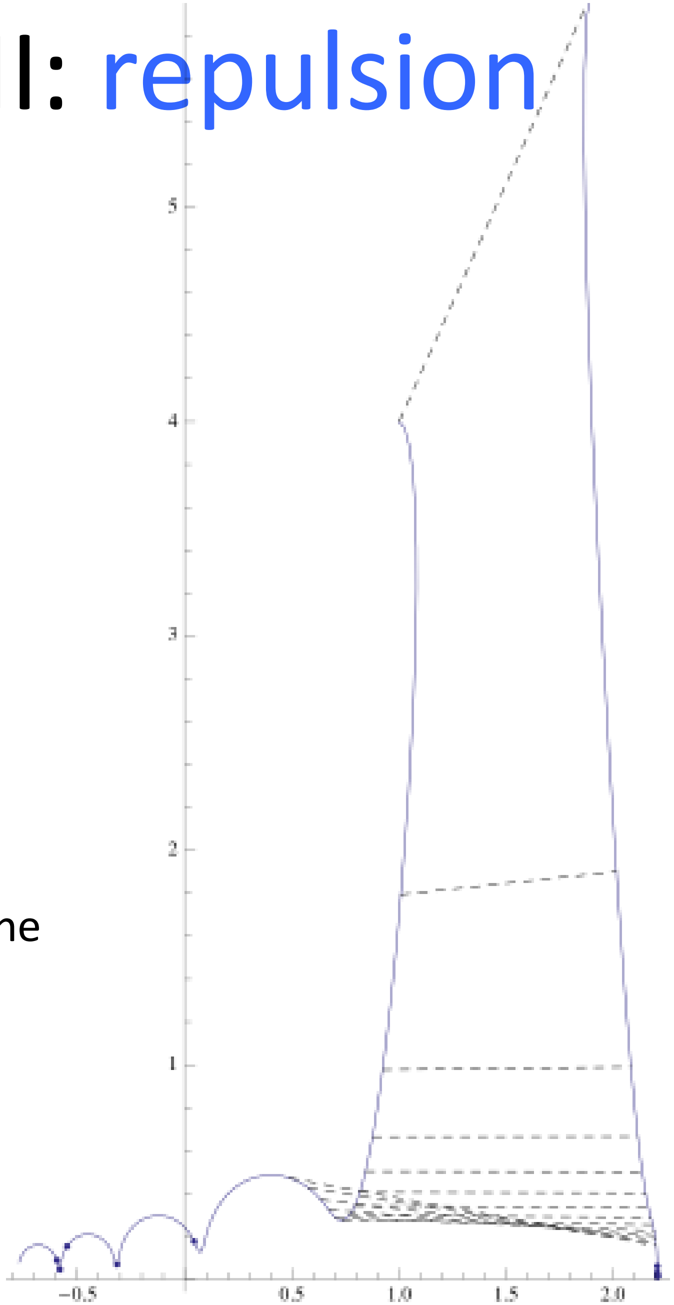
$$S = -m \int \sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}\,dt, \; +V(\{x_i, y_i\})$$



Interaction effect II: repulsion



Mechanism for the hair….?



7. Prediction: Time scales

- Thermalization time < One Dynamical time ,

$$t_{TH} = \sqrt{\frac{1}{r_H^2} - \frac{1}{r_0^2}} = \frac{1}{\pi T} \cdot \sqrt{1 - \frac{(\pi T m)^2}{E^2}}.$$

Soft mode thermalizes first and Hard one later.

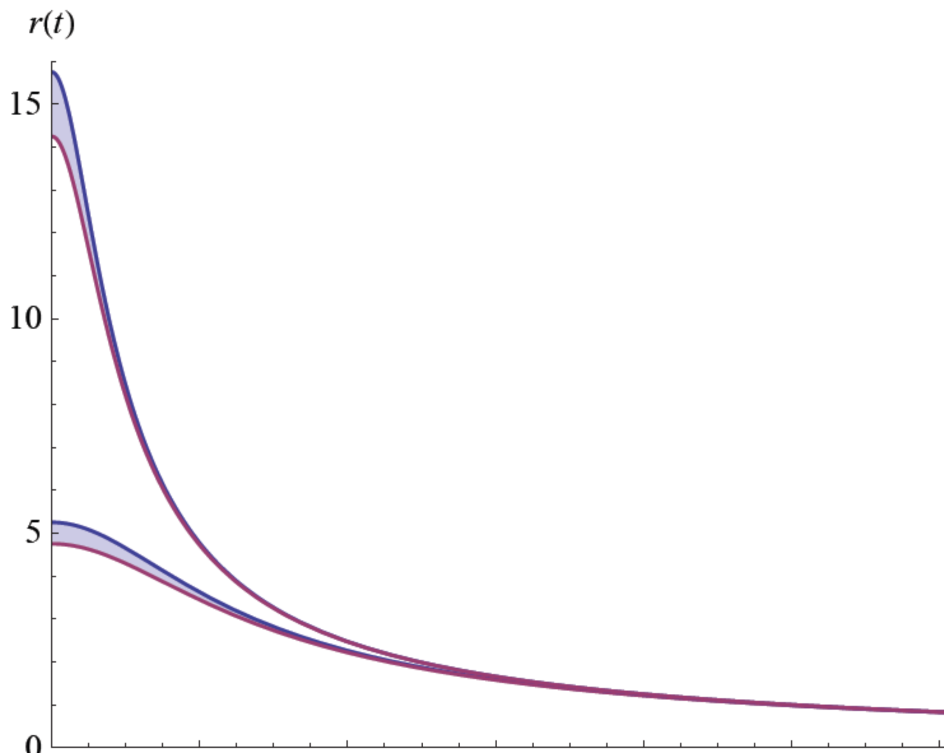
Hydro-nization (Isotropization):

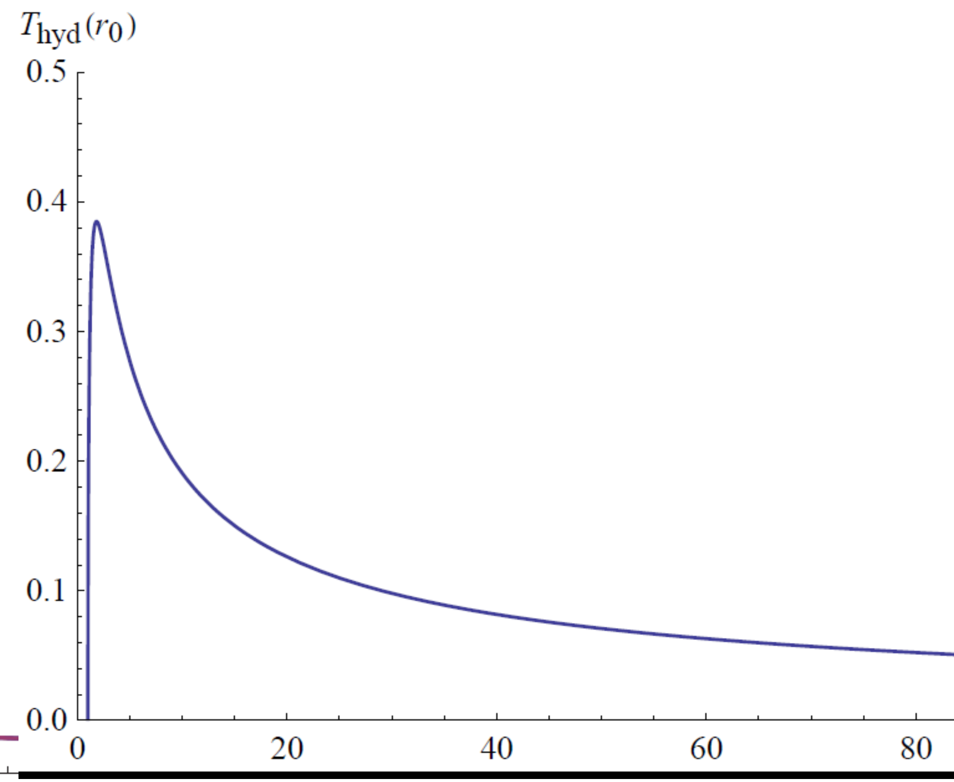
Hard first and then Soft.

higher particles rapidly catches up lower ones. But never takeover.

$$t_{hy} = \frac{(m/\pi^2)^{1/3}}{E^{1/3} T^{2/3}}$$

Speed of Hydronization





8. Where is irreversibility and entropy generation?

- All are classical motion, but …..

- the moment of the horizon formation

- 9. back reaction of gravity? It is regime of numerical relativity.

- 10. Is this mechanism universal for other background? YES!

- 11. Why not Shock wave?
  It is not a competing with our senario.
  It may give a reasonable BC.
  Particle creation is not encoded.

Fall in Poincare Patch of AdS ~ Those in near horizon limit of Dp

For D_p brane case one can show that

$$r(t) = (\beta/t)^{\beta} + \cdots, \quad with \;\; \beta = \frac{2}{5-p}, \quad \text{for } p < 5. \tag{3.8}$$

The difference in the initial condition will decay away with power law. Using the hypergeometric function, we can express the exact solution:

$$t = \frac{1}{\alpha - 1}\left[\frac{1}{r^{\alpha-1}}\, {}_2F_1\left(\frac{1}{\alpha} - 1, \frac{1}{2}, \frac{1}{\alpha}; \frac{r^{\alpha}}{r_0^{\alpha}}\right) - \frac{\Gamma(\frac{1}{\alpha})\Gamma(\frac{1}{2})}{\Gamma(\frac{1}{\alpha} - \frac{1}{2})}\frac{1}{r_0^{\alpha-1}}\right],$$

where $\alpha = \frac{7-p}{2}$ and $r_0^{\alpha} = \epsilon^2$. For p=5, we can find an simple exact solution

$$r(t) = r_0/\sinh^2(t/2), \tag{3.9}$$

12. Can we understand Cascade in wave collapse?

1 scalar field collapse → turbulant instability
Wave is hard to localize!
This in particle QM is uncertainly principle.

2 One of the corollary of our lemma is that
Thick shell → Thin shell.
Each time the shell falls with oscillation, it becomes thiner and thiner. It is not reversible due to the interaction.

Conclusion

- Agravity can give an understanding why rapid thermalization is canonical for strongly interacting system.
- This mechanism is likely universal for any gravity dual background.
  I checked this for near horizon geometry of Dp, M2,M5 branes.
- Future problem: Can we use this to understand the Chaos?

Thank you