Two-loop corrections to $(g-2)_\mu$ in SM und MSSM

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with
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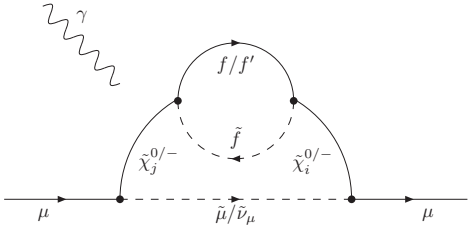


TU Dresden

KIAS Seoul, 12. November 2013

Motivation

$$a_\mu^{E821} = (11659208.9 \pm 6.3) \times 10^{(-10)}$$ [Bennett et al. '06]

New experiment: moving from BNL to Fermilab
E969
$0.54 \rightarrow 0.14$ ppm

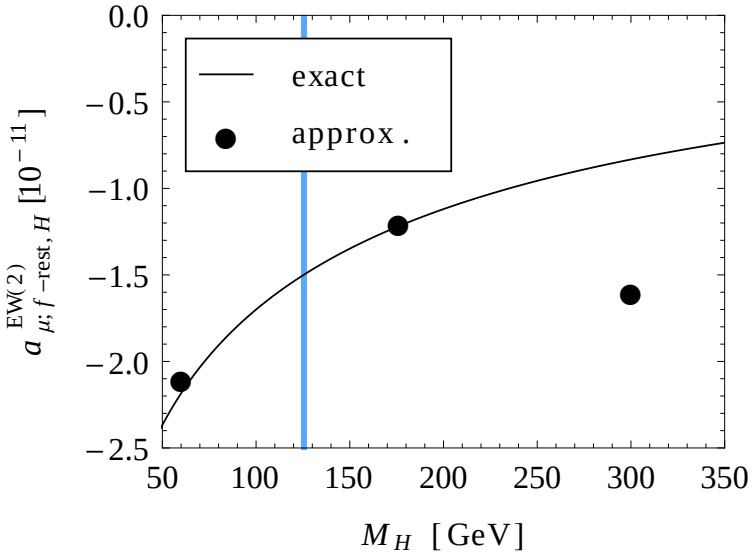
current accuracy of a_μ^{SM}: 0.42 ppm [Davier et al. '10]

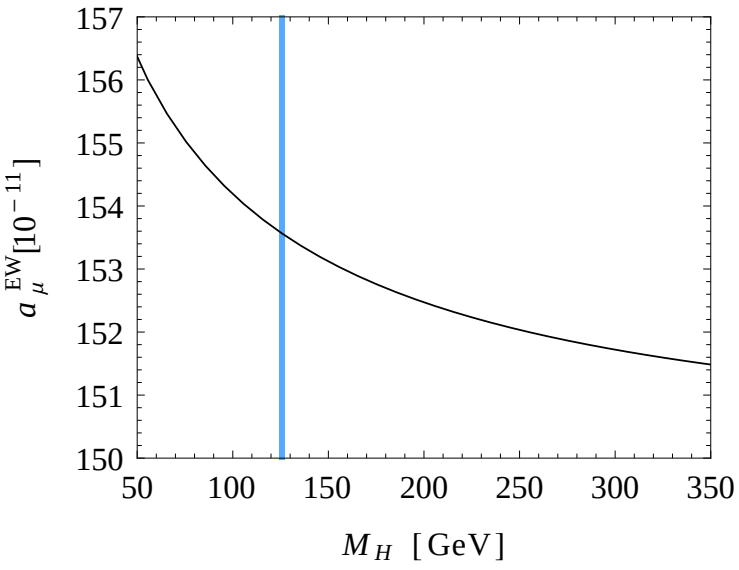
motivated to improve the accuracy of all aspects of the theory prediction, in SM and SUSY

Outline

Recent SM theory progress

- QED 5-loop calculation completed [Aoyama, Hayakawa, Kinoshita, Nio '12]
- Convergence of hadronic contributions [Davier et al., Hagiwara et al., Benayoun et al.]
- Electroweak (full 2-loop) contributions with $M_H = 126\mathrm{GeV}$:
  $a_\mu^{EW} = (153.6 \pm 1.0) \times 10^{-11}$ [Gnendiger, Stöckinger, S-K '13]





approx.[Czarnecki, Krause, Marciano]

a_μ as important constraint on SUSY

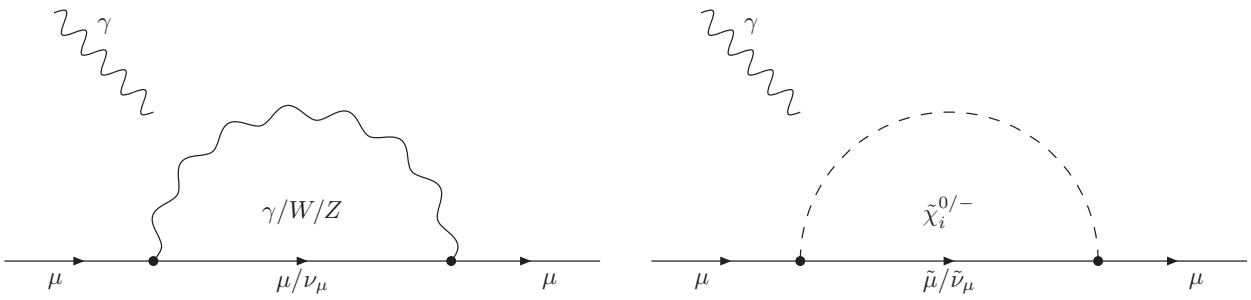
$$\Delta a_\mu(\mathrm{E821}-\mathrm{SM}) = \begin{cases} (28.7 \pm 8.0) \times 10^{-10} & \text{[Davier et al.]} \\ (26.1 \pm 8.0) \times 10^{-10} & \text{[Hagiwara et al.]} \end{cases}$$

$\boxed{\sim 3\sigma}$ deviation

tension between LHC-limits and a_μ

- a_μ is an important constraint on SUSY
- some scenarios (e.g. Constrained MSSM) cannot explain a_μ deviation
- still, large contributions possible, e.g. if sleptons $<<$ squarks (non-traditional models)

Already known SUSY corrections



- SUSY one-loop diagram: $\mu, M_1, M_2, M_E, M_L, \tan\beta$

  [Fayet '80]...[Moroi '96]

- SUSY two-loop corrections to SM 1L diagrams

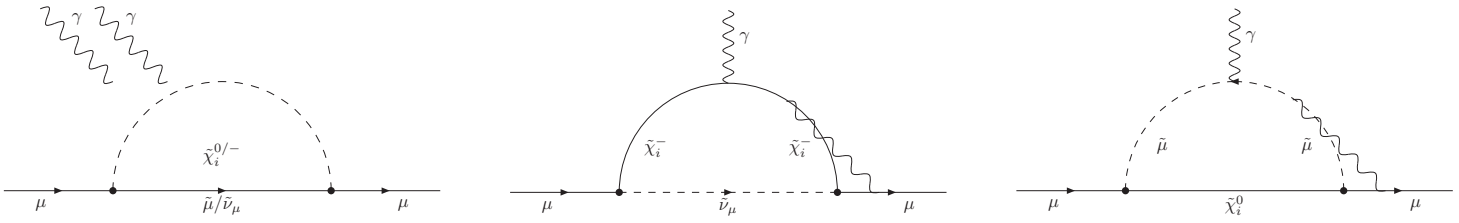
  [Chen, Geng '01],[Arhib, Baek '02],[Heinemeyer, Stöckinger, Weiglein '03, '04]

- Photonic corrections to SUSY 1L diagrams: $\mu, M_1, M_2, M_E, M_L, \tan\beta$

  [v. Weitershausen, Schäfer, Stöckinger, S-K '10]

estimated resulting MSSM theory error $\sim 3 \times 10^{-10}$ [Stöckinger '06]

MSSM photonic two-loop corrections

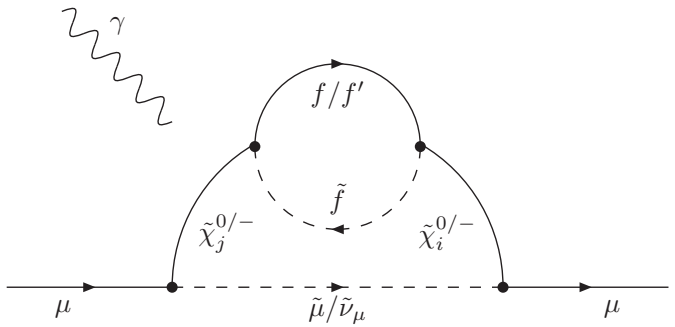


[v. Weitershausen, Schäfer, Stöckinger, S-K '10]

- exact results of photonic two-loop corrections
- same parameter dependence as SUSY one-loop diagram
- reproduces the leading QED-logarithms: $\frac{4\alpha}{\pi}\log\frac{m_\mu}{M_{SUSY}}a_\mu^{1\mathrm{L}SUSY}$ [Degrassi, Giudice '98]
- - (7...9)% corrections for $100 < M_{SUSY} < 1000\,\mathrm{GeV}$

MSSM fermion/sfermion two-loop corrections

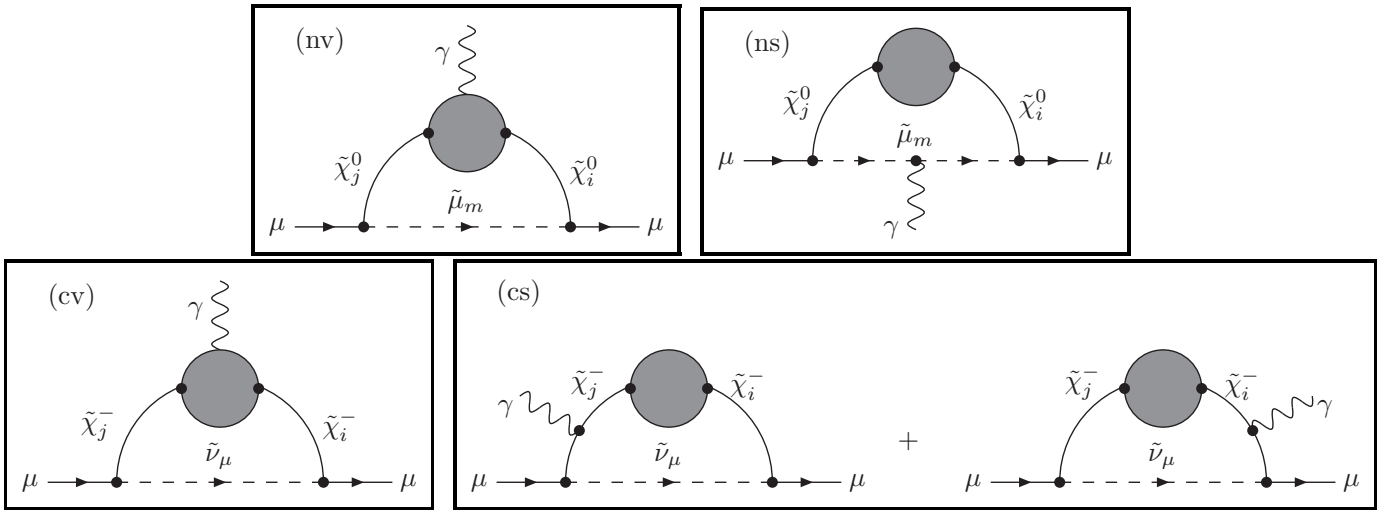
[Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]



- remaining class with dependence on squarks
- $(f\,,\tilde{f}) \in \{(\nu,\tilde{\nu}),(l,\tilde{l}),(u,\tilde{u}),(d,\tilde{d})\}$,
  $(f',\tilde{f}) \in \{(l,\tilde{\nu}),(\nu^c,\tilde{l}^\dagger),(d,\tilde{u}),(u^c,\tilde{d}^\dagger)\}$
- maximum complexity: 5 heavy + 2 light scales
- computed exactly, including renormalization
- additional parameter dependence:
  $M_{Q_i}, M_{U_i}, M_{D_i}, M_{L_i}, M_{E_i}, \qquad i \in \{1,2,3\}$

MSSM fermion/sfermion two-loop corrections

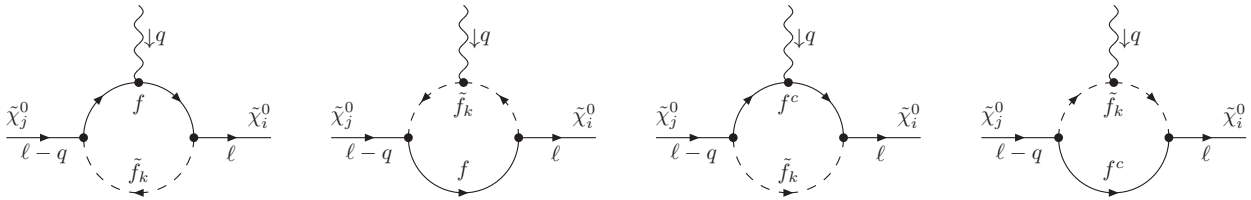
[Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]



- iterated one-loop calculation
- 5-mass-scale two-loop diagrams and their counterterm diagrams
- the sum of the inner loops build up compact vertices: $i\Gamma_{ji}^{0/-}$, $i\Sigma_{ii}$

MSSM fermion/sfermion two-loop corrections: Γ^0_{ji}

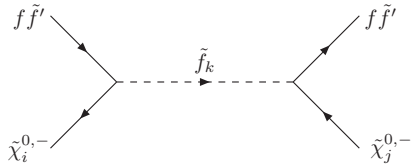
[arXiv:1311.1775][Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]



$$\Gamma^{0\mu}_{ij\tilde{f}_k}(\ell) = \frac{1}{16\pi^2} e Q_f \int_0^1 \frac{\mathrm{d}w}{2} \left[\left(\overline{\mathcal{A}}^{n+}_{ij\tilde{f}_k} - \overline{\mathcal{A}}^{n-}_{ij\tilde{f}_k} \gamma^5 \right) \frac{\not{\ell}\not{q}\gamma^\mu - \not{\ell} q^\mu + \not{q}\ell^\mu - (\ell \cdot q)\gamma^\mu}{\mathcal{D}_{f\tilde{f}_k}(\ell)} + \left(\overline{\mathcal{B}}^{n+}_{ij\tilde{f}_k} - \overline{\mathcal{B}}^{n-}_{ij\tilde{f}_k} \gamma^5 \right) \frac{m_f}{w} \frac{\not{q}\gamma^\mu - q^\mu}{\mathcal{D}_{f\tilde{f}_k}(\ell)} \right].$$

$$\mathcal{A}^{z\pm}_{ij\tilde{f}_k} \equiv z^L_{i\tilde{f}_k} z^{L*}_{j\tilde{f}_k} \pm z^R_{i\tilde{f}_k} z^{R*}_{j\tilde{f}_k}, \qquad \overline{\mathcal{A}}^{n\pm}_{ij\tilde{f}_k} \equiv \mathcal{A}^{n\pm}_{ij\tilde{f}_k} \mp \mathcal{A}^{n\pm *}_{ij\tilde{f}_k},$$

$$\mathcal{B}^{z\pm}_{ij\tilde{f}_k} \equiv z^L_{i\tilde{f}_k} z^{R*}_{j\tilde{f}_k} \pm z^R_{i\tilde{f}_k} z^{L*}_{j\tilde{f}_k}, \qquad \overline{\mathcal{B}}^{n\pm}_{ij\tilde{f}_k} \equiv \mathcal{B}^{n\pm}_{ij\tilde{f}_k} \mp \mathcal{B}^{n\pm *}_{ij\tilde{f}_k}, \qquad z \in \{c, n\}$$



MSSM fermion/sfermion two-loop corrections: sample analytic result

[arXiv:1311.1775][Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]

$$a^{(nv)}_{\mu\, ijm\tilde{f}_k} = \int_0^1 \mathrm{d}w \left[\mathcal{A}^{n+}_{ji\tilde{\mu}_m} \left(\overline{\mathcal{A}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{nv+}_{AA} + \overline{\mathcal{B}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{nv+}_{AB} \right) + \mathcal{B}^{n+}_{ji\tilde{\mu}_m} \left(\overline{\mathcal{A}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{nv+}_{BA} + \overline{\mathcal{B}}^{n+}_{ij\tilde{f}_k} \mathcal{T}^{nv+}_{BB} \right) \right.$$
$$\left. + \mathcal{A}^{n-}_{ji\tilde{\mu}_m} \left(\overline{\mathcal{A}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{nv-}_{AA} + \overline{\mathcal{B}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{nv-}_{AB} \right) + \mathcal{B}^{n-}_{ji\tilde{\mu}_m} \left(\overline{\mathcal{A}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{nv-}_{BA} + \overline{\mathcal{B}}^{n-}_{ij\tilde{f}_k} \mathcal{T}^{nv-}_{BB} \right) \right]$$

where

$$\mathcal{T}^{nv\pm}_{AA} = \left(\frac{1}{16\pi^2} \right) \frac{N_C Q_f}{4} \frac{m_\mu^2}{m^2_{\tilde{\mu}_m}} \frac{m_i}{m_i \mp m_j}$$
$$\times \left[\frac{1 - N_{f\tilde{f}_k} + l_{f\tilde{f}_k} N^2_{f\tilde{f}_k}}{(1 - N_{f\tilde{f}_k})^2 (N_{f\tilde{f}_k} - N_i)} - \frac{1 - N_i + l_i N_i^2}{(1 - N_i)^2 (N_{f\tilde{f}_k} - N_i)} \right] + (i \leftrightarrow j)$$

and

$$N_{i/j} \equiv \frac{m^2_{i/j}}{m^2_{\tilde{\mu}_m}}, \quad N_{f\tilde{f}_k} \equiv \frac{m^2_{f\tilde{f}_k}(w)}{m^2_{\tilde{\mu}_m}}, \quad m^2_{f\tilde{f}_k}(w) \equiv \frac{m_f^2}{w} + \frac{m^2_{\tilde{f}_k}}{1-w}, \quad l_z \equiv \log N_z$$

MSSM fermion/sfermion two-loop corrections: sample analytic result

[arXiv:1311.1775][Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]

$$
\begin{aligned}
\Gamma^{-\mu}_{ij\tilde{f}_k}(\ell) = \frac{1}{16\pi^2} e Q_{f'} \int_0^1 \frac{\mathrm{d}w}{2} \Bigg\{ & \left(\mathcal{A}^{c\,+}_{ij\tilde{f}_k} - \mathcal{A}^{c\,-}_{ij\tilde{f}_k} \gamma_5 \right) \frac{\not{\ell}\not{q}\gamma^\mu - \not{\ell} q^\mu + \not{q}\ell^\mu - (\ell\cdot q)\gamma^\mu}{\mathcal{D}_{f'\tilde{f}_k}(\ell)} \\
& + \left(\mathcal{B}^{c\,+}_{ij\tilde{f}_k} - \mathcal{B}^{c\,-}_{ij\tilde{f}_k} \gamma_5 \right) \frac{m_{f'}}{w} \frac{\not{q}\gamma^\mu - q^\mu}{\mathcal{D}_{f'\tilde{f}_k}(\ell)} \Bigg\} \\
- \frac{1}{16\pi^2} e Q_{\tilde{\chi}^-} \int_0^1 \frac{\mathrm{d}w}{2} \Bigg\{ & \left(\mathcal{A}^{c\,+}_{ij\tilde{f}_k} - \mathcal{A}^{c\,-}_{ij\tilde{f}_k} \gamma_5 \right) w \Bigg[\left(\frac{1}{\epsilon} - L(m^2_{\tilde{f}_k}) \right) \gamma^\mu - 2 \frac{(\ell\cdot q)\,\not{\ell}\,\ell^\mu}{\mathcal{D}^2_{f'\tilde{f}_k}(\ell)} \\
& + \frac{\not{q}\ell^\mu + \not{\ell} q^\mu + (\ell\cdot q)\gamma^\mu - 2\not{\ell}\,\ell^\mu + g_w(\ell^2, m^2_{f'}, m^2_{\tilde{f}_k})\gamma^\mu/2}{\mathcal{D}_{f'\tilde{f}_k}(\ell)} \Bigg] \\
& + \left(\mathcal{B}^{c\,+}_{ij\tilde{f}_k} - \mathcal{B}^{c\,-}_{ij\tilde{f}_k} \gamma_5 \right) m_{f'} \left[-2 \frac{(\ell\cdot q)\ell^\mu}{\mathcal{D}^2_{f'\tilde{f}_k}(\ell)} + \frac{q^\mu - 2\ell^\mu}{\mathcal{D}_{f'\tilde{f}_k}(\ell)} \right] \Bigg\}
\end{aligned}
$$

MSSM fermion/sfermion two-loop corrections: sample analytic result

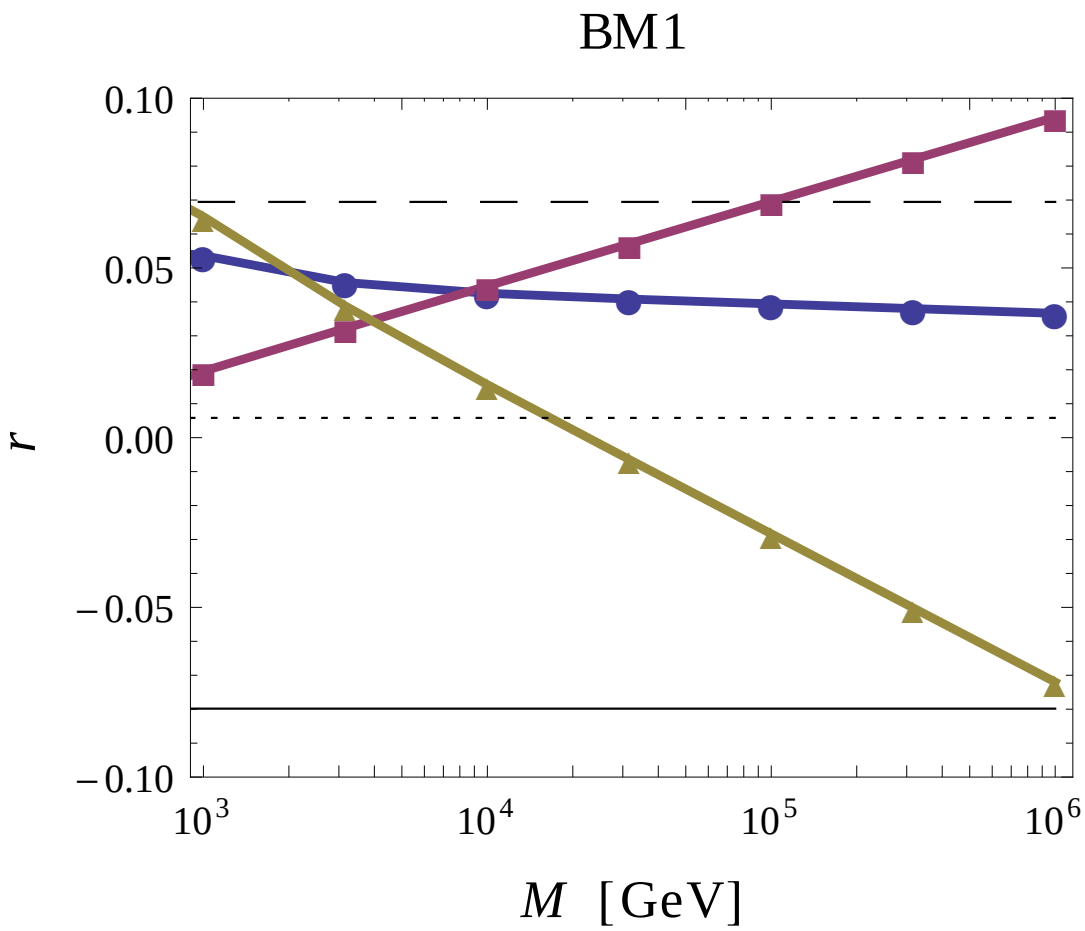
[arXiv:1311.1775][Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]

$$a^{cv}_{\mu,ij\tilde{f}_k} = \int_0^1 \mathrm{d}w \Big[\mathcal{A}^{c+}_{ji\tilde{\nu}_\mu} \left(\mathcal{A}^{c+}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv+}_{AA} + \mathcal{B}^{c+}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv+}_{AB} \right) + \mathcal{B}^{c+}_{ji\tilde{\nu}_\mu} \left(\mathcal{A}^{c+}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv+}_{BA} + \mathcal{B}^{c+}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv+}_{BB} \right)$$
$$+ \mathcal{A}^{c-}_{ji\tilde{\nu}_\mu} \left(\mathcal{A}^{c-}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv-}_{AA} + \mathcal{B}^{c-}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv-}_{AB} \right) + \mathcal{B}^{c-}_{ji\tilde{\nu}_\mu} \left(\mathcal{A}^{c-}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv-}_{BA} + \mathcal{B}^{c-}_{ij\tilde{\nu}_\mu} \mathcal{T}^{cv-}_{BB} \right) \Big]$$

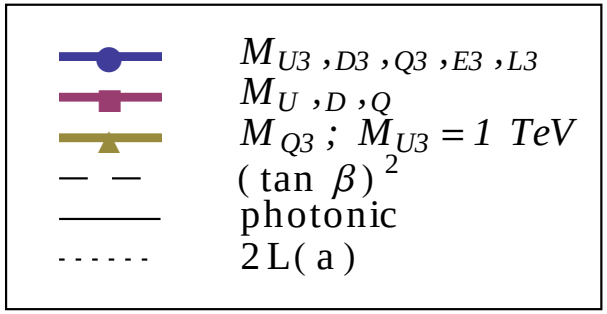
$$\mathcal{T}^{cv2}_{AA}(C_i) = \left[-12L(m_{\tilde{\nu}_\mu}) - 6l_{\tilde{f}_k} - 3l_i + 5 - \frac{6}{C_i} \right] \frac{C_i \left(2 - 2C_i + 3l_i C_i - l_i C_i^2\right)}{144\,(1 - C_i)^3}$$
$$+ \frac{1 + 2l_i C_i - C_i^2}{12\,(1 - C_i)^3}$$

Non-decoupling behavior

[Physics Letters B 726 (2013)][arXiv:1309.0980][Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]



- can be largest 2L contribution $\mathcal{O}(10\%...30\%)$
- $\mu = 350,\ \tan\beta = 40$
- $M_2 = 2M_1 = 300\,\mathrm{GeV}$
  $m_{\tilde{\mu}_{R,L}} = 400\,\mathrm{GeV}$



Non-decoupling behavior

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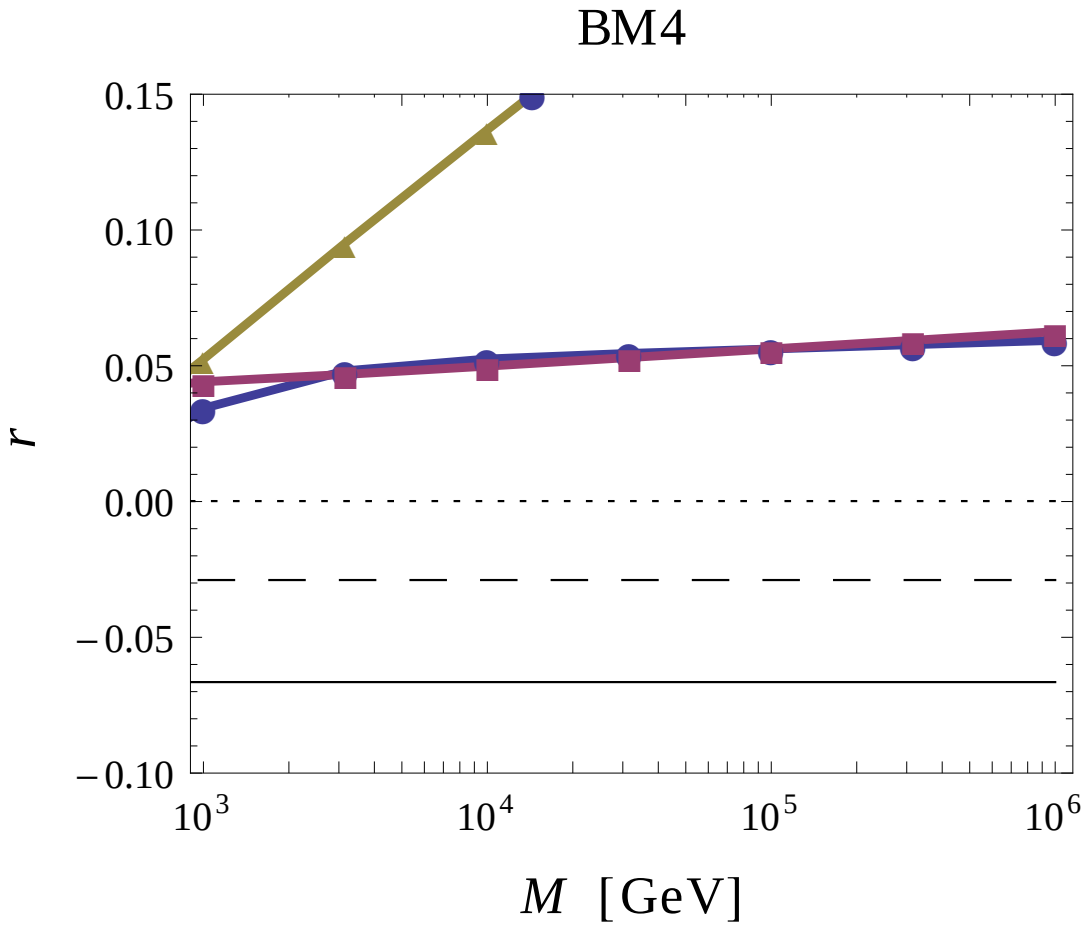
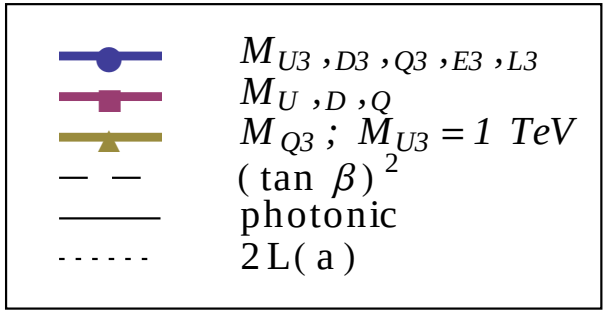


- can be largest 2L contribution $\mathcal{O}(10\%...30\%)$
- $\mu = -160,\ \tan\beta = 40$
- $M_1 = 140\mathrm{GeV}$
  $m_{\tilde{\mu}_R} = 200\mathrm{GeV}$
  $M_2 = m_{\tilde{\mu}_L} = 2000\mathrm{GeV}$



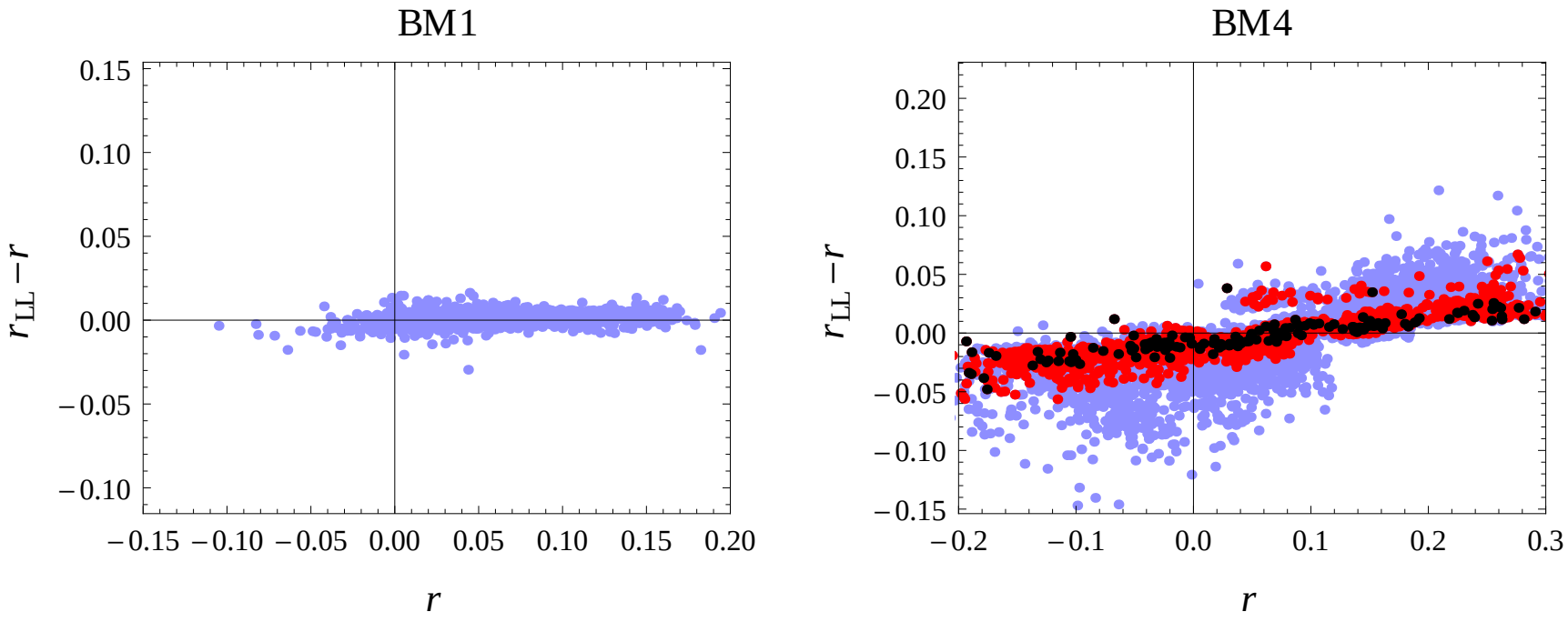
Leading logarithmic approximation

[http://iktp.tu-dresden.de/index.php?id=theory-software]

$$
\begin{aligned}
a_\mu^{\rm 1LSUSY,M.I.} &= a_\mu^{\rm 1L}(\tilde{W}\text{–}\tilde{H},\tilde{\nu}_\mu) + a_\mu^{\rm 1L}(\tilde{W}\text{–}\tilde{H},\tilde{\mu}_L) + a_\mu^{\rm 1L}(\tilde{B}\text{–}\tilde{H},\tilde{\mu}_L) \\
&\quad + a_\mu^{\rm 1L}(\tilde{B}\text{–}\tilde{H},\tilde{\mu}_R) + a_\mu^{\rm 1L}(\tilde{B},\tilde{\mu}_L\text{–}\tilde{\mu}_R) \\
a_\mu^{\rm 1L}(\tilde{W}/\tilde{B},\ldots) &\propto g_{2/1}^2 \mu \tan\beta \\
a_\mu^{\rm 2L,f\tilde{f}LL} &= a_\mu^{\rm 1L}(\tilde{W}\text{–}\tilde{H},\tilde{\nu}_\mu)\left(\Delta_{g_2} + \Delta_{\tilde{H}} + \Delta_{\tilde{W}\tilde{H}} + \Delta_{t_\beta} + 0.015\right), \\
&\quad + a_\mu^{\rm 1L}(\tilde{W}\text{–}\tilde{H},\tilde{\mu}_L)\left(\Delta_{g_2} + \Delta_{\tilde{H}} + \Delta_{\tilde{W}\tilde{H}} + \Delta_{t_\beta} + 0.015\right), \\
&\quad + a_\mu^{\rm 1L}(\tilde{B}\text{–}\tilde{H},\tilde{\mu}_L)\left(\Delta_{g_1} + \Delta_{\tilde{H}} + \Delta_{\tilde{B}\tilde{H}} + \Delta_{t_\beta} + 0.015\right), \\
&\quad + a_\mu^{\rm 1L}(\tilde{B}\text{–}\tilde{H},\tilde{\mu}_R)\left(\Delta_{g_1} + \Delta_{\tilde{H}} + \Delta_{\tilde{B}\tilde{H}} + \Delta_{t_\beta} + 0.04\right), \\
&\quad + a_\mu^{\rm 1L}(\tilde{B},\tilde{\mu}_L\text{–}\tilde{\mu}_R)\left(\Delta_{g_1} + \Delta_{t_\beta} + 0.03\right) \\
\Delta_{\tilde{H}} &= \frac{1}{16\pi^2}\frac{1}{2}\left(3y_t^2 \log\frac{M_{U3}}{m_{\rm SUSY}} + 3y_b^2 \log\frac{M_{D3}}{m_{\rm SUSY}}\right. \\
&\quad \left. +3(y_t^2 + y_b^2)\log\frac{M_{Q3}}{m_{\rm SUSY}} + y_\tau^2 \log\frac{M_{E3}}{m_{\rm SUSY}} + y_\tau^2 \log\frac{M_{L3}}{m_{\rm SUSY}}\right)
\end{aligned}
$$

Leading logarithmic approximation

[Physics Letters B 726 (2013)][arXiv:1309.0980][Fargnoli, Gnendiger, Paßehr, Stöckinger, S-K '13]



[GeV]	BM1	BM4
μ	[100,200]	[-200,-100]
M_1	[100,200]	[100,200]
M_2	[200,400]	[1000,3000]
M_E	[200,500]	[100,300]
M_L	[200,500]	[1000,3000]

$r \equiv a_\mu^{2L,f\tilde{f}}/a_\mu^{1L}$, $r_{\rm LL} \equiv a_\mu^{2L,f\tilde{f},LL}/a_\mu^{1L}$,
red/black min$(M_{SUSY}) > 150/180 GeV$

invalid when tan β or M_{SUSY} is very small,

$M_{SUSY} > 200\mathrm{GeV}$

Summary

- $a_\mu^{\rm SM}$: a_μ^{EW} contributions depending on M_H
  $\rightarrow$ this uncertainty has been eliminated
- a_μ still viable, complementary constraint on SUSY
- $a_\mu^{\rm 2L,\gamma}$: $-(7...9)\%$ corrections to $a_\mu^{\rm 1L,SUSY}$
- fermion/sfermion two-loop contributions:
  - first full calculation of a_μ^{SUSY} two-loop contributions: 5-heavy-mass-scale diagrams
  - leading logarithmic approximation: invalid when $\tan\beta$ or M_{SUSY} is very small
  - $\ln(m_{\tilde{f}})$ enhanced, non-decoupling behavior for (very) heavy squarks: $10...30\%$ corrections to $a_\mu^{\rm 1L,SUSY}$ possible