Towards realistic model of Quarks and Leptons and leptonic CP violation and $0\nu\beta\beta$-decay

Yang-Hwan, Ahn (KIAS)

Collaboration with Paolo Gondolo (Univ. of Utah)

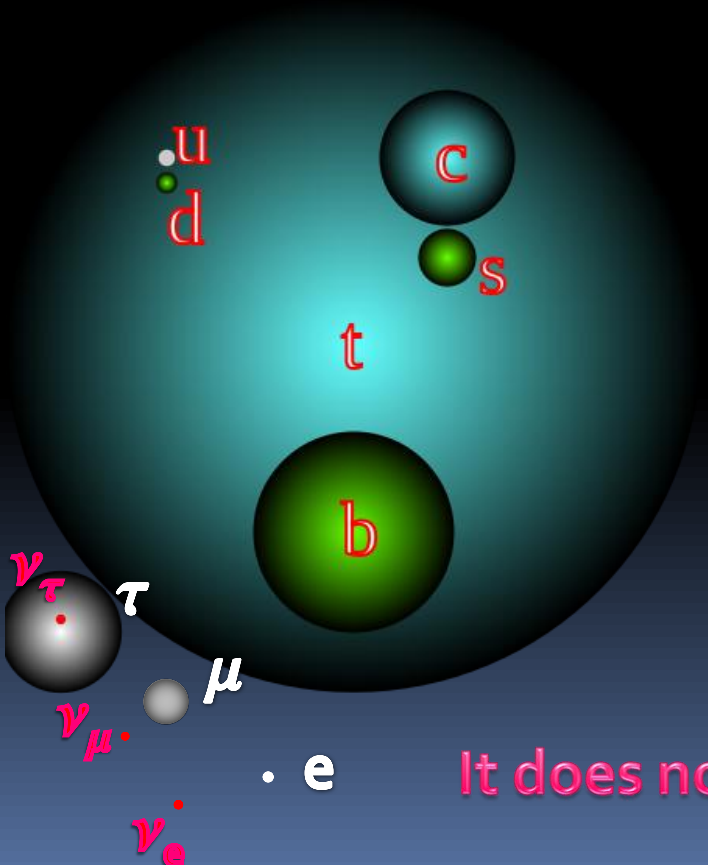
Appear to 1311.xxxxx

The 3rd KIAS workshop on Particle physics and Cosmology

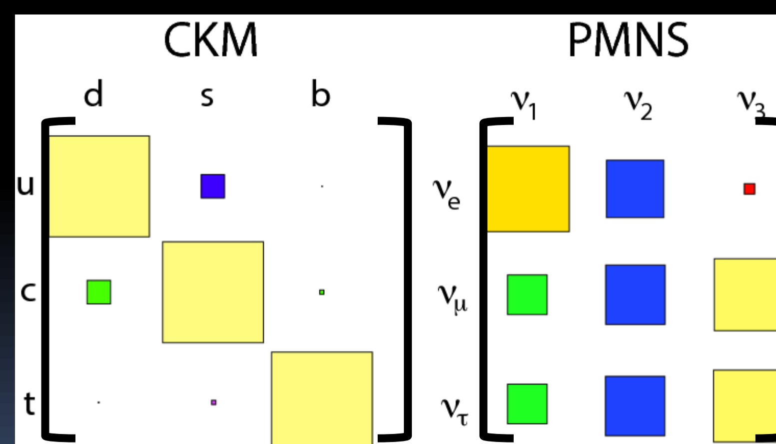
The SM as an effective theory

- Several theoretical arguments (inclusion of gravity, instability of he Higgs potential, neutrino masses,….) and cosmological evidence (dark matter, inflation, cosmological constant,…) point toward the existence of physics beyond the SM.

- The outstanding puzzles in the SM are enormously various hierarchies spanned by the fermion masses and their mixing patterns :



$m_t/m_u=10^5$, $m_\tau/m_e\sim10^3$,……
$m_t/m_\nu\lesssim10^{12}$ (Planck, KATRIN….)



It does not look "accidental"

Questions

Why lepton mixing angles are so different from those of the quark sector ?

- ✓ **The bi-large mixing angles of leptons may be telling us about some new symmetries not presented in the quark sector and provide a clue to the nature of the quark-lepton physics beyond the SM.**

Why neutrino masses are so small, compared with the charged fermion masses ?

- ✓ **The huge smallness of neutrino masses compared to the charged fermion masses is related to the existence of a new fundamental scale, and thus new physics beyond the SM. [SEESAW Mechanism]**

Why the hierarchy of neutrino masses are so mild, while those of charged fermion masses are so strong ?

- ✓ **The large ratios between the masses of fermions of successive generations may be due to higher order effects. [Froggatt-Nielsen Mechanism]**

What determines the observed pattern of masses and mixings of quarks and leptons ?

Our Idea

The symmetric way
[main road of particle physics so far]

Anarchy + Anthropic selection
[chance and necessity]

- ✓ No clear direction for future searches
- ✓ large number of independent $\mathcal{O}(1)$ parameters

Anarchy
UV - theory

Λ

Flavor symmetric
Effective - theory

In a complete model, the mass scale suppression can be identified as the masses of the messenger fields

A new advanced approach for the flavor puzzle

It consists of the introduction of family symmetries with higher order effects, which constrains the flavor structure of Yukawa couplings and lead to predictions for fermion masses and mixings :

$$\mathcal{OP}_3 \cdot (\mathcal{F})^1 + \mathcal{OP}_4 \cdot (\mathcal{F})^0 + \mathcal{OP}_4 \cdot \left(\frac{\mathcal{F}}{\Lambda}\right)^1 + \mathcal{OP}_4 \cdot \left(\frac{\mathcal{F}}{\Lambda}\right)^2 + \mathcal{OP}_4 \cdot \left(\frac{\mathcal{F}}{\Lambda}\right)^3 + \ldots$$

The gauge singlet flavon field $\mathcal{F}$ is activated to dimension-4(3) operators with different orders

Non-Abelian flavor symmetry + anomalous U(1) global symmetry are imposed

The coefficients are all of order one

Λ is the scale of flavor dynamics, and is a mass scale for the FN fields which are integrated out

Since the Yukawa couplings are eventually responsible for the fermion masses they must be related in a very simple way at a large scale in order for intermediate scale physics to produce all the interesting structure in the fermion mass matrices

Where Do we Stand ?

(arXiv: 1209, 3023 Gonzalez-Garcia, Maltoni, Salvado and Schwetz)

	$\theta_{13}[°]$	$\delta_{CP}[°]$	$\theta_{12}[°]$	$\theta_{23}[°]$	$\Delta m^2_{\rm Sol}[10^{-5}{\rm eV}^2]$	$\Delta m^2_{\rm Atm}[10^{-3}{\rm eV}^2]$
Best-fit	8.66	300	33.36	$40.0 \oplus 50.4$	7.50	2.437 (N), 2.427 (I)
$3\ \sigma$	$7.19 \to 9.96$	$0 \to 360$	$31.09 \to 35.89$	$35.8 \to 54.8$	$7.00 \to 8.09$	$2.276 \to 2.695$ (N) $2.242 \to 2.649$ (I)

Large mixings (solar & Atm) ⇨ *New flavor symmetry*

Large deviations from maximality of Atm. at best-fits

⇨ Start to disfavor QLC on θ_{23}

Relatively large θ_{13} can give constraint δ_{CP}
(LBL experiments T2K and NoνA)

However, the large uncertainty on θ_{23} is now limiting the information which can be extracted from ν_e appearance measurements

Precise measurements all the mixings are needed to maximize sensitivity to CP violation

Summary of unknown

Absolute neutrino mass Scale is unknown

Ordering (either normal or inverted) not unknown yet

Violation of total lepton number Not yet established

Nothing is known about all 3 CP-violating phases

Where Do we Stand ?

There was a claim of a signal in $0\nu\beta\beta$-decay in ^{76}Ge

$$\text{HdM}: \quad T^{0\nu}_{1/2}(^{76}\text{Ge}) = 2.23^{+0.44}_{-0.31} \times 10^{25} \text{ yr at } 68\% \text{ C.L.}$$

$$\Rightarrow |m_{ee}| = 0.32 \pm 0.03 \text{ eV}$$

There are two claims:

$$\left.\begin{array}{l} T^{0\nu}_{1/2}(^{76}\text{Ge}) > 2.1 \times 10^{25} \text{ yr at } 90\% \text{ C.L.}: \ \text{GERDA-I} \\ T^{0\nu}_{1/2}(^{76}\text{Ge}) > 1.6 \times 10^{25} \text{ yr at } 90\% \text{ C.L.}: \ \text{IGEX} \\ T^{0\nu}_{1/2}(^{76}\text{Ge}) > 1.9 \times 10^{25} \text{ yr at } 90\% \text{ C.L.}: \ \text{HdM} \end{array}\right\}$$

$$\Rightarrow \underline{T^{0\nu}_{1/2}(^{76}\text{Ge}) > 3.0 \times 10^{25} \text{yr at } 90\% \text{ C.L.} \Leftrightarrow |m_{ee}| < 0.2 - 0.4 \text{ eV}}$$

$$\left.\begin{array}{l} T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.9 \times 10^{25} \text{yr at } 90\% \text{ C.L.}: \ \text{KLZ} \\ T^{0\nu}_{1/2}(^{136}\text{Xe}) > 1.6 \times 10^{25} \text{ yr at } 90\%: \quad \text{EXO-200} \end{array}\right\}$$

$$\Rightarrow \underline{T^{0\nu}_{1/2}(^{136}\text{Xe}) > 3.4 \times 10^{25} \text{ yr at } 90\% \text{ C.L.} \Leftrightarrow |m_{ee}| < 0.12 - 0.25 \text{ eV}}$$

This new result excludes the KK claim independent of the physical mechanism for $0\nu\beta\beta$-decay

$$\text{Planck I}: \quad \sum_i m_{\nu i} < 0.23 \text{ eV at } 95\% \text{ C.L.}$$

$$\text{Planck II}: \quad \sum_i m_{\nu i} < 0.66 \text{ eV at } 95\% \text{ C.L.}$$

stronger than the direct experimental bound coming from Tritium-β decay $\lesssim$ 2eV

A4 Symmetry (Smallest group for three-families)

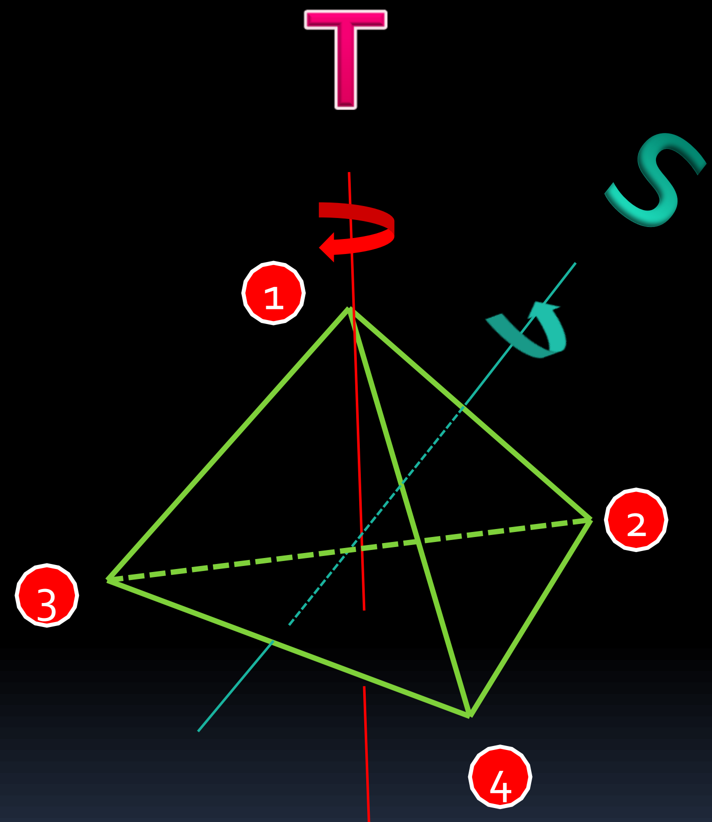
Even permutations of 4 objects : $S^2=T^3=(ST)^3=I$

$S^2=I$ ⇔ Z2 symmetry

$T^3=I$ ⇔ Z3 symmetry

Finite group of		S,	TST^2,	T^2ST
12 elements in 4	T,	TS,	ST,	STS
conjugacy classes	T^2,	ST^2,	T^2S,	TST

There are 4 irreducible representation : 1, 1′, 1″, 3



Why A4 (Discrete & non-Abelian) ?

- ✓ A4 is the smallest discrete group that has 3-dimensional irreducible representation
- ✓ A4 flavor symmetry can give a μ−τ symmetric pattern for experimental data

- ✓ subgroup itself ➡ large mixings
- ✓ mismatch between Z2 and Z3 symmetry

$|U_{e3}|\approx 8.7°$

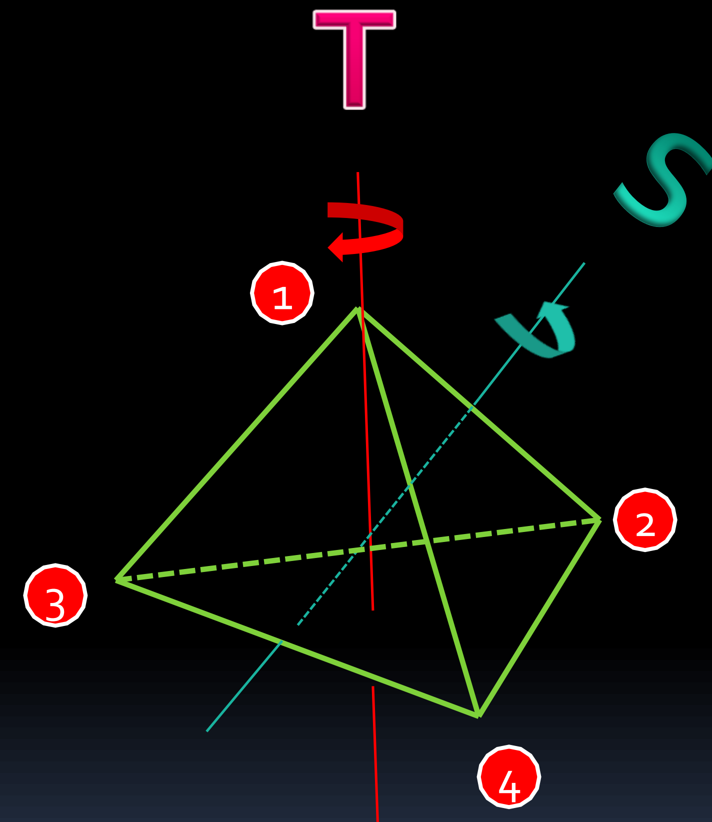
A4 Symmetry (Smallest group for three-families)

Even permutations of 4 objects : $S^2=T^3=(ST)^3=I$

$S^2=I$ ⟺ Z2 symmetry

$T^3=I$ ⟺ Z3 symmetry

Finite group of 12 elements in 4 conjugacy classes				
		S,	TST^2,	T^2ST
	T,	TS,	ST,	STS
	T^2,	ST^2,	T^2S,	TST



There are 4 irreducible representation : 1, 1′, 1″, 3

Finite-dimensional unitary irreducible representations

$$\mathbf{1}: \quad S = 1,\ T = 1$$

$$\mathbf{1'}: \quad S = 1,\ T = \omega$$

$$\mathbf{1''}: \quad S = 1,\ T = \omega^2$$

$$\mathbf{3}: \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix},\ T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\omega^3 = 1, \quad \omega = e^{2i\pi/3}$$

- ✓ subgroup itself ➡ large mixings
- ✓ mismatch between Z2 and Z3 symmetry

$|U_{e3}| \approx 8.7°$

Fermionic tetrahedral Symmetry A_4

SM left-handed leptons

SM right-handed leptons

SM left-handed quarks

3 new Higgs singlet

1 Higgs singlelet

Field	L_e, L_μ, L_τ	e_R, μ_R, τ_R	N_R	D_R	u_R, c_R, t_R	$Q_{L_1}, Q_{L_2}, Q_{L_3}$	χ	Θ	η	Φ
A_4	$\mathbf{1},\ \mathbf{1}',\ \mathbf{1}''$	$\mathbf{1},\ \mathbf{1}',\ \mathbf{1}''$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1},\ \mathbf{1},\ \mathbf{1}''$	$\mathbf{1},\ \mathbf{1},\ \mathbf{1}''$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
$SU(2)_L \times U(1)_Y$	$2_{-\frac{1}{2}}$	1_{-1}	1_0	$1_{-\frac{2}{3}}$	$1_{\frac{4}{3}}$	$2_{\frac{1}{3}}$	1_0	1_0	$2_{\frac{1}{2}}$	$2_{\frac{1}{2}}$

3 (heavy) right-handed singlet neutrinos

SM right-handed quarks

1 Higgs doublet

3 Higgs doublets

Fermionic tetrahedral Symmetry A_4

Comparison with Ma, Rajasekaran (2001), Babu, Ma, Valle (2003)……

Ahn, Gondolo

Field	L_e, L_μ, L_τ	e_R, μ_R, τ_R	N_R	D_R	u_R, c_R, t_R	$Q_{L_1}, Q_{L_2}, Q_{L_3}$	χ	Θ	η	Φ
A_4	$\mathbf{1}, \mathbf{1}', \mathbf{1}''$	$\mathbf{1}, \mathbf{1}', \mathbf{1}''$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}, \mathbf{1}, \mathbf{1}''$	$\mathbf{1}, \mathbf{1}, \mathbf{1}''$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
$SU(2)_L \times U(1)_Y$	$2_{-\frac{1}{2}}$	1_{-1}	1_0	$1_{-\frac{2}{3}}$	$1_{\frac{4}{3}}$	$2_{\frac{1}{3}}$	1_0	1_0	$2_{\frac{1}{2}}$	$2_{\frac{1}{2}}$

Non-zero θ_C , θ_{13}

See also, *Ahn, Baek, Gondolo (2012)*

Ma, Rajasekaran (2001)

$$(\nu_i, l_i)_L \sim (\underline{3}, 1), \qquad l_{3R} \sim (\underline{1}'', 1), \qquad \eta = (\eta^+, \eta^0) \sim (\underline{1}, -1).$$
$$l_{1R} \sim (\underline{1}, 1), \qquad N_{iR} \sim (\underline{3}, 0),$$
$$l_{2R} \sim (\underline{1}', 1), \qquad \Phi_i = (\phi_i^+, \phi_i^0) \sim (\underline{3}, 0),$$

θ_C , θ_{13}=0

Babu, Ma, Valle (2003)

$$\widehat{Q}_i = (\hat{u}_i, \hat{d}_i), \qquad \widehat{L}_i = (\hat{\nu}_i, \hat{e}_i) \sim \underline{3}, \qquad \hat{\phi}_{1,2} \sim \underline{1},$$
$$\hat{u}_1^c,\ \hat{d}_1^c,\ \hat{e}_1^c \sim \underline{1}, \qquad \hat{u}_2^c,\ \hat{d}_2^c,\ \hat{e}_2^c \sim \underline{1}', \qquad \hat{u}_3^c,\ \hat{d}_3^c,\ \hat{e}_3^c \sim \underline{1}''.$$
$$\widehat{U}_i,\ \widehat{U}_i^c,\ \widehat{D}_i,\ \widehat{D}_i^c,\ \widehat{E}_i,\ \widehat{E}_i^c,\ \widehat{N}_i^c,\ \hat{\chi}_i \sim \underline{3},$$

Leptonic tetrahedral Symmetry A_4

η appears in Yukawa terms coupling for the charged leptons

$$
\begin{aligned}
-\mathcal{L}^{\ell\nu}_{\rm Yuk} &= y_e \bar{L}_e \eta\, e_R + y_\mu \bar{L}_\mu \eta\, \mu_R + y_\tau \bar{L}_\tau \eta\, \tau_R \\
&+ y_1^\nu \bar{L}_e (\tilde{\Phi} N_R)_{\mathbf{1}} + y_2^\nu \bar{L}_\mu (\tilde{\Phi} N_R)_{\mathbf{1}'} + y_3^\nu \bar{L}_\tau (\tilde{\Phi} N_R)_{\mathbf{1}''} \\
&+ \frac{y_R}{2} (\overline{N_R^c} N_R)_{\mathbf{1}} \Theta + \frac{y_R^\nu}{2} (\overline{N_R^c} N_R)_{\mathbf{3}_s} \chi + \text{h.c.},
\end{aligned}
$$

Renormalizable

χ and Θ appears in Yukawa terms coupling between RH neutrinos

Φ appears in Yukawa terms coupling SM to RH neutrinos

- Minimal Yukawa couplings !! ➡ 5 parameters

- Each Dirac-like ν and charged-lepton sector has three independent Yukawa terms
- A non-degenerate Dirac-neutrino Yukawa matrix
- Heavy νs acquire mass terms induced by χ and Θ fields
- The three leptons e, μ, τ are eigenstates of T with eigenvalues 1, ω, ω^2 respectively : $L_e(e_R) \sim 1$ $L_\mu(\mu_R) \sim \omega$ $L_\tau(\tau_R) \sim \omega^2$

As a consequence, the charged lepton mass matrix automatically diagonal.

Quark tetrahedral Symmetry A_4

$$
\begin{aligned}
-\mathcal{L}^{\rm Quark}_{\rm Yuk} &= y_u \bar{Q}_{L_1}\tilde{\eta}\, u_R + y_c \bar{Q}_{L_2}\tilde{\eta}\, c_R + y_t \bar{Q}_{L_3}\tilde{\eta}\, t_R \\
&+ y_d \bar{Q}_{L_1}(\Phi D_R)_{\mathbf{1}} + y_s \bar{Q}_{L_2}(\Phi D_R)_{\mathbf{1}} + y_b \bar{Q}_{L_3}(\Phi D_R)_{\mathbf{1}''} + \text{h.c.}
\end{aligned}
$$

Renormalizable

$\Downarrow$

$V_{CKM} \neq I$

Φ and η appears in the usual SM Yukawa terms for quarks

- Each flavor of up-(down-)type quarks has three independent Yukawa terms; the down-type terms involve the A4 triplets Φ and D_R, and singlets Q_L, while the up-type terms involve the A4 singlet η and singlets Q_L.

- The right-handed up-type quarks are eigenstates of T with eigenvalues 1, ω^2, ω respectively : $\omega=\exp(2\pi i/3)$

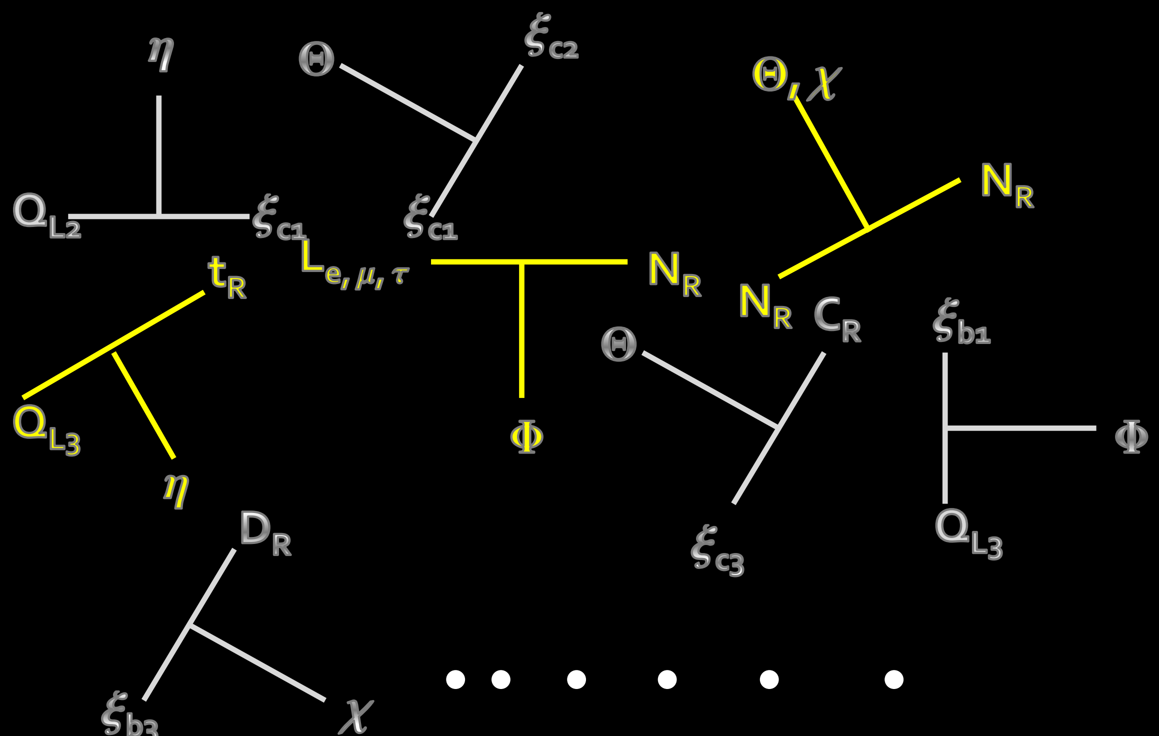
  $u_R \sim 1$, $c_R \sim \omega$, $t_R \sim \omega^2$

$<\Phi> \sim v_\Phi(1,1,1)$

$$
\mathcal{M}_d = \frac{v_\Phi}{\sqrt{2}} \begin{pmatrix} y_d & y_d & y_d \\ y_s & y_s & y_s \\ y_b & \omega y_b & \omega^2 y_b \end{pmatrix} \Rightarrow \mathcal{M}_d \mathcal{M}_d^\dagger = \frac{3 v_\Phi^2}{2} \begin{pmatrix} |y_d|^2 & y_d y_s^* & 0 \\ y_s y_d^* & |y_s|^2 & 0 \\ 0 & 0 & |y_b|^2 \end{pmatrix}
$$

$\theta_C = |m_d/m_s| \approx \lambda$

$U(1)_X$ Hierarchy : **Strong & Mild**



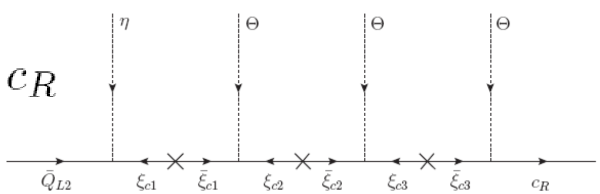
$$\mathcal{M}_\nu^0 \sim \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

$$\mathcal{M}_Q^0 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

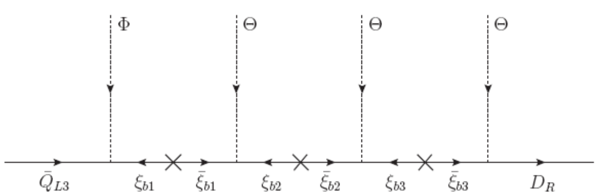
Λ

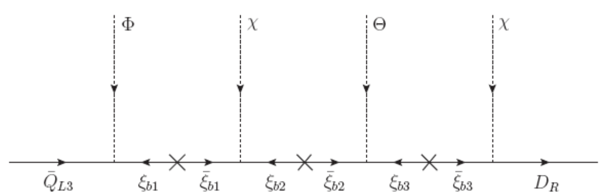
Example

$$y_c \bar{Q}_{L_2} \tilde{\eta}\ c_R$$



$$y_b \bar{Q}_{L_3} (\Phi D_R)_{\mathbf{1}''}$$

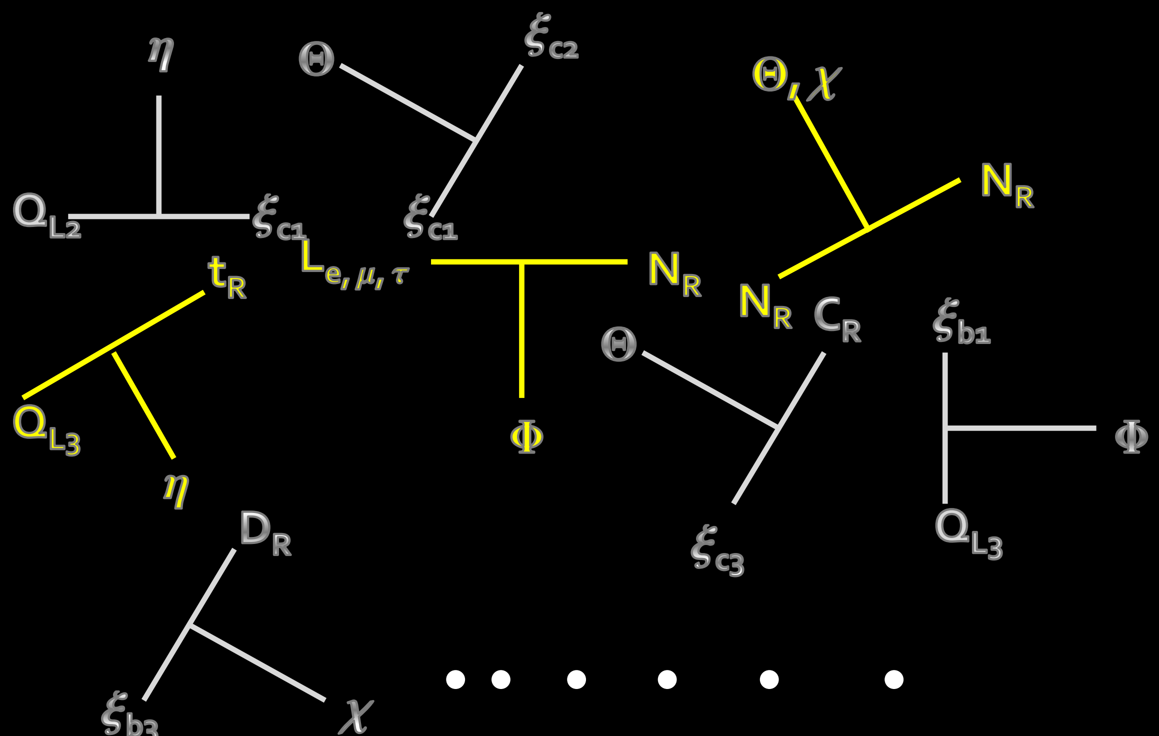




$$Y_{ij}^{\nu,t}(\mathcal{F}) = \mathcal{O}(1)$$

$$Y_{ij}^{u,d,\ell}(\mathcal{F}) = \sum_{n=0}^{\infty} \hat{y}_{ij}^{(n)} \left(\frac{\mathcal{F}}{\Lambda}\right)^n$$

$U(1)_X$ Hierarchy : **Strong & Mild**



$$\mathcal{M}_\nu^0 \sim \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

$$\mathcal{M}_Q^0 \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Λ

Example

$$y_c \bar{Q}_{L_2} \tilde{\eta}\ c_R$$

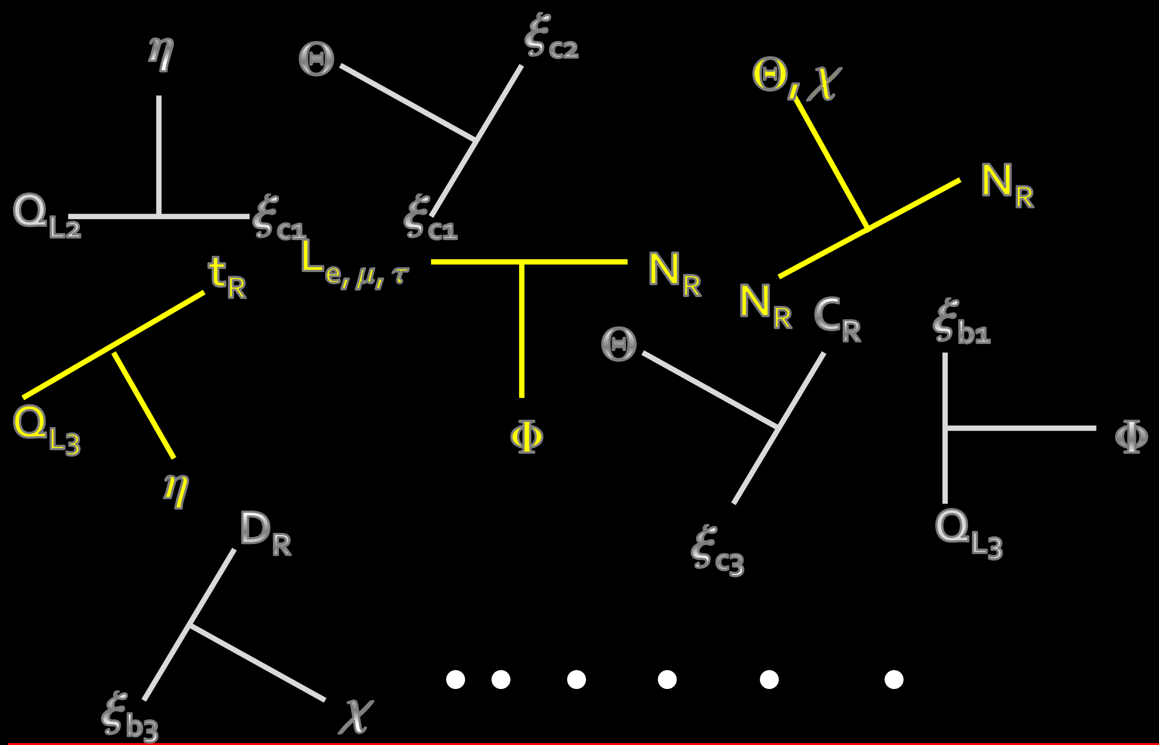
$$y_b \bar{Q}_{L_3} (\Phi D_R)_{\mathbf{1}''}$$

The mass scale suppression,
required for the UV completion of each non-renormalizable terms,
have absorbed into the re-definition of coefficients

$$Y_{ij}^{\nu,t}(\mathcal{F}) = \mathcal{O}(1)$$

$$Y_{ij}^{u,d,\ell}(\mathcal{F}) = \sum_{n=0}^{\infty} \hat{y}_{ij}^{(n)} \left(\frac{\mathcal{F}}{\Lambda}\right)^n$$

$U(1)_X$ Hierarchy : **Strong & Mild**



$$\mathcal{M}^0_\nu \sim \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(1) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

$$\mathcal{M}^0_Q \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Λ

After U(1)x symmetry breaking,
non-renormalizable operators generate Renormalizable operators

$U(1)_X$ Hierarchy : Strong & Mild

The given $U(1)_X$ symmetry allows more terms but restricted in the down-type quark sector, at leading order

$$-\Delta\mathcal{L}^d_{\rm Yuk} = x_d^{s(a)}\bar{Q}_{L_1}(\Phi D_R\hat{\chi})_{\mathbf{1}} + x_s^{s(a)}\bar{Q}_{L_2}(\Phi D_R\hat{\chi})_{\mathbf{1}} + x_b^{s(a)}\bar{Q}_{L_3}(\Phi D_R\hat{\chi})_{\mathbf{1}'} + {\rm h.c.}.$$

Provide off-diagonal entries in the matrix and lead to the correct CKM

hierarchies

$$v_\Theta = \lambda\,\Lambda,\ \ v_\chi = \kappa\ v_\Theta \gg v_\Phi = v_\eta = 123{\rm GeV}$$

$$\begin{aligned}
y_e &= \hat{y}_e\lambda^8, & y_\mu &= \hat{y}_\mu\lambda^5, & y_\tau &= \hat{y}_\tau\lambda^3,\\
y_u &= \hat{y}_u\lambda^7, & y_c &= \hat{y}_c\lambda^3, & y_t &= 2,\\
y_d &= \lambda^6\sum_{n=0}^{3}\hat{y}_d^{(n)}\kappa^{2n}, & y_s &= \lambda^5\sum_{n=0}^{2}\hat{y}_s^{(n)}\kappa^{2n}, & y_b &= \lambda^3\sum_{n=0}^{1}\hat{y}_b^{(n)}\kappa^{2n},\\
x_d^{s(a)} &= \lambda^5\sum_{n=0}^{2}\hat{x}_d^{s(a)(n)}\kappa^{2n}, & x_s^{s(a)} &= \lambda^4\sum_{n=0}^{2}\hat{x}_s^{s(a)(n)}\kappa^{2n}, & x_b^{s(a)} &= \lambda^2\sum_{n=0}^{1}\hat{x}_b^{s(a)(n)}\kappa^{2n}.
\end{aligned}$$

Recalling that the coefficients $\hat{y}_f^{(n)}$ and $\hat{x}_f^{(n)}$ are complex numbers and of $\mathcal{O}(1)$.

Spontaneously-broken Leptonic A_4 Symmetry

$\langle\eta\rangle \sim$ 100 GeV gives
Dirac masses to the charged leptons

$$
\begin{aligned}
-\mathcal{L}^{\ell\nu}_{\rm Yuk} &= y_e \bar{L}_e \eta\, e_R + y_\mu \bar{L}_\mu \eta\, \mu_R + y_\tau \bar{L}_\tau \eta\, \tau_R \\
&+ y_1^\nu \bar{L}_e (\tilde{\Phi} N_R)_{\mathbf{1}} + y_2^\nu \bar{L}_\mu (\tilde{\Phi} N_R)_{\mathbf{1}'} + y_3^\nu \bar{L}_\tau (\tilde{\Phi} N_R)_{\mathbf{1}''} \\
&+ \frac{y_R}{2} (\overline{N_R^c} N_R)_{\mathbf{1}} \Theta + \frac{y_R^\nu}{2} (\overline{N_R^c} N_R)_{\mathbf{3}_s} \chi + \text{h.c.},
\end{aligned}
$$

$\langle\Theta\rangle \sim 10^{12}$ GeV and $\langle\chi\rangle$ give
Majorana masses to the (heavy) neutrinos

$\langle\Phi\rangle \sim$ 100 GeV gives
Dirac masses to the neutrinos

5 parameters : $y_1^\nu \;,\quad y_2^\nu \;,\quad y_3^\nu \;,\quad y_R v_\Theta \;,\quad y_R^\nu\, v_\chi$

Spontaneously-broken Leptonic A_4 Symmetry

$$
\begin{aligned}
-\mathcal{L}_{\ell W} &= \frac{1}{2}\overline{N_R^c}M_R N_R + \overline{\nu_L} m_D N_R + \overline{\ell_L}\mathcal{M}_\ell \ell_R + \frac{g}{\sqrt{2}} W_\mu^- \overline{\ell_L}\gamma^\mu \nu_L + \text{h.c.} \\
&= \frac{1}{2}\begin{pmatrix} \overline{\nu_L} & \overline{N_R^c} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \overline{\ell_L}\mathcal{M}_\ell \ell_R + \frac{g}{\sqrt{2}} W_\mu^- \overline{\ell_L}\gamma^\mu \nu_L + \text{h.c.}
\end{aligned}
$$

$$
\mathcal{M}_\ell = \frac{v_\eta}{\sqrt{2}} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix},
$$

Charged lepton mass

$$
m_D = \frac{v_\Phi}{\sqrt{2}} Y_\nu = \frac{v_\Phi}{\sqrt{2}} \begin{pmatrix} y_1^\nu & y_1^\nu & y_1^\nu \\ y_2^\nu & \omega^2 y_2^\nu & \omega y_2^\nu \\ y_3^\nu & \omega y_3^\nu & \omega^2 y_3^\nu \end{pmatrix} = \frac{v_\Phi y_1^\nu}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U_\omega^\dagger
$$

Dirac neutrino mass

$$
M_R = \begin{pmatrix} y_R v_\Theta & 0 & 0 \\ 0 & y_R v_\Theta & y_R^\nu v_\chi \\ 0 & y_R^\nu v_\chi & y_R v_\Theta \end{pmatrix} = M \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \tilde{\kappa} e^{i\phi} \\ 0 & \tilde{\kappa} e^{i\phi} & 1 \end{pmatrix}
$$

HEAVY NEUTRINO MASS

After seesawing

$$\mathcal{M}_\nu = -m_D\, M_R^{-1}\, m_D^T = m_0 \begin{pmatrix} 1+2F & (1-F)y_2 & (1-F)y_3 \\ (1-F)y_2 & (1+\frac{F-3G}{2})y_2^2 & (1+\frac{F+3G}{2})y_2y_3 \\ (1-F)y_3 & (1+\frac{F+3G}{2})y_2y_3 & (1+\frac{F-3G}{2})y_3^2 \end{pmatrix}$$

$$m_0 = \frac{v_\Phi^2 y_1^{\nu 2}}{2M}, \qquad F = \frac{1}{1+\tilde{\kappa}e^{i\phi}}, \qquad G = \frac{1}{1-\tilde{\kappa}e^{i\phi}}$$

- In the limit $y_2 \to y_3$ the above matrix goes to μ-τ symmetry leading to $\theta_{13}=0°$ and $\theta_{23}=45°$
- In the limit $y_2 = y_3 \to 1$ ⇨ TBM: $\theta_{13}=0°$, $\theta_{23}=45°$ and $\theta_{12}=\sin^{-1}(1/\sqrt{3})$

$$\boxed{m_1^\nu \simeq \mathcal{O}(m_0), \qquad m_2^\nu \simeq \mathcal{O}(m_0), \qquad m_3^\nu \simeq \mathcal{O}(m_0)}$$

mild

$$U_{\rm PMNS} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{CP}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{CP}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} Q_\nu$$

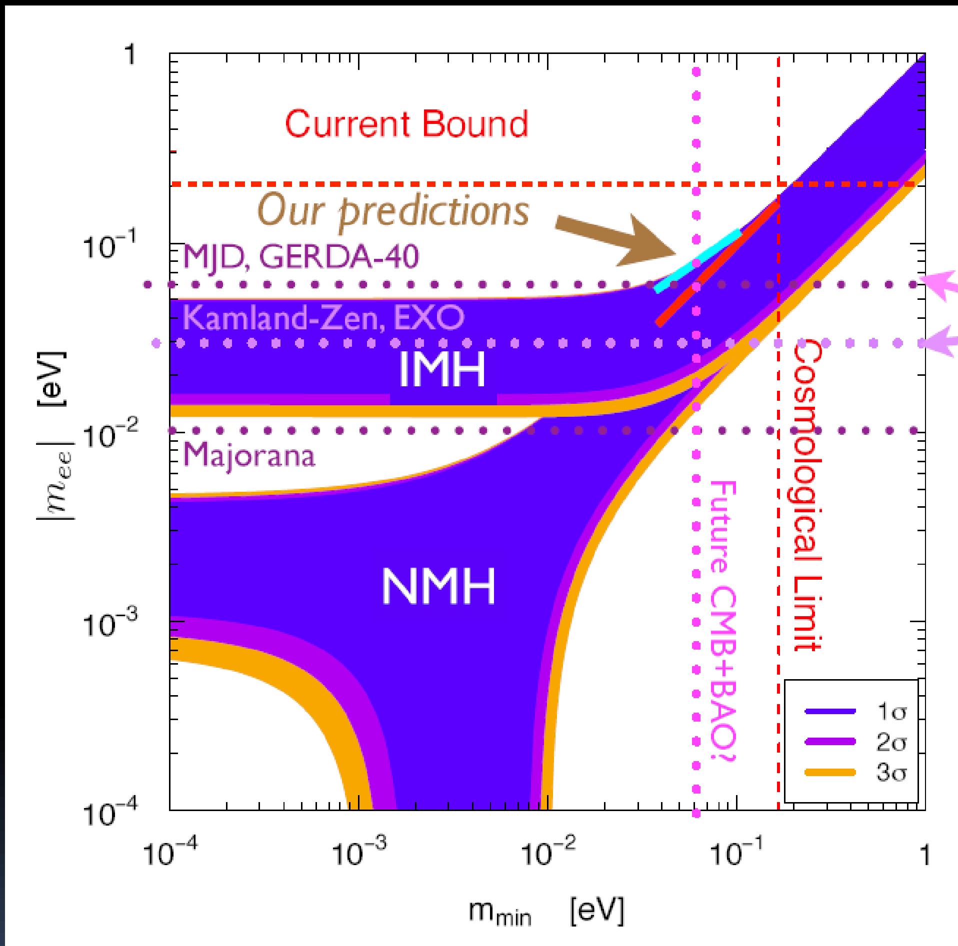
Non-zero θ_{13} requires deviations of y_2, y_3 from unit,
in turn implying a possibility of Leptonic CP violation

Phenomenology of light neutrino

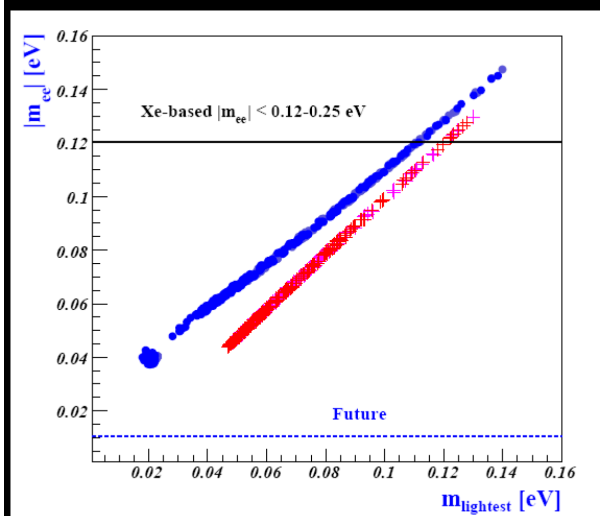
5 parameters	5 measured quantities
$m_0 = v_\Phi^2 y_1^{\nu 2}/(2M)$	θ_{12}
y_2	θ_{23}
y_3	θ_{13}
$\tilde{\kappa}$	$\Delta m^2_{\mathrm{Sol}}$
ϕ	$\Delta m^2_{\mathrm{Atm}}$

Model prediction, CP-violating phases and $0\nu\beta\beta$-decay

Neutrinoless double beta decay

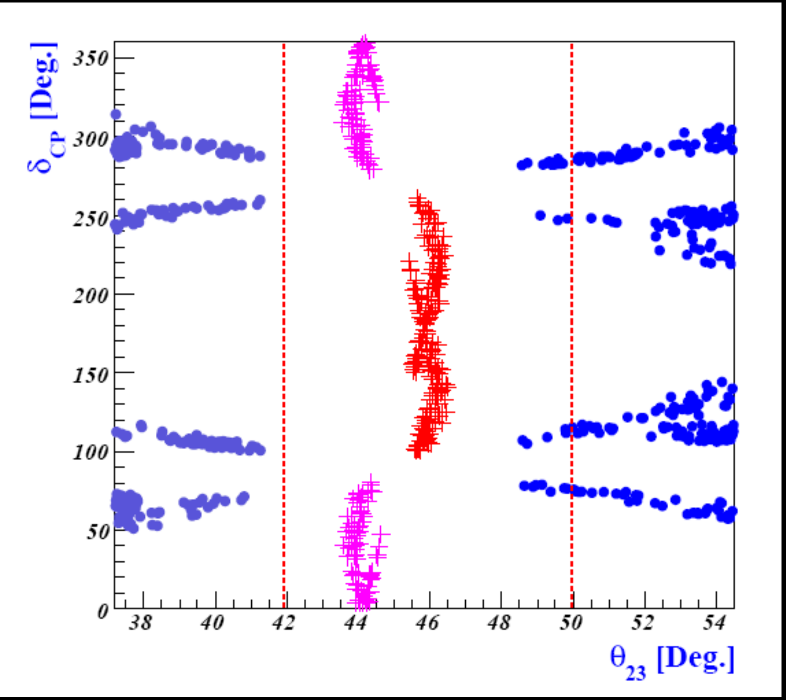


Fully testable in current experiments



Backgound from M.Drewes (1303.6912)

Leptonic Dirac CP-phase vs Atm. angle



NMO : +

IMO : ●

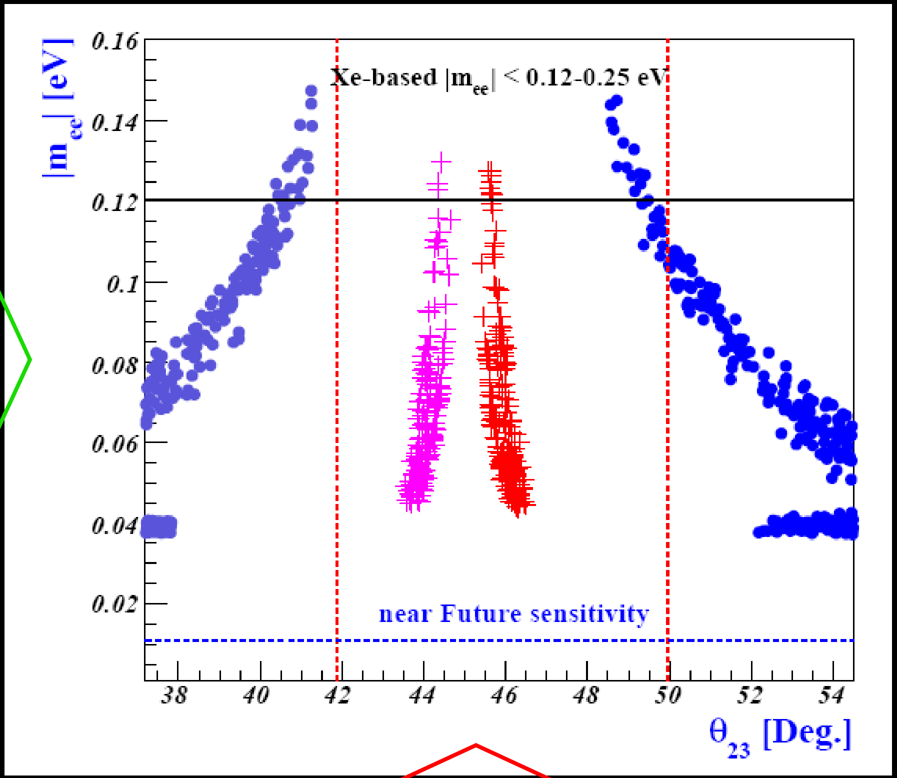
NMO favors $\theta_{23}>45°$ and $\theta_{23}<45°$ (but small deviations from the maximality),

while IMO favors $\theta_{23}<45°$ and $\theta_{23}>45°$ with large deviations from maximality.

Our model can be testable
in the near future experiments

$0\nu\beta\beta$-decay $|m_{ee}|$ vs Atm. mixing θ_{23}

GERDA-II
MAJORANA
CUORE, ...



NMO : +

IMO : •

T2K, NOνA, ...

Our model can be tested in the very near future neutrino oscillation experiments and/or $0\nu\beta\beta$-decay experiments

Conclusion

We have proposed an economical model based on $SU(2)_L \times U(1)_Y \times A_4 \times U(1)_X$ in a seesaw frame work, in which the Yukawa couplings are functions of flavon fields responsible for heavy neutrino masses. We have shown that it can naturally explain both large (small) mixing angles in the lepton (quark) sector and the enormously various hierarchies spanned by the fermion masses in terms of successive powers of the flavon fields. An important point is that our model depicts why the hierarchy of light neutrino masses is so mild, while the hierarchy of the charged fermions is strong.

Our model can be testable in the very near future through on-going LBL neutrino oscillation experiments and/or $0\nu\beta\beta$-decay experiments

The Higgs sector

- We have to make $\Phi\chi$ terms "sufficiently small", which would destroy the vacuum stability
- the extended Higgs sector can spontaneously break CP through a phase in the VEV of the gauge-singlet scalar field

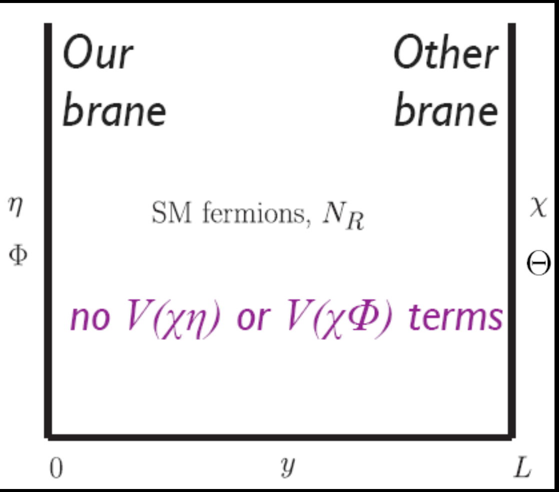
One example:

use extra dimensions

$$V = V_{y=0} + V_{y=L},$$

$$V_{y=0} = V(\Phi) + V(\eta) + V(\eta\Phi),$$

$$V_{y=L} = V(\Theta) + V(\chi) + V(\Theta\chi)$$



$$\begin{aligned}
V(\Theta) &= \mu_\Theta^2 \Theta^*\Theta + \lambda^\Theta \Theta^*\Theta^*\Theta\Theta \ , \\
V(\chi) &= \mu_\chi^2 (\chi\chi^*)_{\mathbf{1}} + \lambda_1^\chi (\chi\chi)_{\mathbf{1}}(\chi^*\chi^*)_{\mathbf{1}} + \tilde\lambda_1^\chi (\chi^*\chi)_{\mathbf{1}}(\chi^*\chi)_{\mathbf{1}} + \lambda_2^\chi (\chi\chi)_{\mathbf{1}'}(\chi^*\chi^*)_{\mathbf{1}''} \\
&+ \tilde\lambda_2^\chi (\chi^*\chi)_{\mathbf{1}'}(\chi^*\chi)_{\mathbf{1}''} + \lambda_3^\chi (\chi\chi)_{\mathbf{3}_s}(\chi^*\chi^*)_{\mathbf{3}_s} + \tilde\lambda_3^\chi (\chi^*\chi)_{\mathbf{3}_s}(\chi^*\chi)_{\mathbf{3}_s} \ , \\
V(\Theta\chi) &= \lambda_1^{\chi\Theta} (\chi\chi^*)_{\mathbf{1}}\Theta^*\Theta + \left\{ \lambda_2^{\chi\Theta} (\chi\chi)_{\mathbf{1}}\Theta^*\Theta^* + \lambda_3^{\chi\Theta} (\chi\chi\chi^*)_{\mathbf{1}}\Theta^* + \text{h.c.} \right\} \ ,
\end{aligned}$$

$$\begin{aligned}
V(\Phi) &= \mu_\Phi^2 (\Phi^\dagger\Phi)_{\mathbf{1}} + \lambda_1^\Phi (\Phi^\dagger\Phi)_{\mathbf{1}}(\Phi^\dagger\Phi)_{\mathbf{1}} + \lambda_2^\Phi (\Phi^\dagger\Phi)_{\mathbf{1}'}(\Phi^\dagger\Phi)_{\mathbf{1}''} + \lambda_3^\Phi (\Phi^\dagger\Phi)_{\mathbf{3}_s}(\Phi^\dagger\Phi)_{\mathbf{3}_s} \\
&+ \lambda_4^\Phi (\Phi^\dagger\Phi)_{\mathbf{3}_a}(\Phi^\dagger\Phi)_{\mathbf{3}_a} + i\lambda_5^\Phi (\Phi^\dagger\Phi)_{\mathbf{3}_s}(\Phi^\dagger\Phi)_{\mathbf{3}_a} \ , \\
V(\eta) &= \mu_\eta^2 (\eta^\dagger\eta) + \lambda^\eta (\eta^\dagger\eta)^2 \ , \\
V(\eta\Phi) &= \lambda_1^{\eta\Phi} (\Phi^\dagger\Phi)_{\mathbf{1}}(\eta^\dagger\eta) + \lambda_2^{\eta\Phi} [(\eta^\dagger\Phi)(\Phi^\dagger\eta)]_{\mathbf{1}} + \left\{ \lambda_3^{\eta\Phi} [(\eta^\dagger\Phi)(\eta^\dagger\Phi)]_{\mathbf{1}} + \text{h.c.} \right\} \\
&+ \left\{ \lambda_4^{\eta\Phi} (\Phi^\dagger\Phi)_{\mathbf{3}_s}(\Phi^\dagger\eta) + \text{h.c.} \right\} + \left\{ \lambda_5^{\eta\Phi} (\Phi^\dagger\Phi)_{\mathbf{3}_a}(\Phi^\dagger\eta) + \text{h.c.} \right\} \ .
\end{aligned}$$