How conventional measures overestimate EW fine-tuning in SUSY theory, with implications for LHC and ILC

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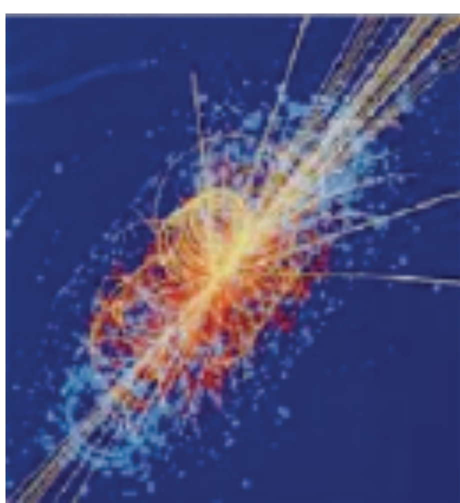
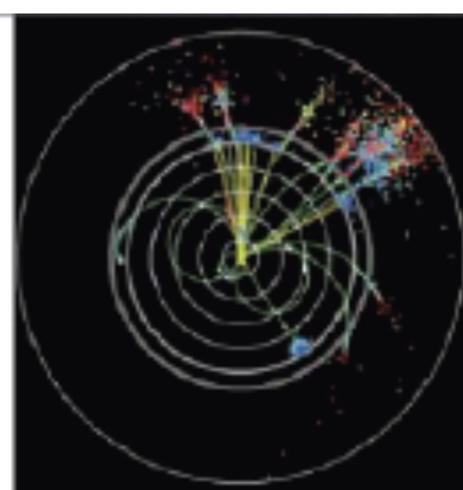
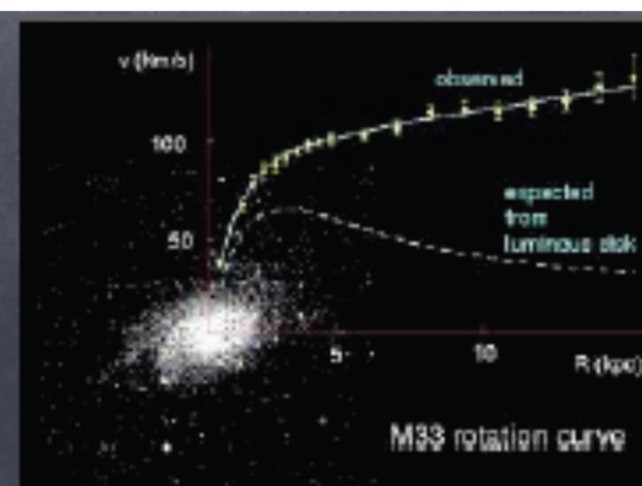
(For dark matter implications,
see talk by Kyu Jung Bae, Friday)

Discovery of Higgs at LHC vindicates SM!

but....

- Scalar particles contain quadratic divergences
- No dark matter candidates
- no valid baryogenesis mechanism

- h(125.5+-0.5 GeV) discovered at LHC
- scalars need protective symmetry: SUSY
- m(h)~125.5 GeV falls within narrow MSSM expectation
- m(h) requires highly mixed TeV-scale stops
- LHC: no SUSY: m(glno)>1.3 TeV, m(sqrk)>1.7 TeV, t1 limits
- impression: then MSSM EW fine-tuned at .1%
- SUSY as expected likely wrong?
- needs new features or new model?

- How does this perception arise?
- Why is it wrong?
- Overestimate of EWFT
- What does SUSY look like?
- How can we tell? Need ILC

Naturalness in the Standard Model

SM case: invoke a single Higgs doublet

$$V = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$m_h^2 = m_h^2|_{tree} + \delta m_h^2|_{rad}$$

$$m_h^2|_{tree} = \sqrt{2}\mu^2 \qquad\qquad \delta m_h^2|_{rad} = \frac{c}{16\pi^2}\Lambda^2$$

$m_h^2|_{tree}$ and $\delta m_h^2|_{rad}$ are *independent*,

$$\Delta_{SM} \equiv \delta m_h^2|_{rad}/(m_h^2/2)$$

$$\Delta_{SM} < 1 \Rightarrow \ \Lambda \sim 1\, TeV$$

MSSM case:

$$m_h^2 \simeq \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2|_{rad}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2}\left(-\frac{3}{5}g_1^2M_1^2 - 3g_2^2M_2^2 + \frac{3}{10}g_1^2S + 3f_t^2X_t\right) \qquad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S, mHu and running;
then we can integrate from mSUSY to Lambda

$$\delta m_{H_u}^2|_{rad} \sim -\frac{3f_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_t^2)\ln\left(\Lambda^2/m_{SUSY}^2\right)$$

$\Delta \equiv \delta m_{H_u}^2/(m_h^2/2) \lesssim 10$ then $m_{\tilde{t}_{1,2}}, m_{\tilde{b}_1} \lesssim 200$ GeV and $m_{\tilde{g}} \lesssim 600$ GeV

almost certainly in violation of LHC constraints!

What's wrong with this argument?

In zeal for simplicity, have neglected that, unlike case of SM, for SUSY

$m_{H_u}^2$ and $\delta m_{H_u}^2|_{rad}$ are not independent

the larger the value of $m_{H_u}^2(\Lambda)$, then the larger is the cancelling correction $\delta m_{H_u}^2|_{rad}$

The dependent terms should be grouped together

$$m_h^2|_{phys} = \mu^2 + \left(m_{H_u}^2(\Lambda) + \delta m_{H_u}^2\right)$$

where instead both μ^2 and $(m_{H_u}^2 + \delta m_{H_u}^2)$ should be comparable to $m_h^2|_{phys}$.

Such a re-grouping is used in Barbieri-Giudice measure:

$$\Delta_{BG} \equiv max_i\,[c_i] \quad \text{where} \quad c_i = \left|\frac{\partial \ln m_Z^2}{\partial \ln a_i}\right|$$

Here, the a_i are parameters of the theory

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \mu^2$$

↑

express weak scale value in terms of high scale parameters

Express m(Z) in terms of high scale parameters:

$$m_Z^2 \simeq -2m_{H_u}^2 - 2\mu^2$$

$$-2\mu^2(m_{SUSY}) = -2.18\mu^2$$

$$\begin{aligned} -2m_{H_u}^2(m_{SUSY}) = \; & 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & +0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & -0.025M_1A_t + 0.22A_t^2 + 0.004m_3A_b \\ & -1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & +0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & +0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & +0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned}$$

Abe, Kobayashi, Omura; S. P. Martin

For generic parameter choices, Δ_{BG} *is large*

But if: $m_{Q_{1,2}} = m_{U_{1,2}} = m_{D_{1,2}} = m_{L_{1,2}} = m_{E_{1,2}} \equiv m_{16}(1,2)$ then $\sim 0.007m_{16}^2(1,2)$

Even better: $m_{H_u}^2 = m_{H_d}^2 = m_{16}^2(3) \equiv m_0^2$ => $-0.017m_0^2$

For correlated parameters, EWFT collapses in 3rd gen. sector!

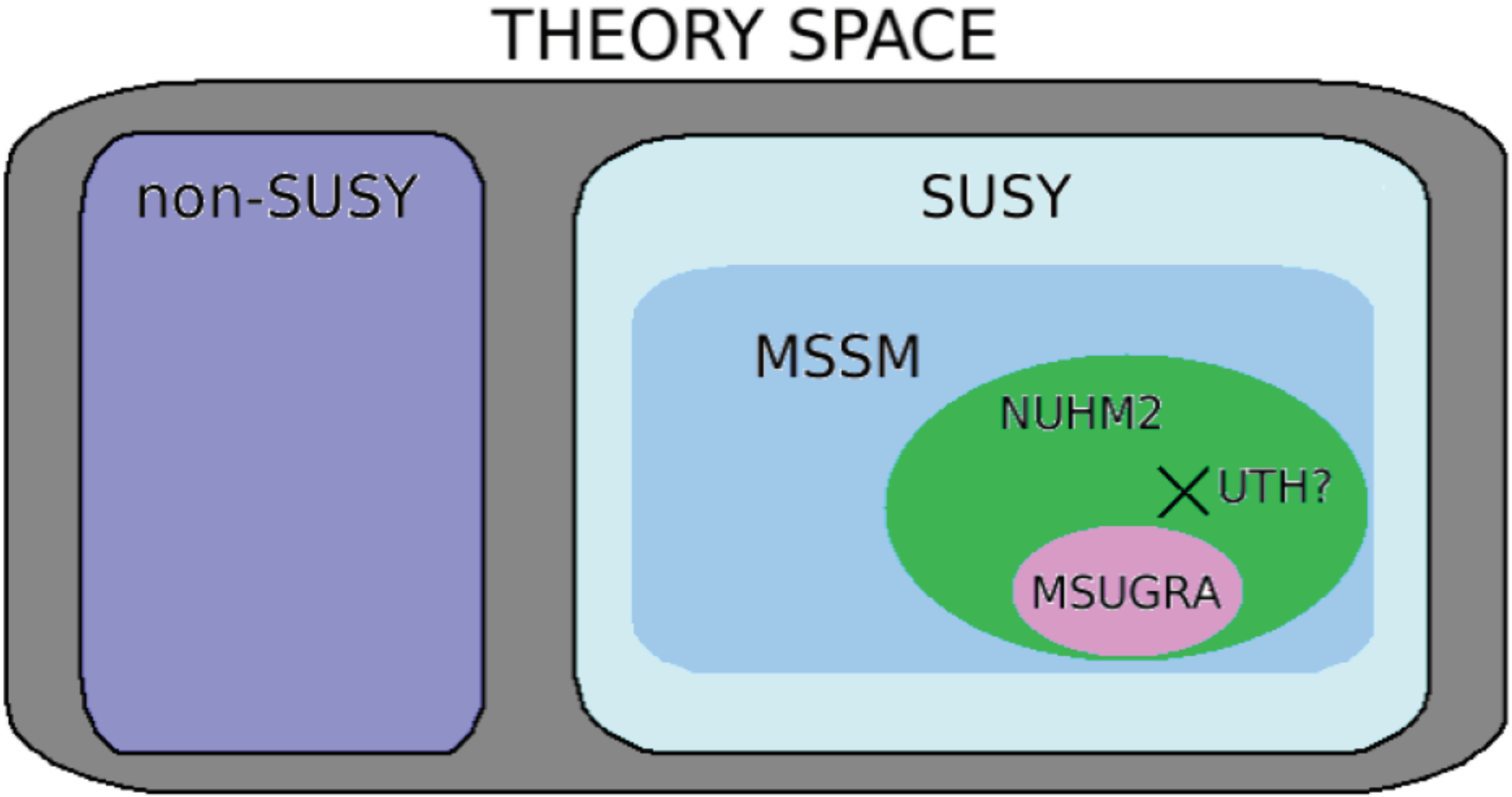
model	c_{m_0}	$c_{m_{1/2}}$	c_{A_0}	c_μ	c_{H_u}	c_{H_d}	Δ_{BG}
mSUGRA	156	762	1540	-25.1	−−	−−	1540
NUHM2	16041	762	1540	-25.1	-15208	-643.6	16041

Table 1: Sensitivity coefficients and Δ_{BG} for mSUGRA and NUHM2 model with $m_0 = 9993.4$ GeV, $m_{1/2} = 691.7$ GeV, $A_0 = -4788.6$ GeV and $\tan\beta = 10$. The mSUGRA output values of $\mu = 309.7$ GeV and $m_A = 9859.9$ GeV serve as NUHM2 inputs so that the two models have exactly the same weak scale spectra.

Lesson: the BG measure determines fine-tuning within particular effective theories.
Its value changes from theory to theory,
i.e. it is highly model-dependent,
as it must be since it depends on parameters.

Interpretation of BG in terms of UTH:

- most theorists hypothesize existence of an ultimate theory which describes nature
- perhaps MSSM with all correlated parameters is low E effective theory: UTH
- hope is that UTH is contained within more general multi-parameter effective theories which are popular in literature: mSUGRA, nuhm2,...
- The Δ_{BG} measures EWFT in the multi-parameter effective theories instead of UTH: for large number of parameters, it loses parameter correlations: this leads leads to overestimate
- example: mSUGRA serves as toy UTH for NUHM2 which contains more parameters
- need an EWFT measure which gives same value for effective theories as for UTH (i.e. model-independent)


THEORY SPACE
non-SUSY
SUSY
MSSM
NUHM2
UTH?
MSUGRA

What we really want to know is:
is nature is fine-tuned,
(and by implication the UTH which describes it),
and not whether-or-not
the more general effective theories
(which might contain the UTH)
are fine-tuned

Are we then to give up on naturalness
as a guide to SUSY models?

Model-independent EWFT measure: Δ_{EW}

No large uncorrelated cancellations in m(Z) or m(h)

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

a weak scale relation!

$\Delta_{EW} \equiv max_i \left|C_i\right| / (m_Z^2/2)$ with $C_{H_u} = -m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)$ etc.

Since Δ_{EW} model-independent (within MSSM),

expect same value for Eff. theory as for UTH!

In order to achieve low Δ_{EW}, it is necessary that $-m_{H_u}^2$, μ^2 and $-\Sigma_u^u$ all be nearby to $m_Z^2/2$ to within a factor of a few[12, 13]:

1. μ is required to lie in the $100 - 300$ GeV range,
2. a value of $m_{H_u}^2(m_{GUT}) \sim (1.3 - 2.5)m_0$ may be chosen so that $m_{H_u}^2$ is driven radiatively to slightly negative at the weak scale, leading to $m_{H_u}^2(weak) \sim -m_Z^2/2$, and
3. with large stop mixing from $A_0 \sim \pm 1.6 m_0$, the top-squark radiative corrections are softened while m_h is raised to the ~ 125 GeV level.

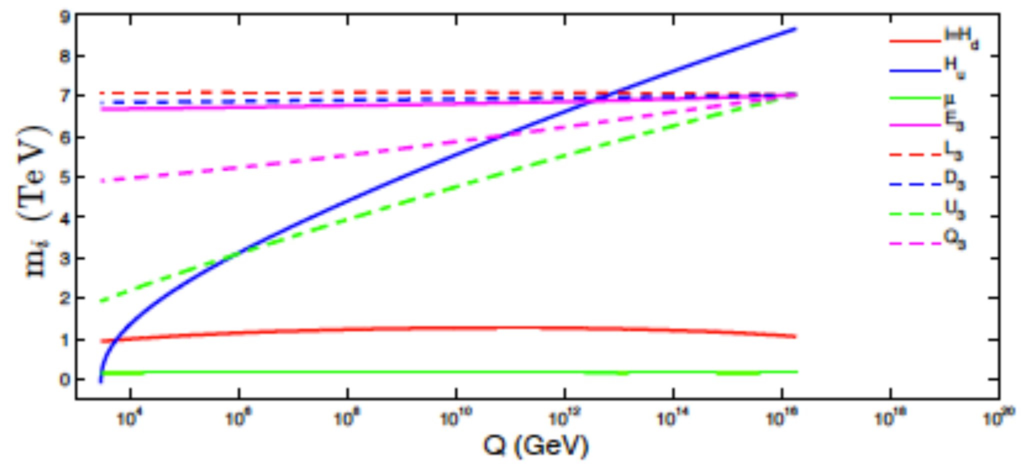
Model-independent EWFT measure: Δ_{EW}

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

For the top squark contributions, we find

$$\Sigma_u^u(\tilde t_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde t_{1,2}}^2)\left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8 g_Z^2 (\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde t_2}^2 - m_{\tilde t_1}^2}\right]$$

where $\Delta_t = (m_{\tilde t_L}^2 - m_{\tilde t_R}^2)/2 + M_Z^2 \cos 2\beta (\frac{1}{4} - \frac{2}{3}x_W)$ and $x_W \equiv \sin^2\theta_W$



where

$$F(m^2) = m^2 \left(\log \frac{m^2}{Q^2} - 1 \right)$$

with the optimized scale choice $Q^2 = m_{\tilde t_1} m_{\tilde t_2}$.

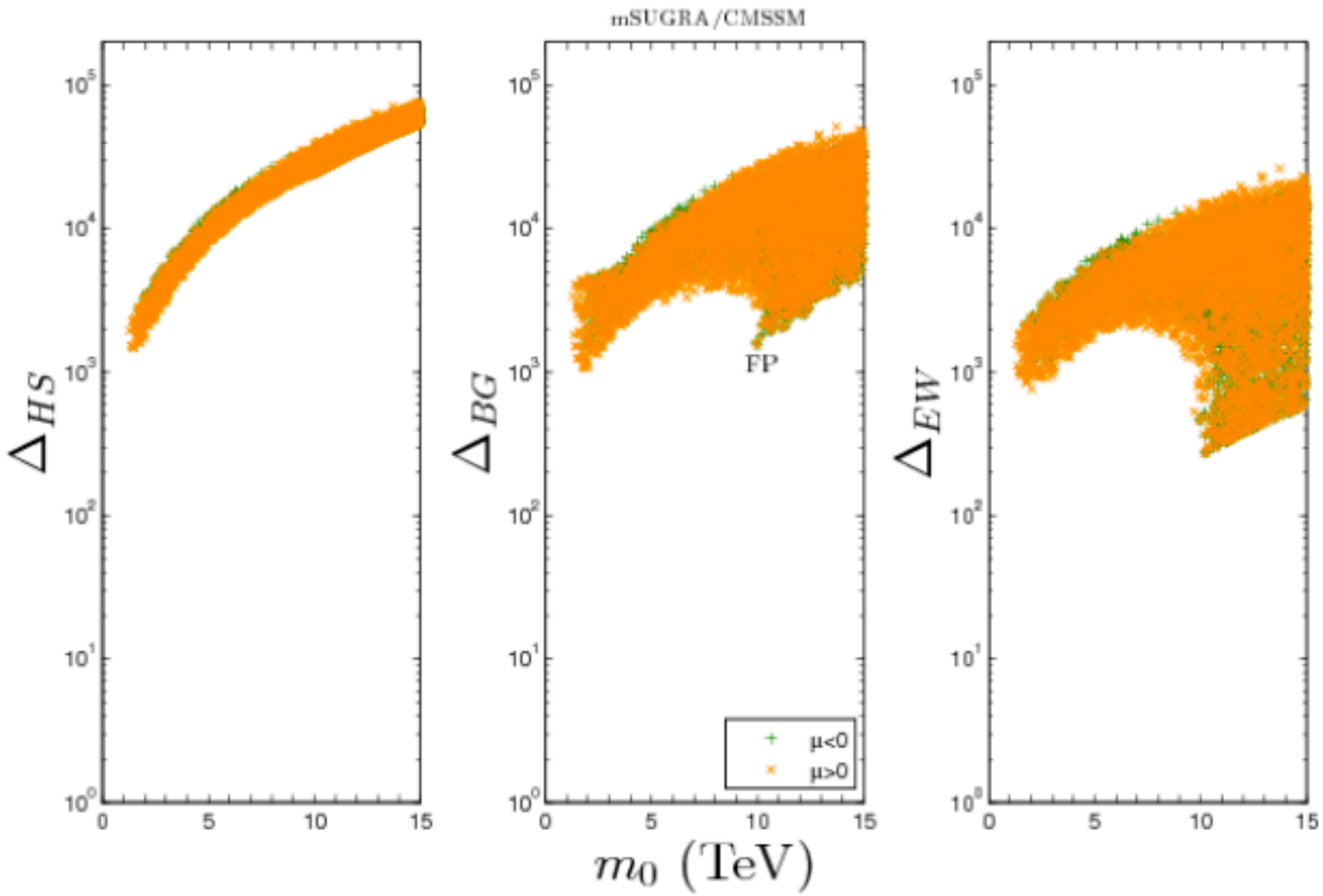
Large At drives down $\Sigma_u^u(\tilde t_{1,2})$ while lifting m(h) to 125 GeV!

Radiatively-driven natural SUSY (RNS):

H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, *Phys. Rev. Lett.* **109** (2012) 161802.

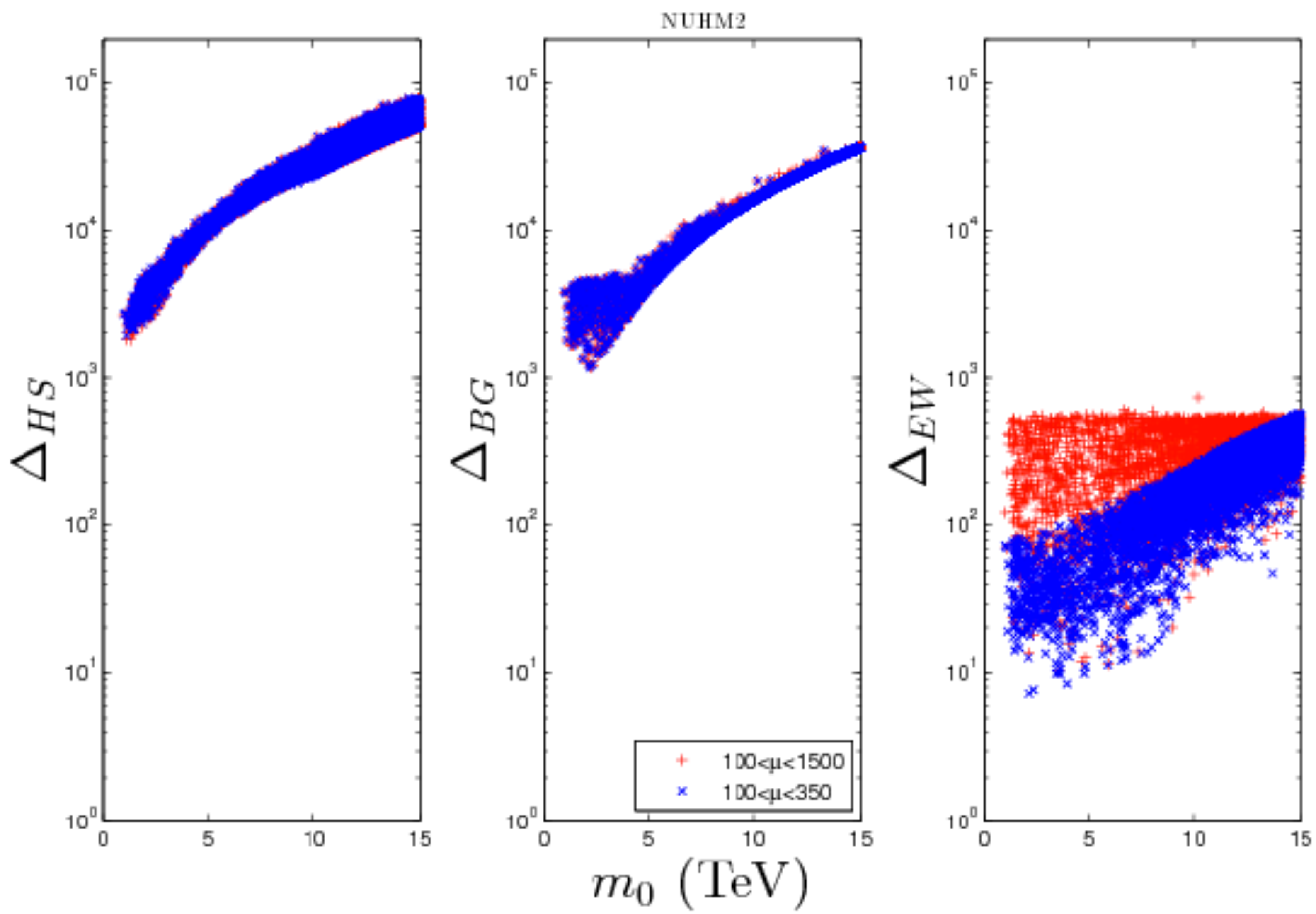
H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, *Phys. Rev.* **D 87** (2013) 115028 [arXiv:1212.2655 [hep-ph]].

Scan over mSUGRA/CMSSM model:
is fine-tuned under all three measures:
this effective theory is unlikely to contain the UTH



HB, Barger, Mickelson: arXiv:1309.2984

The NUHM2 model allows for not-too-heavy stops at 1-3 TeV with large mixing and m(h)~125 GeV while maintaining low mu~100-200 GeV:
it allows for EWFT at just ~10% level,
thus it may well contain the UTH



HB, Barger, Mickelson: arXiv:1309.2984

How conventional measures overestimate electroweak fine-tuning in supersymmetric theory

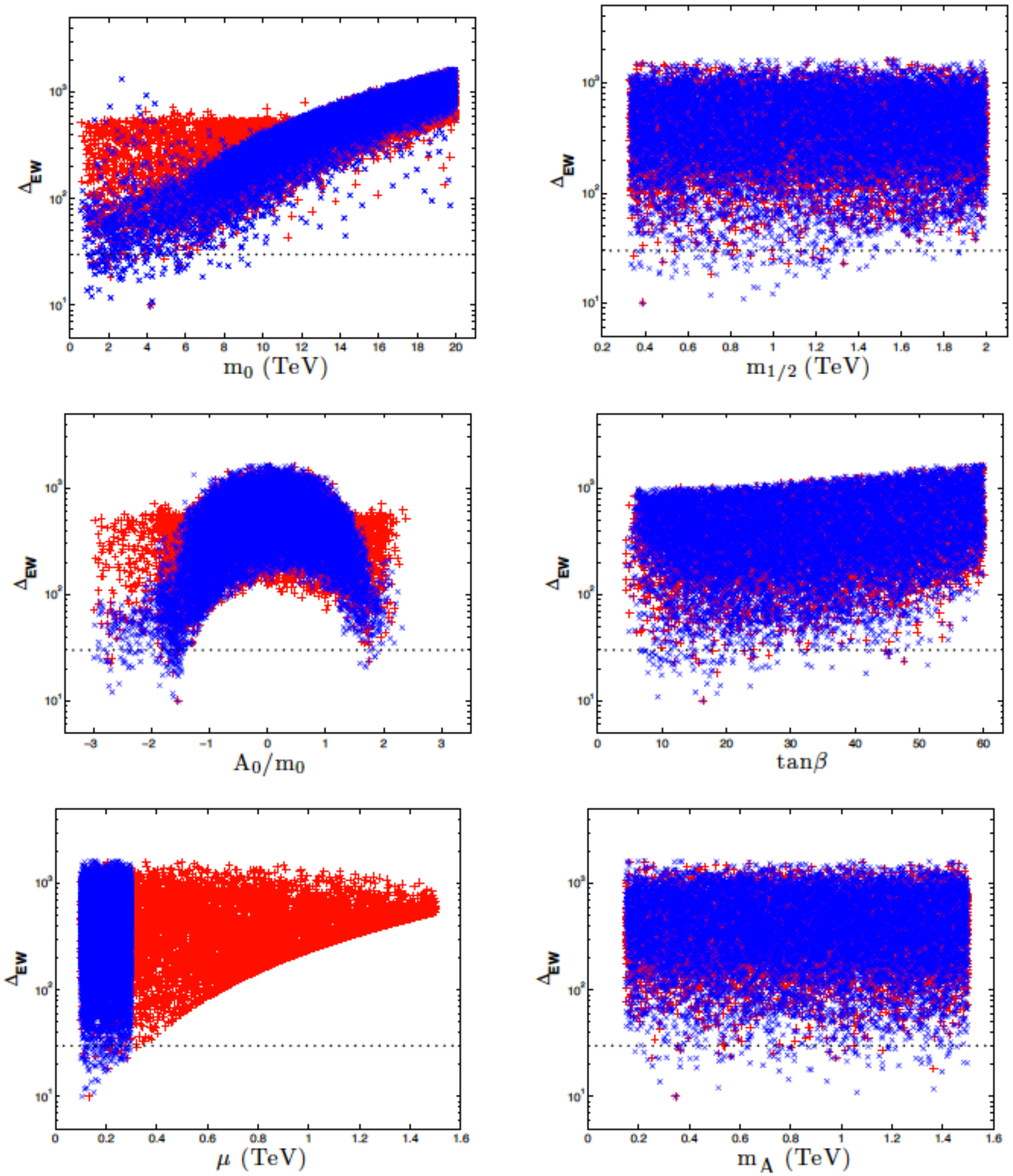
HB, Barger, Mickelson: arXiv:1309.2984

model	Δ_{HS}	Δ_{BG}	Δ_{EW}
mSUGRA	24302	1540	462
NUHM2	24302	16041	462
pMSSM	462	462	462

Table 2: Values of Δ_{HS}, Δ_{BG} and Δ_{EW} for the mSUGRA/CMSSM, NUHM2 and pMSSM models. For mSUGRA, we take $m_0 = 9993.4$ GeV, $m_{1/2} = 691.7$ GeV, $A_0 = -4788.6$ GeV and $\tan\beta = 10$. The mSUGRA output values of $\mu = 309.7$ GeV and $m_A = 9859.9$ GeV serve as NUHM2 inputs. The weak scale outputs of mSUGRA and NUHM2 serve as pMSSM inputs so that all three models have exactly the same weak scale spectra.

$$\Delta_{EW} < \Delta_{BG} \lesssim \Delta_{HS}$$

Which parameter choices lead to low EWFT and how low can Δ_{EW} be?



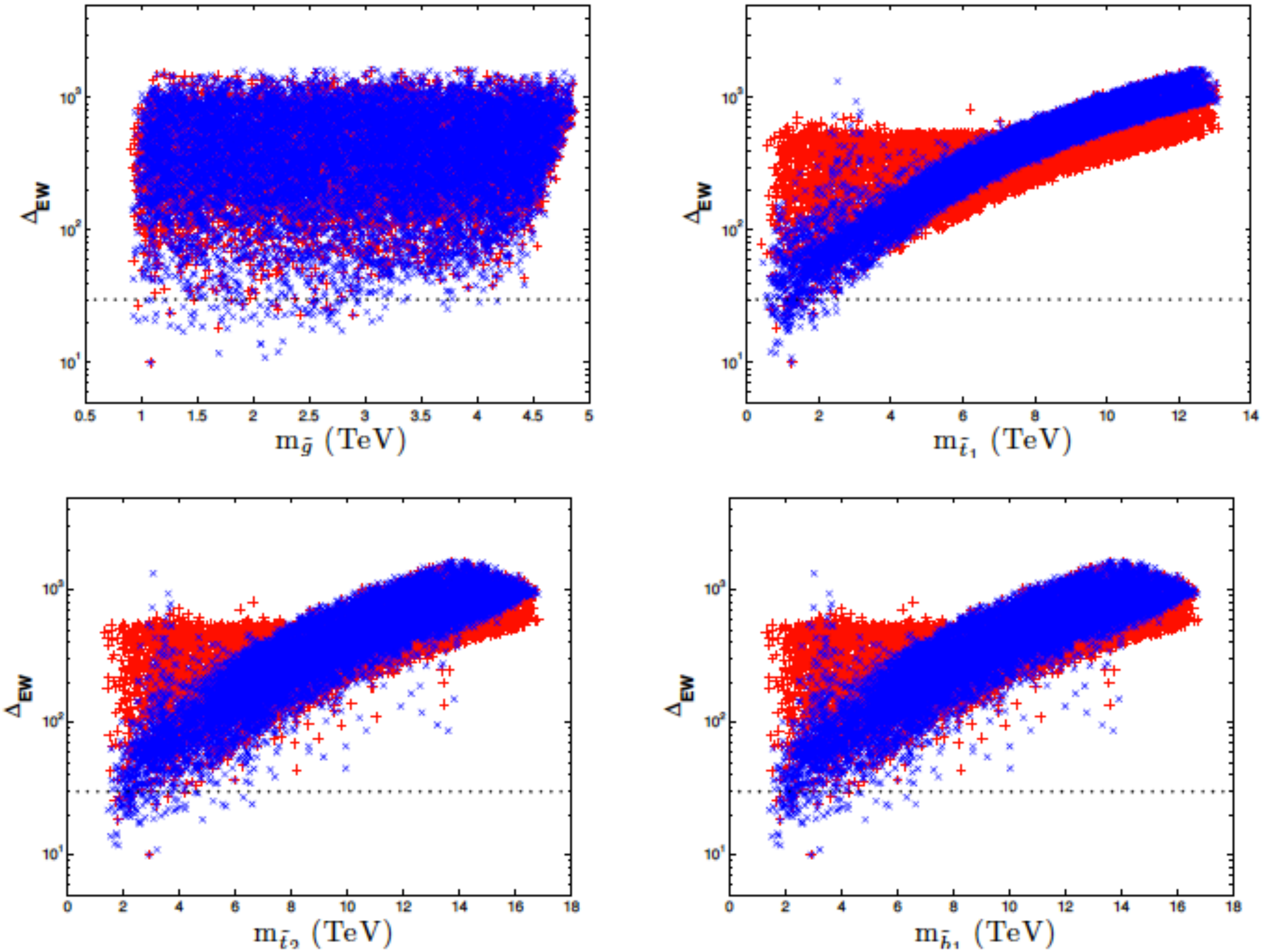
$$\Delta_{EW} \sim 10 \text{ or } 10\%\ EWFT$$

High-scale models with low Δ_{EW}:

Radiatively-driven natural SUSY, or RNS

HB, Barger, Huang, Mickelson, Mustafayev, Tata, arXiv:1212.2655

Sparticle masses:

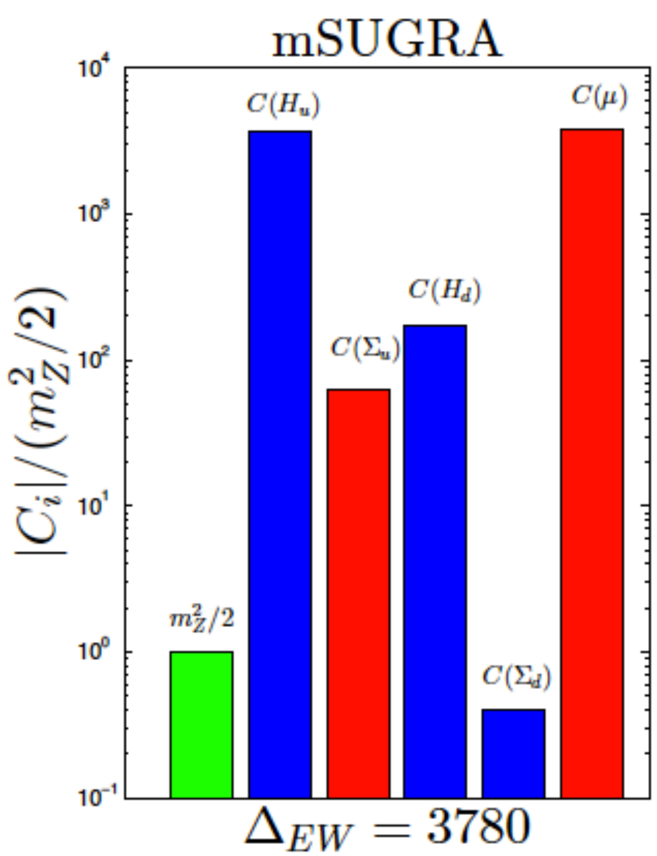


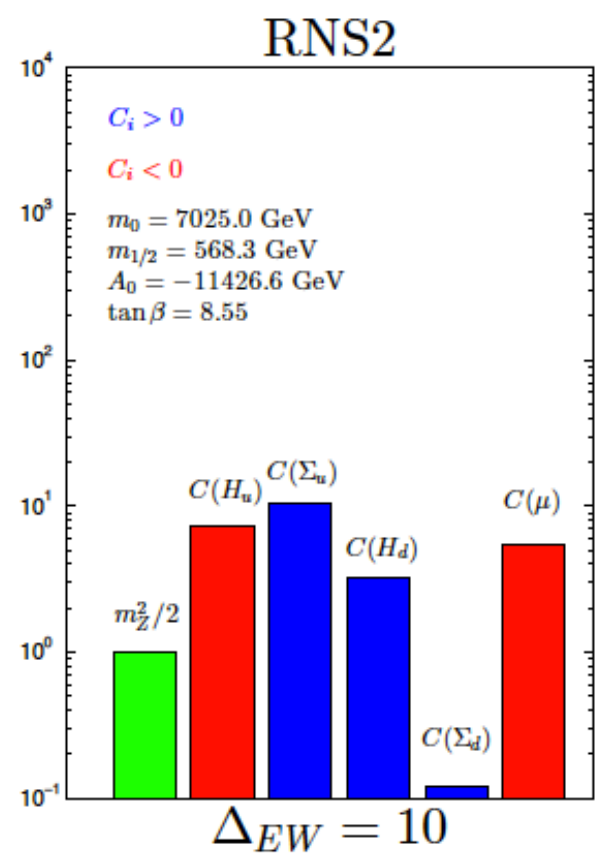
m(t1)~1-3 TeV
m(t2,b1)~2-4 TeV
m(glno)~1-4 TeV

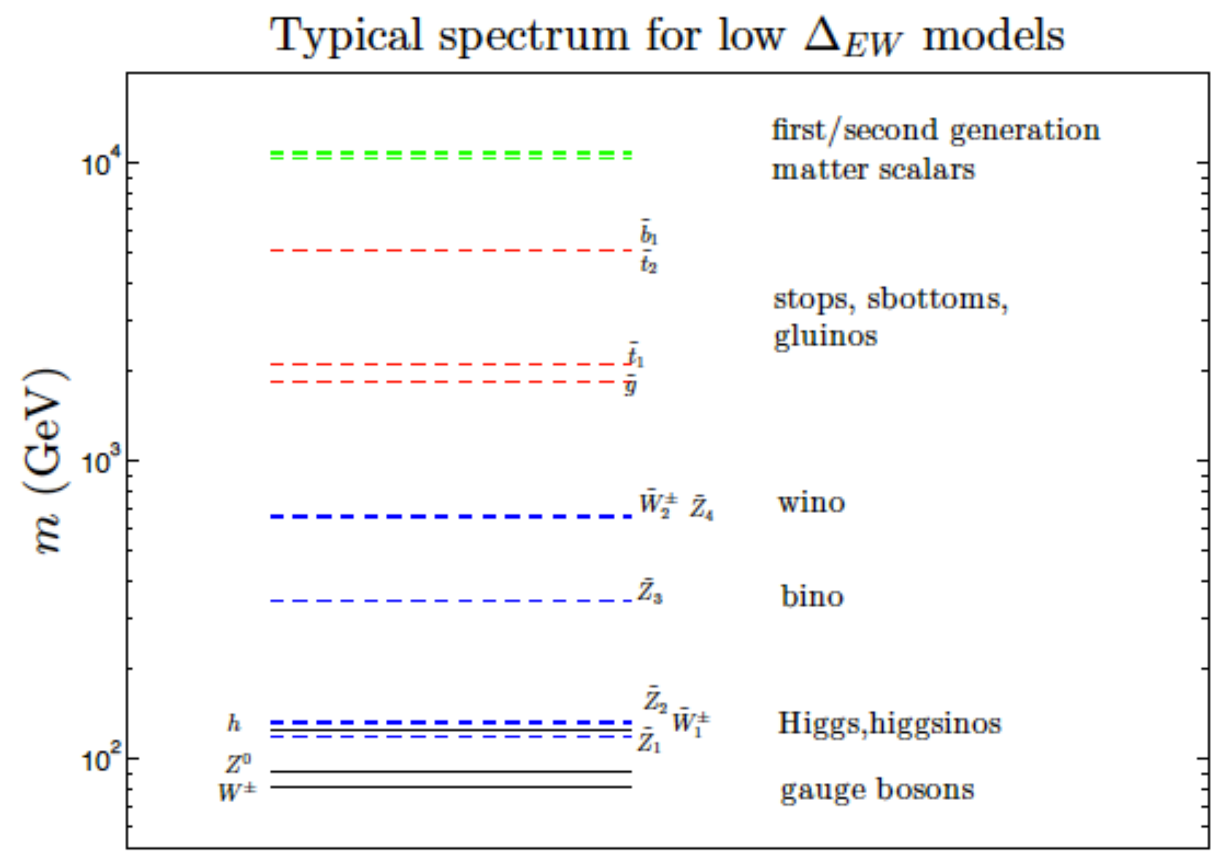
heavier than earlier NS models,
allows for m(h)~125 GeV,
BF(b->s gamma) OK; evade stop searches
within MSSM

All contributions to m(Z) and m(h) are comparable to m(Z) and m(h):

model is natural in EW sector!







There is a Little Hierarchy, but it is no problem

dark matter in natural SUSY

- see talk by KJ Bae: Bae, HB, Chun, arXiv:1309.0519
- thermal WIMP (higgsino) abundance low by 10-15
- solve ``strong fine-tuning" via axion
- tame SUSY mu problem via Kim-Nilles/DFSZ
- get 90-95% axion CDM plus 5-10% higgsinos over bulk of parameter space
- reduced abundance of higgsinos still seeable at ton-scale WIMP detectors
- may see axion as well, e.g. ADMX

Can radiatively-driven natural SUSY be discovered at LHC?

To check, create an RNS model line with variable gluino mass:

$$
\begin{aligned}
m_0 &= 5 \text{ TeV}, \\
m_{1/2} &: \quad \text{variable between } 0.3 - 2 \text{ TeV}, \\
A_0 &= -1.6 m_0, \\
\tan\beta &= 15, \\
\mu &= 150 \text{ GeV}, \\
m_A &= 1 \text{ TeV}.
\end{aligned}
$$

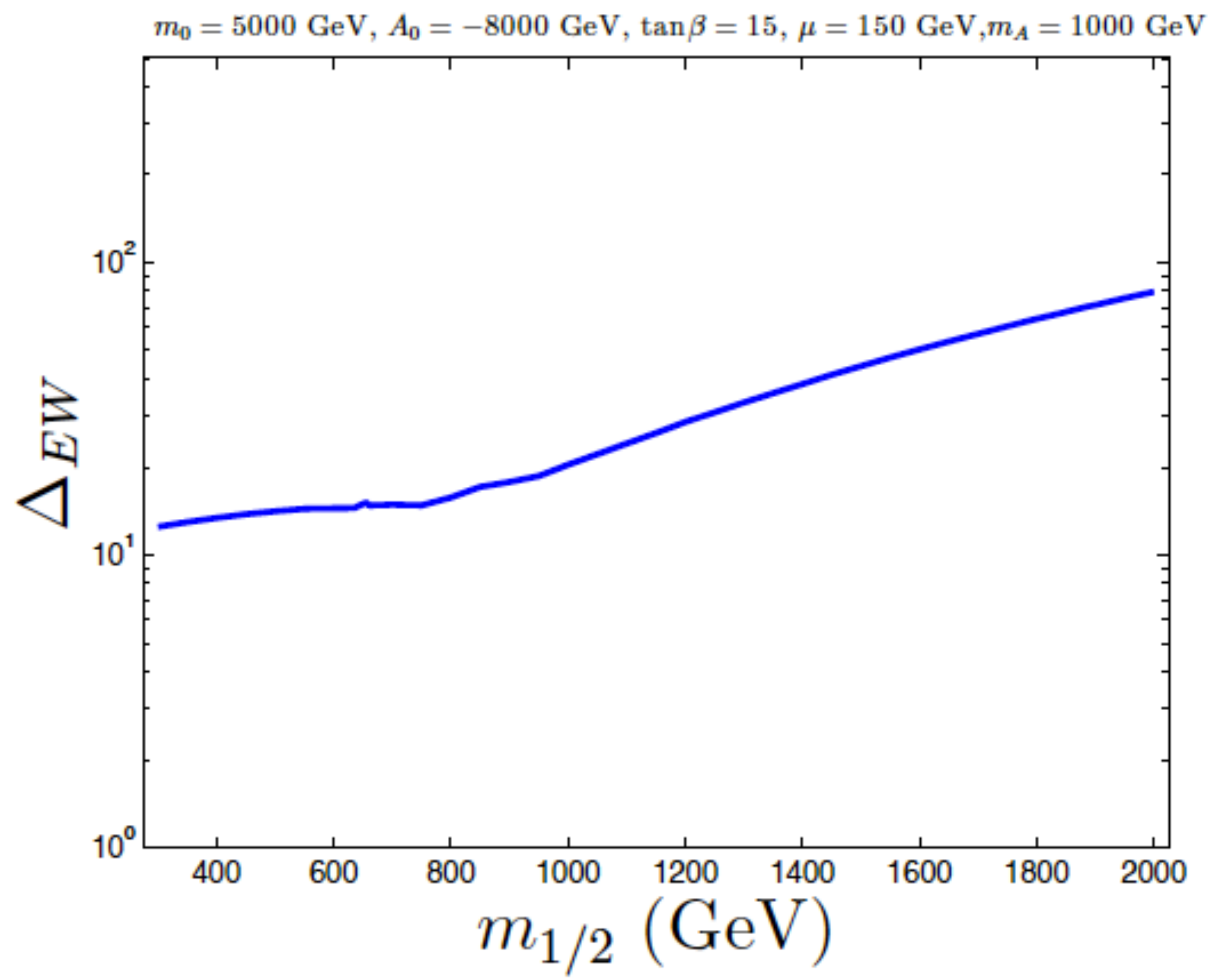


Figure 1: Plot of $\Delta_{\rm EW}$ versus $m_{1/2}$ along the RNS model line.

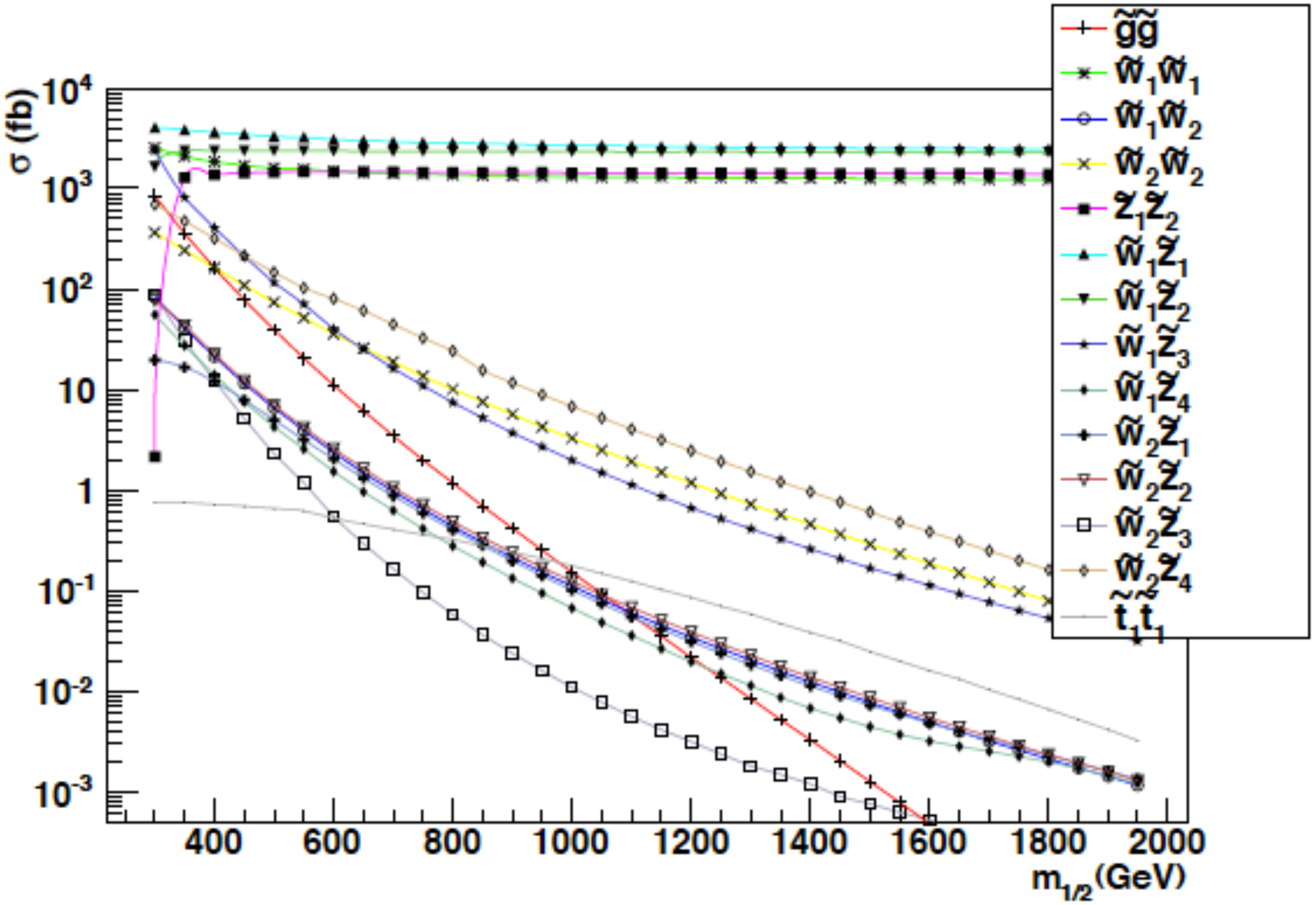
10%-2% EWFT

(Split generation model allows m0(1,2)~20-30 TeV)

H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, arXiv:1310.4858

Sparticle production along RNS model-line:

LHC14

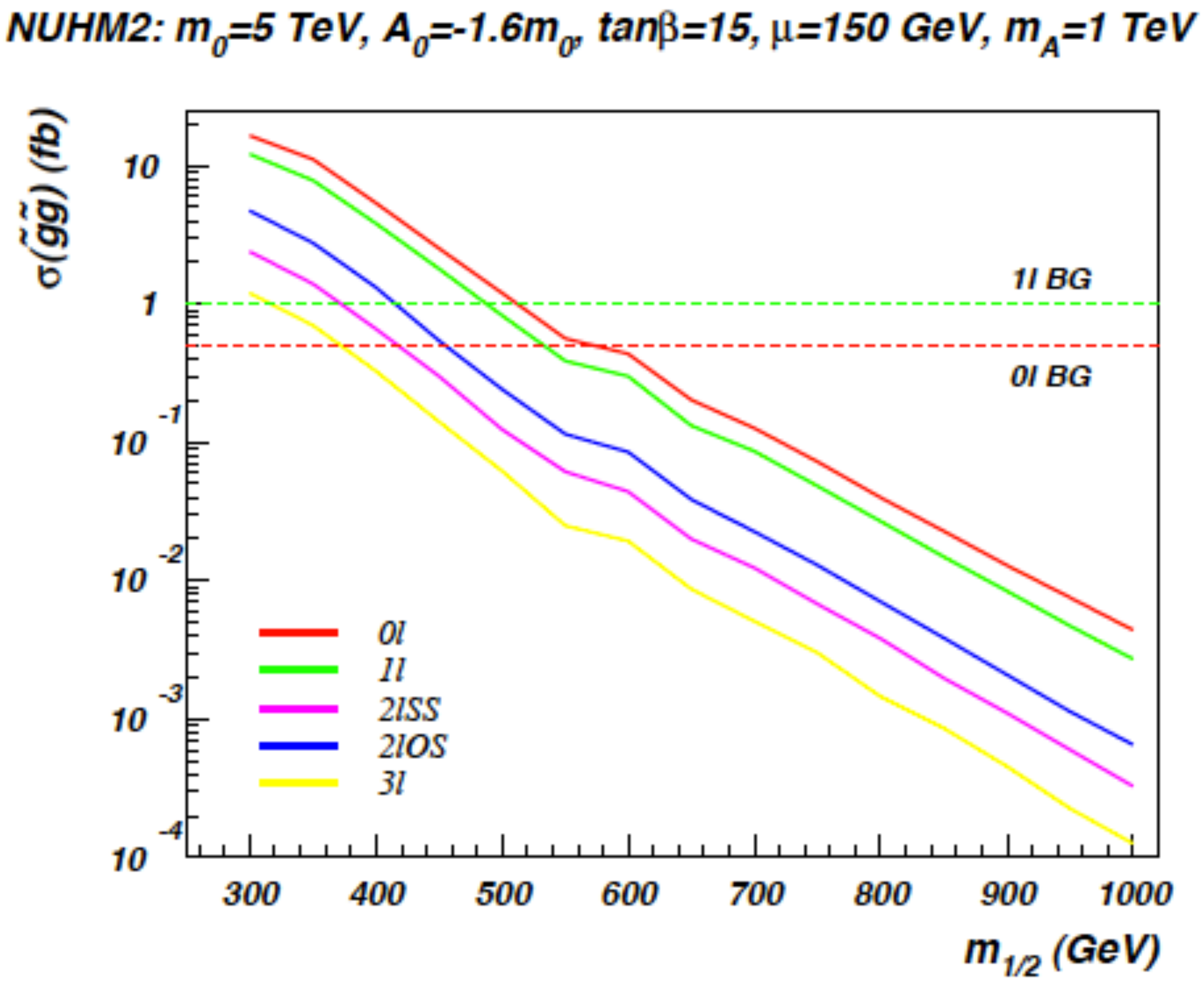


higgsino pair production dominant-but only soft visible energy release from higgsino decays

largest visible cross section: wino pairs

gluino pairs sharply dropping

gluino pair cascade decay signatures



Particle	dom. mode	BF
$\tilde{g}$	$\tilde{t}_1 t$	$\sim 100\%$
$\tilde{t}_1$	$b\widetilde{W}_1$	$\sim 50\%$
$\widetilde{Z}_2$	$\widetilde{Z}_1 f\bar{f}$	$\sim 100\%$
$\widetilde{Z}_3$	$\widetilde{W}_1^{\pm}W^{\mp}$	$\sim 50\%$
$\widetilde{Z}_4$	$\widetilde{W}_1^{\pm}W^{\mp}$	$\sim 50\%$
$\widetilde{W}_1$	$\widetilde{Z}_1 f\bar{f}'$	$\sim 100\%$
$\widetilde{W}_2$	$\widetilde{Z}_i W$	$\sim 50\%$

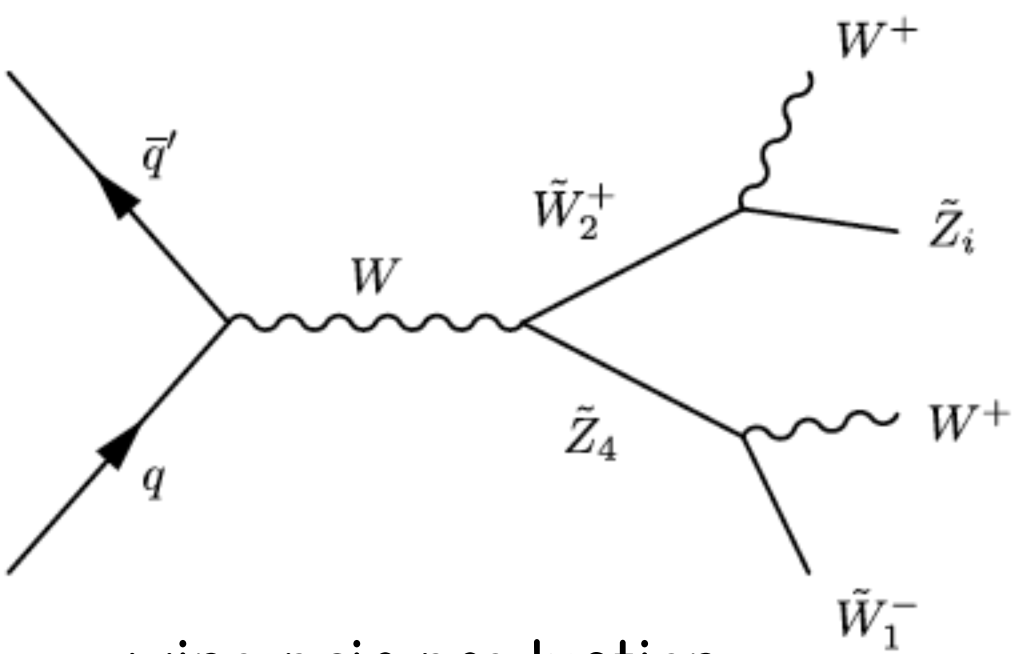
Table 1: Dominant branching fractions of various sparticles along the RNS model line for $m_{1/2} =$ 1 TeV.

Int. lum. (fb^{-1})	$\tilde{g}\tilde{g}$
10	1.4
100	1.6
300	1.7
1000	1.9

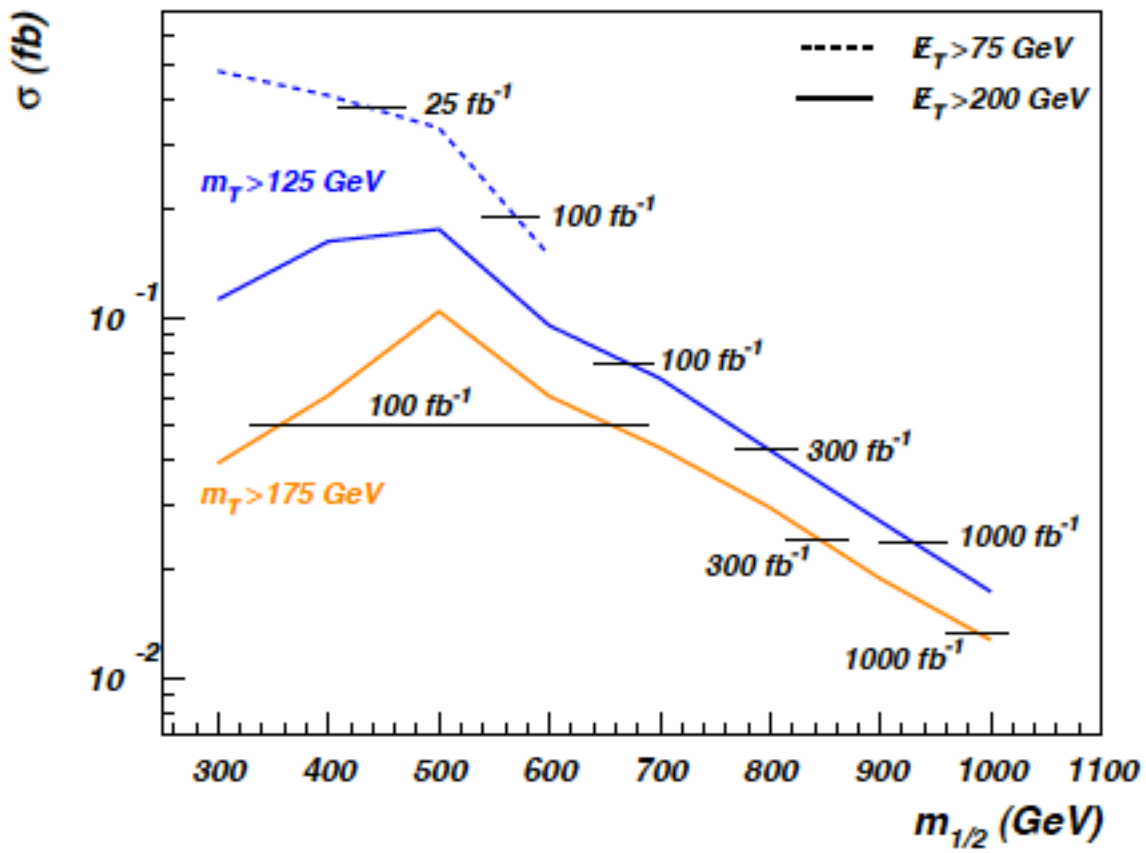
LHC14 reach in m(gluino) (TeV)

since m(gluino) extends to ~5 TeV, LHC14 can see about half the low EWFT parameter space in these modes

Characteristic same-sign diboson (SSdB) signature from SUSY models with light higgsinos:



wino pair production

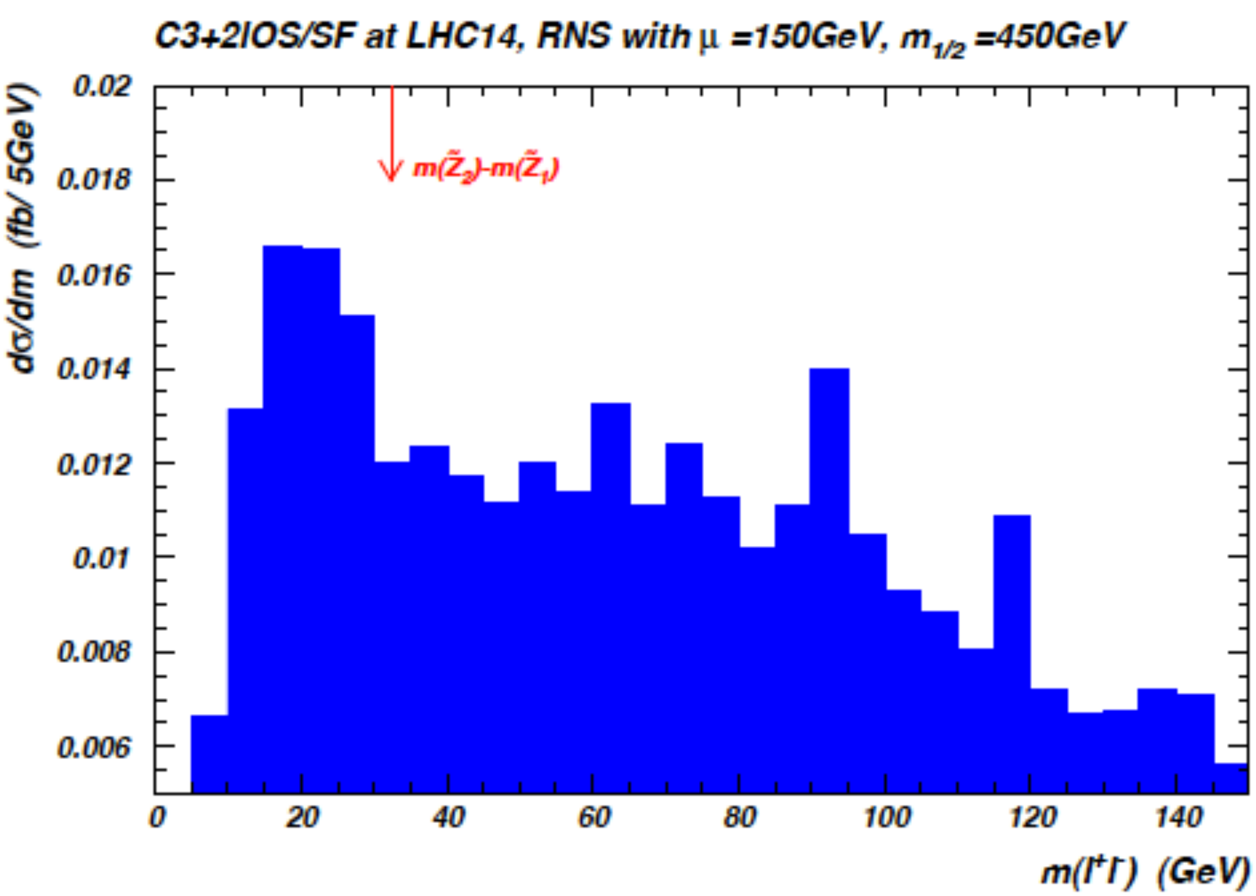


H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, *Phys. Rev. Lett.* **110** (2013) 151801.

This channel offers best reach of LHC14 for RNS; it is also indicate of wino-pair prod'n followed by decay to higgsinos

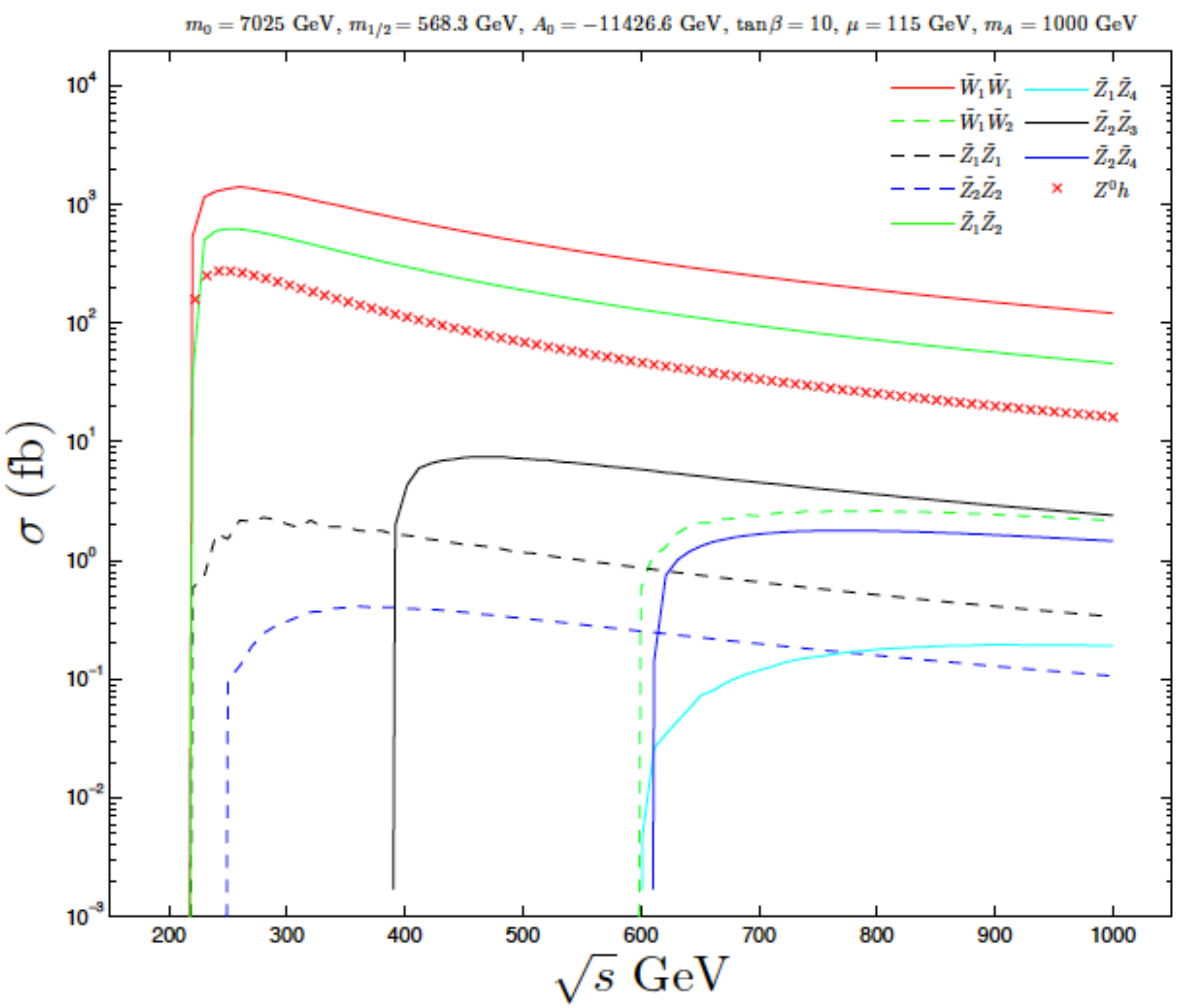
LHC14 has some reach for RNS; if a signal is seen, should be characteristic

Int. lum. (fb^{-1})	$\tilde{g}\tilde{g}$	SSdB	$WZ \to 3\ell$	4ℓ
10	1.4	–	–	–
100	1.6	1.6	–	~ 1.2
300	1.7	2.1	1.4	$\gtrsim 1.4$
1000	1.9	2.4	1.6	$\gtrsim 1.6$



OS/SF dilepton mass edge apparent from cascade decays with z2->z1+l+lbar

Smoking gun signature:
light higgsinos at ILC:
ILC is Higgs/higgsino factory!



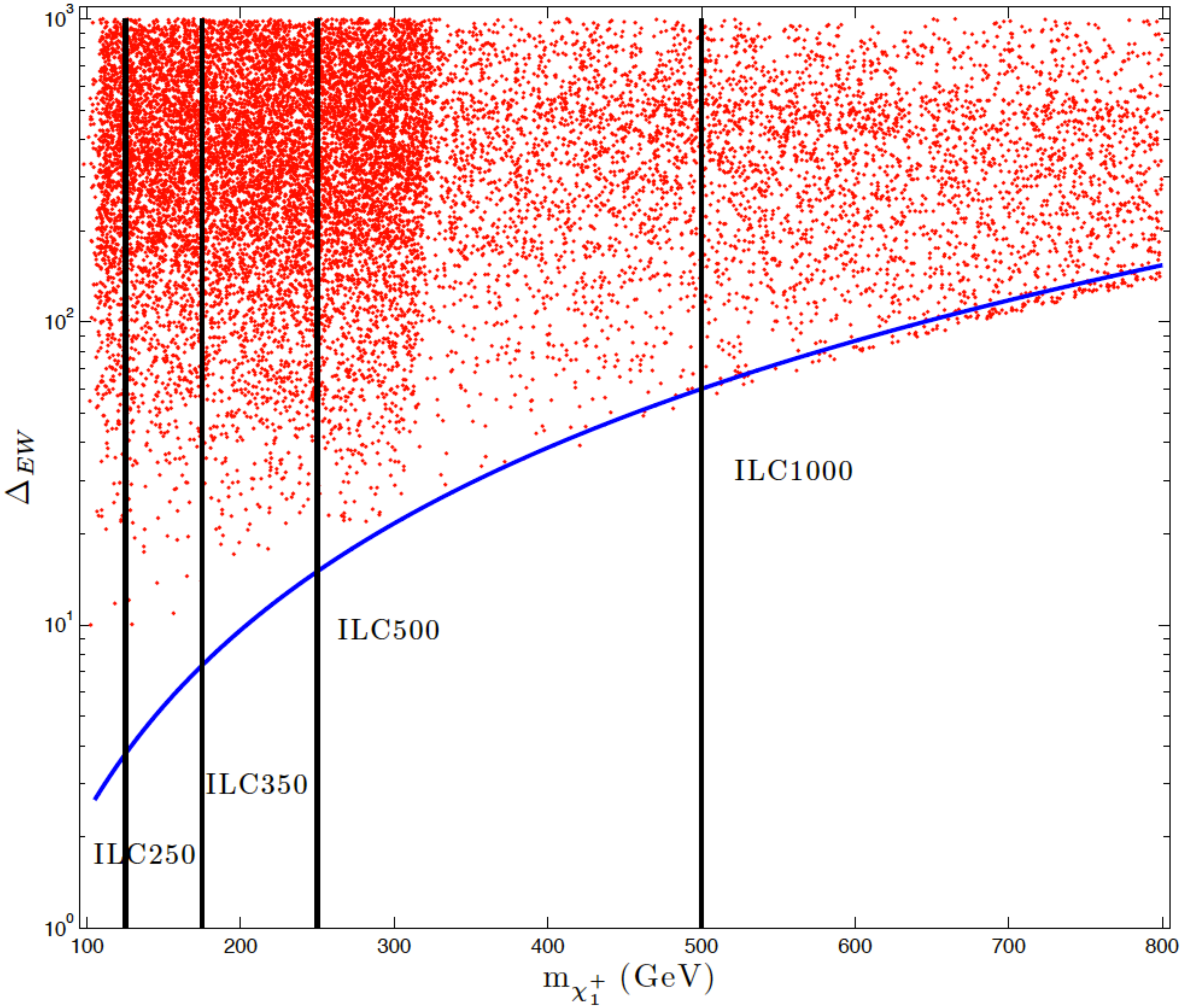
$$\sigma(higgsino) \gg \sigma(Zh)$$

10-15 GeV higgsino mass
gaps no problem
in clean ILC environment

ILC either sees light higgsinos or natural SUSY dead

Smoking gun signature: 4 light higgsinos at ILC!

$$e^+e^- \to \tilde{W}_1^+\tilde{W}_1^-,\ \tilde{Z}_1\tilde{Z}_2$$



$$m_{\tilde{W}_1^\pm},\ m_{\tilde{Z}_{1,2}}$$

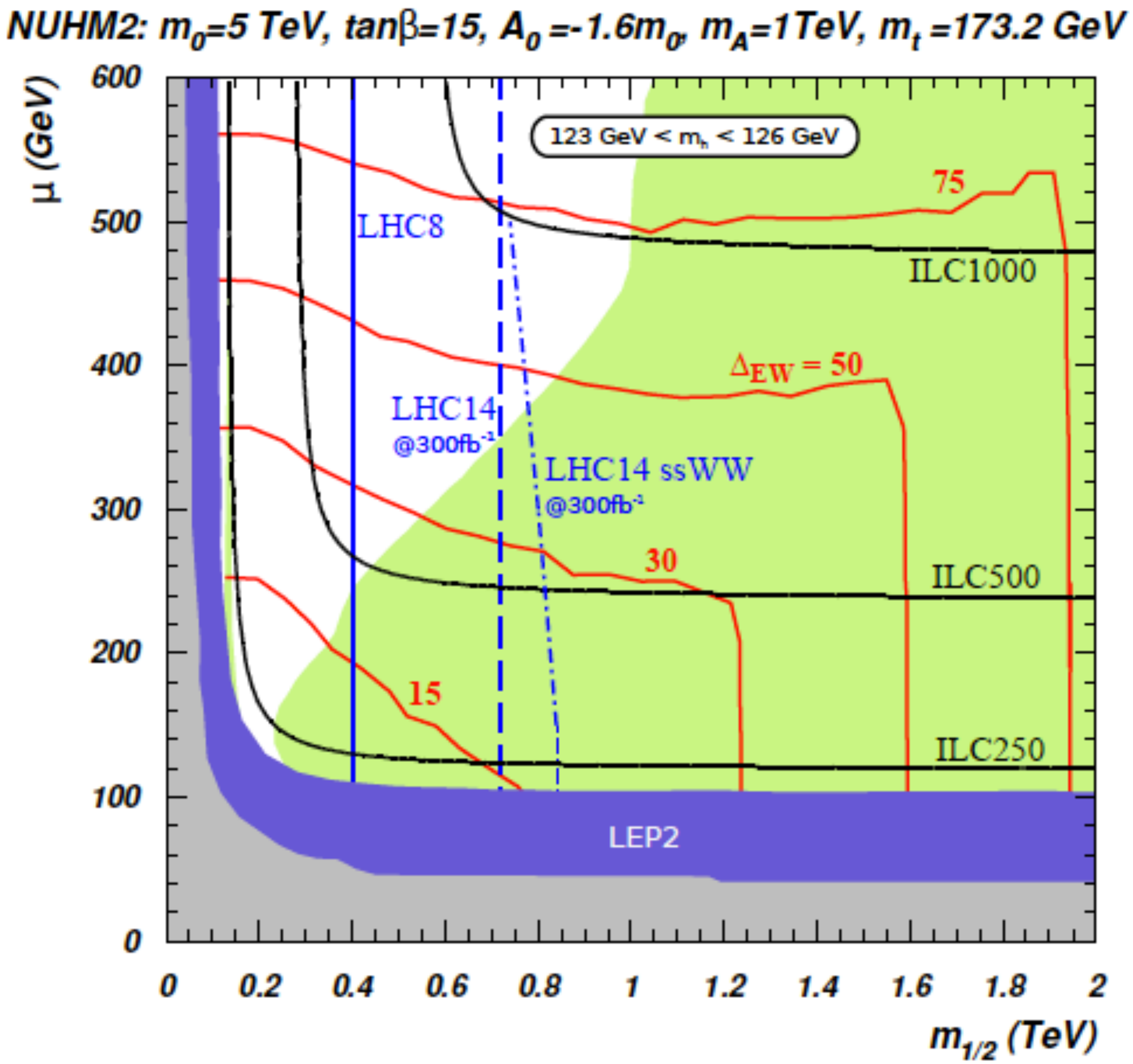
$$\sqrt{s} \sim \sqrt{2\Delta_{EW}}m_Z$$

ILC/CLIC have capability to measure SUSY parameters and actually reconstruct

$$\Delta_{EW}$$

measure and check if nature is EWFT'd?

LHC/ILC complementarity

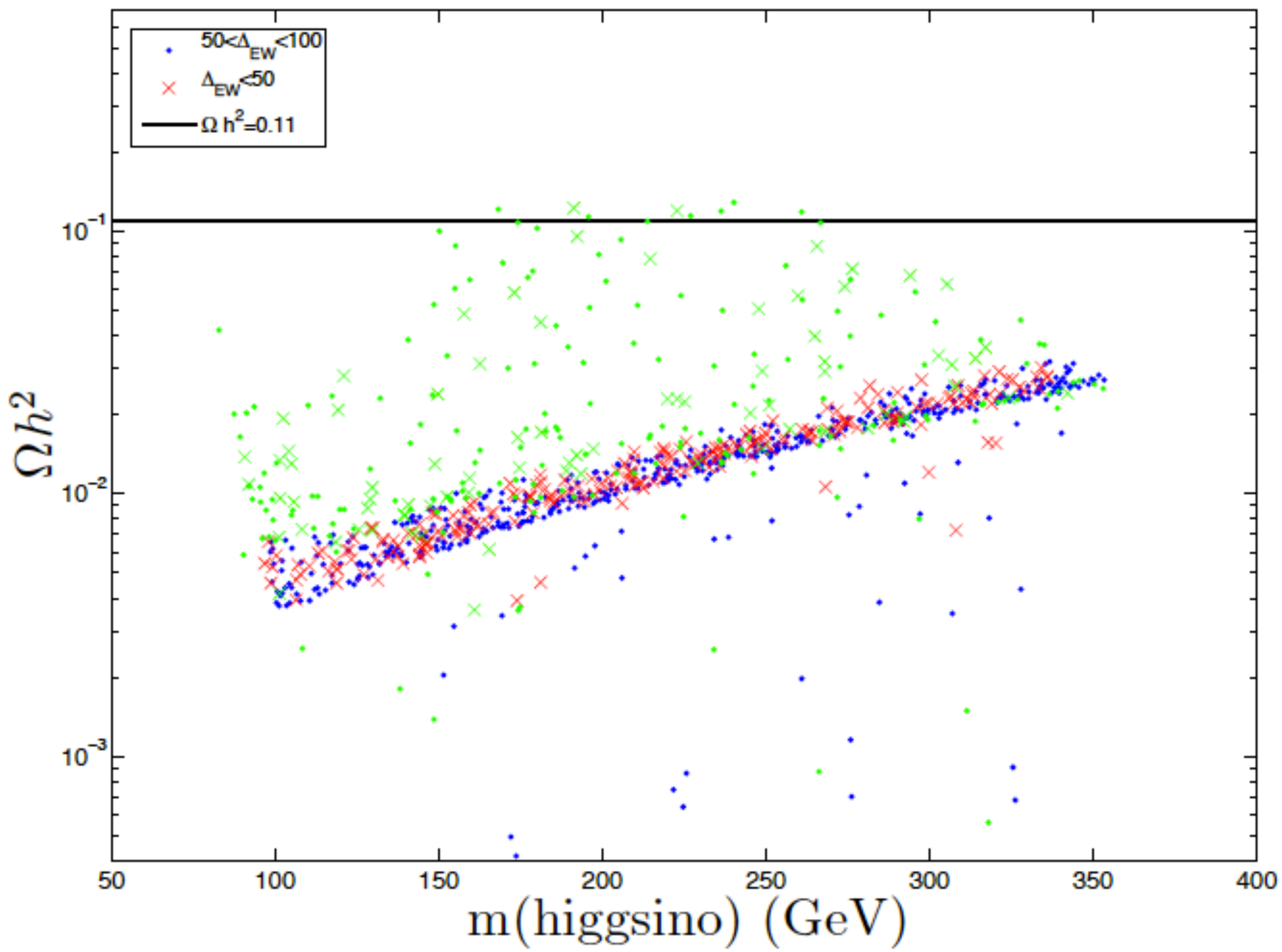


When to give up on naturalness in SUSY?
If ILC(500-600 GeV) sees no light higgsinos

Conclusions: status of SUSY post LHC8

- SUSY EWFT non-crisis: EWFT allowed at 10% level in radiatively-driven natural SUSY
- RNS spectra characterized by mainly higgsino-like WIMP: standard relic underabundance
- Also address strong CP problem via axion-axino-saxion
- DFSZ invisible axion model: solves mu problem while allowing for mu~m(Z)
- Expect mainly axion CDM with 5-10% higgsino-like WIMPs over much of p-space
- Direct detect both axion and higgsino-like WIMP
- Gamma-ray signal below bounds by 10-20: ways to go...
- LHC14 w/ 300 fb^-1 can see about half of RNS parameter space
- e+e- collider with sqrt(s)~500-600 GeV needed to find predicted light higgsino states

Mainly higgsino-like WIMPs thermally underproduce DM



green: excluded;
red/blue:allowed

Factor of 10-15 too low

But so far we have addressed only Part 1
of fine-tuning problem:

In QCD sector, the term $\frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu} \tilde{F}_A^{\mu\nu}$ must occur

But neutron EDM says it is not there: strong CP problem

(frequently ignored by SUSY types)

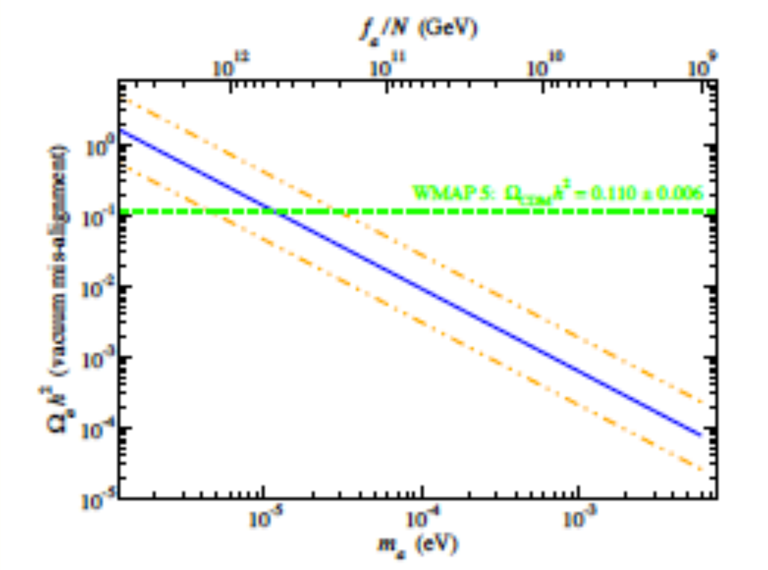
Best solution after 35 years: PQWW invisible axion

In SUSY, axion accompanied by axino and saxion

Changes DM calculus:
expect mixed WIMP/axion DM (2 particles)

Axion cosmology

- ★ Axion field eq'n of motion: $\theta = a(x)/f_a$
  - $\ddot{\theta} + 3H(T)\dot{\theta} + \frac{1}{f_a^2}\frac{\partial V(\theta)}{\partial \theta} = 0$
  - $V(\theta) = m_a^2(T) f_a^2 (1 - \cos\theta)$
  - Solution for T large, $m_a(T) \sim 0$: $\theta = const.$
  - $m_a(T)$ turn-on ~ 1 GeV
- ★ a(x) oscillates, creates axions with $\vec{p} \sim 0$: production via vacuum mis-alignment
- ★ $\Omega_a h^2 \sim \frac{1}{2}\left[\frac{6\times10^{-6} eV}{m_a}\right]^{7/6} \theta_i^2 h^2$
- ★ astro bound: stellar cooling $\Rightarrow f_a \gtrsim 10^9 GeV$



Axino/saxion decays

Decays very model-dependent;
also depend on KSVZ or DFSZ model

axino-> particle+sparticle: augment LSP abundance but
also provide late-time entropy injection

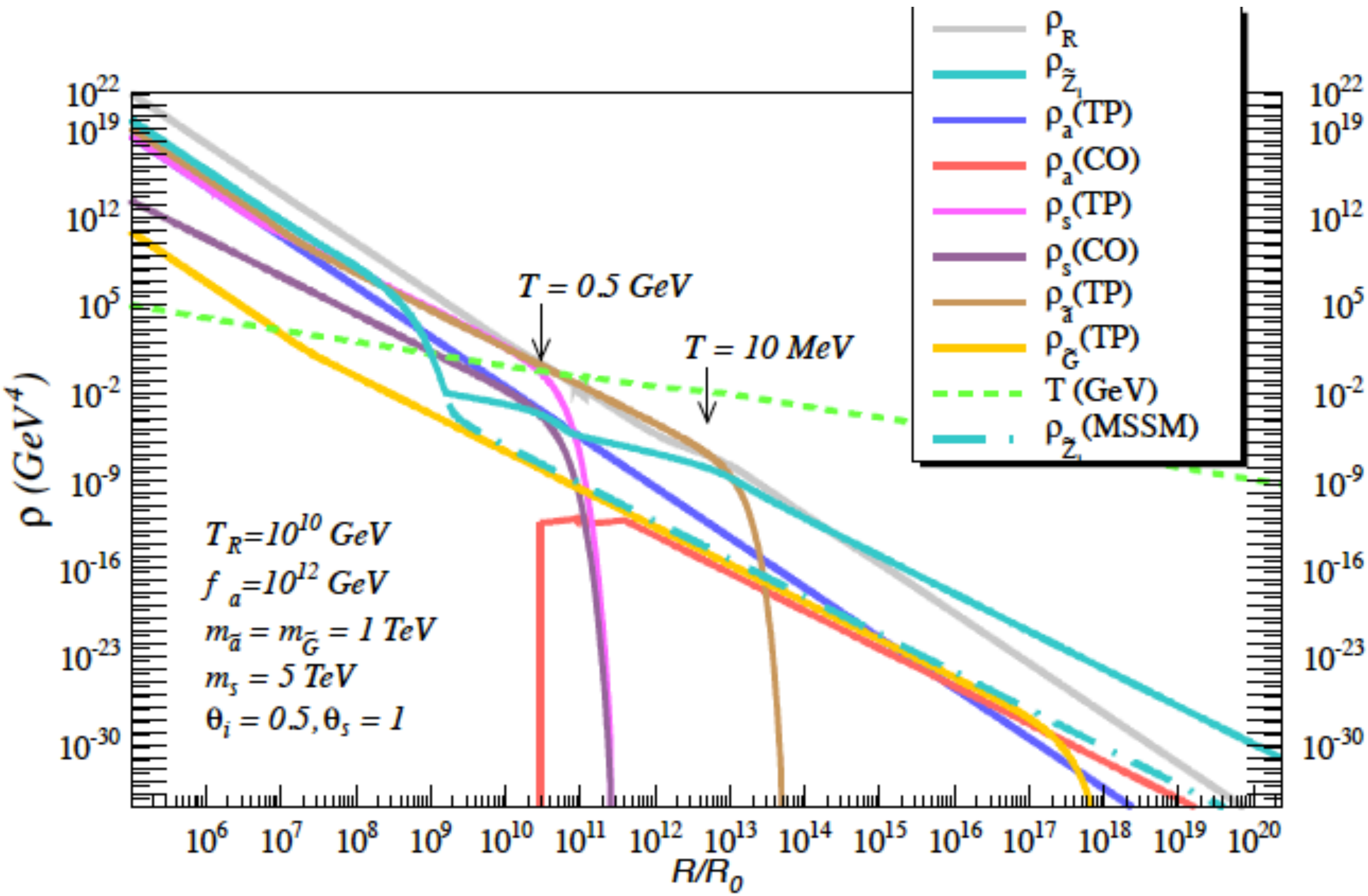
saxion-> gg, hh, etc SM particles (entropy dilution)

saxion-> glno+gno, hgno+hgno, etc (SUSY particles, augment)

saxion->aa, dark radiation, $\Delta N_{eff}\ bounds$

Bae, HB, Lessa, JCAP1304 (2013) 041

Coupled Boltzmann KSVZ $\xi = 0$



HB, Lessa, Sreethawong

mu problem: why is mu~m(weak) and not Mp?

$$W_{MSSM} \ni \mu H_u H_d$$

Kim-Nilles solution to SUSY mu problem

SUSY DFSZ model

(Dine-Fischler-Srednicki-Zhitnitsky, 1983)

Higgs fields Hu and Hd carry PQ charge: mu term forbidden

Field S carries PQ charge and contains axion-axino-saxion

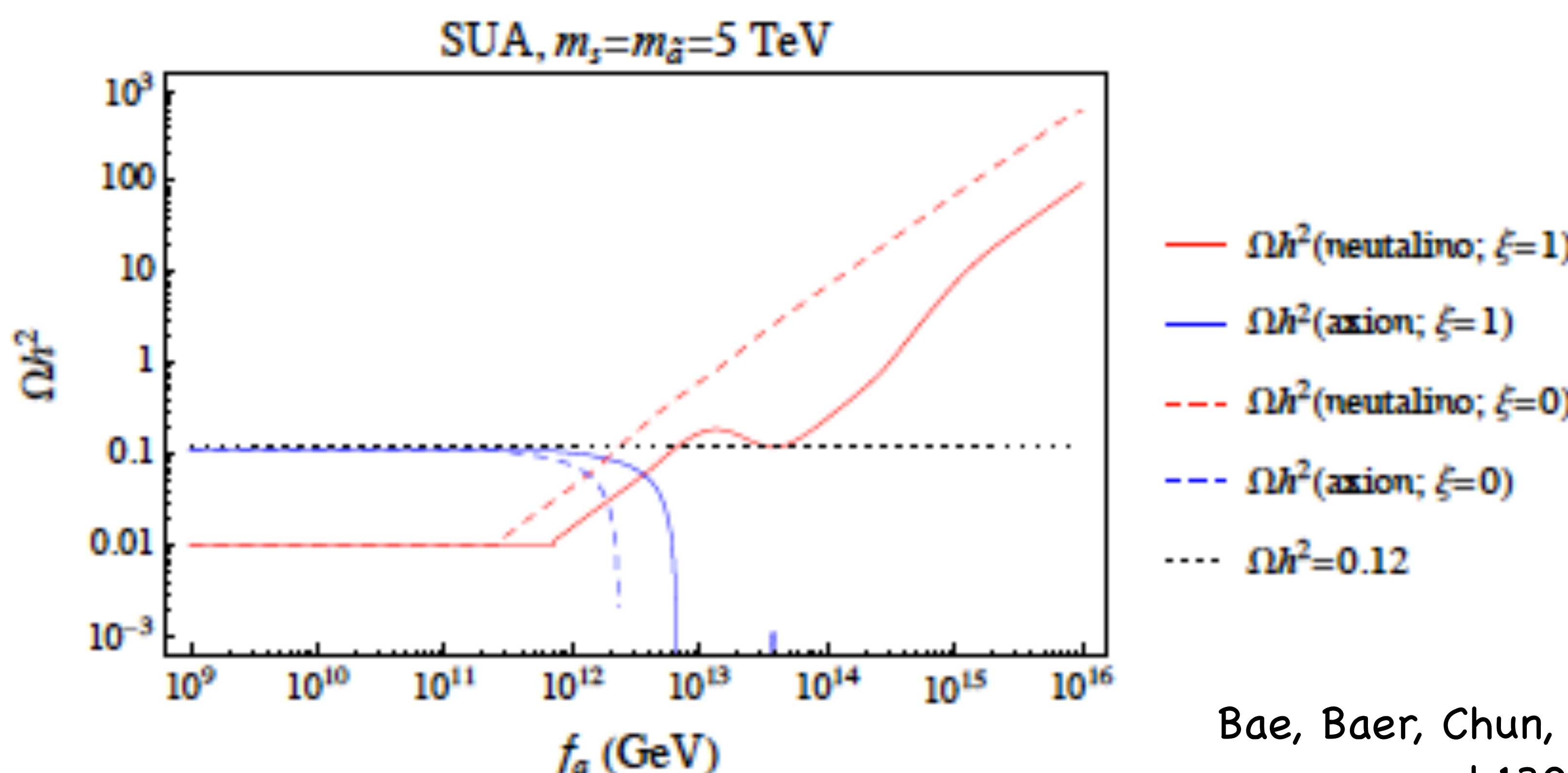
$$W_{\rm DFSZ} \ni \lambda \frac{S^2}{M_P} H_u H_d$$

If S develops vev ~ f_a, then weak scale mu generated!

$$\mu \sim \lambda f_a^2 / M_P \sim \lambda m_{3/2}$$

Tree level axion superfield couplings to higgs/higgsinos: axino/saxion decay before WIMP freezeout for f_a<10^12 GeV

Then usual WIMP abundance obtains but supplemented by axion CDM!



Bae, Baer, Chun, arXiv:1309.0519 and 1309.5365

Get 90-95% axion CDM plus 5-10% higgsino-like WIMPs

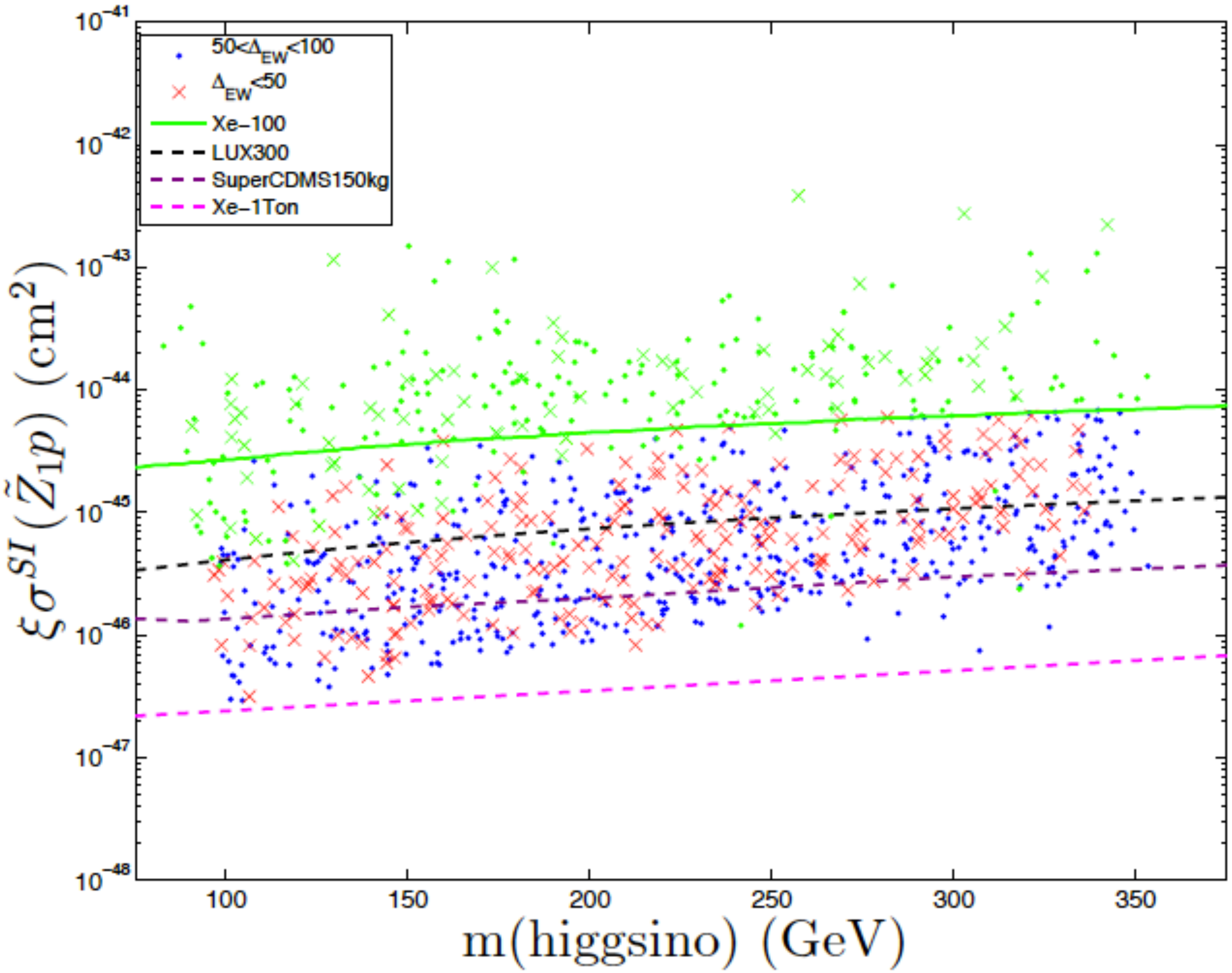
Detection of mixed a/Z1 DM in natural SUSY with DFSZ axion

detection of axion as usual: range of PQ scale f_a:
10^9-10^12 favored in SUSY DFSZ

detection of WIMPs same as usual but theory projections should be scaled to account for WIMPs making only a fraction of total DM density

use Bottino, Fornengo et al. $\xi \equiv \Omega_\chi h^2/0.12$ rescaling factor

Direct higgsino detection rescaled for minimal local abundance



HB, Barger, Mickelson
arXiv:1303.3816

$$\mathcal{L} \ni -X_{11}^h \overline{\widetilde{Z}}_1 \widetilde{Z}_1 h$$

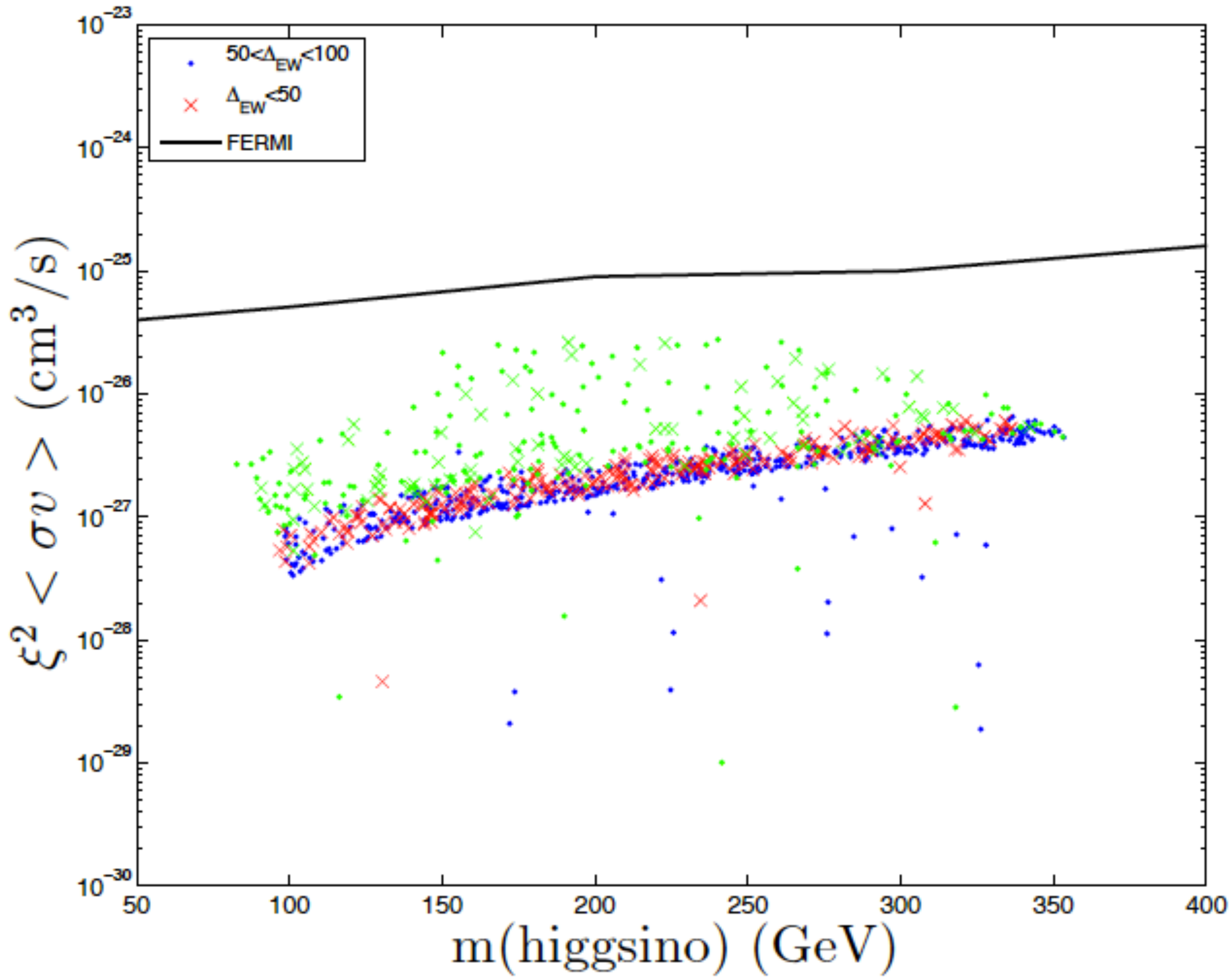
$$X_{11}^h = -\frac{1}{2}\left(v_2^{(1)}\sin\alpha - v_1^{(1)}\cos\alpha\right)\left(g v_3^{(1)} - g' v_4^{(1)}\right)$$

new LUX results

Deployment of Xe-1ton coming soon!

Can test completely with ton scale detector or equivalent (subject to minor caveats)

Higgsino detection via halo annihilations:



green: excluded by Xe-100

annihilation rate is high but rescaling is squared

Gamma-ray sky signal is factor 10-20 below current limits

Historical aside:
large log SUSY EWFT I believe arose with
Kitano, Nomura e.g. PRD73 (2006) 095004

The dominant contribution to $m_{H_u}^2|_{\rm rad}$ arises from top-stop loop:

$$m_{H_u}^2|_{\rm rad} \simeq -\frac{3y_t^2}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + |A_t|^2)\ln\left(\frac{M_{\rm mess}}{m_{\tilde{t}}}\right), \qquad (4)$$

$$m_{\tilde{t}}^2 \lesssim \frac{2\pi^2}{3y_t^2}\frac{M_{\rm Higgs}^2}{\left(1+\frac{x^2}{2}\right)\Delta^{-1}\ln\frac{M_{\rm mess}}{m_{\tilde{t}}}} \approx (700\ {\rm GeV})^2\frac{1}{1+\frac{x^2}{2}}\left(\frac{20\%}{\Delta^{-1}}\right)\left(\frac{3}{\ln\frac{M_{\rm mess}}{m_{\tilde{t}}}}\right)\left(\frac{M_{\rm Higgs}}{200\ {\rm GeV}}\right)^2, \qquad (5)$$

but, see footnote!

[1]In the case that $\ln(M_{\rm mess}/m_{\tilde{t}})$ is large, for example in gravity mediated models, the expression in Eq. (4) is not reliable and we should sum up the leading logarithms using renormalization group equations. This case will be addressed in the next subsection.

$$\frac{m_Z^2}{2} = \frac{(m_{H_d}^2(\Lambda) + \delta m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - (\mu^2(\Lambda) + \delta\mu^2)$$

$$\Delta_{HS} \equiv max_i|B_i|/(m_Z^2/2)$$