Non-perturbative Renormalization of Bilinear Operators with Staggered Fermions

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Introduction

- We present matching factors for the bilinear operators obtained using the non-perturbative renormalization method (NPR) for improved staggered fermions on the MILC asqtad lattices($N_f = 2 + 1$).
- We obtain the wave function renormalization factor Z_q from the conserved vector and axial currents. Also we obtain the mass renormalization factor Z_m from scalar and pseudo-scalar bilinear operators.
- We also calculate the renormalization factor of other bilinear operators.

Bilinear Operator Renormalization

- $\tilde{p}$ is the momentum in reduced Brillouin zone.

$$p \in (-\frac{\pi}{a}, \frac{\pi}{a}]^4, \qquad \tilde{p} \in (-\frac{\pi}{2a}, \frac{\pi}{2a}]^4, \qquad p = \tilde{p} + \pi_B$$

  where $\pi_B (\equiv \frac{\pi}{a} B)$ is cut-off momentum in hypercube.
- a : lattice spacing.
- B : vector in hypercube. Each element is 0 or 1
  ex) $B = (0, 0, 1, 1)$

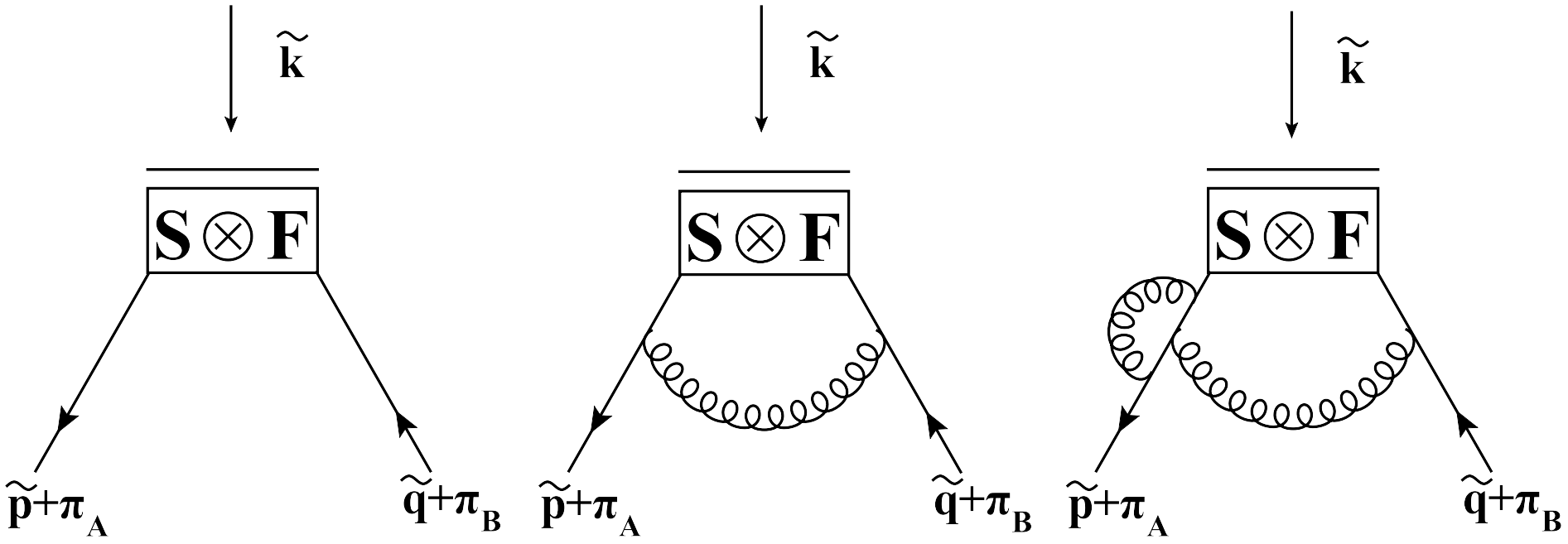
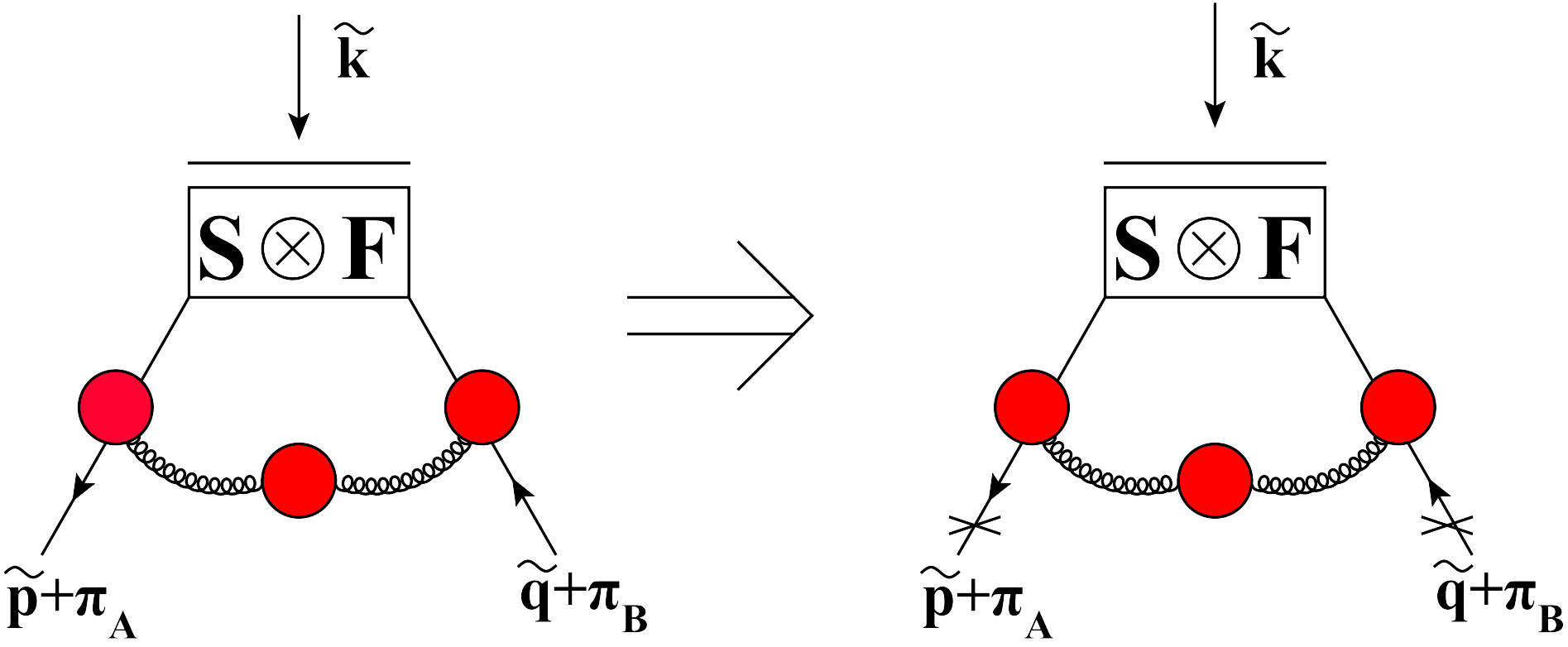


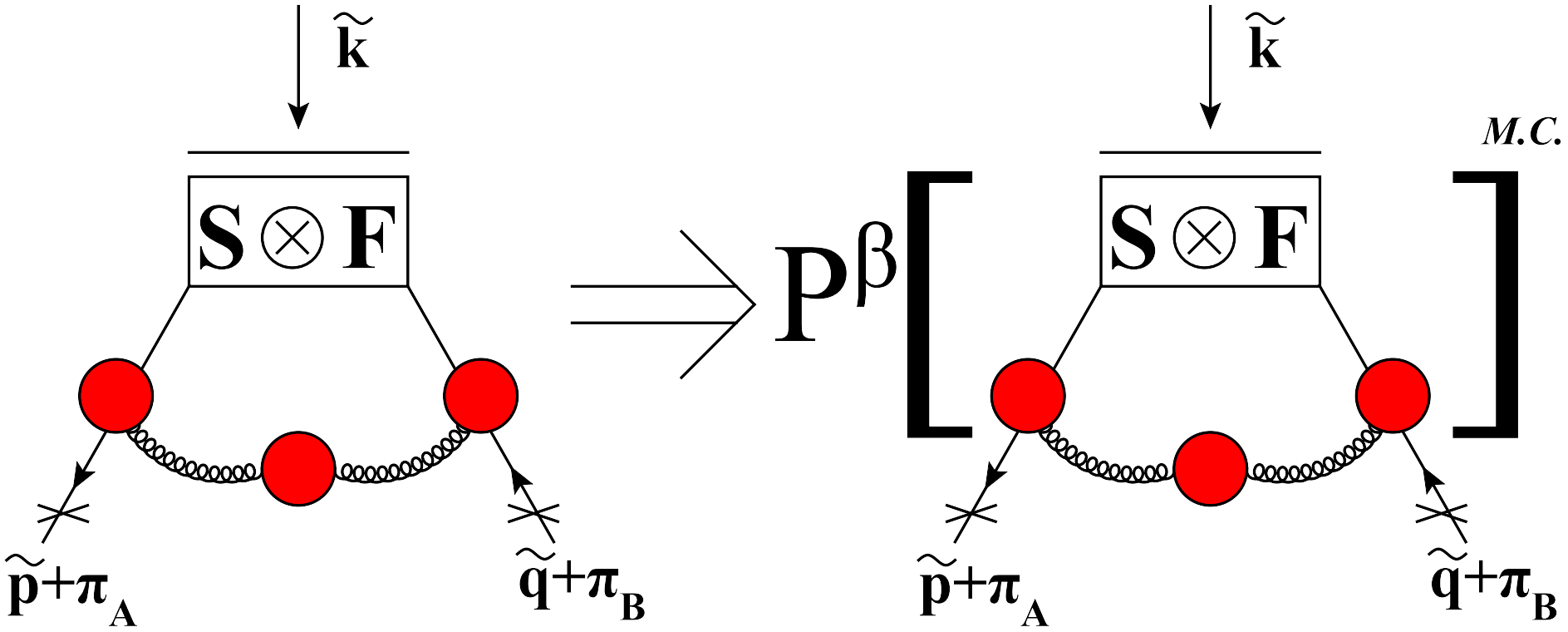
Figure: The Green's functions of bilinear operator : The diagrams that contribute to bilinear operator



Unamputated Green's function
$\widetilde{H}^{\alpha}$

Amputated Green's function
$\widetilde{\Lambda}^{\alpha}$

Red Circle : 1PI Diagram.



Amputated Green's function

$\tilde{\Lambda}^\alpha$

Projected amputated Green's function

$\Gamma^{\alpha\beta}$

- α, β : the indices to represent different operators.
  ex) $\alpha = (\gamma_\mu \otimes 1)$, $\beta = (1 \otimes 1)$
- M.C. : momentum conservation condition. $\tilde{p} = \tilde{q} + \tilde{k}$

The projection operator is

$$\hat{\mathbb{P}}^{\beta}_{BA;c_2c_1} = \frac{1}{48}\overline{\overline{(\gamma^{\dagger}_{S'} \otimes \xi^{\dagger}_{F'})}}_{BA}\delta_{c_2c_1}$$

The renormalization of $\Gamma(\tilde{p}, \tilde{q})$ is

$$\Gamma^{\alpha\sigma}_{R}(\tilde{p}, \tilde{q}) = \sum_{\beta} Z_q^{-1} Z_O^{\alpha\beta} \Gamma^{\beta\sigma}_{B}(\tilde{p}, \tilde{q})$$

- A, B: hypercube index
- c : color index
- α, β, σ: the indices to represent different operators.
- Γ_B : bare projected amputated Green's function
- Γ_R : renormalized projected amputated Green's function
- Z_q : the wave function renormalization factor for quark fields
- $Z_O^{\alpha\beta}$: the renormalization factor matrix element which represents the mixing between the α and β operators.

The RI-MOM scheme prescription is

$$\Gamma_R^{\alpha\sigma}(\tilde{p},\tilde{p}) = \Gamma_{tree}^{\alpha\sigma}(\tilde{p},\tilde{p}) = \delta^{\alpha\sigma} \ ,$$

where $\Gamma_{tree}^{\alpha\sigma}(\tilde{p},\tilde{p})$ is the projected amputated Green's function at the tree level.

Therefore, the renormalization factor is obtained from the following equation.

$$Z_O^{\alpha\beta} = Z_q \cdot [\Gamma_B^{-1}(\tilde{p},\tilde{p})]^{\alpha\beta}$$

Simulation Detail

- $20^3 \times 64$ MILC asqtad lattice ($a \approx 0.12fm,\ am_\ell/am_s = 0.01/0.05$).
- HYP smeared staggered fermions as valence quarks.
- The number of configurations is 30.
- 5 valence quark masses (0.01, 0.02, 0.03, 0.04, 0.05)
- 14 external momenta in the units of $(\frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_t})$.
- We do the uncorrelated fitting and use the jackknife resampling method to estimate statistical errors.

n(x, y, z, t)	$a\tilde{p}$	GeV	n(x, y, z, t)	$a\tilde{p}$	GeV
$(1,0,1,3)$	0.5330	0.8835	$(1,2,2,4)$	1.0210	1.6922
$(1,1,1,2)$	0.5785	0.9588	$(2,1,2,6)$	1.1114	1.8420
$(1,1,1,3)$	0.6187	1.0254	$(2,2,2,7)$	1.2871	2.1332
$(1,1,1,4)$	0.6710	1.1122	$(2,2,2,8)$	1.3421	2.2243
$(1,1,1,5)$	0.7328	1.2146	$(2,2,2,9)$	1.4018	2.3233
$(1,1,1,6)$	0.8019	1.3291	$(2,3,2,7)$	1.4663	2.4302
$(1,2,1,5)$	0.9128	1.5128	$(3,3,3,9)$	1.8562	3.0764

Wave Function Renormalization Factor

For the conserved vector current, the renormalization factor $Z_O^{\alpha\beta} = 1$.

Therefore

$$Z_q^{\text{RI-MOM}} = \Gamma_0^{\alpha\beta}(\tilde{p}, \tilde{p}) \,,$$

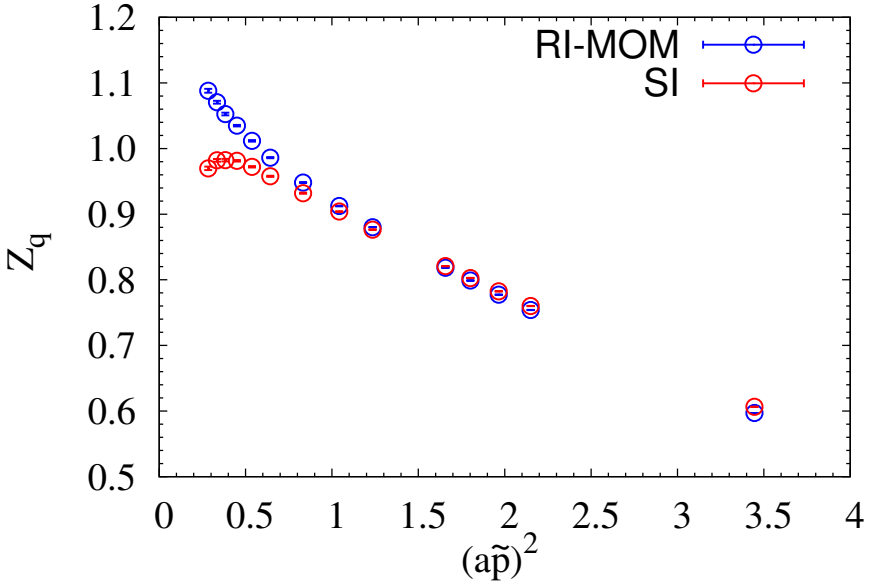
where $\alpha = \beta = (\gamma_\mu \otimes 1)$.
The superscript RI-MOM denotes that the wave function renormalization factor Z_q is defined in the RI-MOM scheme.

RI-MOM scheme to SI scheme

We convert the raw data in the RI-MOM scheme into the scale-invariant(SI) data by removing the scale-dependent part of the RG evolution matrix as follows.

$$Z_q^{\text{SI}} = \frac{c(\alpha_s(\mu_0))}{c(\alpha_s(\mu))} Z_q^{\text{RI-MOM}}(\mu), \qquad (\mu_0 = 2\text{GeV}, \quad \mu^2 = \tilde{p}^2)$$

This Wilson coefficient $c(x)$ is calculated using four-loop anomalous dimension.

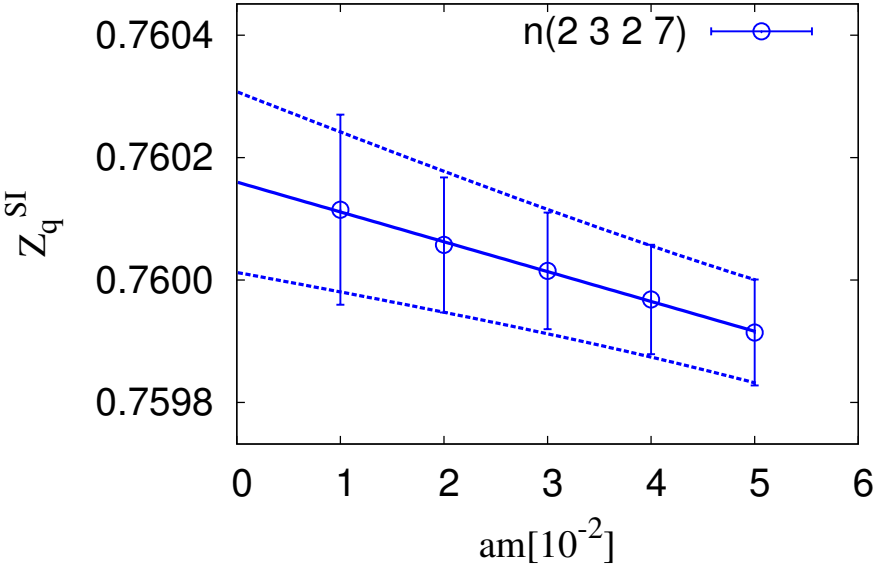


m-fit

We fit the data with respect to quark mass for a fixed momentum to the linear function f_{Z_q}.

$$f_{Z_q}(m, a, \tilde{p}) = a_1 + a_2 \cdot am,$$

where a_i is a function of $\widetilde{p}$. We call this m-pit. After m-fit, we take the chiral limit values which corresponds to $a_1(a, \widetilde{p})$.



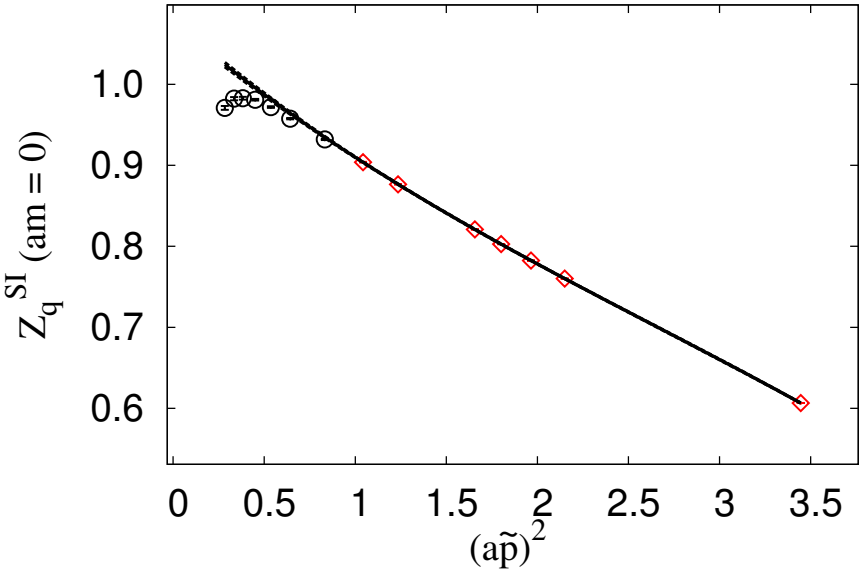
a_1	a_2
0.76016(15)	-0.0049(21)
χ^2/dof	
0.0024(62)	

p-fit

We fit $a_1(a, \widetilde{p})$ to the following fitting function.

$$f_{Z_q}(am = 0, a\tilde{p}) = b_1 + b_2(a\tilde{p})^2 + b_3((a\tilde{p})^2)^2 + b_4((a\tilde{p})^2)^3$$

To avoid non-perturbative effects at small $(a\tilde{p})^2$, we choose the momentum window as $(a\tilde{p})^2 > 1$. Because we assume that those terms of $\mathcal{O}((a\widetilde{p})^2)$ and higher order are pure lattice artifacts, we take the b_1 as Z_q at $\mu = 2$GeV in the RI-MOM scheme.



b_1	b_2
1.0764(44)	-0.1908(69)
b_3	b_4
0.0279(33)	-0.00350(49)
χ^2/dof	
0.06(16)	

Mass Renormalization Factor

By the Ward identity, the mass renormalization factor is

$$Z_m = \frac{1}{Z_{S\otimes S}}$$

where $Z_{S\otimes S}$ is a renormalization factor of scalar bilinear operator with scalar taste. Therefore

$$(Z_q \cdot Z_m)^{\text{RI-MOM}} = (\frac{Z_q}{Z_{S\otimes S}})^{\text{RI-MOM}} = \Gamma_{S\otimes S}(\tilde{p}, \tilde{p}) \,,$$

where $Z_{S\otimes S} \equiv Z_O^{\alpha\beta}$ with $\alpha = \beta = (S \otimes S)$
and $\Gamma_{S\otimes S} \equiv \Gamma_B^{\alpha\beta}$ with $\alpha = \beta = (S \otimes S)$.

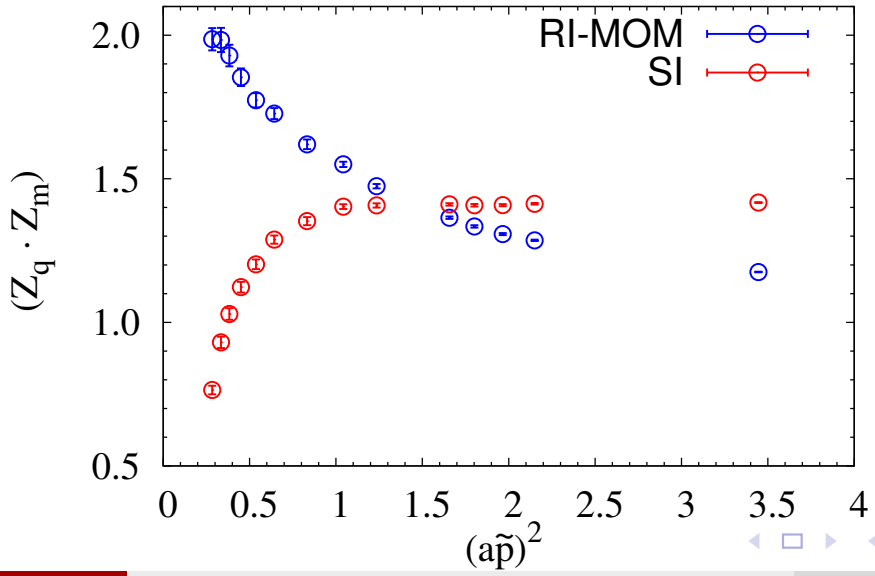
RI-MOM scheme to SI scheme

To obtain the scale-invariant(SI) quantity, we divide $(Z_q \cdot Z_m)^{\text{RI-MOM}}$ by the RG running factor.

$$(Z_q \cdot Z_m)^{\text{SI}} = \frac{c(\alpha_s(\mu_0))}{c(\alpha_s(\mu))} \cdot \frac{d(\alpha_s(\mu_0))}{d(\alpha_s(\mu))} (Z_q \cdot Z_m)^{\text{RI-MOM}}(\mu)$$

$$(\mu_0 = 2\text{GeV}, \quad \mu^2 = \tilde{p}^2)$$

where $d(x)$ is the Wilson coefficient calculated using the quark mass anomalous dimension at the four-loop level.

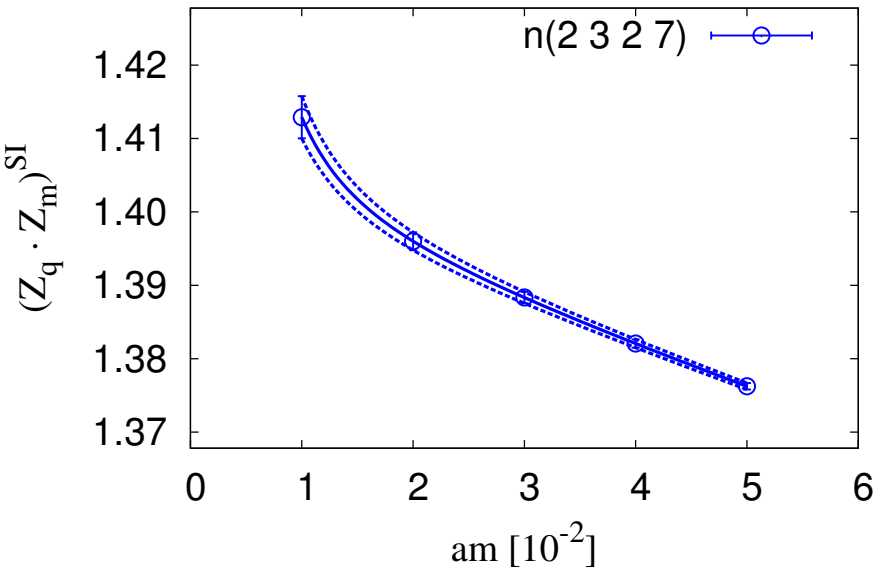


m-fit

We use the following fitting function:

$$f_{Z_q \cdot Z_m}(m, a, \tilde{p}) = c_1 + c_2(am) + c_3(am)^2 + c_4 \frac{1}{(am)^2} ,$$

where m is the valence quark mass. the c_4 term comes from the chiral behavior of the chiral condensate which is proportional to $1/m^2$ due to zero mode. Because of the sea quark determinant contributions to the chiral condensate, the c_4 term contribution vanishes in the chiral limit. After m-fit, we take the chiral limit values which corresponds to c_1.

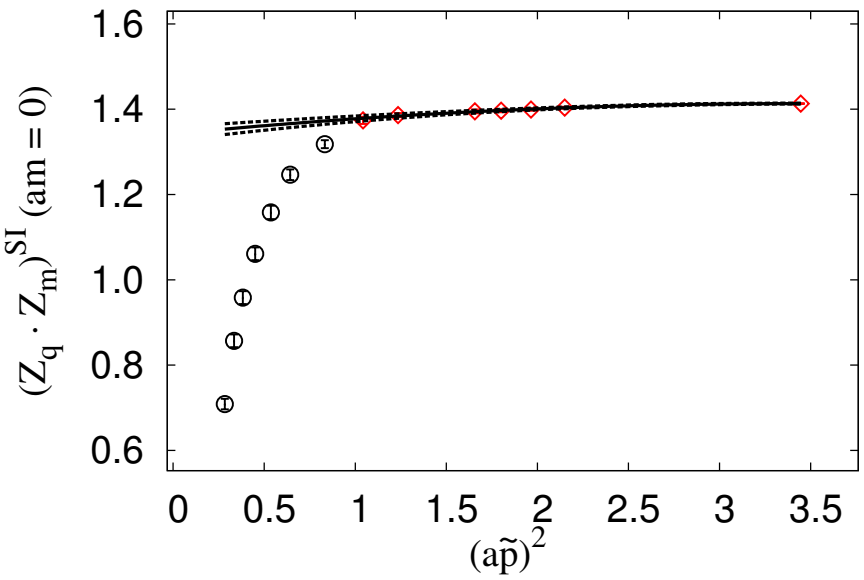


c_1	c_2
1.4036(22)	-0.573(72)
c_3	c_4
0.28(67)	0.00000150(26)
χ^2/dof	
0.00008(51)	

p-fit

We fit the $c_1(a, \widetilde{p})$ to the following fitting function.

$$f_{Z_q \cdot Z_m}(am = 0, a\tilde{p}) = d_1 + d_2(a\tilde{p})^2 + d_3((a\tilde{p})^2)^2$$



d_1	d_2
1.342(16)	0.041(12)
d_3	χ^2/dof
-0.0060(21)	0.18(28)

Preliminary Result

α	Z_q	χ^2/dof (m-fit)	χ^2/dof (p-fit)
$(V_\mu \otimes S)$	1.0764(44)	0.0024(62)	0.06(16)
$(A_\mu \otimes P)$	1.075(32)	0.0003(27)	0.12(28)

α	Z_m	χ^2/dof (m-fit)	χ^2/dof (p-fit)
$(S \otimes S)$	1.246(15)	0.00008(51)	0.18(28)
$(P \otimes P)$	1.255(18)	0.0000008(36)	0.06(19)

α	$Z_O^{\alpha\alpha}$	χ^2/dof (m-fit)	χ^2/dof (p-fit)
$[S \times P]$	1.079(18)	0.00004(23)	0.19(48)
$[P \times A_\mu]$	0.8947(66)	0.00218(25)	0.032(74)
$[V_\mu \times V_\mu]$	0.982(11)	0.000003(17)	0.17(40)
$[A_\mu \times A_\nu]$	1.115(27)	0.0000006(33)	0.007(47)
$[T_{\mu\nu} \times T_{\rho\sigma}]$	1.293(16)	0.0000035(72)	0.008(42)
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Conclusion

- We obtain the wave function renormalization factor Z_q from conserved vector and axial current and mass renormalization factor Z_m from scalar and pseudo-scalar bilinear operators.
- Also we calculate the renormalization factor of other bilinear operators.
- We are in the middle of analysis of systematic errors.