Flavor and CP Violations

– as Probes of BSM Physics (i.e., SUSY, in this talk) –

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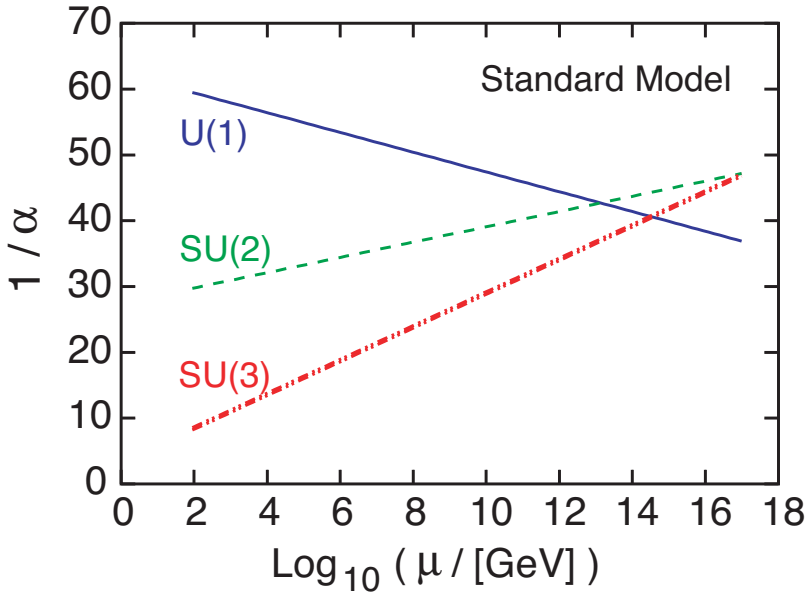
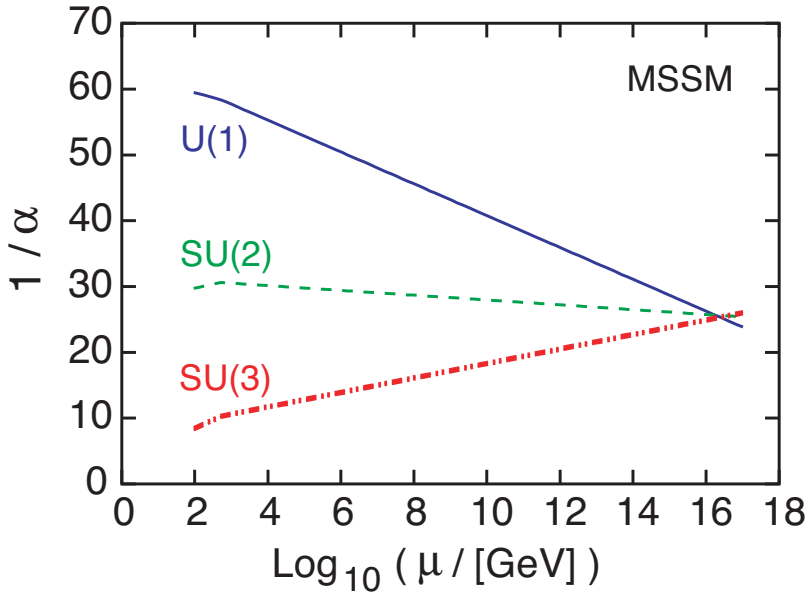
KIAS, Korea, November, 2013

1. Introduction

Supersymmetry: good & bad

- Gauge coupling unification can be realized





- Dark matter candidate exists (neutralino, gravitino, $\cdots$)
- SUSY particles have not been discovered yet

  $\Rightarrow$ For e.g., $m_{\tilde{q}} \gtrsim 1.8$ TeV & $m_{\tilde{g}} \gtrsim 1.4$ TeV in mSUGRA

- $m_h \simeq 125 - 126$ GeV looks heavier than naive expectation

One possibility: Heavy sfermions: $m_{\tilde{f}} \sim O(10 \text{ TeV})$

- Focus-point

  [Feng, Matchev & TM ('99)]

- (Some class of) anomaly-mediation

  [Giudice, Luty, Murayama & Rattazzi ('99)]

- Split SUSY

  [Giudice & Romanino ('04); Arkani-Hamed, Dimopoulos ('04); Wells ('04)]

- Minimal gravity mediation

  [Ibe, TM & Yanagida ('06); Ibe & Yanagida ('11)]

- $\cdots$

LHC may not find the SUSY signals in heavy SUSY scenario

$\Rightarrow$ Flavor and CP physics may be useful

Today, I will talk about:

- Flavor and CP violations in SUSY model
- Issues related to muon $g-2$ (if I have time)

Outline

1. Introduction
2. K^0-$\bar{K}^0$ Mixing
3. Leptonic Flavor and CP Violations
4. Muon Magnetic Dipole Moment
5. Summary

2. K^0-$\bar{K}^0$ Mixing

K^0-$\bar{K}^0$ system: CP and Hamiltonian eigenstates differ

- $|K_L\rangle \simeq |K_{CP=-}\rangle - \epsilon_K |K_{CP=+}\rangle$
- $|K_S\rangle \simeq |K_{CP=+}\rangle + \epsilon_K |K_{CP=-}\rangle$

$$CP|K^0\rangle = |\bar{K}^0\rangle \quad \Rightarrow \quad |K_{CP=\pm}\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \pm |\bar{K}^0\rangle)$$

Hamiltonian and ϵ_K parameter

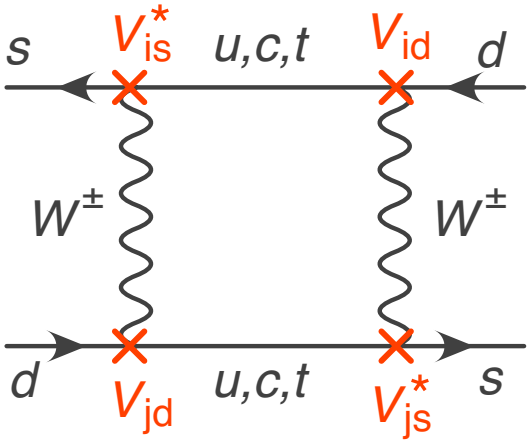
$$\mathcal{H}_K = \begin{pmatrix} \langle K^0|\hat{\mathcal{H}}|K^0\rangle & \langle K^0|\hat{\mathcal{H}}|\bar{K}^0\rangle \\ \langle \bar{K}^0|\hat{\mathcal{H}}|K^0\rangle & \langle \bar{K}^0|\hat{\mathcal{H}}|\bar{K}^0\rangle \end{pmatrix} = \begin{pmatrix} M_{11} + \frac{i}{2}\Gamma_{11} & M_{12} + \frac{i}{2}\Gamma_{12} \\ M_{12}^* + \frac{i}{2}\Gamma_{12}^* & M_{11} + \frac{i}{2}\Gamma_{11} \end{pmatrix}$$

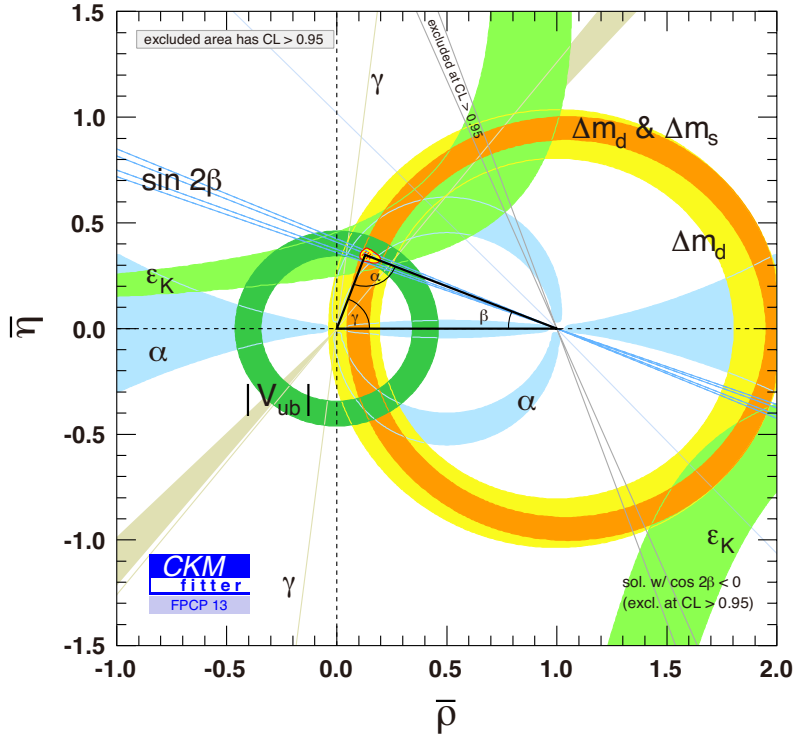
We take Wu-Yang phase convention: $\langle (2\pi)_{I=0}|K_{CP=-}\rangle = 0$

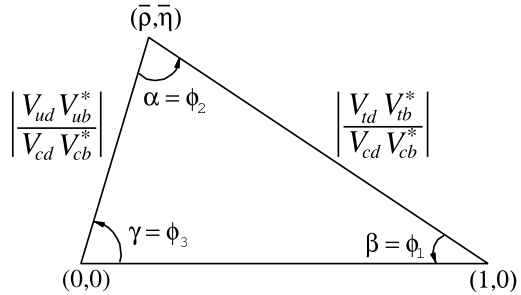
$$\Rightarrow \epsilon_K \simeq \frac{i}{2}\frac{\Im M_{12} - i\Im\Gamma_{12}/2}{\Re M_{12} - i\Re\Gamma_{12}/2}$$

$\Rightarrow$ CP violation, if $\Im M_{12} \neq 0$ or $\Im\Gamma_{12} \neq 0$

In the SM, K^0-$\bar{K}^0$ mixing originates from $W^\pm$-boson loop







[CKMfitter ('13)]

- $|\epsilon_K^{(\text{exp})}| = (2.228 \pm 0.011) \times 10^{-3}$
- $|\epsilon_K^{(\text{exp})}| - |\epsilon_K^{(\text{SM})}| = (3.8 \pm 2.7) \times 10^{-4} \;\Rightarrow\; |\epsilon_K^{(\text{extra})}| < 9.2 \times 10^{-4}\ (2\sigma)$

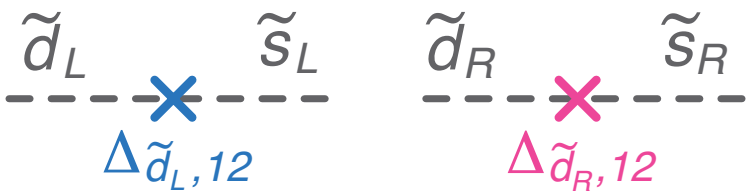
In SUSY model, new sources of flavor and CP violations exist

$\Rightarrow$ Off-diagonal elements of sfermion mass matrices

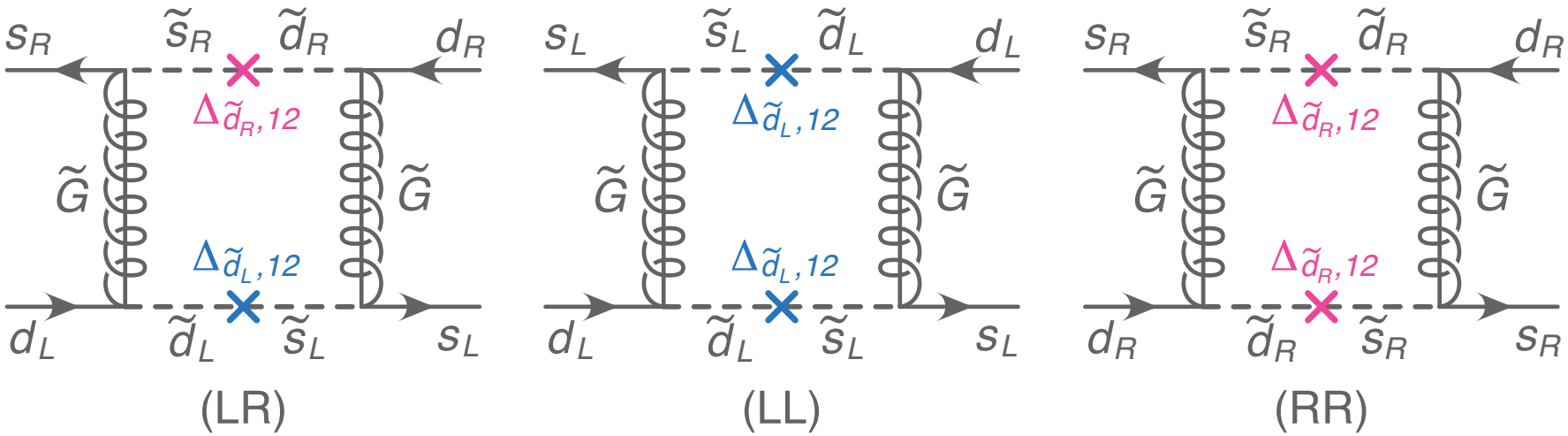
$$\mathcal{M}^2_{\tilde{Q}} = m^2_{\tilde{q}} \begin{pmatrix} \Delta_{\tilde{Q},11} & \Delta_{\tilde{Q},12} & \Delta_{\tilde{Q},13} \\ \Delta_{\tilde{Q},21} & \Delta_{\tilde{Q},22} & \Delta_{\tilde{Q},23} \\ \Delta_{\tilde{Q},31} & \Delta_{\tilde{Q},32} & \Delta_{\tilde{Q},33} \end{pmatrix} \qquad \tilde{Q} = \tilde{u}_L, \tilde{u}^{c*}_R, \tilde{d}_L, \tilde{d}^{c*}_R$$

$$\mathcal{M}^2_{\tilde{L}} = m^2_{\tilde{l}} \begin{pmatrix} \Delta_{\tilde{L},11} & \Delta_{\tilde{L},12} & \Delta_{\tilde{L},13} \\ \Delta_{\tilde{L},21} & \Delta_{\tilde{L},22} & \Delta_{\tilde{L},23} \\ \Delta_{\tilde{L},31} & \Delta_{\tilde{L},32} & \Delta_{\tilde{L},33} \end{pmatrix} \qquad \tilde{L} = \tilde{e}_L, \tilde{e}^{c*}_R, \tilde{\nu}_L$$

Mass terms: $\mathcal{L}_{\text{mass}} = -[\mathcal{M}^2_{\tilde{d}_L}]_{ij} \tilde{d}^*_{L,i} \tilde{d}_{L,j} - [\mathcal{M}^2_{\tilde{d}^c_R}]_{ij} \tilde{d}^c_{R,i} \tilde{d}^{c*}_{R,j} - \cdots$



Diagrams contributing to ϵ_K & Δm_K



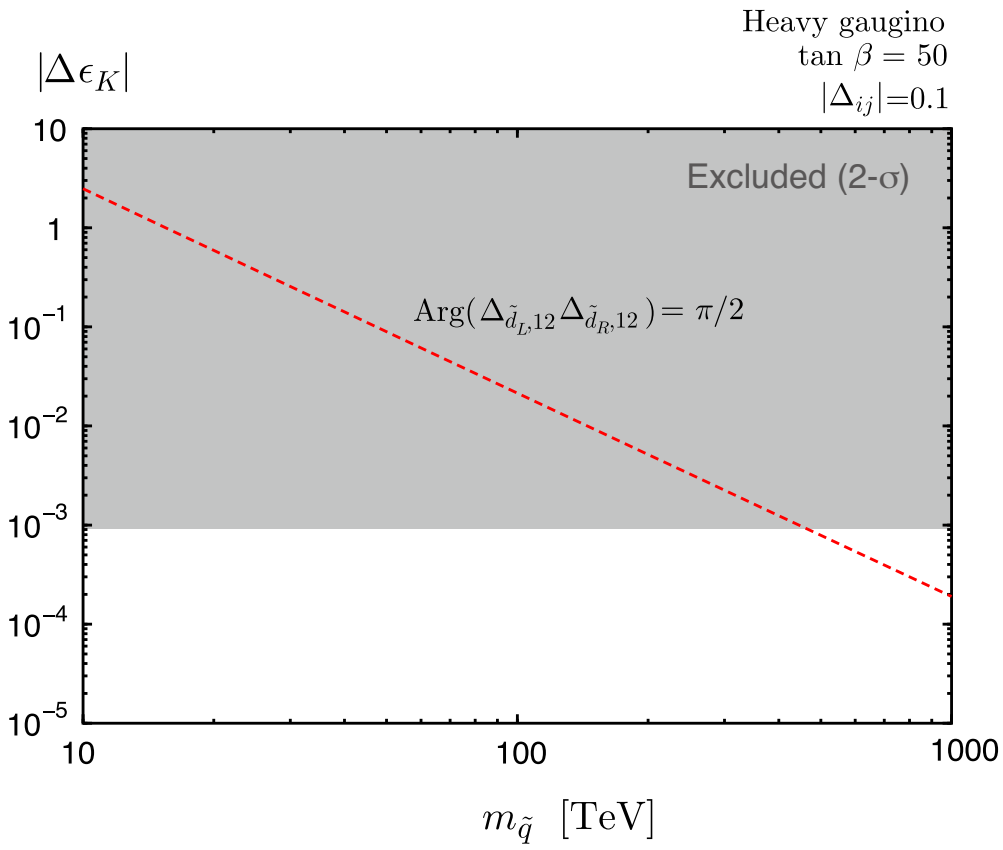
$\Rightarrow$ Important parameters: $\Delta_{\tilde{d}_L,12}$ & $\Delta_{\tilde{d}_R,12}$

K-$\bar{K}$ mixing gives serious constraints on the MSSM

$\Rightarrow$ It often requires $m_{\text{SUSY}} \gtrsim O(100\ \text{TeV})$, if off-diagonal elements of the squark mass matrices are sizable

[Gabbiani, Gabrielli, Masiero & Silvestrini ('96)]

Numerical result



- $M_3 = m_{\tilde{q}}$
- $\Delta_{\tilde{d},12} = 0.1$
- $\mathrm{Arg}(\Delta_{\tilde{d}_L,12}\Delta_{\tilde{d}_R,12}) = \pi/2$

$\Rightarrow$ ϵ_K often gives a serious constraint on the MSSM

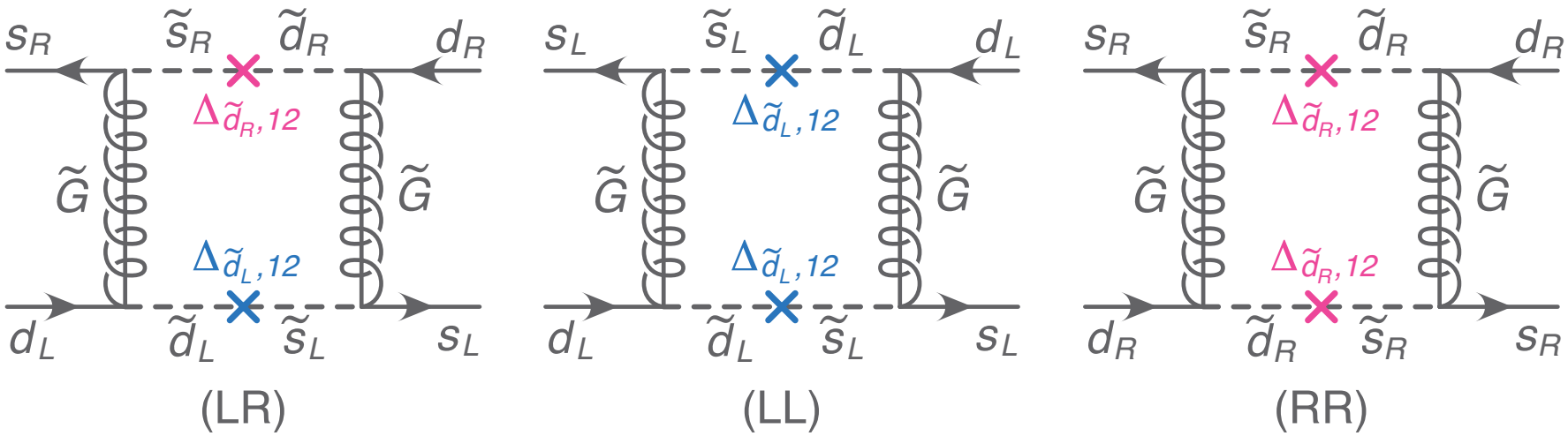
$\Rightarrow$ Constraints from Δm_K and Δm_B are less stringent (assuming maximal CP violation)

$\epsilon_K^{(\mathrm{SUSY})}$ may be suppressed, if

- mSUGRA, gauge mediation, $\cdots$
- Alignment (flavor symmetry, accidental, $\cdots$)
- $SO(10)$-like relation: $\mathcal{M}^2_{\tilde{d}_L} \simeq \mathcal{M}^{2\,*}_{\tilde{d}_R} \;\Rightarrow\; \Delta_{\tilde{d}_L} \simeq \Delta^*_{\tilde{d}_R}$

  [TM & Nagai ('13)]

If $\mathcal{M}^2_{\tilde{d}_L} \simeq \mathcal{M}^{2\,*}_{\tilde{d}_R}$, sum of the following diagrams becomes real



Lagrangian (the most important part):

$$\mathcal{L} = -i\sqrt{2}g_3(\tilde{d}_L^\dagger T^a d_L)\tilde{G}^a + i\sqrt{2}g_3(\tilde{d}_R^c T^a \bar{d}_R^c)\bar{\tilde{G}}^a$$
$$+ \tilde{d}_{L,I}^\dagger \mathcal{M}^2_{\tilde{d}_L,IJ}\tilde{d}_{L,J} + \tilde{d}_{R,I}^c \mathcal{M}^2_{\tilde{d}_R,IJ}\tilde{d}_{R,J}^{c\dagger} + \cdots$$

If $\mathcal{M}^2_{\tilde{d}_L} = \mathcal{M}^{2\,*}_{\tilde{d}_R}$, C invariance approximately holds

- $\tilde{d}_L \leftrightarrow \tilde{d}_R^c$
- $d_L \leftrightarrow d_R^c$

ϵ_K is suppressed due to this (approximate) C invariance

$\Rightarrow \epsilon_K \neq 0$, due to:

- Violation of $\mathcal{M}^2_{\tilde{d}_L} = \mathcal{M}^{2\,*}_{\tilde{d}_R}$
- $SU(2)_L$ and $U(1)_Y$ interactions

The relation $\mathcal{M}^2_{\tilde{d}_L} = \mathcal{M}^{2\,*}_{\tilde{d}_R}$ is affected by RG effects

Effects of third-generation Yukawas may be important

Case with (almost) universal scalar mass at the GUT scale

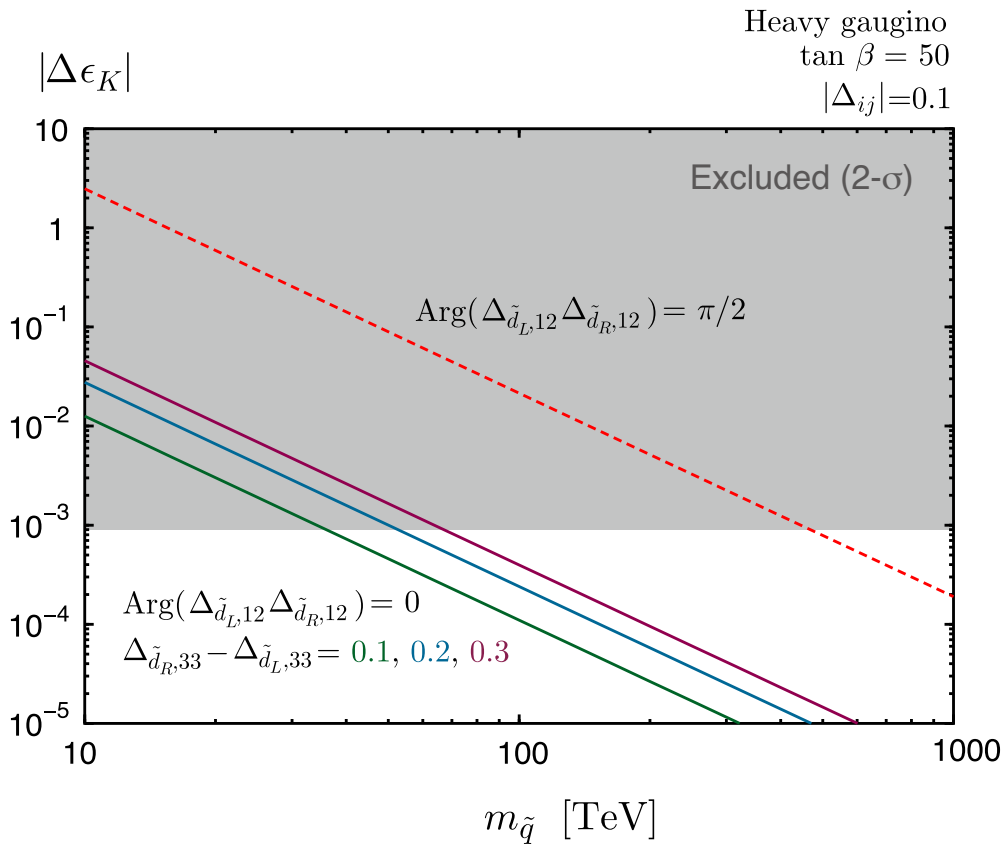
$$\Rightarrow \mathcal{M}^2_{\tilde{d}_L} \sim m^2_{\tilde{q}} \begin{pmatrix} 1 & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & 1 & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta^L_{33} \end{pmatrix}, \quad \mathcal{M}^2_{\tilde{d}_R} \sim m^2_{\tilde{q}} \begin{pmatrix} 1 & \Delta^*_{12} & \Delta^*_{13} \\ \Delta^*_{21} & 1 & \Delta^*_{23} \\ \Delta^*_{31} & \Delta^*_{32} & \Delta^R_{33} \end{pmatrix}$$

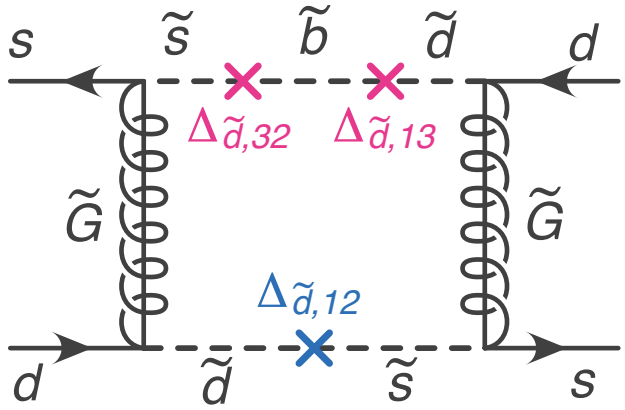
$\Rightarrow$ Breaking of the $SO(10)$-like relation at the MSSM scale

$\tan\beta$	Δ^L_{33}	Δ^R_{33}
10	0.7	1.0
30	0.7	0.8
50	0.5	0.5

$\Delta^{L,R}_{33}(\text{GUT}) = 1$

$\epsilon_K^{(\mathrm{SUSY})}$ for the case with $\mathcal{M}^2_{\tilde{d}_L} \simeq \mathcal{M}^{2\,*}_{\tilde{d}_R}$ (except for 33 elements)





$\Rightarrow$ Constraint from ϵ_K may be relaxed

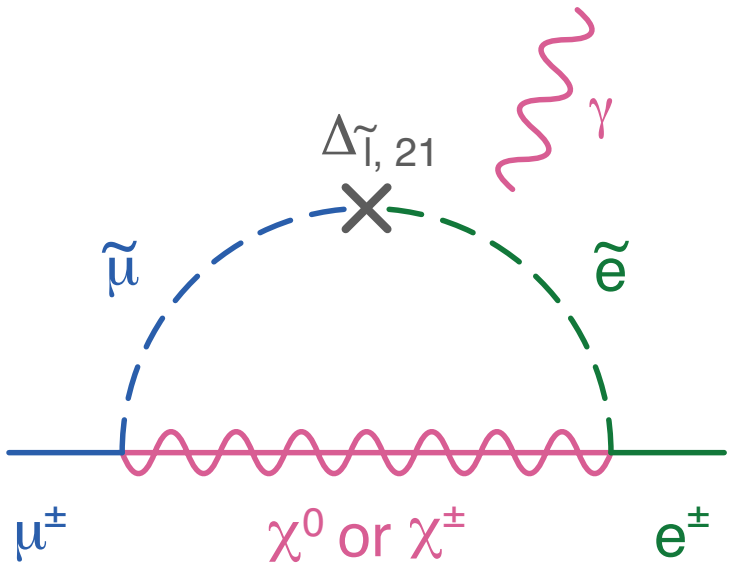
$\Rightarrow$ Other observables are also important (like LFV)

3. Leptonic Flavor and CP Violations

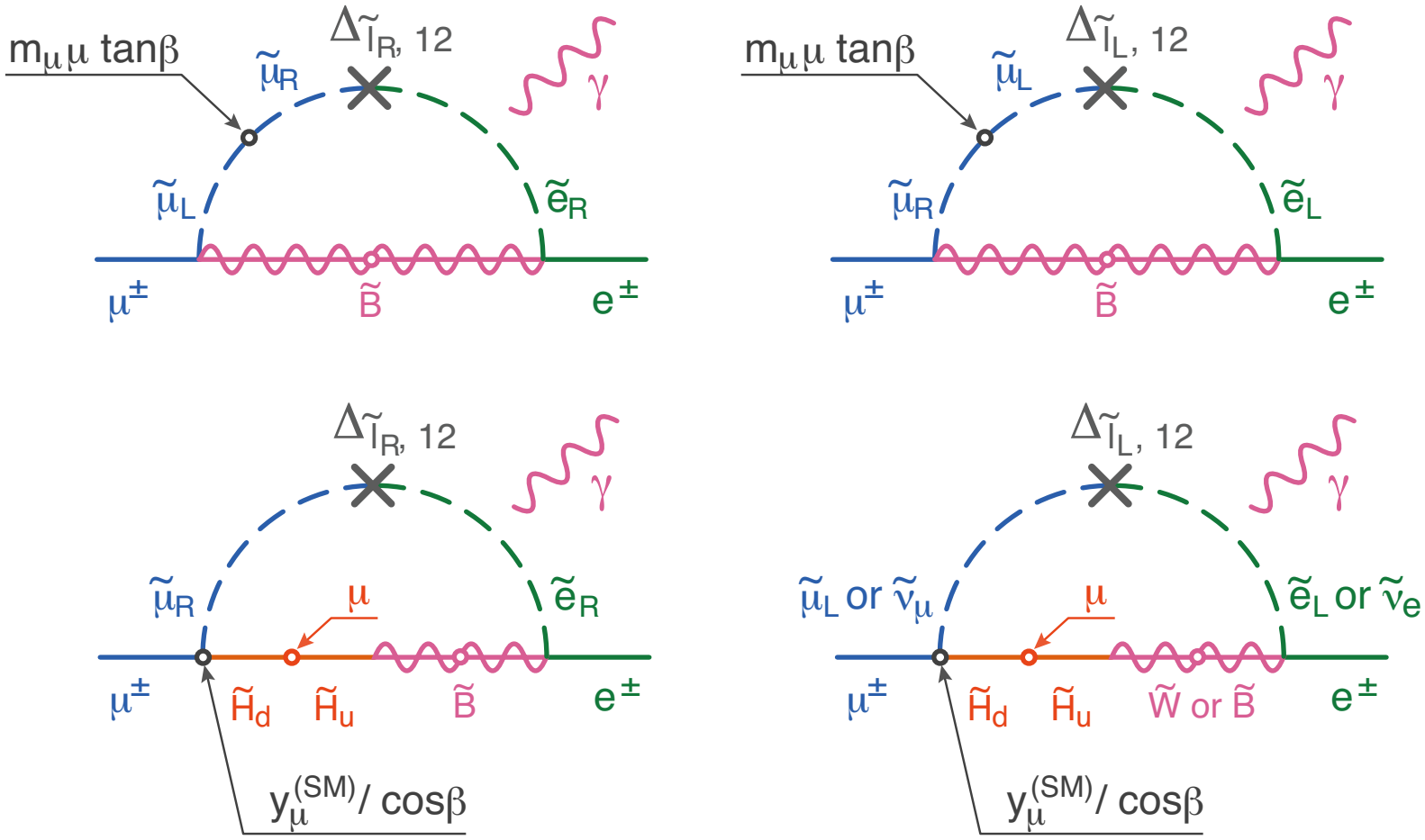
LFVs in SUSY models: (probably) due to $\Delta_{\tilde{l}_L}$ and $\Delta_{\tilde{l}_R}$

$$\mathcal{M}^2_{\tilde{l}_{L,R}} = m^2_{\tilde{l}_{L,R}} \begin{pmatrix} \Delta_{\tilde{l}_{L,R},11} & \Delta_{\tilde{l}_{L,R},12} & \Delta_{\tilde{l}_{L,R},13} \\ \Delta_{\tilde{l}_{L,R},21} & \Delta_{\tilde{l}_{L,R},22} & \Delta_{\tilde{l}_{L,R},23} \\ \Delta_{\tilde{l}_{L,R},31} & \Delta_{\tilde{l}_{L,R},32} & \Delta_{\tilde{l}_{L,R},33} \end{pmatrix}$$

Off-diagonal elements of the mass matrices induce LFVs

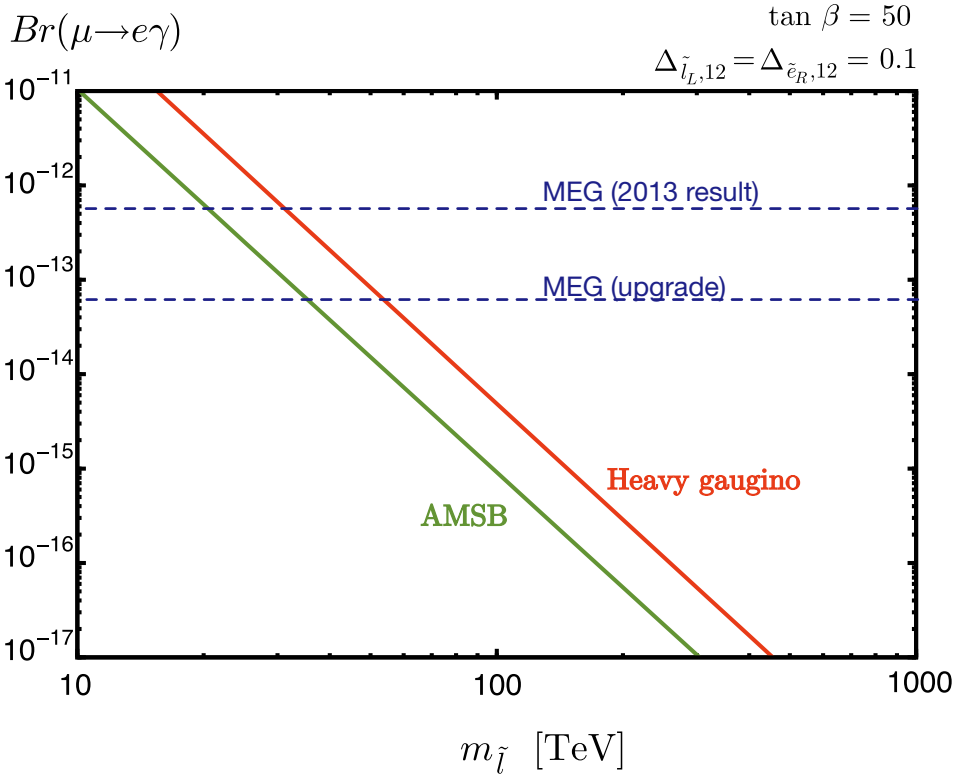


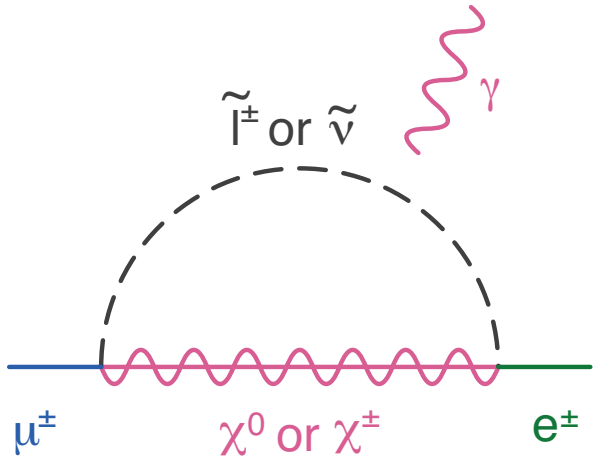
In the MSSM, LFV processes are enhanced by $\tan\beta$



$\Rightarrow$ LFV rates are (approximately) proportional to $\tan^2\beta$

$\mu \to e\gamma$ (with $\Delta^L_{e\mu} = \Delta^R_{e\mu} = 0.1,\ \tan\beta = 50,\ \cdots$)





[TM & Nagai ('13)]

- MEG (current bound): $Br < 5.7 \times 10^{-13}$

  [MEG experiment ('13)]

- MEG upgrade: $Br \lesssim 6 \times 10^{-14}$

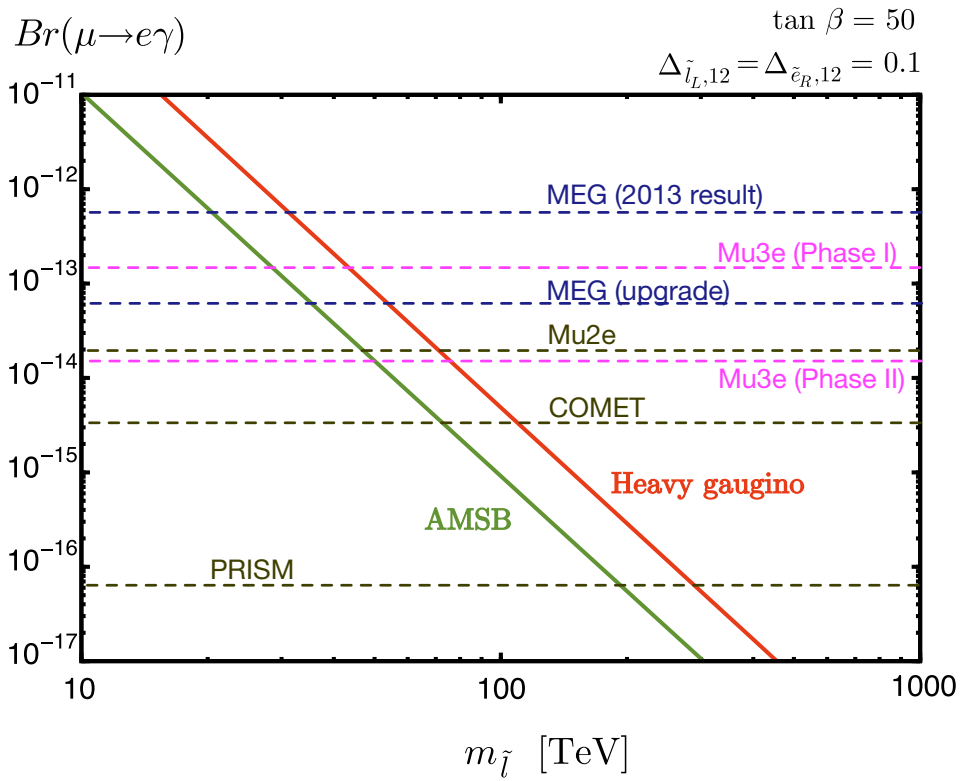
Of course, other LFV processes may occur

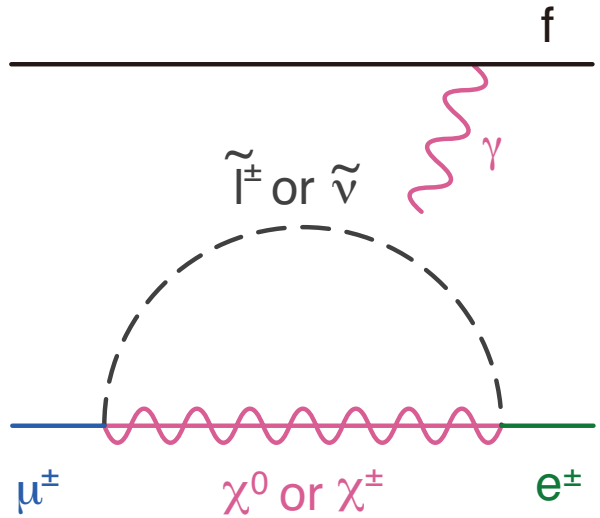
- $\mu \to 3e$
  - Current bound: $Br < 1 \times 10^{-12}$
  - Mu3e (Phase I): $Br \lesssim 10^{-15}$
  - Mu3e (Phase II): $Br \lesssim 10^{-16}$
- μ-e conversion
  - Current bound: $R_{\mu e} < 7 \times 10^{-13}$ (with Au)
  - Mu2e: $R_{\mu e} \lesssim 6 \times 10^{-17}$ (with Al)
  - COMET: $R_{\mu e} \lesssim 10^{-17}$ (with Al)
  - PRISM: $R_{\mu e} \lesssim 10^{-19}$

In the case of dipole dominance:

- $\mu \to 3e$: $Br(\mu \to 3e) \simeq 7 \times 10^{-3} \times Br(\mu \to e\gamma)$
- μ-e conversion: $R_{\mu e} \sim$ (a few) $\times 10^{-3} \times Br(\mu \to e\gamma)$

Comparison (with the naive rescalings)

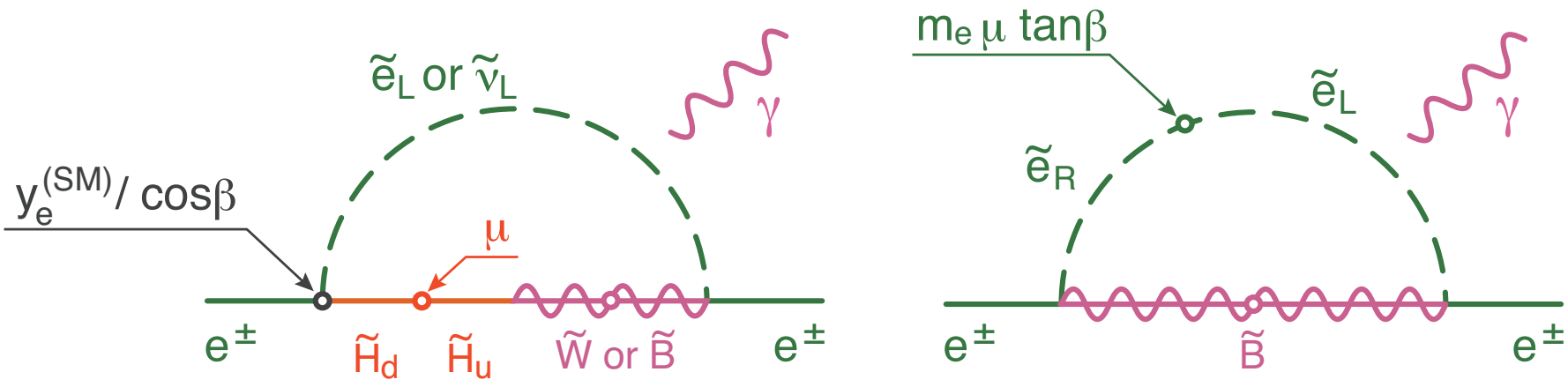




Another possible probe of heavy SUSY scenario

Electric dipole moments (EDMs)

Electron EDM, for example

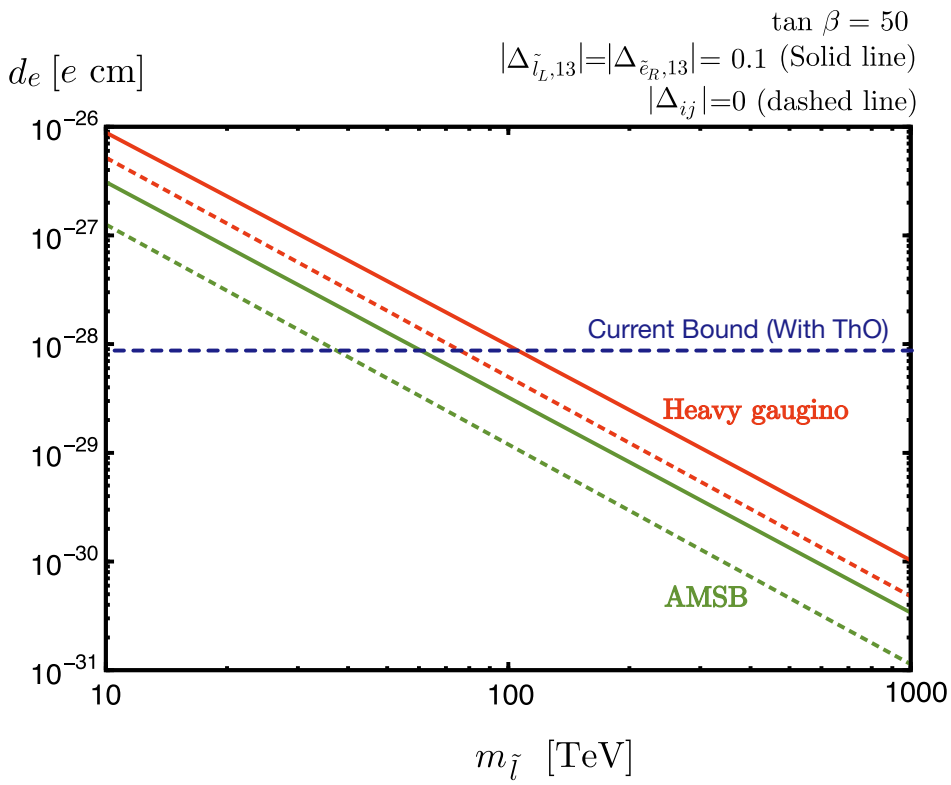


- $\mu \tan\beta$ can be complex

  $\Rightarrow d_e^{(\mathrm{SUSY})}$ is approximately proportional to $\Im(\mu \tan\beta)$

- Phases in the sfermion mass matrices may also contribute

Numerical result with $\mu = m_{\tilde{l}}$ & $\arg(\mu) = \pi/2$:



- Current bound

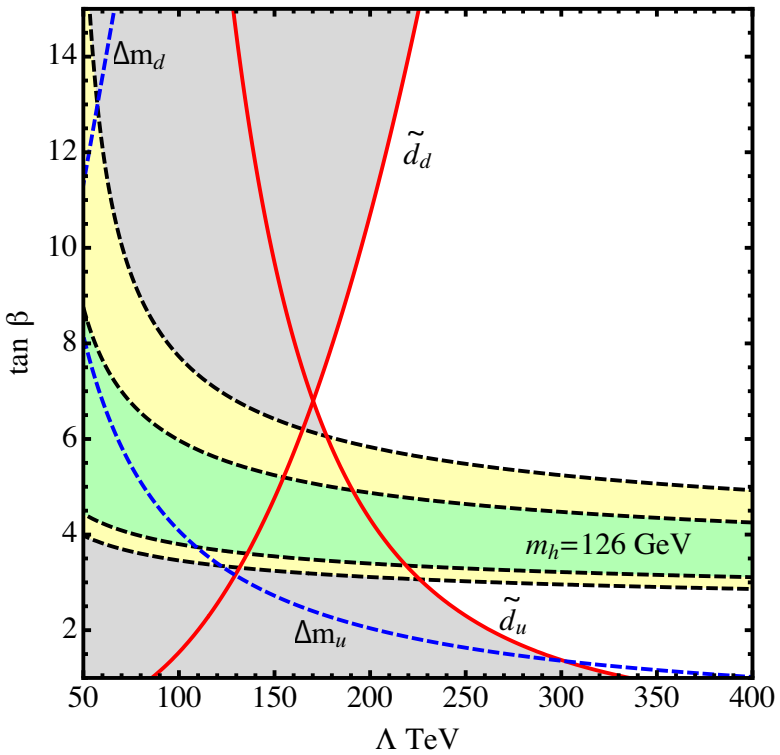
$$|d_e| < 8.7 \times 10^{-29} e \text{ cm}$$

[ACME collaboration ('13)]

[TM & Nagai ('13)]

$\Rightarrow$ Electron EDM covers the sfermion mass of ~ 100 TeV

EDMs of hadrons



Gray region:

Constraint on CEDMs (Hg)

$(|\tilde{d}_u - \tilde{d}_d| < 5.75 \times 10^{-27}$ cm)

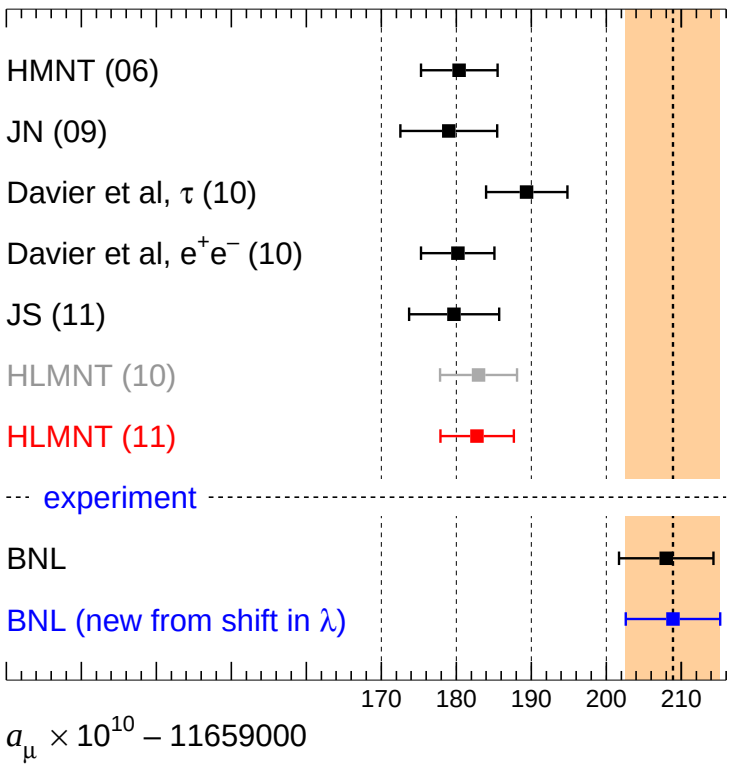
[McKeen, Pospelov & Ritz ('13)]

$\Rightarrow$ Hadronic EDMs are also important

4. Muon Magnetic Dipole Moment

Muon Magnetic Dipole Moment

$$\mathcal{L}_{\text{MDM}} = \frac{e}{4m_\mu} a_\mu \bar{\mu}\sigma_{\alpha\beta}\mu F^{\alpha\beta} \to \frac{e}{m_\mu} a_\mu \vec{S}\vec{B} \qquad a_\mu = \frac{1}{2}(g_\mu - 2)$$

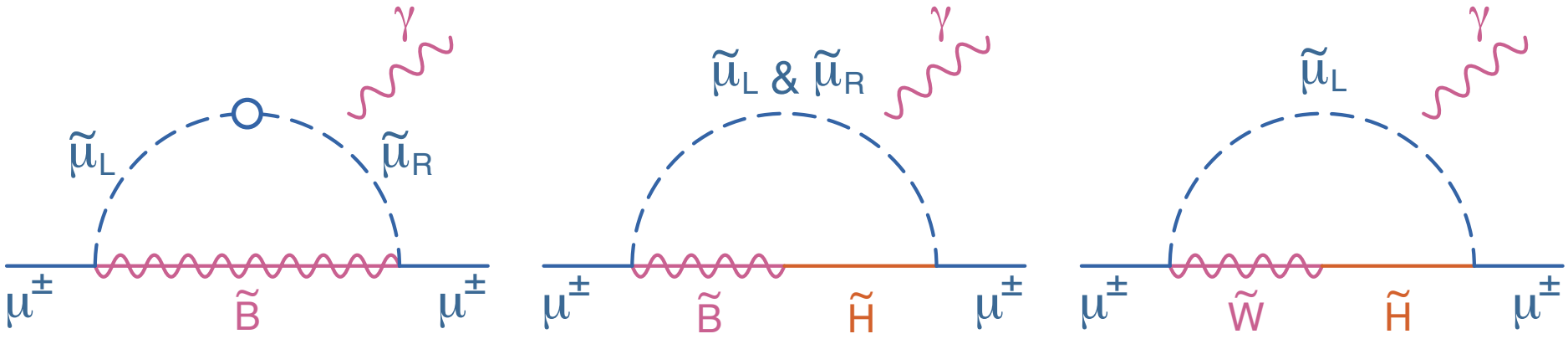


- $a_\mu^{(\text{exp})} = (11\ 659\ 208.9 \pm 6.3) \times 10^{-10}$
- $a_\mu^{(\text{SM})} = (11\ 659\ 182.8 \pm 5.0) \times 10^{-10}$
- $a_\mu^{(\text{exp})} - a_\mu^{(\text{SM})} = (26.1 \pm 8.0) \times 10^{-10}$

$\Rightarrow \sim 3\text{-}\sigma$ discrepancy

[Hagiwara, Liao, Martin, Nomura & Teubner]

SUSY contribution may be the origin of muon $g-2$ anomaly



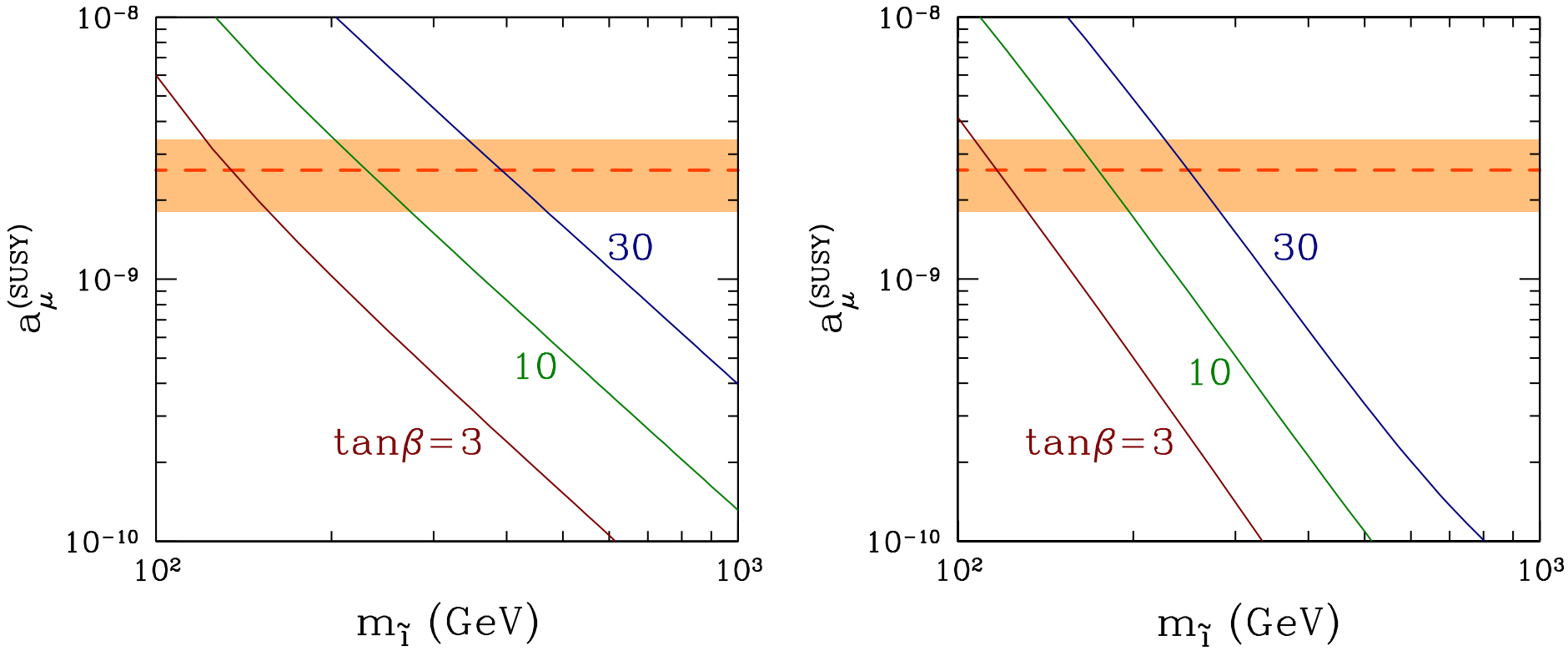
$\Rightarrow a_\mu^{(\mathrm{SUSY})}$ is approximately proportional to $\tan\beta$

A possibility: Heavy colored / light non-colored superparticles

[Ibe, Matsumoto, Yanagida & Yokozaki ('13); Akula & Nath ('13)]

- Sleptons (and Bino or Wino) should be relatively light to realize $a_\mu^{(\mathrm{SUSY})} \sim 2.6 \times 10^{-9}$
- Colored superparticles can be heavy enough to avoid LHC constraints

$a_\mu^{(\mathrm{SUSY})}$: SUSY contribution to muon MDM



- Left: $m_{\tilde{\mu}_L} = m_{\tilde{\mu}_R} = \mu = M_1 = M_2$
- Right: $m_{\tilde{\mu}_L} = m_{\tilde{\mu}_R} = M_1,\ \mu = 1$ TeV, $M_2 =$ large

Interesting possibility:

Reconstructing the SUSY contribution to a_μ at ILC

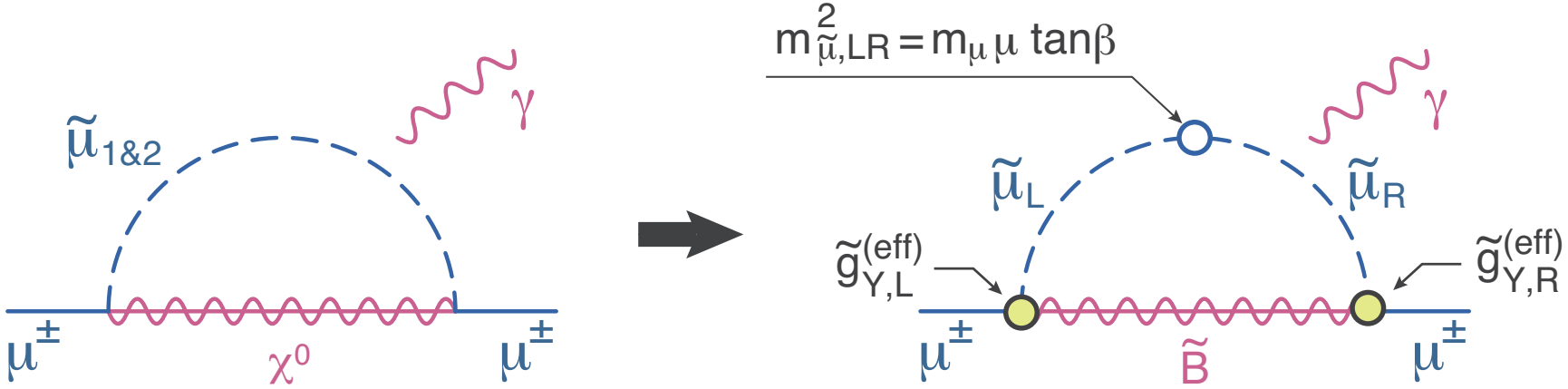
ILC (International e^+e^- linear collider)

- $E_{\text{CM}} = 250 - 500$ GeV (or maybe up to ~ 1 TeV)
- $\mathcal{L} \gtrsim 500$ fb^{-1}
- Polarized e^- and e^+ beams will be available
- Precise study of the sleptons is possible (if produced)

Parameters necessary to reconstruct $a_\mu^{(\text{SUSY})}$ may be measured

- Masses of superparticles
- Coupling and mixing parameters

Case with Bino-diagram dominance



Strategy (case with Bino-diagram dominance)

- Masses of superparticles in the loop can be measured by endpoint and/or threshold analysis
- Left-right mixing may be determined by studying $\tilde{\tau}$ (if A-parameters are negligible)
- Coupling parameters may be determined from selectron production cross section

Reconstruction on our sample point (with $\mathcal{L} \sim 500$ fb^{-1})

[Endo, Hamaguchi, Iwamoto, Kitahara & TM]

X	Input	δX	$\delta_X a_\mu^{(\tilde{B})}$	Process
$\mu \tan\beta$	6100 GeV	$12\,\%$	$13\,\%$	$e^+e^- \to \tilde{\tau}^+\tilde{\tau}^-$
$m_{\tilde{\mu}1}$	126 GeV	200 MeV	$0.3\,\%$	$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-$
$m_{\tilde{\mu}2}$	200 GeV	200 MeV	$0.3\,\%$	$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-$
$m_{\tilde{\chi}_1^0}$	90 GeV	100 MeV	$< 0.1\,\%$	$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-/\tilde{e}^+\tilde{e}^-$
$\tilde{g}_{1,L}^{(\text{eff})}$		a few %	a few %	$e^+e^- \to \tilde{e}_L^+\tilde{e}_R^-$
$\tilde{g}_{1,R}^{(\text{eff})}$		$1\,\%$	$0.9\,\%$	$e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$
$a_\mu^{(\text{SUSY})}$	2.6×10^{-9}	$13\,\%$		

$\Rightarrow$ Error in the reconstruction of $a_\mu^{(\text{SUSY})}$

$$\frac{\delta a_\mu^{(\tilde{B})}}{a_\mu^{(\tilde{B})}} \equiv \frac{1}{a_\mu^{(\tilde{B})}} \sqrt{\sum_X \left(\frac{\partial a_\mu^{(\tilde{B})}}{\partial X} \delta X \right)^2} = 13\,\%$$

5. Summary

Today, I have discussed that:

- There exist new sources of flavor and CP violations in the MSSM
- We may see a signal of BSM physics in flavor and CP violating processes
  - LFVs, like $\mu \to e\gamma$, $\mu \to 3e$, μ-e conversion, etc
  - Electron EDM
- Muon $g-2$ anomaly is still still in controversy
  - The muon $g-2$ anomaly may be due to SUSY contribution
  - Such a scenario may be tested if ILC will become available

Backups

Flavor violation in the SM:

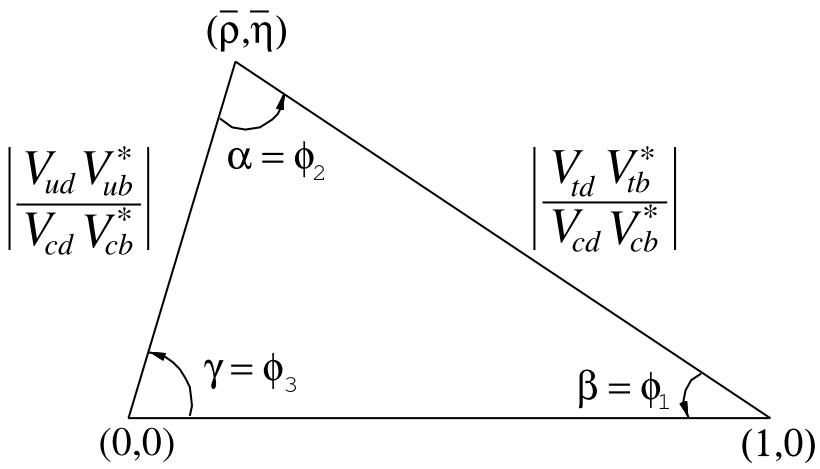
$$\mathcal{L}_W = \frac{g_2}{\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma^\mu W_\mu^+ P_L V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} + \text{h.c.}$$

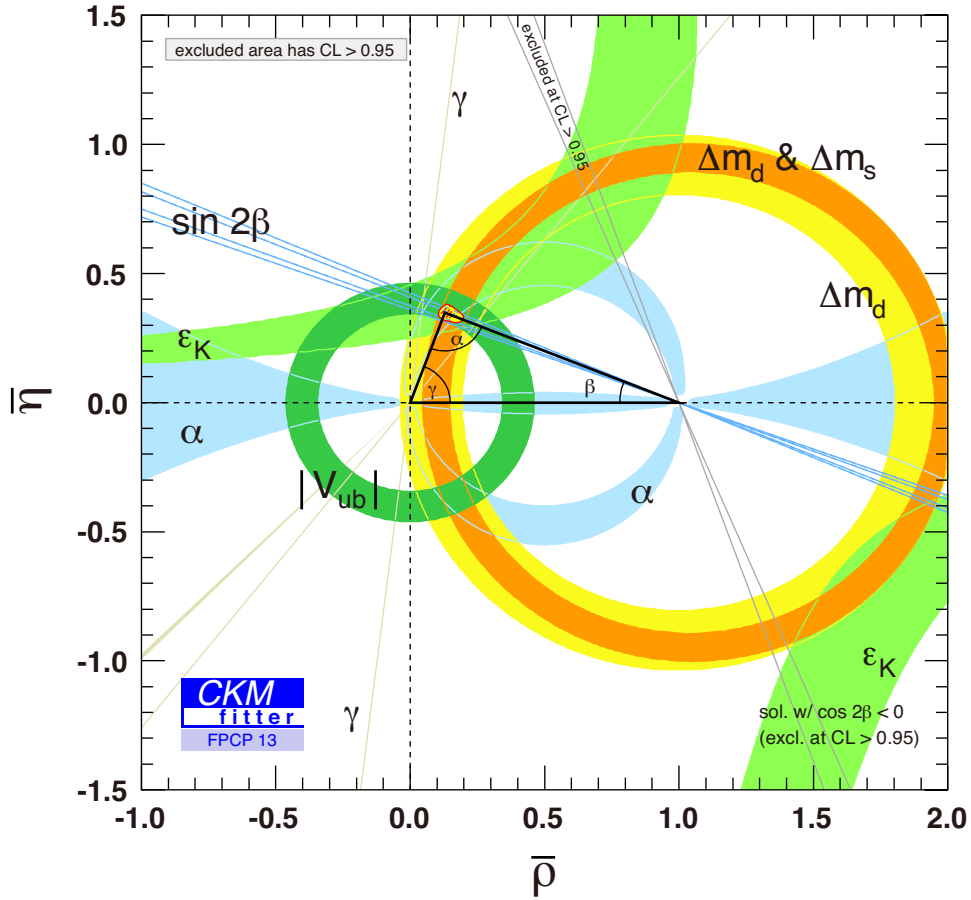
CKM matrix

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

CKM triangle: $[V_{\text{CKM}}^{\dagger} V_{\text{CKM}}]_{bd} = 0$





$\Rightarrow$ Flavor and CP violation seem to be explained well by the CKM mechanism (so far)

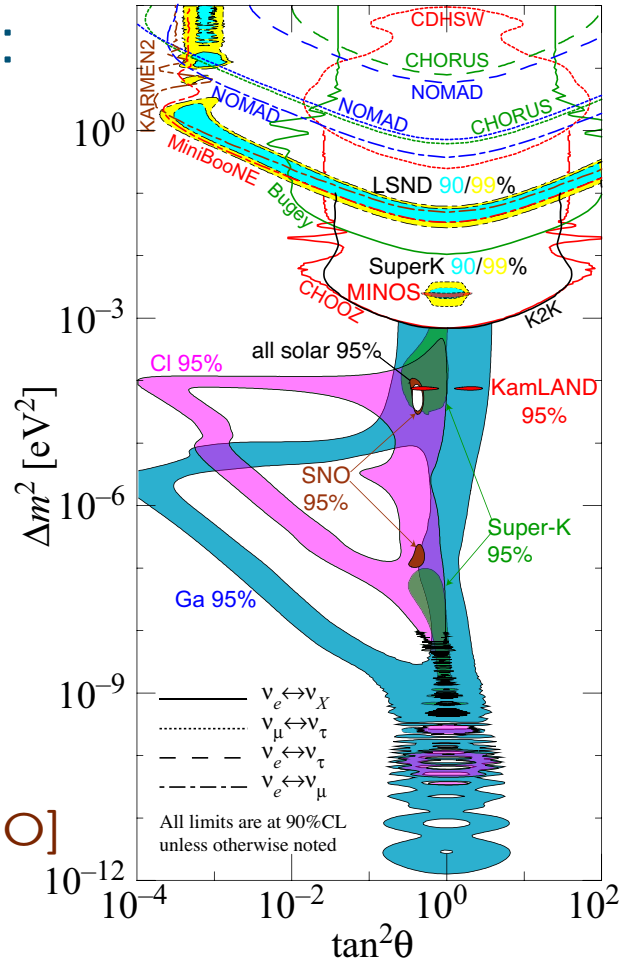
Lepton sector: Neutrino oscillation is observed

$\Leftrightarrow$ In the SM, no leptonic flavor violation ($m_\nu = 0$)

Neutrino masses and mixings (a la PDG):

- $\Delta m^2_{\text{solar}} = (7.58^{+0.22}_{-0.26}) \times 10^{-5}$ eV^2
- $\Delta m^2_{\text{atm}} = (2.35^{+0.12}_{-0.09}) \times 10^{-3}$ eV^2
- $\sin^2\theta_{12} = 0.312^{+0.018}_{-0.015}$
- $\sin^2\theta_{23} = 0.42^{+0.08}_{-0.03}$
- $\sin^2 2\theta_{13} = 0.096 \pm 0.013$

[Machado, Minakata, Nunokawa & Funchal ('13), w/ T2K, MINOS, Double Chooz, Daya Bay & RENO]

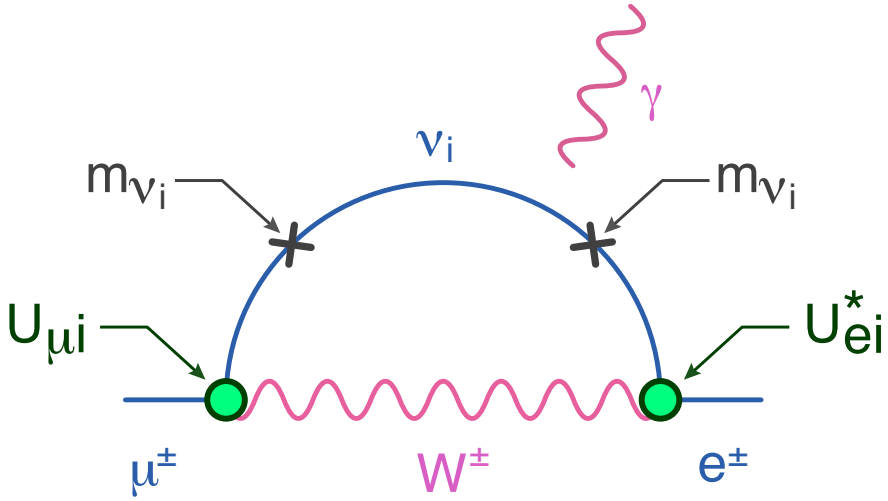


http://hitoshi.berkeley.edu/neutrino

We have already seen LFVs: Neutrino oscillations

$\Rightarrow$ How large are the LFV rates in charged sector?

$\Rightarrow$ For e.g., $Br(\mu \to e\gamma)|_{\text{SM}+\nu} \sim 10^{-55}$ (in SM with massive ν)



$\Leftrightarrow$ Current bound: $Br(\mu \to e\gamma) < 5.7 \times 10^{-13}$

[MEG experiment ('13)]

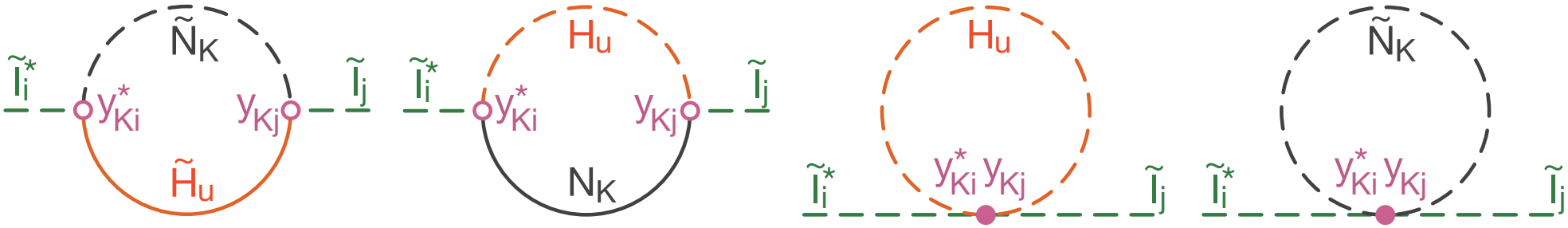
$\Rightarrow$ No hope to see LFVs in the SM (+ neutrino mass)

Off-diagonal elements may be generated by RG effects

$\Rightarrow$ Even if scalar masses are universal at the GUT scale, off-diagonal elements may be generated

In particular, the right-handed (s)neutrinos are important

[Borzumati & Masiero ('86); Hisano, TM, Tobe & Yamaguchi ('95)]

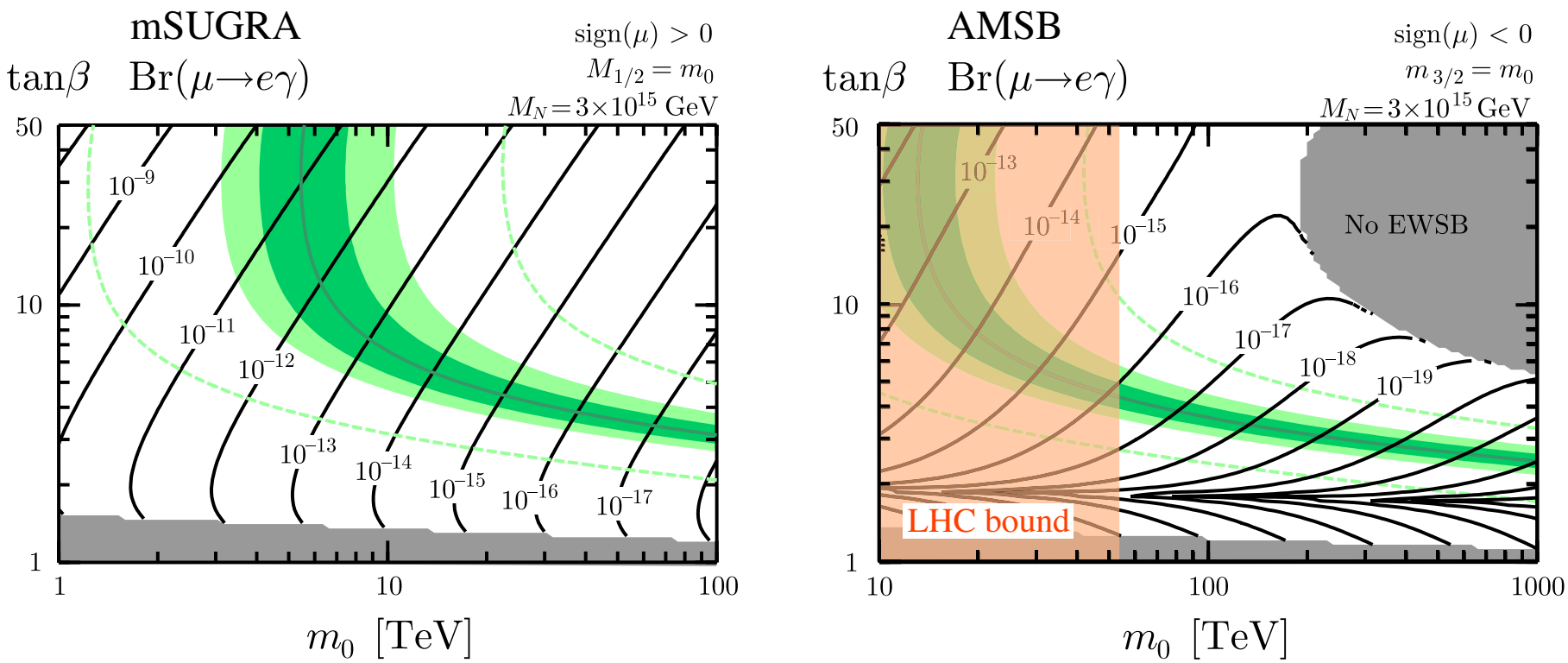


$$\Rightarrow \Delta\mathcal{M}^2_{\tilde{l}_L,ij} \simeq -\frac{6m_0^2}{16\pi^2}(y_\nu^\dagger y_\nu)_{ij} \ln\frac{M_{\mathrm{GUT}}}{M_N}$$

$$y_{\nu,Ij} \simeq \frac{\sqrt{2M_N m_{\nu_L,I}}[U_{\mathrm{PMNS}}]_{Ij}}{v\sin\beta} \quad (\text{when } M_{N,IJ} = M_N\delta_{IJ})$$

$Br(\mu \to e\gamma)$ in SUSY model with right-handed neutrinos

- Universal scalar mass at the GUT scale is assumed



[TM, Nagai & Yanagida ('13)]

- Dark green: $125 \leq m_h \leq 127$ GeV
- Light green: $124 \leq m_h \leq 128$ GeV

Muon MDM: Theoretical calculation (1)

- QED loops: up to α^5

  [Aoyama, Hayakawa, Konoshita & Nio ('12)]

  $$\Rightarrow a_\mu^{(\text{QED})} = (11\ 658\ 471.8951 \pm 0.0080) \times 10^{-10}$$

- EW loops: up to 2 loop

  [Gnendiger, Stočkinger & Stočkinger-Kim ('13); see also Czarnecki, Marciano & Vainshtein ('03)]

  $$\Rightarrow a_\mu^{(\text{EW})} = (15.36 \pm 0.10) \times 10^{-10}$$

  $m_h = 125.6 \pm 1.5$ GeV is used

  Error: 3-loop contributions & hadronic uncertainties

Muon MDM: Theoretical calculation (2)

- Hadronic contribution: vacuum polarization (LO)

$$\Rightarrow a_\mu^{(\text{had,v.p.,LO})} = (692.3 \pm 4.2) \times 10^{10}$$

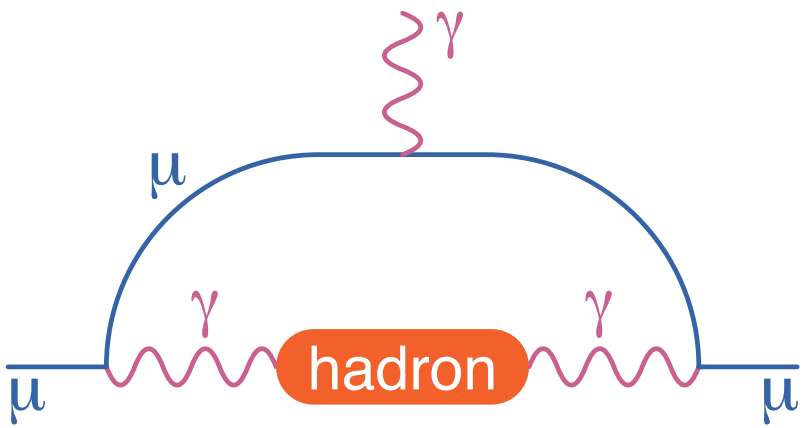
[Davier, Hoecker, Malaescu & Zhang ('11)]

$$\Rightarrow a_\mu^{(\text{had,v.p.,LO})} = (690.75 \pm 4.72) \times 10^{10}$$

[Jegerlehner & Szafron ('11)]

$$\Rightarrow a_\mu^{(\text{had,v.p.,LO})} = (694.91 \pm 4.27) \times 10^{10}$$

[Hagiwara, Liao, Martin, Nomura & Teubner ('11)]



Muon MDM: Theoretical calculation (3)

- Hadronic contribution: vacuum polarization (NLO)

$$\Rightarrow a_\mu^{(\text{had,v.p.,NLO})} = (-9.84 \pm 0.07) \times 10^{10}$$

[Hagiwara, Liao, Martin, Nomura & Teubner]

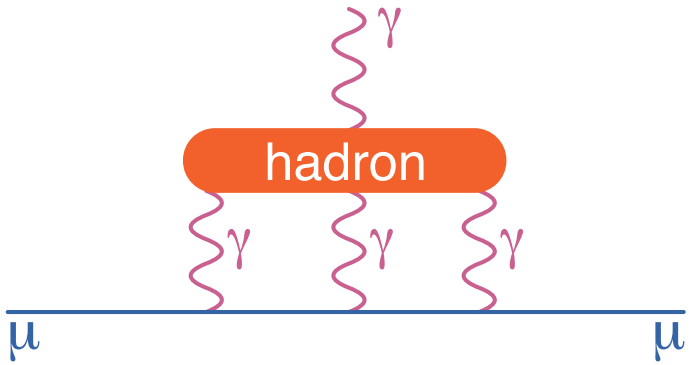
- Hadronic contribution: light-by-light

$$\Rightarrow a_\mu^{(\text{had,lbl})} = (10.5 \pm 2.6) \times 10^{10}$$

[Prades, Rafael & Vainshtein ('09)]

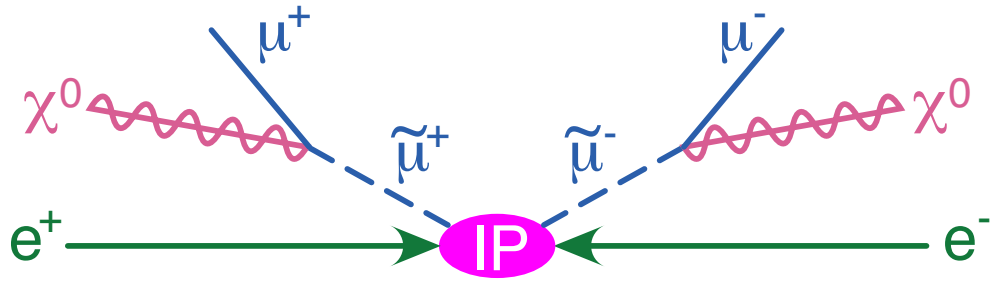
$$\Rightarrow a_\mu^{(\text{had,lbl})} = (11.6 \pm 4.0) \times 10^{10}$$

[Nyffeler ('09)]



Mass determination via the endpoint study

⇒ Example: $e^+e^- \to \tilde{\mu}^+\tilde{\mu}^-$, followed by $\tilde{\mu}^\pm \to \chi_1^0\mu^\pm$

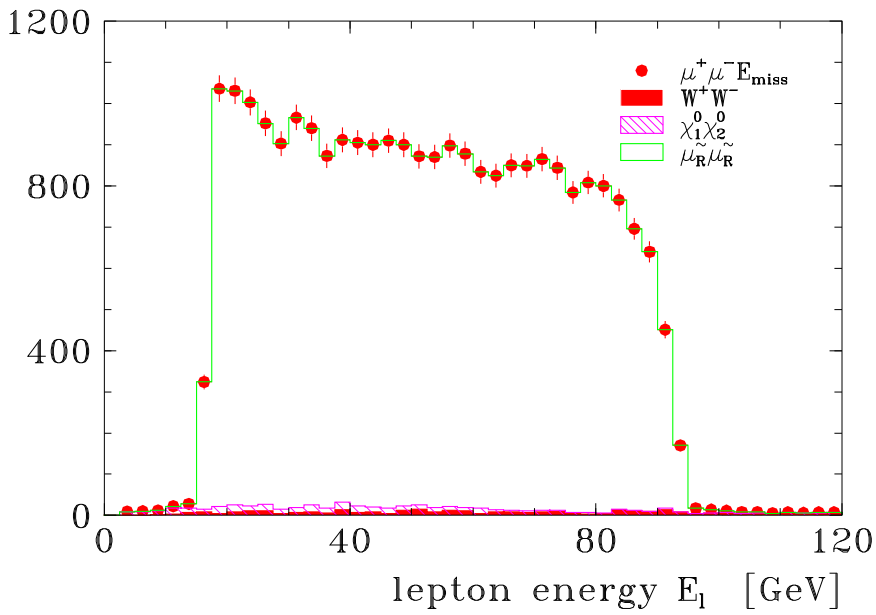


⇒ Energy of $\mu^\pm$: $E_\mu^{(-)} \leq E_\mu \leq E_\mu^{(+)}$

$$E_\mu^{(\pm)} \equiv \frac{1}{2}E_{\text{beam}}\left[1-\left(\frac{m^2_{\chi_1^0}}{m^2_{\tilde{\mu}}}\right)\right]\left[1\pm\left(1-\frac{m^2_{\tilde{\mu}}}{E^2_{\text{beam}}}\right)^{1/2}\right]$$

⇒ We obtain the information about smuon and neutralino masses from the endpoints

Result of MC (SPS1a, with $\sqrt{s} = 400$ GeV & $\mathcal{L} = 200$ fb^{-1}):



$m_{\chi_1^0} = 96.0$ GeV

$m_{\tilde{\mu}_R} = 143.0$ GeV

[Martyn ('04)]

$\Rightarrow \delta m_{\tilde{\ell}}, \delta m_{\chi_1^0} \lesssim 200$ MeV

Left-right mixing may be determined by studying $\tilde{\tau}$

$$\mathcal{M}^2_{\tilde{\ell}} = \begin{pmatrix} m^2_{\tilde{\ell}LL} & m_\ell(\mu \tan\beta + A_\ell) \\ m_\ell(\mu \tan\beta + A_\ell) & m^2_{\tilde{\ell}RR} \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta_{\tilde{\ell}} & \sin\theta_{\tilde{\ell}} \\ -\sin\theta_{\tilde{\ell}} & \cos\theta_{\tilde{\ell}} \end{pmatrix} \begin{pmatrix} m_{\tilde{\ell}_1} & 0 \\ 0 & m_{\tilde{\ell}_2} \end{pmatrix} \begin{pmatrix} \cos\theta_{\tilde{\ell}} & -\sin\theta_{\tilde{\ell}} \\ \sin\theta_{\tilde{\ell}} & \cos\theta_{\tilde{\ell}} \end{pmatrix}$$

$$\Rightarrow m^2_{\tilde{\ell}LR} \equiv m_\ell(\mu \tan\beta + A_\ell) = \frac{1}{2}(m^2_{\tilde{\ell}1} - m^2_{\tilde{\ell}2}) \sin 2\theta_{\tilde{\ell}}$$

Slepton production cross section depends on θ_ℓ

$\Rightarrow$ Mixing angle in the stau sector can be sizable

$\Rightarrow$ $\sigma(e^+e^- \to \tilde{\tau}_i\tilde{\tau}_j)$ is sensitive to $\theta_{\tilde{\tau}}$, and can be used to determine $\mu \tan\beta$ (assuming $A_\ell \ll \mu \tan\beta$)

Gaugino couplings may deviate from the gauge coupling

[Hikasa & Nakamura ('96); Nojiri, Fujii & Tsukamoto ('96); Cheng, Feng & Polonsky ('97); Katz, Randall & Su ('98)]

$$\tilde{g}_{Y,L} \simeq g_Y \left[1 + \frac{1}{4\pi} \left(4\alpha_Y \ln \frac{m_{\tilde{q},H}}{m_{\tilde{l}}} - \frac{1}{6}\alpha_Y \ln \frac{m_{\tilde{H}}}{m_{\tilde{l}}} + \frac{9}{4}\alpha_2 \ln \frac{m_{\tilde{W}}}{m_{\tilde{l}}} \right) \right]$$

$$\tilde{g}_{Y,R} \simeq g_Y \left[1 + \frac{1}{4\pi} \left(4\alpha_Y \ln \frac{m_{\tilde{q},H}}{m_{\tilde{l}}} - \frac{1}{6}\alpha_Y \ln \frac{m_{\tilde{H}}}{m_{\tilde{l}}} \right) \right]$$

$\tilde{g}_{Y,L}$ and $\tilde{g}_{Y,R}$ can be measured using $\sigma(e^+e^- \to \tilde{e}^+\tilde{e}^-)$