Coleman-Weinberg inflation and Higgs mass in light of PLANCK data

Hyun Min Lee
(Chung-Ang University)

Ref. w/ G. Barenboim, E. J. Chun, 1309.1695 [hep-ph];

The 3rd KIAS Workshop
on Particle Physics and Cosmology
Nov 15, 2013
Outline

• Introduction
• Coleman-Weinberg inflation
• Fully radiative EWSB and CW inflation
• Conclusions
Slow-roll inflation

\[ \dot{\phi} + 3H\phi + \frac{\partial V}{\partial \phi} = 0 \]

\[ H^2 = \frac{1}{3M_p^2} \left[ \frac{\dot{\phi}^2}{2} + V(\phi) \right] \]

\[ 3H\dot{\phi} \approx -\frac{\partial V}{\partial \phi}, \]

\[ H^2 \approx \frac{V(\phi)}{3M_p^2} : \]

\[ ds^2 \approx -dt^2 + e^{2Ht} \delta_{ij} dx^i dx^j \]

• Solves horizon, homogeneity, flatness and structure formation; inflaton perturbations generate CMB anisotropies.
Planck and inflation

- Almost scale-invariant CMB
  \[ \langle R(k)R(k') \rangle = \delta^3(k-k') \frac{2\pi^2}{k^3} \mathcal{P}_R(k), \]
  \[ n_s = 1 - 6\epsilon + 2\eta, \quad \epsilon = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad \eta = M_P^2 \frac{V''}{V} \ll 1. \]

- Tensor perturbation (gravity-wave) in CMB less than 10%.

- Almost Gaussian: \( f_{NL}^{\text{local}} = 2.7 \pm 5.8 \)

\[ n_s = 0.9603 \pm 0.0073 \]
\[ r_{0.002} < 0.12 \]
(Planck+WP)
Higgs inflation

- Higgs scalar is the obvious option for inflaton.

$V(h) = \frac{1}{4} \lambda h^4 \left(1 + \frac{\xi h^2}{M_P^2}\right)^2$

$\xi = 0$  \hspace{1cm} $\xi \neq 0$

Inflation

SM vacuum

$H$ inflation

$\mathcal{L} = \frac{1}{2} M_P^2 \mathcal{R} + \mathcal{L}_{\text{SM}} + \xi |H|^2 \mathcal{R}, \quad \xi \sim 10^4.$

Predictions in agreement with Planck data.

But, there is unitarity problem.

[Bezrukov, Shaposhnikov (2007)]

[Barbon, Espinosa (2009); Hertzberg(2010)]
Unitarity problem

- Large non-minimal coupling lead to unitarity problem.

\[ A(WW \rightarrow WW) \sim \frac{E^2}{v^2} (1 - a^2) \sim \frac{E^2}{\Lambda^2}. \]

\[ a = 1 - \frac{3v^2}{\Lambda^2}; \quad b = 1 - \frac{12v^2}{\Lambda^2} \]

- W/Z bosons decoupled from Higgs during inflation:

\[ M_W^2 \sim \frac{g^2 M_P^2}{2 \xi}, \quad g_{WW} = -g^2 \langle h \rangle \left( \frac{M_P^2}{\xi \langle h \rangle^2} \right)^2. \]  

[Bezrukov et al (2010)]

Cutoff scale becomes \( M_P/\sqrt{\xi} \gg H: R^{2+n}/M^{2n} \ll 1. \)

- But, uncontrolled higher order terms could spoil Higgs inflation.

Beyond Higgs boson.
Coleman-Weinberg inflation
Coleman-Weinberg mechanism

- **No mass corrections assumed**, e.g. in DR.

- **Quantum corrections** break gauge symmetry spontaneously.

\[
V = \frac{1}{4} \lambda \phi^4 + \frac{\phi^4}{64\pi^2}(10\lambda^2 + 3e^4\phi) \left( \ln \frac{\phi^2}{M^2} - \frac{25}{6} \right) \sim \beta \lambda \phi > 0
\]

* minimization condition: \( \lambda(\langle \phi \rangle) \approx \frac{11}{16\pi^2} e^4(\langle \phi \rangle) \) @ \( M = \langle \phi \rangle \).

- **Dimensional transmutation.**

\[
\langle \phi \rangle = \Lambda_{UV} \exp\left( \frac{11}{6} \right) \exp\left( - \frac{8\pi^2}{3} \frac{\lambda(\Lambda_{UV})}{e^4(\Lambda_{UV})} \right).
\]

\[
\Lambda_{QCD} = \Lambda_{UV} e^{-\frac{8\pi^2}{b^2(\Lambda_{UV})}}
\]

\[
\lambda(\Lambda_{UV}) \sim e^4(\Lambda_{UV}) \quad \Rightarrow \quad \langle \phi \rangle \ll \Lambda_{UV}.
\]
Small-field CW inflation

- General CW potential (at minimum, $V(v_\phi) = 0, \ V'(v_\phi) = 0$):
  \[ V(\phi) = A\phi^4 \left( \ln \frac{\phi}{v_\phi} - \frac{1}{4} \right) + \frac{1}{4} Av_\phi^4. \]

- Slow-roll parameters
  \[
  \begin{align*}
  \epsilon &= 8 \left( \frac{4M_P}{v_\phi} \right)^2 \left( \frac{\phi}{v_\phi} \right)^6 \ln^2 \left( \frac{\phi}{v_\phi} \right), \\
  \eta &= \left( \frac{4M_P}{v_\phi} \right)^2 \left( \frac{\phi}{v_\phi} \right)^2 \left( 3 \ln \left( \frac{\phi}{v_\phi} \right) + 1 \right)
  \end{align*}
  \]

$\epsilon, |\eta| \ll 1, v_\phi < M_P \quad \Rightarrow \quad \phi_* \ll v_\phi :$ small-field inflation

$\phi :$ a SM-singlet scalar
Correlated observables

- Predictions

\[ N \approx \frac{3}{1 - n_s}, \]

\[ \frac{dn_s}{d \ln k} \approx -\frac{1}{3} (1 - n_s)^2, \]

\[ r \approx \frac{16}{27} \left( \frac{v_\phi}{4M_P} \right)^4 \frac{(1 - n_s)^3}{|\ln(\phi/v_\phi)|} \]

\[ P_R(k) \approx A \frac{72}{\pi^2} \frac{|\ln(\phi/v_\phi)|}{(1 - n_s)^3}. \]

- Measurements

\[ n_s = 0.9603 \pm 0.0073 \quad [68\% \text{ C.L.}] \quad \text{(Planck+WP)} \]

\[ \frac{dn_s}{d \ln k} = -0.013 \pm 0.009 \]

\[ r < 0.12 \quad [95\% \text{ C.L.}] \]

\[ P_R \approx 2.2 \times 10^{-9} \]

- \( n_s \approx 0.96 \) requires \( N \approx 75. \)

\[ N \approx 61 - \ln \left( \frac{10^{16} \text{GeV}}{V_*^{1/4}} \right) + \ln \left( \frac{V_*^{1/4}}{V_{\text{end}}^{1/4}} \right) - \frac{1}{3} \ln \left( \frac{V_*^{1/4}}{\rho_{\text{reh}}^{1/4}} \right) \]

But, the condition for horizon re-entry needs \( N \lesssim 60. \)
Numerical results

- “Sub-Planckian” inflaton fields assumed.
- Small quartic coupling: \( A \sim 10^{-15} - 10^{-14} \).
- Small inflation value: \( \phi_* / v_\phi \lesssim 10^{-3} \).

But, the obtained spectral index with \( N=60 \) deviates from Planck data more than 2σ.

\[2\sigma\]

\[3\sigma\]

cf. For trans-Planckian inflaton field values, N. Okada, Q. Shafi, 1311.0921 [hep-ph].
Cosmology on the brane

- Matter lives on a brane in extra dimension.

\[ G_{AB} = \kappa^2 \tilde{T}_{AB}, \]
\[ \tilde{T}^A_B = \tilde{T}^A_B|_{\text{bulk}} + T^A_B|_{\text{brane}}, \]

- Friedman equation on the brane in the RS model.

\[ H^2 = \frac{1}{3M_P^2} \rho \left(1 + \frac{\rho}{2\Lambda_b}\right), \quad \Rightarrow \quad H^2 \approx \frac{V}{3M_P^2} \left(1 + \frac{V}{2\Lambda_b}\right). \]
\[ \dot{\rho} + 3H(\rho + p) = 0, \quad \Rightarrow \quad 3H\dot{\phi} \approx -\partial V/\partial \phi. \]

BBN bound: \(|\Lambda_b|^{1/4} \gg 1\text{ MeV}.\)
Larger number of efoldings

- Modified Friedman equation makes Hubble radius smaller, cosmological scale leaves horizon earlier, requiring a larger \( N \).

\[
N_{B,*} \approx 61 + \frac{1}{2} \ln \left(1 + \tilde{V}\right) - \ln \left(\frac{10^{16} \text{GeV}}{V_*^{1/4}}\right) - \frac{1}{3} \left(\frac{V_*^{1/4}}{\rho_{reh}}\right).
\]

\( \Lambda_b^{1/4} = \infty, 10^2, 1, 10^{-2} \text{ GeV} \)

\( V_*^{1/4} = 10^5 \text{ GeV} \)
Brane CW inflation

- Correlations maintained under modified parameters.

\[
\epsilon_B = \epsilon \cdot \frac{1 + 2\tilde{V}}{(1 + \tilde{V})^2}, \quad N_B = -\frac{1}{M_P^2} \int_{\phi_*}^{\phi_f} \frac{V}{\tilde{V}} (1 + \tilde{V}) \, d\phi.
\]

\[
\eta_B = \frac{1}{1 + \tilde{V}}, \quad \mathcal{P}_{R,B}(k) = \mathcal{P}_R(k) \cdot (1 + \tilde{V})^3
\]

\[
r_B = 16\epsilon_B \frac{1}{1 + 2\tilde{V}},
\]

- Need a small brane tension scale:

\[
M_B = (2|\Lambda_b|)^{1/4} \lesssim 10^{10-11} \text{ GeV} \ll v_\phi \sim 10^{16} \text{ GeV}.
\]
Conditions for inflation

- Quartic coupling remains small: \( A \sim 10^{-15} - 10^{-14} \).
- “Medium-field” inflation: \( \phi_* / v_\phi \gtrsim 0.1 \).

cf. Einstein gravity : \( \phi_* / v_\phi \lesssim 10^{-3} \).
Fully radiative EWSB and CW inflation
126 GeV Higgs boson

- EBH mechanism.

\[ V(H) = -\mu^2 |H|^2 + \lambda_H |H|^4 \]

\[ \lambda_H = \frac{m_H^2}{2v^2} = 0.131 \]

; \(|\mu| = 89\text{ GeV}\)

What is the origin of the small Higgs mass?

- Supersymmetry?
- Composite Higgs?

\{ \text{No evidence at the LHC.} \}

- Coleman-Weinberg mechanism? \rightarrow \text{Beyond the SM.}
Vanishing Higgs potential

- Higgs quartic coupling vanishes at high scale.

[Degrassi et al (2013)]

- SM vacuum may be metastable or Higgs potential remains zero at high scales ("conformality").
B-L gauge symmetry

• Neutrino masses requires new physics.

\[ \mathcal{L} = -\frac{c v^2}{M_N} \nu \nu, \quad m_\nu = \frac{c v^2}{M_N} \lesssim 0.1 \text{eV} : \quad M_N/c \gtrsim 10^{14} \text{GeV}. \]

• U(1) B-L gauge symmetry: anomaly-free with SM + 3 right-handed (RH) neutrinos.

<table>
<thead>
<tr>
<th></th>
<th>SU(3)_c</th>
<th>SU(2)_L</th>
<th>U(1)_Y</th>
<th>U(1)_{B-L}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q^i_L )</td>
<td>3</td>
<td>2</td>
<td>+1/6</td>
<td>+1/3</td>
</tr>
<tr>
<td>( u^i_R )</td>
<td>3</td>
<td>1</td>
<td>+2/3</td>
<td>+1/3</td>
</tr>
<tr>
<td>( d^i_R )</td>
<td>3</td>
<td>1</td>
<td>-1/3</td>
<td>+1/3</td>
</tr>
<tr>
<td>( \ell^L )</td>
<td>1</td>
<td>2</td>
<td>-1/2</td>
<td>-1</td>
</tr>
<tr>
<td>( \nu^i_R )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>( e^i_R )</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>( H )</td>
<td>1</td>
<td>2</td>
<td>-1/2</td>
<td>0</td>
</tr>
<tr>
<td>( \Phi )</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>+2</td>
</tr>
</tbody>
</table>

“B-L Higgs”

• B-L breaking generates RH neutrino masses.
Vanishing B-L potential

[Chun, Jung, HML (2013, erratum)]

- Generalized charges \( X = Y_{B-L} - xY \) with \( 0.43 < x < 1.13 \).

  Fully radiative generation of the B-L potential.

[Alternative: Hashimoto et al (2013)]

\[
(4\pi)^2 \beta_{\lambda_\phi} = 20\lambda_\phi^2 - 16\text{Tr}(y_N^4) + 96g_X^4 + 8\lambda_\phi \text{Tr}(y_N^2) - 48\lambda_\phi g_X^2 + 2\lambda^2_{H\phi}, \quad (4\pi)^2 \beta_{y_{N_i}} = y_{N_i} \left( 4y_{N_i}^2 + 2\text{Tr}(y_N^2) - 6g_X^2 \right).
\]
**B-L CW mechanism**

- **B-L potential in the IR.**
  
  \[
  V_X(\phi) = \frac{1}{4} \lambda_\Phi \phi^4 + \frac{\phi^4}{64\pi^4} \left(10\lambda_\Phi^2 + 48g_X^2 - 8\sum_{i=1}^{3} y_{N_i}^4 \right) \left(\ln \frac{\phi^2}{M^2} - \frac{25}{6}\right)
  \]
  
  \[\sim \beta_{\lambda_\Phi} > 0\]

- **CW breaking of B-L symmetry.**

  \[
  \lambda_\Phi(v_\phi) = \frac{11}{48\pi^2} \left(10\lambda_\Phi^2 + 48g_X^4 - 8y_N^4\right)(v_\phi),
  \]

  \[
  v_\phi \sim M_* e^{\frac{11}{8}} \exp \left( -\frac{\pi^2}{6} \frac{\lambda_\Phi(M_*)}{g_{B-L}(M_*) - \frac{16}{96}y_N^4(M_*)} \right)
  \]

- **Masses:**
  
  \[
  M_\phi^2 = \frac{6}{11} \lambda_\Phi(v_\phi)v_\phi^2, \quad M_X^2 = 4g_X^2(v_\phi)v_\phi^2, \quad M_N = \sqrt{2}y_N v_\phi
  \]

- **B-L Higgs inflation:**

  \[
  V_X(\phi) = \frac{3\lambda_\Phi}{22} \phi^4 \left(\ln \left(\frac{\phi}{v_\phi}\right) - \frac{1}{4}\right) + \frac{3\lambda_\Phi}{88} v_\phi^4
  \]

  \[\rightarrow \quad A = \frac{3}{22} \lambda_\Phi.\]
**Fully radiative EWSB**

- **Higgs mass** is generated from Higgs portal coupling.
  \[ m_H^2 = \frac{1}{2} \lambda_{H\Phi} v^2 < 0. \]

- **Higgs quartic coupling** vanishes at higher scale than in SM.

- **Higgs potential** is fully generated by radiative corrections.

\[ \lambda_H = 0, \lambda_\Phi = 0, \lambda_{H\Phi} = 0, m_\Phi^2 = 0, m_H^2 = 0, \text{ and } \tilde{g} = 0 \] in the UV.
Numerical results

- One parameter family of solutions.

\[ M_f = 2 \times 10^{11} \text{GeV} \]

\[ y_N(v_\phi) \]

\[ \lambda_\phi(v_\phi) \]

\[ v_\phi(\text{GeV}) \]

- B-L quartic coupling is so small that a tuning in the dimensionless parameters is necessary.

\[ \lambda_\phi \approx \frac{11}{\pi^2} \left( g_X^4 - \frac{1}{6} y_N^4 \right) \sim 10^{-7} (10^{-14}) \quad \text{for} \quad g_X \sim 0.1 (0.01) \]
B-L pheno.

- $Z'$ mass $\sim$ RH neutrino mass $>3$ TeV chosen.
- Gauge kinetic mixing is small; Higgs mixing is consistent with LEP and Higgs data.
- B-L scalar mass is relatively light as 0.1-8 GeV.

"search strategies"- $Z' \rightarrow \phi \phi$, $\phi \rightarrow \tau \bar{\tau}$, $c \bar{c}$.  

![Graphs and diagrams showing the relationship between $M_Z$ and $g_x(v_{\phi})$, and other variables.](image)
B-L inflation & EWSB

• Planck 1σ limits: $\nu_\phi > 0.6-3 \times 10^7$ GeV for $M_B > 1, 10$ MeV.

• B-L breaking + EWSB

Parameter space favored by Planck is consistent with Higgs mass.
Conclusions

• Coleman-Weinberg inflation in standard cosmology is not favored by Planck data at $> 2\sigma$.

• CW inflation on the brane can be consistent with Planck data, because smaller horizon radius allows for a larger number of efoldings.

• Successful Coleman-Weinberg inflation and EWSB can be achieved consistently with a generalized B-L symmetry.

• Fully radiative symmetry breaking leads to a light singlet scalar of 0.1-8 GeV, which can be searched for at the LHC.