New routes of reactions by a long-lived negatively charged massive particle during big bang nucleosynthesis

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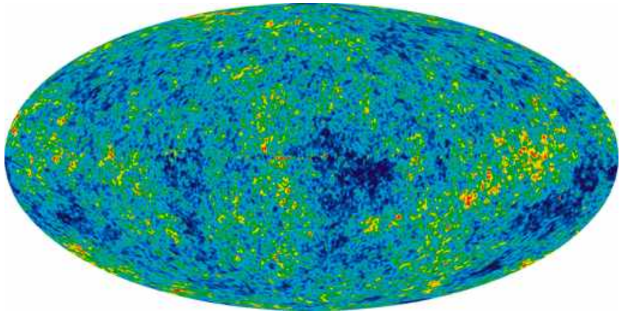
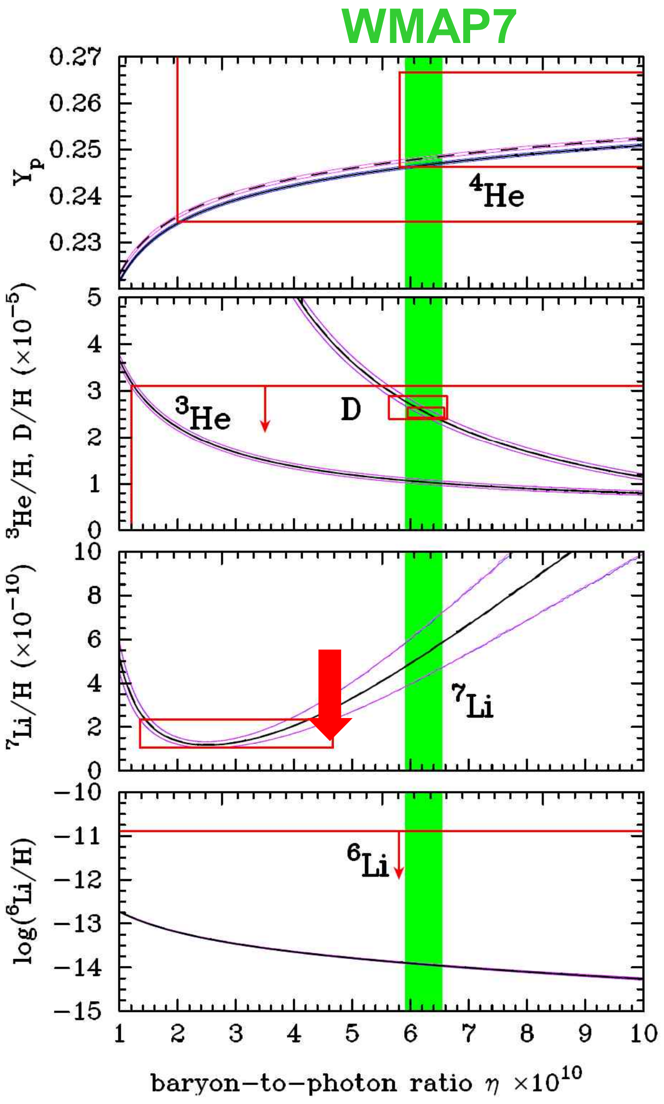
Introduction

1. Light element abundances

- Standard big bang nucleosynthesis (BBN)
  parameter: baryon-to-photon ratio η
- Observation of CMB →constraint on η



WMAP Science Team

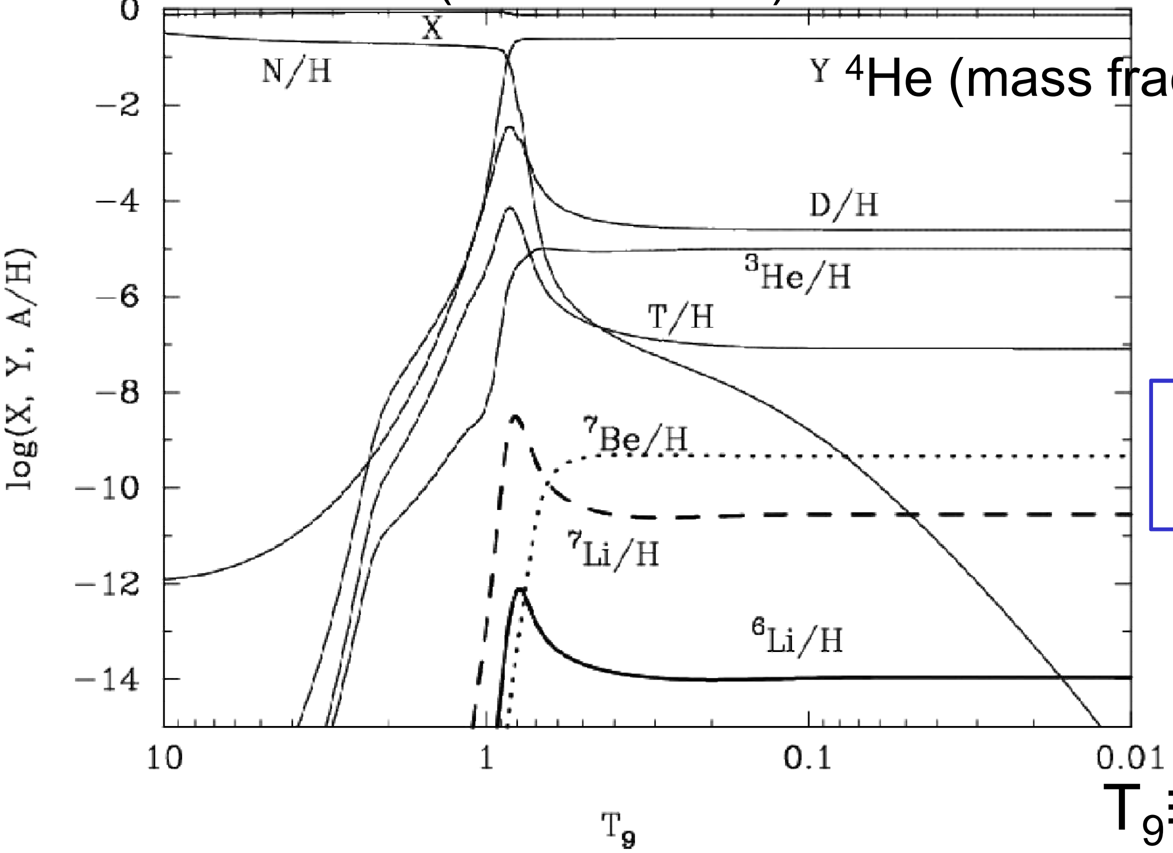


- Observation of metal-poor stars
  - ✓^{7}Li abundance is smaller than theory by a factor of ~**3**

Signature of new physics?

- Primordial abundances of ^{9}Be, ^{10,11}B, …
  are not detected yet.

2. Standard Big Bang nucleosynthesis (SBBN)



^{1}H (mass fraction)

^{4}He (mass fraction)

$\eta = n_b/n_\gamma = 6.3 \times 10^{-10}$

Recombination
$\rightarrow e^-$ capture ($^7Be \rightarrow {}^7Li$)

$T_9 \equiv T/(10^9 K)$

3. Li problem

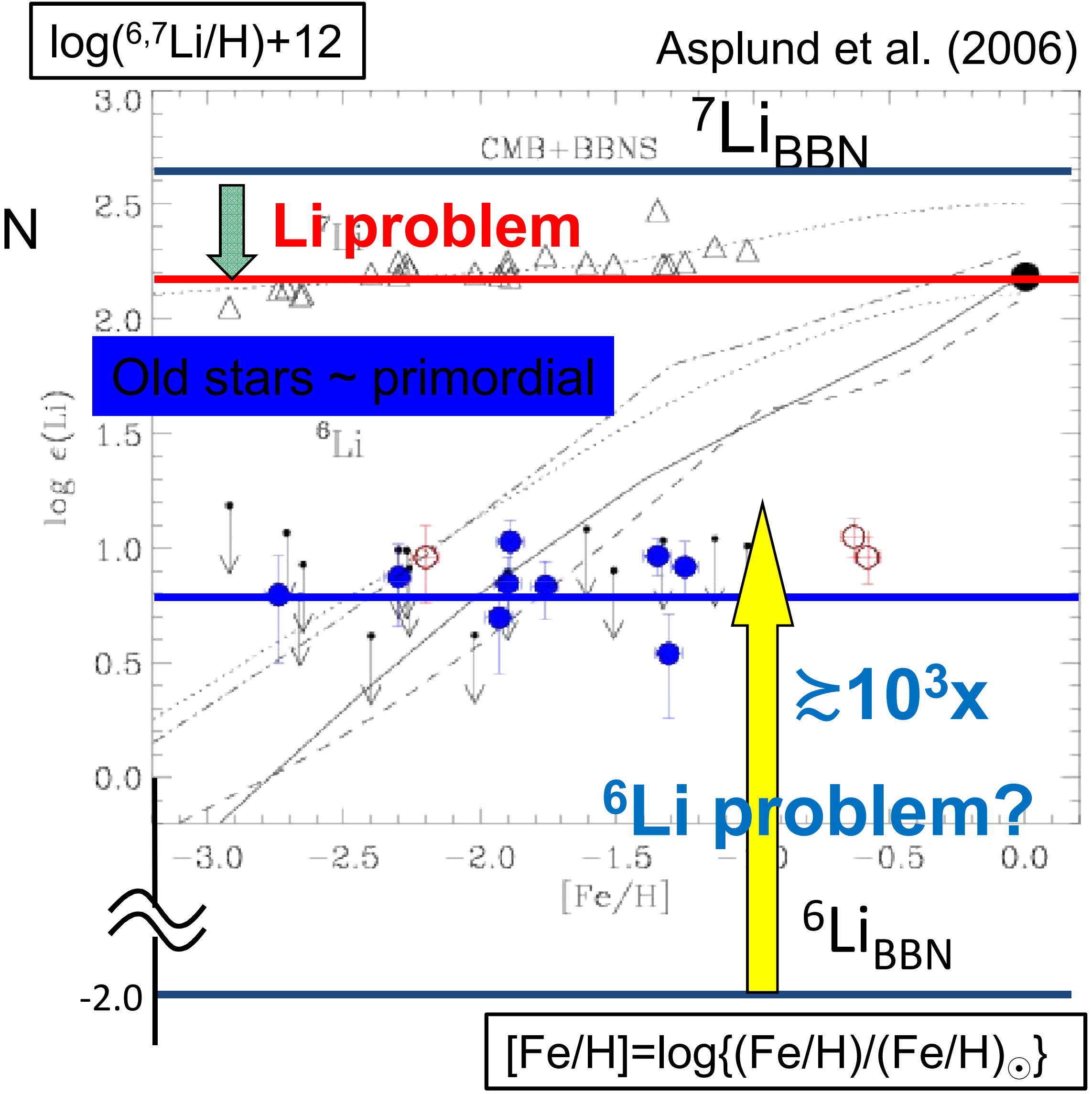
- ^{7}Li abundance observed in metal-poor stars are a factor~3 smaller than SBBN

  $^{7}\mathrm{Li/H}=(1.1\text{-}1.5)\times 10^{-10}$

- Some of the stars have large abundances of ^{6}Li

  $^{6}\mathrm{Li/H} \approx 6\times 10^{-12}$

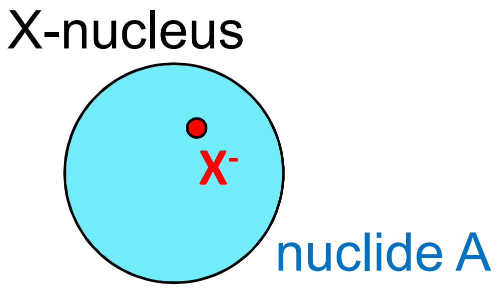
- But, Lind et al. perform 3D NLTE analysis for atmosphere, and found **no detection**



News: Lind et al. Astron. Astrophys. (2013)

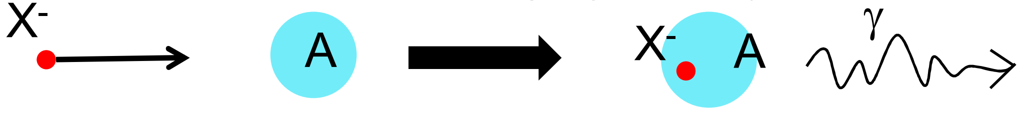
4. Effect of long-lived negatively charged massive particle (CHAMP) on BBN

➢CHAMP X^- recombines with nuclide A, and X-nuclide (A_X) forms (Cahn & Glashow 1981)

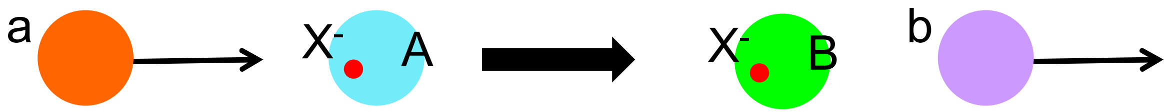


➢exotic atoms form →exotic nucleosynthesis
(Pospelov 2007, Kohri & Takayama 2007, Kawasaki et al. 2007-, Hamaguchi et al. 2007, Jedamzik 2008-, MK et al. 2007-)

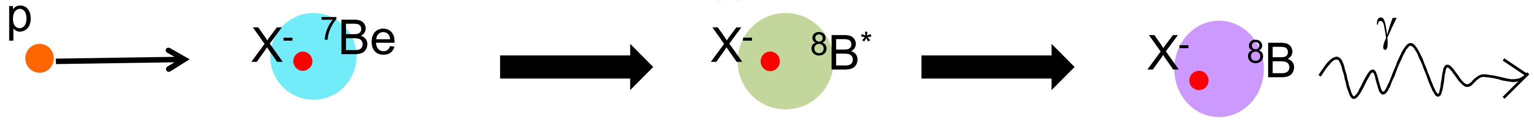
✓Recombination of X^- and nuclide (e. g. De Rujula et al. 1990, Dimopoulos et al 1990)



✓Nuclear reaction of X-nuclide (e.g. Kohri & Takayama 2007)



✓resonant reaction via exotic atoms of $^8B_X{}^*$ (Bird et al. 2007; MK et al. 2007)



➢Important reactions related to ^{7}Be abundance

$^7Be(X^-, \gamma)^7Be_X$
$^7Be_X + p \rightarrow {}^8B_X{}^* \rightarrow {}^8B_X + \gamma$

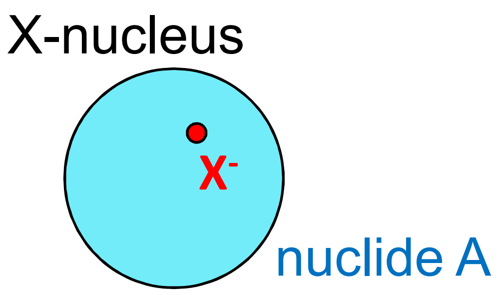
(1) Another route for ^7Be_X formation

$\boxed{^7Be(e^-, \gamma)^7Be^{3+}(X^-, e^-)^7Be_X}$

(MK, Kim, Cheoun, Kajino, Kino, PRD 88, 063514, 2013)

4. Effect of long-lived negatively charged massive particle (CHAMP) on BBN

- CHAMP X^- recombines with nuclide A, and X-nuclide (A_X) forms (Cahn & Glashow 1981)



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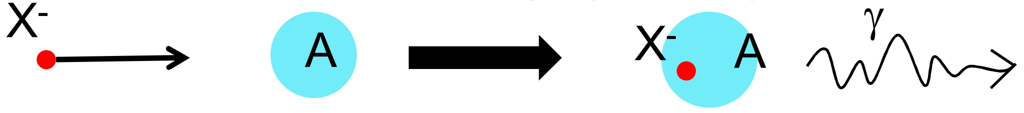

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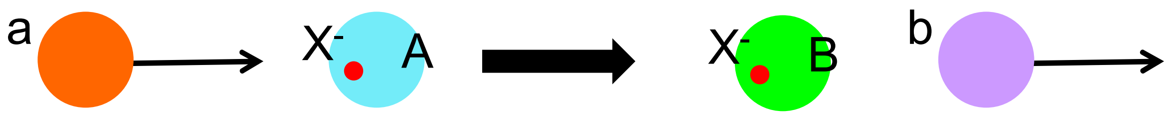

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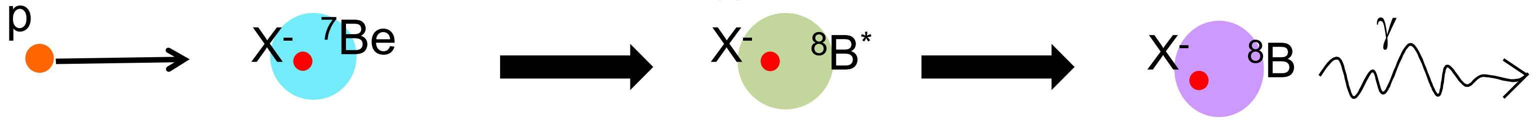


(2) extensive study of XBBN **Goal**

- Precise calculation of recombination rates
- Estimation of dependences of BBN on the mass of X^- (m_X), and the nuclear charge distribution

(1) Recombination rate via $^7\mathrm{Be}(e^-,\gamma)^7\mathrm{Be}^{3+}(X^-,e^-)^7\mathrm{Be}_X$

$$\Gamma_{\mathrm{rec}} = \underline{\frac{n_{\mathrm{Be}^{3+}}}{n_{\mathrm{Be}^{4+}}}} \left[\underline{\Gamma_{\mathrm{Be}^{3+}\to \mathrm{Be}_X^*}} \ \underline{\frac{\Gamma_{\mathrm{Be}_X^*,\mathrm{tr}}}{\Gamma_{\mathrm{Be}_X^*,\mathrm{de}} + \Gamma_{\mathrm{Be}_X^*,\mathrm{tr}}}} \right]$$

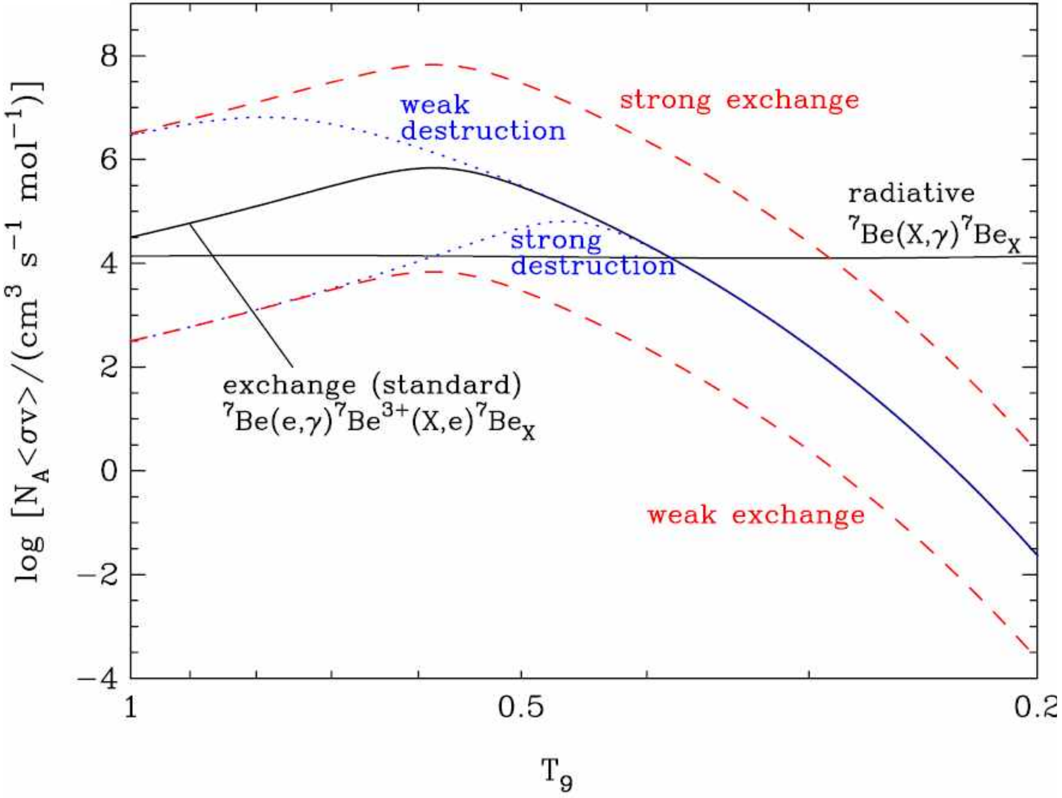
①number ratio of Be^{3+} and Be^{4+} is given by equilibrium value (Saha eq.)

②Reaction rate of ^{7}Be^{3+} via $^7\mathrm{Be}^{3+}(X^-, e^-)\,^7\mathrm{Be}_X{}^*$

$$\sigma(E) = \sigma(I(^7\mathrm{Be}^{3+}))\left[\frac{E}{I(^7\mathrm{Be}^{3+})}\right]^{-1/2} H(I(^7\mathrm{Be}^{3+}) - E)$$

$$\sigma(I(^7\mathrm{Be}^{3+})) = 10/(Z_{^7\mathrm{Be}}\alpha m_e)^2 = 1.75\times 10^7 \text{ b.}$$

MK, Kim, Cheoun, Kajino, Kino,
PRD 88, 063514, (2013)



③Probability that ^{7}Be$_X{}^*$ is converted to the GS ^{7}Be$_X$

- ➢ The dominant destruction process:
  Collisional ionization: $A_X{}^* + e^\pm \rightarrow A + X^- + e^\pm$

Cross section is assumed as

$$\sigma(E) = \sigma_{\mathrm{de}} H(E - E_{\mathrm{th}})$$

$$\sigma_{\mathrm{de}} \sim 10^{-2} \times \pi[2n^2/(Z_1 Z_2 \alpha\mu)]^2$$

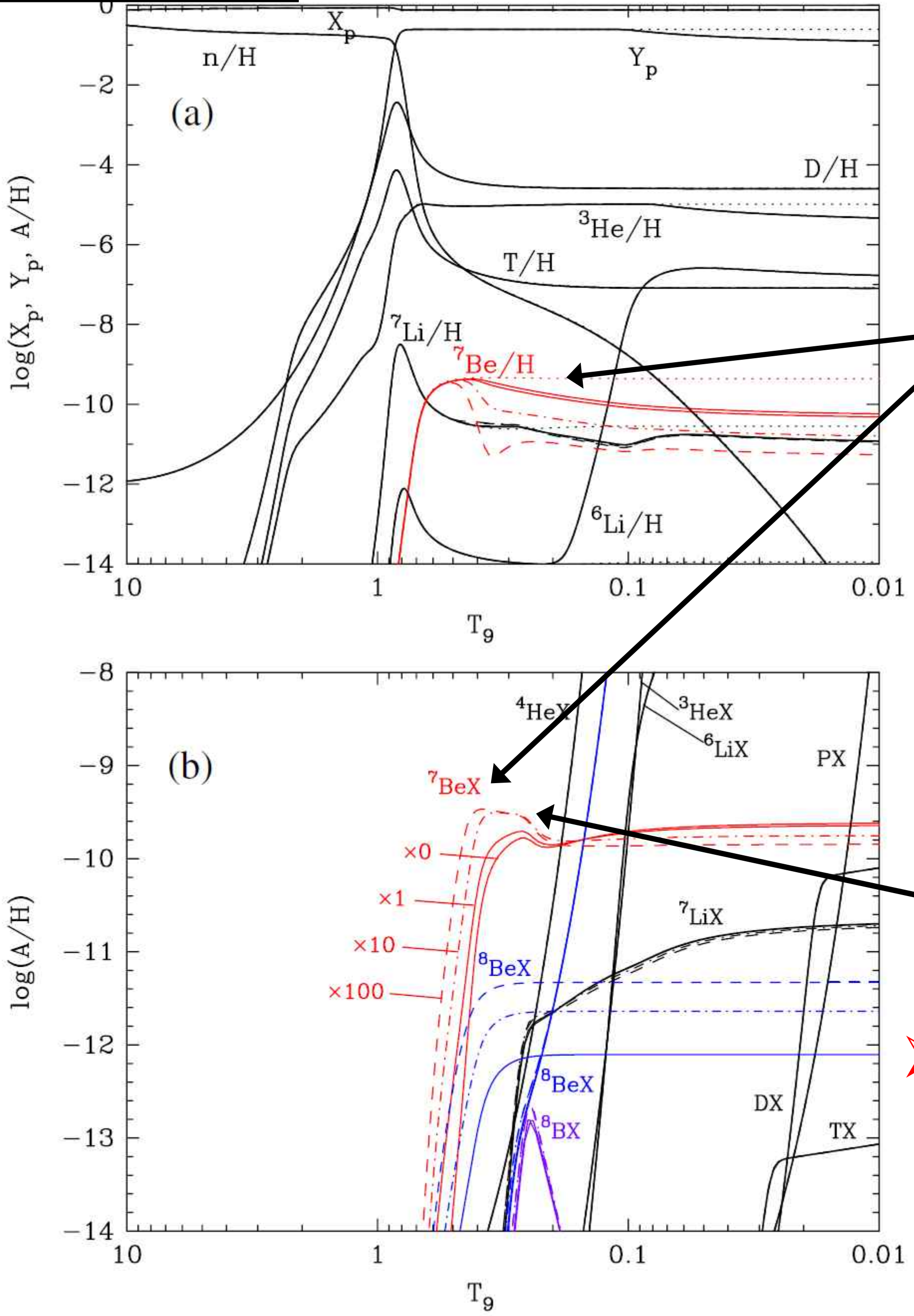
Abundance

Results

MK, Kim, Cheoun, Kajino, Kino,
PRD 88, 063514, (2013)

abundances evolution

$n_X=0.05n_b$, $m_X>>1$ GeV, $\tau_X>>200$ s,
$\eta=6.19\times10^{-10}$ (WMAP 9yr)



^{7}Be recombination

^{7}Be(X$^-$, γ)^{7}Be$_X$

^{7}Be(e$^-$, γ)^{7}Be^{3+}(X$^-$, e$^-$)^{7}Be$_X$

— $\sigma=0$

— $\sigma_s=17.5$ Mb (standard)

-·-·- $\sigma=10\sigma_s$

- - - $\sigma=100\sigma_s$

^{7}Be$_X$+p→^{8}B$_X$^{*a}→^{8}B$_X$+γ

- amount of ^{7}Be destruction depends on the rate of ^{7}Be^{3+}(X$^-$, e$^-$)^{7}Be$_X$*

Temperature $T_9=T/(10^9$ K)

(2) Extensive study on XBBN

Model

1. Binding energy of X-nuclides

[assumption]

- X^-: spin 0, charge -e, mass m_X (parameter)
- **Nuclear charge density**

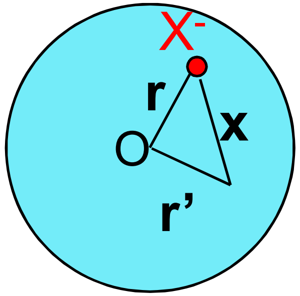
Root mean square charge radius

1) Woods-Saxon $\rho(r') = \frac{ZeC}{1+\exp[(r'-R)/a]}$

2) Gaussian $\rho(r') = Ze(\pi b)^{-3/2}\exp(-r'^2/b^2)$ $b=\sqrt{2/3}\langle r_c{}^2\rangle^{1/2}$

3) homogeneous $\rho(r') = \frac{3Ze}{4\pi r_0{}^3}H(r_0-r')$ $r_0=\sqrt{5/3}\langle r_c{}^2\rangle^{1/2}$

X-nucleus



Nucleus A

Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V(r) - E\right]\psi(\mathbf{r}) = 0$$

$$V(r) = \int_0^\infty \frac{-e\rho(r')}{x}d^3r'$$

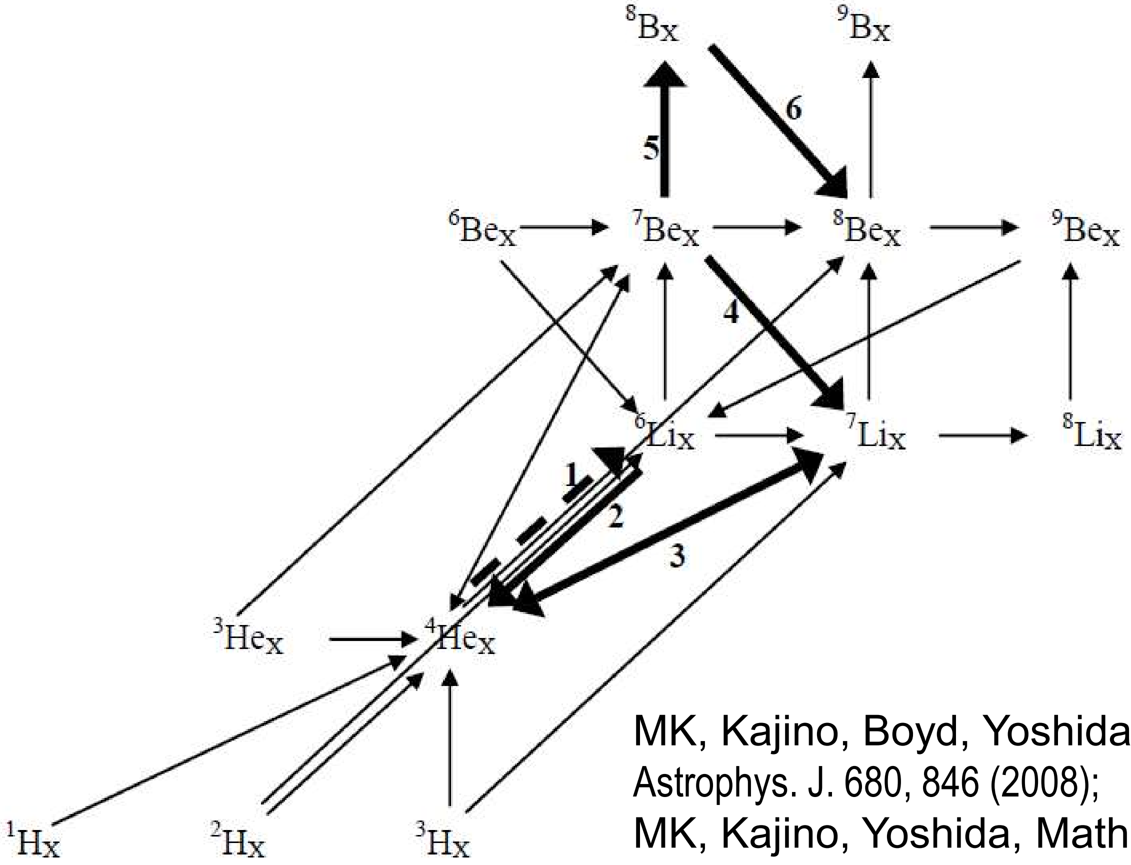
✓Binding energies and wave functions are derived with

1) variational calculation (Gaussian expansion method, c.f. Hiyama et al. 2003)

2) numerical integration (RADCAP, code by Bertulani 2003)

2. Reaction network

- Up to C_X



MK, Kajino, Boyd, Yoshida, Mathews,
Astrophys. J. 680, 846 (2008);
MK, Kajino, Yoshida, Mathews,
PRD 81, 083521 (2010)

Important reactions

1. $^4He_X(d, X^-)^6Li$
2. $^6Li_X(p, {}^3He\ \alpha)X^-$
3. $^4He_X(t, \gamma)^7Li_X$ & $^7Li_X(p, 2\alpha)X^-$
4. $^7Be_X(, X^0)^7Li$
5. $^7Be_X(p, \gamma)^8B_X$
6. $^8B_X(, e^+\nu_e)^8Be_X$

✓X^- recombination: 16
✓X nuclear reaction: 59
(including β-decay: 2)
✓X^- charge transfer: 3
✓X-decay: 11

3. Recombination cross section

- ➢Resonant and nonresonant cross sections for (^{7}Be, ^{7}Li, ^{9}Be, ^{4}He)

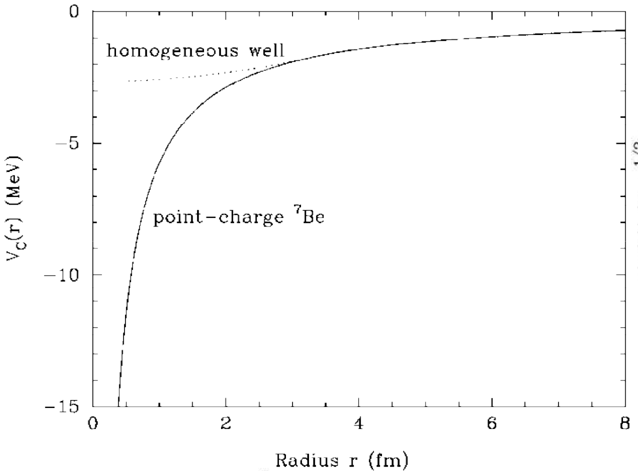
Finite size of nuclear charge

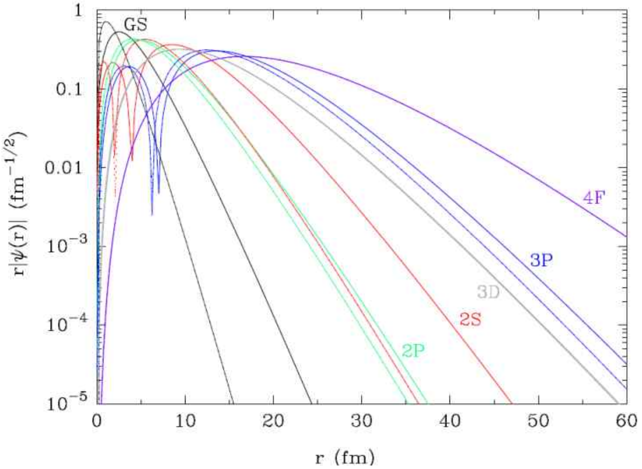
→binding energies of tightly bound states are smaller than those of point-charges

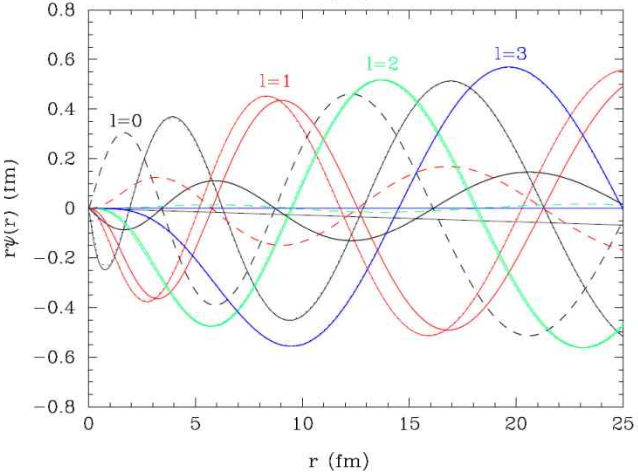
→wave functions and recombination cross sections are also different

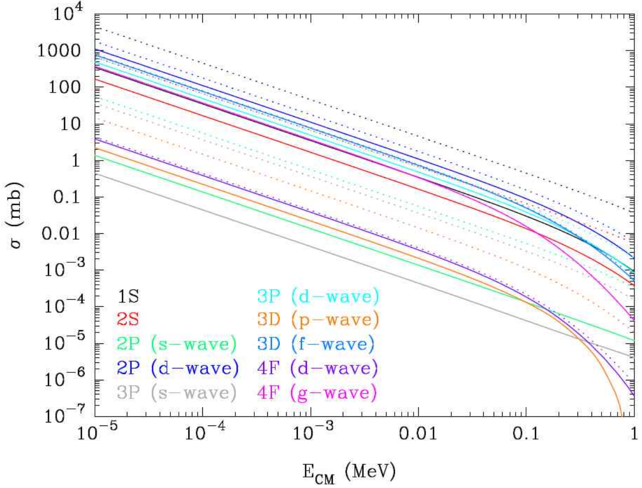
ex) $^7\mathrm{Be}_X$ (m_X=1 TeV) b.s. wfs

s. s. wfs









Nonresonant cross section

- ✓ Smaller binding energies of lower states
  →**The most important transition is**
  d-wave →2P state
  (cf. p-wave → 1S state for electronic ion case)

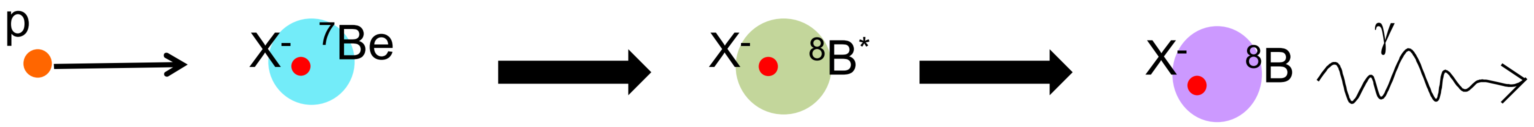
→inclusion of many transitions

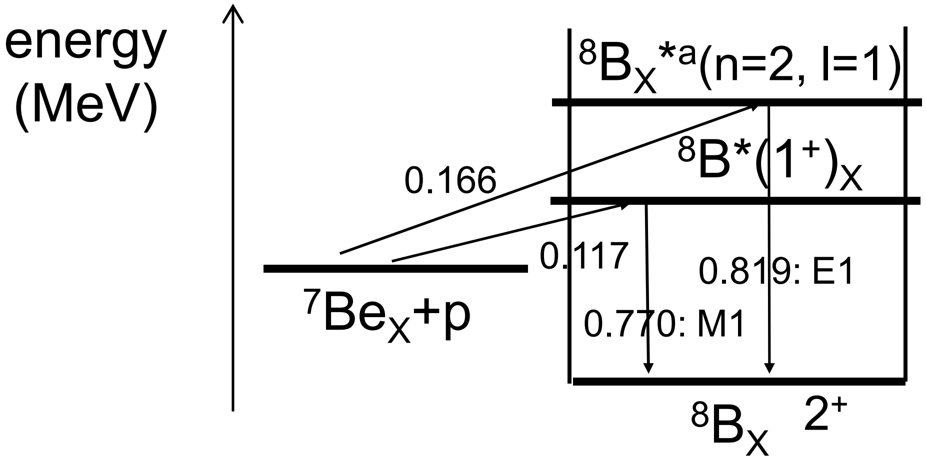
→**rate is 6 times larger than previous result**

4. Nuclear reaction rate

- Binding energies of X-nuclei → reaction Q-values, detailed balance
- Reaction rates of X-nuclei are estimated with those of normal nuclei correcting for charge number and reduced mass
- We adopt cross sections calculated with a quantum three-body model (Hamaguchi et al. 2007, Kamimura et al. 2008)

- Important resonant reaction:





atomic ES
(Bird, Koopmans, Pospelov, 2007)

atomic GS composed of nuclear ES and X^-
(MK, Kajino, Boyd, Yoshida, Mathews, PRD 2007)

$(m_X=\infty)$

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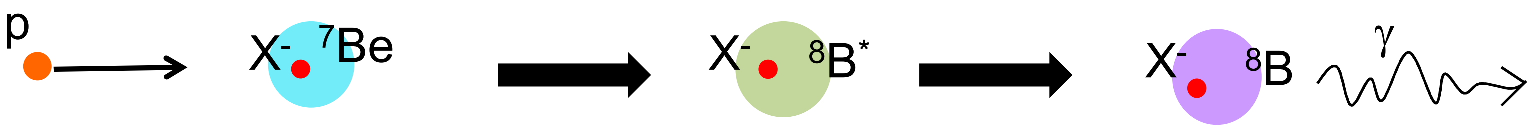
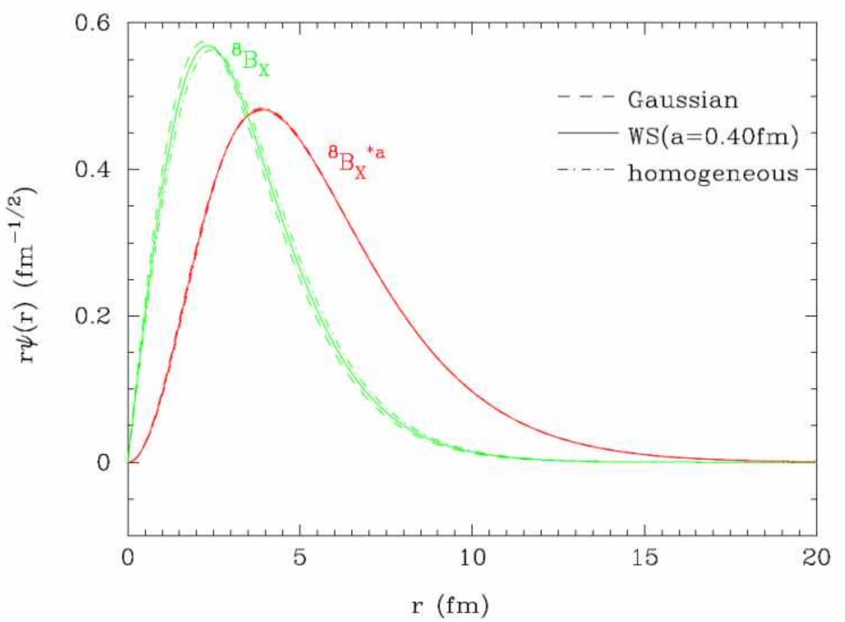
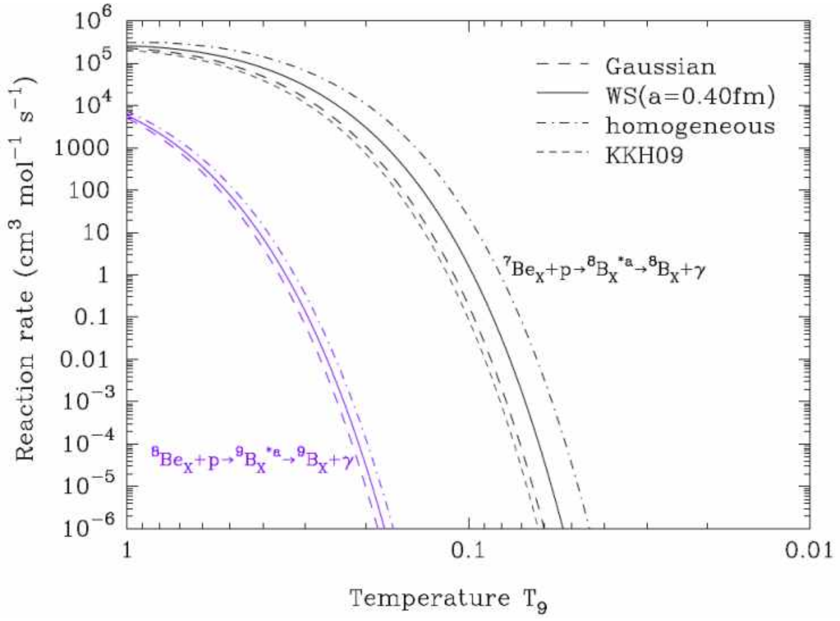


ex) $m_X=\infty$ b.s. wfs (^8B_X)

Thermal reaction rates





Abundance

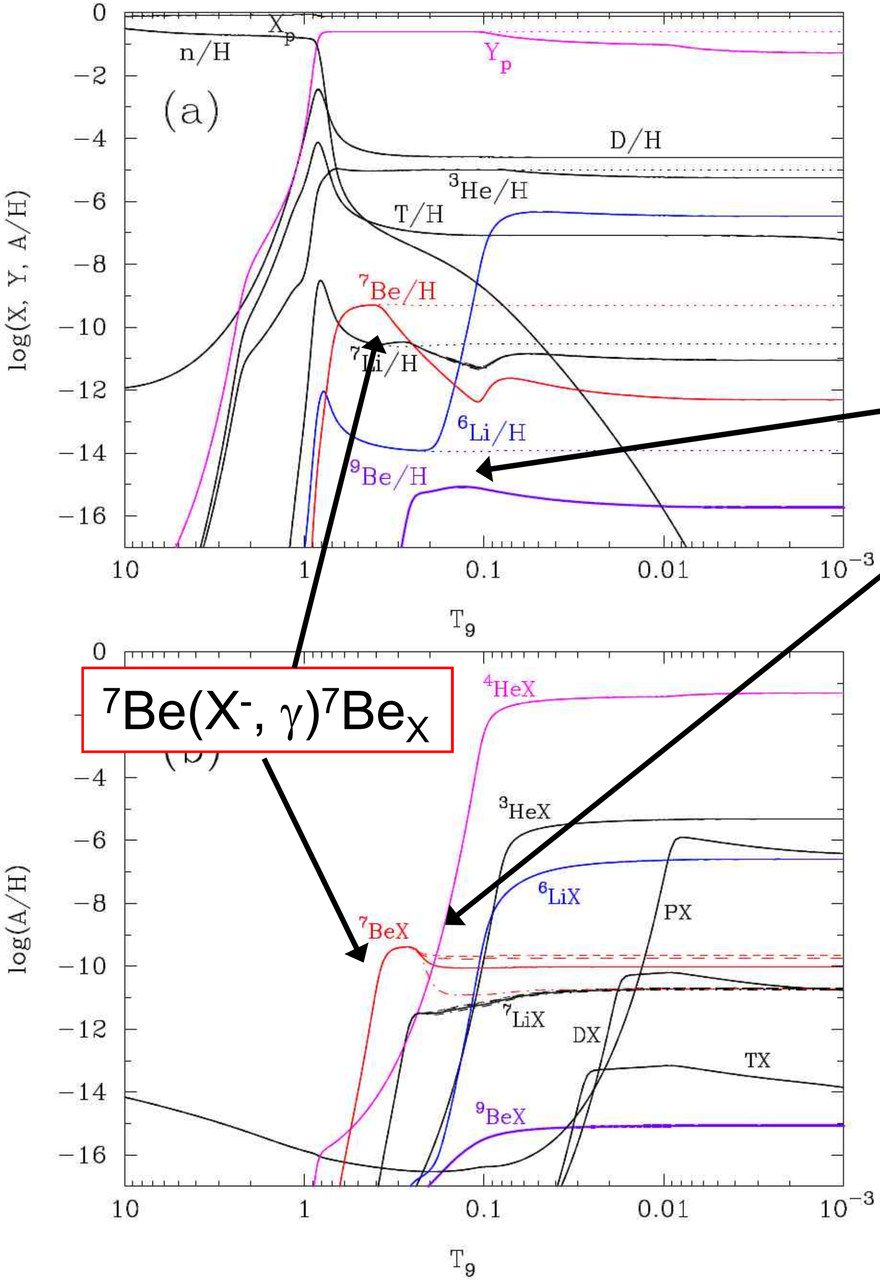
Results

abundances evolution

case 1 (m_X=1 TeV)

n_x=0.05n_b, τ_x>>200 s, η=6.19 × 10^{-10}

(WMAP 9yr)



^{7}Li$_X$(d, X$^-$)^{9}Be (this study)

^{7}Be$_X$+p→^{8}B$_X^{*a}$→^{8}B$_X$+γ (Bird et al. 2008)

^{7}Be(X$^-$, γ)^{7}Be$_X$

– – – – – – Gaussian

—————— **Woods-Saxon**

–·–·–·–·– homogeneous

- the resonance height of ^{8}B$_X^*$ is sensitive to the nuclear charge distribution

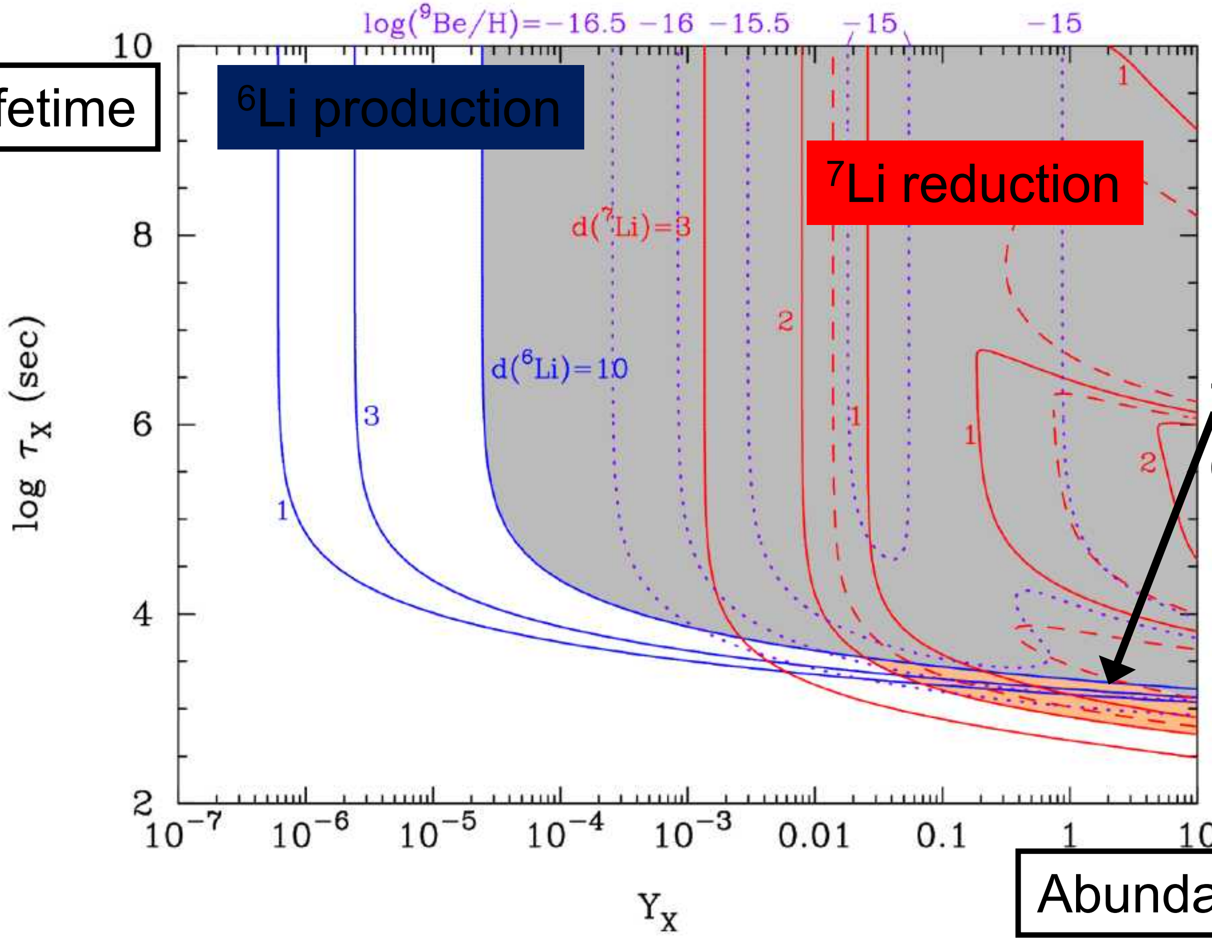
⇩

- **amount of ^{7}Be destruction significantly depends on the charge distribution**

Temperature T_9=T/(10^9 K)

Parameter search: case 1 (m_X=1 TeV)

Contours of calculated Li abundance relative to the observed value: $d(^{A}\mathrm{Li})={}^{A}\mathrm{Li}^{\mathrm{Calc}}/{}^{A}\mathrm{Li}^{\mathrm{Obs}}$



Lifetime

^{6}Li production

^{7}Li reduction

Region for ^{7}Li reduction without ^{6}Li overproduction

Abundance $Y_X=n_X/n_b$

- Realistic parameter region is shifted to **$Y_X \gtrsim 0.02$ and $\tau_X \approx (0.6\text{-}3)\times10^3$s** from the previous one: $Y_X \gtrsim 1$ and $\tau_X \approx (1\text{-}2)\times10^3$ s (MK et al. 2010)

Summary

- We calculated light-element nucleosynthesis during BBN with negatively-charged X^- particles dynamically.

- **New route of ^7Be_X formation** (^7Be exchange between $^7Be^{3+}$ and X^-)

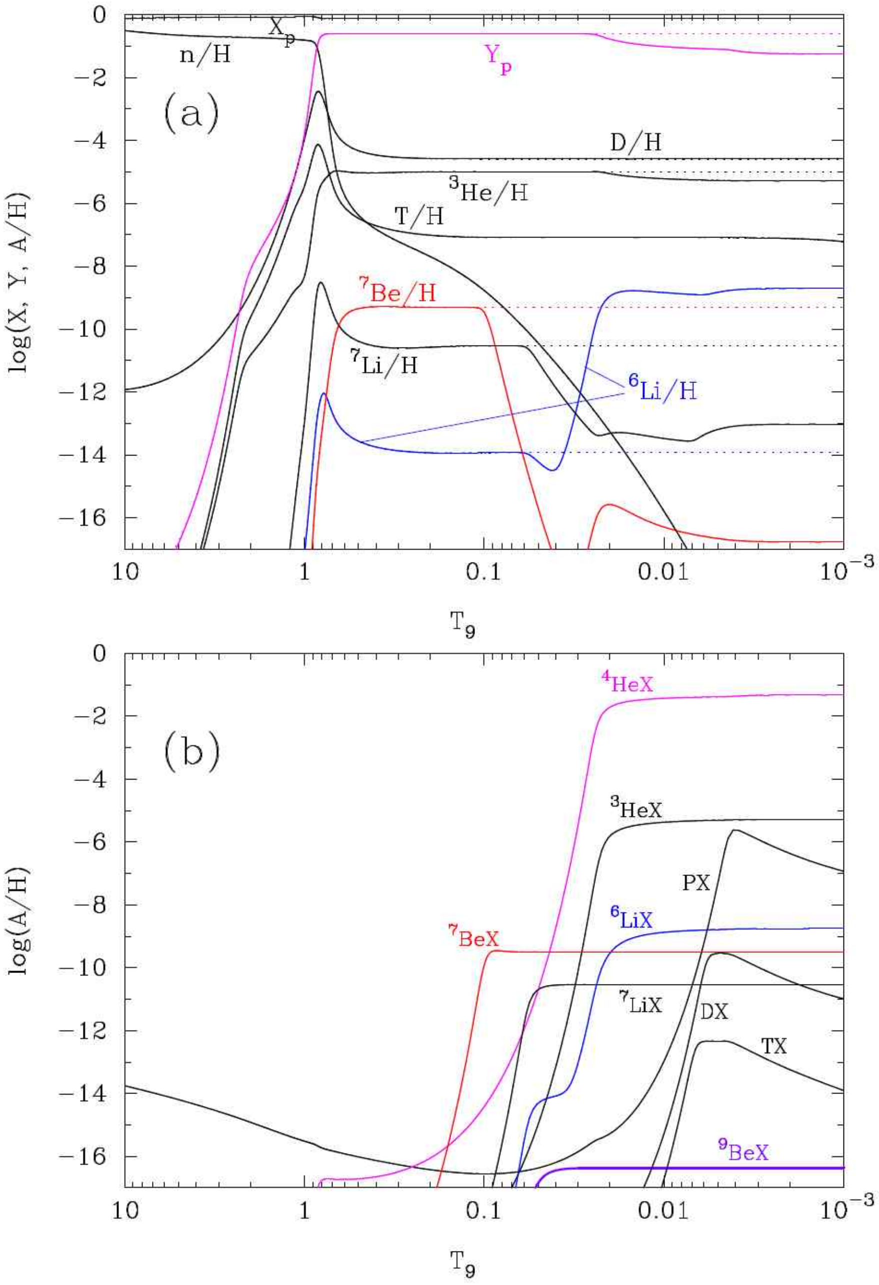
- Rates of recombination of X^- and ^7Be, ^7Li, ^9Be, ^4He are derived.

- **New ^9Be production reaction** [$^7Li_X(d, X^-)^9Be$].

- **Parameter region of ^7Li reduction is moved**
  - ✓ $Y_X \gtrsim 0.02$ and $\tau_X \approx (0.6\text{-}3)\times 10^3$s (for m_X=1 TeV)
    → required abundance of X^- particle is smaller than
    the previous estimate by more than a factor of 10

- **Resulting ^7Li abundance depends significantly on assumed nuclear charge distribution**
  - ✓ Energy levels of X-nuclides are affected by charge distribution
    → resonant reaction rates are also affected

Abundance



abundances evolution

case 2 (m_X=1 GeV)

$n_x=0.05n_b$, $\tau_x>>200$ s, $\eta=6.19\times10^{-10}$
(WMAP 9yr)

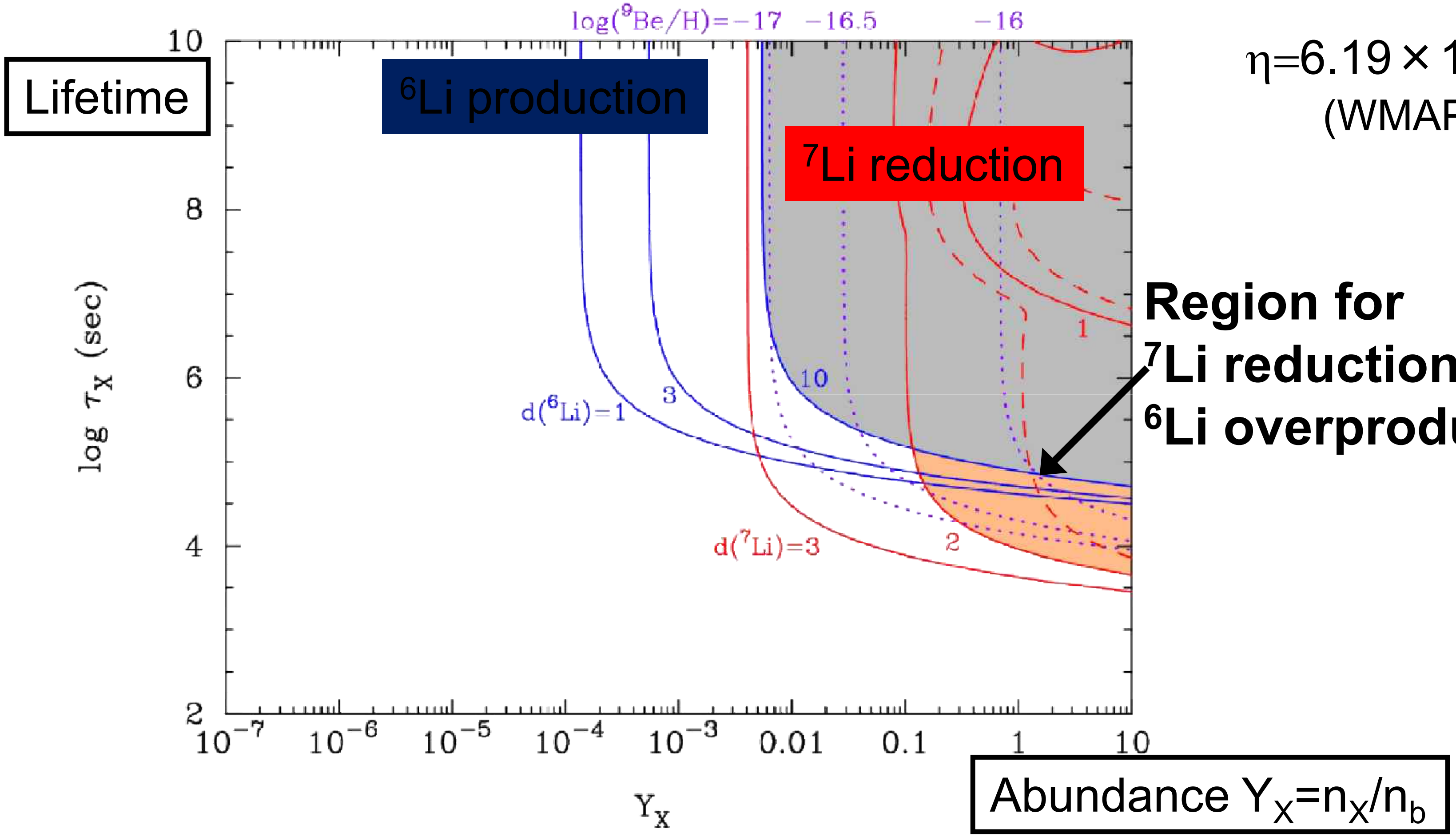
- m_X is small →binding energies are small
  →A_X forms at low temperature

- ^{7}Be destruction and ^{6}Li production are less efficient

temperature $T_9=T/(10^9$ K)

Parameter search: case 2 (m_X=1 GeV)

Contours of calculated Li abundance relative to the observed value: $d(^A\mathrm{Li})={}^A\mathrm{Li}^{\mathrm{Calc}}/{}^A\mathrm{Li}^{\mathrm{Obs}}$



Lifetime

^{6}Li production

^{7}Li reduction

Abundance $Y_X=n_X/n_b$

$\eta=6.19\times10^{-10}$
(WMAP 9yr)

Region for
^{7}Li reduction without
^{6}Li overproduction

- Parameter region for smaller mass cases locates at longer lifetime region:
  $Y_X \gtrsim 0.1$ and $\tau_X \approx 5\mathrm{x}10^3$-$2\mathrm{x}10^5$ s (for m_X=1 GeV)

Processes affecting elemental abundances

	Model	^{7}Li problem solved ?	Signatures on other nuclides ?
Existence of particle [z~10^9]	**sub-SIMP X^0**	✓	**^{6}Li, ^{9}Be**
	SIMP X^0	no	**^{9}Be and/or ^{10}B**
	CHAMP X^- *	✓	^{6}Li, **^{9}Be**
Early stars [z~O(10)]	Early cosmic ray	no	^{6}Li, ^{9}Be & ^{10,11}B

Model

1. Recombination rate via $^7\mathrm{Be}(e^-, \gamma)^7\mathrm{Be}^{3+}(X^-, e^-)^7\mathrm{Be}_X$

$$\Gamma_{\mathrm{rec}} = \underline{\frac{n_{\mathrm{Be}^{3+}}}{n_{\mathrm{Be}^{4+}}}}\left[\underline{\Gamma_{\mathrm{Be}^{3+}\to \mathrm{Be}^*_X}}\,\underline{\frac{\Gamma_{\mathrm{Be}^*_X,\mathrm{tr}}}{\Gamma_{\mathrm{Be}^*_X,\mathrm{de}} + \Gamma_{\mathrm{Be}^*_X,\mathrm{tr}}}}\right]$$

①number ratio of Be^{3+} and Be^{4+}

②Reaction rate of $^7\mathrm{Be}^{3+}$ for the charge exchange reaction $^7\mathrm{Be}^{3+}(X^-, e^-)\,^7\mathrm{Be}_X{}^*$

③Probability that $^7\mathrm{Be}_X{}^*$ excited states (ESs) transit to the $^7\mathrm{Be}_X$ ground state (GS)

$\Gamma_{\mathrm{Be}^*_X,\mathrm{tr}}$ rate for ES → GS $\Gamma_{\mathrm{Be}^*_X,\mathrm{de}}$ destruction rate

2. number ratio $\mathrm{Be}^{3+}/\mathrm{Be}^{4+}$

1) Hydrogen like-ion

$\mathrm{BE}(n)=(Z_1Z_2\alpha)^2\mu/(2n^2)$; $\langle r\rangle \sim n^2/(Z_1Z_2\alpha\mu)=Z_1Z_2\alpha/[2\mathrm{BE}(n)]$

$^7\mathrm{Be}^{3+}$ GS: BE(1) = 218 eV, $\langle r\rangle=1.98\times 10^{-9}$ cm
$^7\mathrm{Be}_X$ (n>>1) states: BE(n) = 2.78 MeV/n^2, $\langle r\rangle\sim 1.04n^2\times 10^{-13}$ cm
→$^7\mathrm{Be}_X$ (n=113) ESs has nearly the same binding energy as the $^7\mathrm{Be}^{3+}$ GS

2) $^7\mathrm{Be}^{3+}$ can be considered as an isolated ionic state ← $\langle r\rangle < l_{\mathrm{ave}}(e^{\pm})$

3) The equilibrium value

$$\frac{n_{\mathrm{Be}^{3+}}}{n_{\mathrm{Be}^{4+}}} = \left(\frac{2\pi}{m_eT}\right)^{3/2} \exp\left[\frac{I(^7\mathrm{Be}^{3+})}{T}\right] n_e \sim 2e^{-m_e/T}$$

3. Reaction rate for the reaction $^{7}\mathrm{Be}^{3+}(X^{-}, e^{-})\,^{7}\mathrm{Be}_X^{*}$

- cross section is assumed
  by analogies of protonium formation and muonic hydrogen formation

$\mathrm{H}+\bar{p}\rightarrow p\bar{p}+e^{-}$ $\mathrm{H}+\mu^{-}\rightarrow p\mu+e^{-}$

$$\sigma(E)=\sigma(I(^{7}\mathrm{Be}^{3+}))\left[\frac{E}{I(^{7}\mathrm{Be}^{3+})}\right]^{-1/2} H(I(^{7}\mathrm{Be}^{3+})-E)$$

$$\sigma(I(^{7}\mathrm{Be}^{3+})) = 10/(Z_{^{7}\mathrm{Be}}\alpha m_e)^2 = 1.75\times 10^{7}\ \mathrm{b}.$$

- binding energies of final states are similar to that of the initial state
  → $^{7}\mathrm{Be}^{3+}$ (n) is converted to $^{7}\mathrm{Be}_X^{*}$ (113n)

4. Probability that $^{7}\mathrm{Be}_X^{*}$ is converted to $^{7}\mathrm{Be}_X$

I) transition

1) Spontaneous emission: $A_X^{*}\rightarrow A_X^{*\prime}+\gamma$
2) Stimulate emission: $A_X^{*}+\gamma\rightarrow A_X^{*\prime}+2\gamma$
3) Photo-absorption: $A_X^{*\prime}+\gamma\rightarrow A_X^{*}$

II) destruction

1) Collisional ionization: $A_X^{*}+e^{\pm}\rightarrow A+X^{-}+e^{\pm}$
2) Charge exchange: $A_X^{*}+e^{-}\rightarrow A^{(Z-1)+}+X^{-}$
3) Photo-ionization: $A_X^{*}+\gamma\rightarrow A+X^{-}$

$\sigma_{des}\sim 10^{-2}\times\pi[2n^2/(Z_1Z_2\alpha\mu)]^2$

$\Gamma_{\mathrm{Be}_X^{*},\mathrm{tr}} \gtrsim \Gamma_{\mathrm{Be}_X^{*},\mathrm{de}}$ →GS $^{7}\mathrm{Be}^{3+}$ is only available path to GS $^{7}\mathrm{Be}_X$

5. Recombination rate via $^7\mathrm{Be}(e^-,\gamma)^7\mathrm{Be}^{3+}(X^-,e^-)^7\mathrm{Be}_X$

$$\Gamma_{\mathrm{rec}} = \underline{\frac{n_{\mathrm{Be}^{3+}}}{n_{\mathrm{Be}^{4+}}}}\left[\underline{\Gamma_{\mathrm{Be}^{3+}\to\mathrm{Be}_X^*}}\,\underline{\frac{\Gamma_{\mathrm{Be}_X^*,\mathrm{tr}}}{\Gamma_{\mathrm{Be}_X^*,\mathrm{de}}+\Gamma_{\mathrm{Be}_X^*,\mathrm{tr}}}}\right]$$

①number ratio of Be^{3+} and Be^{4+} is given by equilibrium value (Saha eq.)

②cross section for charge exchange reaction $^7\mathrm{Be}^{3+}(X^-,e^-)^7\mathrm{Be}_X^*$ is assumed as

$$\sigma(E)=\sigma(I(^7\mathrm{Be}^{3+}))\left[\frac{E}{I(^7\mathrm{Be}^{3+})}\right]^{-1/2}H(I(^7\mathrm{Be}^{3+})-E)$$

$$\sigma(I(^7\mathrm{Be}^{3+})) = 10/(Z_{^7\mathrm{Be}}\alpha m_e)^2 = 1.75\times10^7\ \mathrm{b}.$$

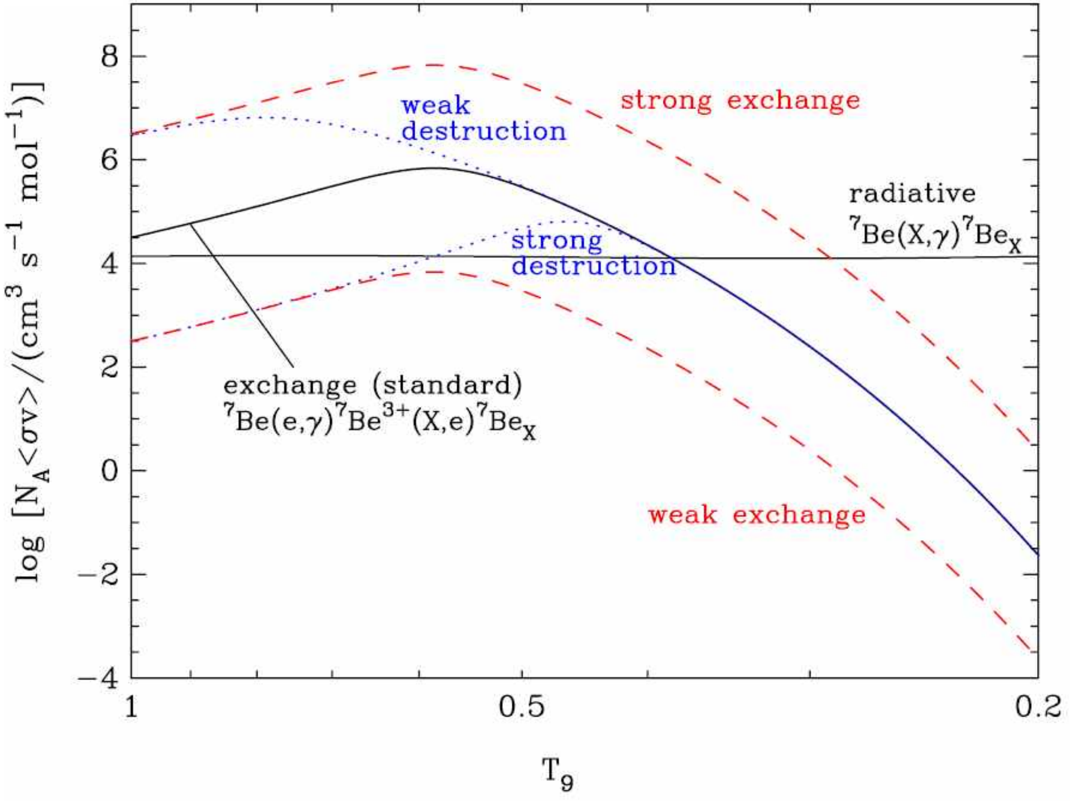
MK, Kim, Cheoun, Kajino, Kino,
PRD 88, 063514, (2013)

③Probability of the GS $^7\mathrm{Be}_X$ formation

- The dominant destruction process: ionization via $A_X^*+e^\pm$ collision

Cross section is assumed as

$$\sigma(E)=\sigma_{\mathrm{de}}H(E-E_{\mathrm{th}})$$



3. Reaction rate for the reaction $^{7}\mathrm{Be}^{3+}(X^{-}, e^{-})\,^{7}\mathrm{Be}_X^*$

- cross section is assumed
  by analogies of <u>protonium formation</u> and <u>muonic hydrogen formation</u>

$\mathrm{H}+\bar{p}\rightarrow p\bar{p}+e^{-}$ $\mathrm{H}+\mu^{-}\rightarrow p\mu+e^{-}$

$$\sigma(E)=\sigma(I(^{7}\mathrm{Be}^{3+}))\left[\frac{E}{I(^{7}\mathrm{Be}^{3+})}\right]^{-1/2} H(I(^{7}\mathrm{Be}^{3+})-E)$$

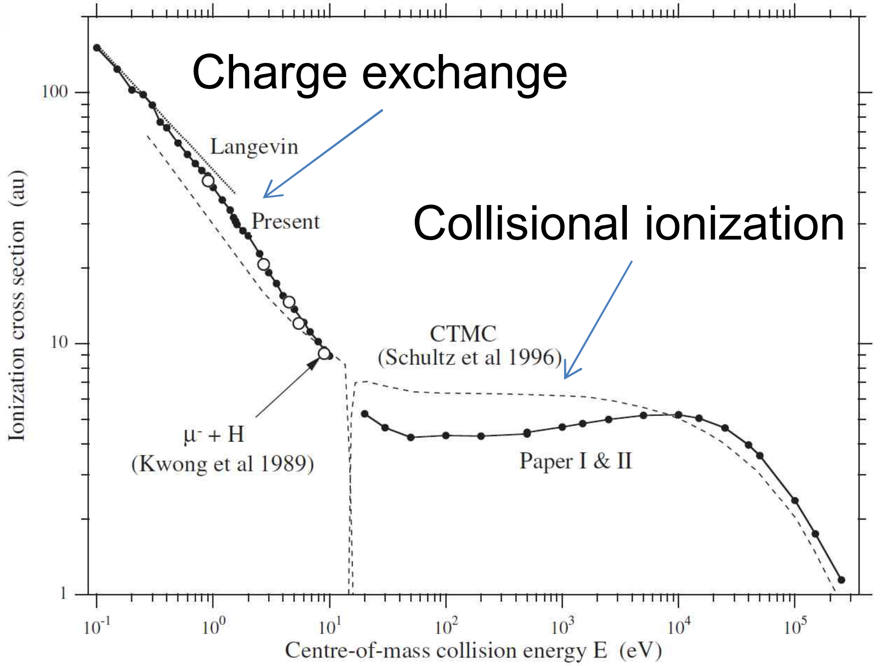
$$\sigma(I(^{7}\mathrm{Be}^{3+})) = 10/(Z_{^{7}\mathrm{Be}}\alpha m_e)^2 = 1.75\times 10^{7}\ \mathrm{b}.$$

- binding energies of final states are
  similar to that of the initial state

→$^{7}\mathrm{Be}^{3+}$ (n) is converted to $^{7}\mathrm{Be}_X^*$ (113n)

$$\begin{aligned}\Gamma_{\mathrm{Be}^{3+}\rightarrow\mathrm{Be}_X^*} &= n_X\langle\sigma v\rangle = n_b Y_X\langle\sigma v\rangle\\ &= 3.96 Y_X\times 10^{4}\ \mathrm{s}^{-1}\left(\frac{h}{0.700}\right)^{2}\left(\frac{\Omega_b}{0.0463}\right)\\ &\quad\times\left(\frac{T_9}{0.4}\right)^{3/2}\left[\frac{\sigma(I(^{7}\mathrm{Be}^{3+}))}{1.75\times 10^{7}\mathrm{b}}\right],\end{aligned}$$

Sakimoto, J. Phys. B (2001)



Charge exchange

Collisional ionization

Figure 9. Ionization cross sections in the energy range $E = 10^{-1}$–2.5×10^{5} eV calculated by the present method of direct numerical solution. The values at $E < I$ are the present results for the protonium formation (1), and the ones at $E > I$ are the results of paper I ($E \leqslant 500$ eV) and paper II ($E \geqslant 500$ eV) for the break-up ionization (2). The CTMC results are given by Schultz *et al* (1996). The μ^{-}+ H results are the capture cross section σ_{cap} calculated by the semiclassical method (Kwong *et al* 1989, Cohen 1998).

4. Probability that ${}^{7}\mathrm{Be}_X^*$ is converted to ${}^{7}\mathrm{Be}_X$

I) transition

1) Spontaneous emission: $A_X^* \rightarrow A_X^{*'} + \gamma$
2) Stimulate emission: $A_X^* + \gamma \rightarrow A_X^{*'} + 2\gamma$
3) Photo-absorption: $A_X^{*'} + \gamma \rightarrow A_X^*$

$$\Gamma^{\gamma}_{u,\mathrm{st}} = \sum_l B_{ul} B_{\nu_{ul}}(T) = \sum_l \frac{2Z_A^2 \alpha}{m_A} \frac{g_l}{g_u} f_{lu} \frac{E_{ul}^2}{\exp(E_{ul}/T) - 1}$$

$$\sim \frac{2N_l Z_A^2 \alpha}{m_A} T \overline{E_{ul}}$$

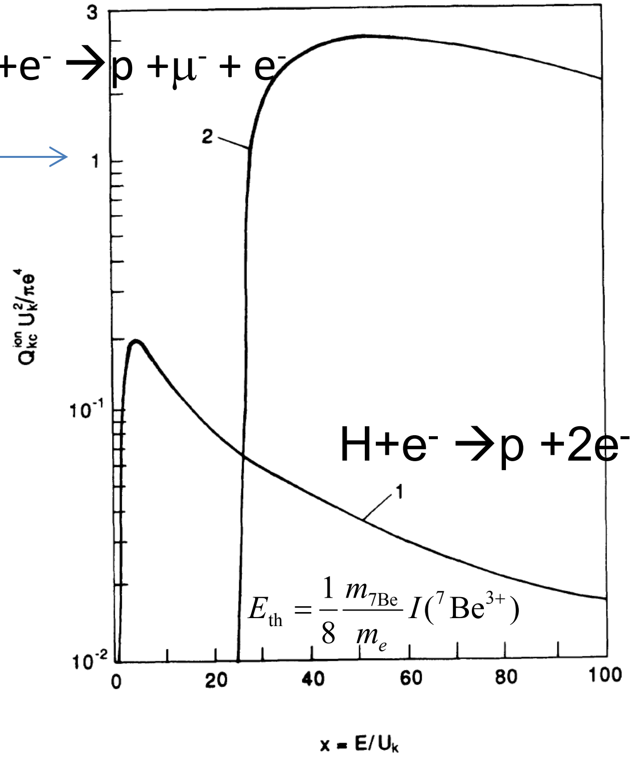
$$= 9.21 \times 10^{13}\ \mathrm{s}^{-1} \left(\frac{N_l}{226}\right)\left(\frac{\overline{E_{ul}}}{218\ \mathrm{eV}}\right)\left(\frac{T_9}{0.4}\right)\left(\frac{Z_A}{4}\right)^2 \left(\frac{m_A}{6.53\ \mathrm{GeV}}\right)^{-1}$$

$$\overline{E_{ul}} \equiv \textstyle\sum_l [(g_l/g_u) f_{lu} E_{ul}]/N_l$$

II) destruction

1) Collisional ionization: $A_X^* + e^{\pm} \rightarrow A + X^- + e^{\pm}$
2) Charge exchange: $A_X^* + e^- \rightarrow A^{(Z-1)+} + X^-$
3) Photo-ionization: $A_X^* + \gamma \rightarrow A + X^-$

Kunc, Phys. Rev. A (1994)



$p\mu + e^- \rightarrow p + \mu^- + e^-$

$4 \times (\text{Atomic radius})^2$
$= \pi[2n^2/(Z_1 Z_2 \alpha\mu)]^2$

$H + e^- \rightarrow p + 2e^-$

$$E_{\mathrm{th}} = \frac{1}{8}\frac{m_{7\mathrm{Be}}}{m_e} I({}^{7}\mathrm{Be}^{3+})$$

$$\Gamma_{\mathrm{Be}^*_X,\mathrm{de}} = (n_{e^-} + n_{e^+})\langle \sigma v \rangle$$

$$= 2.79 \times 10^{10}\ \mathrm{s}^{-1} \left(\frac{T_9}{0.4}\right)^2 \frac{\mathrm{e}^{-m_e/T}}{2.97 \times 10^{-7}}$$

$$\times \frac{(1 + E_{\mathrm{th}}/T)}{11.1} \frac{\mathrm{e}^{-E_{\mathrm{th}}/T}}{4.13 \times 10^{-5}} \left(\frac{n}{113}\right)^4, \qquad (29)$$

GS ${}^{7}\mathrm{Be}^{3+}$ is only available path

4. Probability that $^7\mathrm{Be}_X^*$ is converted to $^7\mathrm{Be}_X$

I) transition

1) Spontaneous emission: $A_X^* \rightarrow A_X^{*\prime} + \gamma$
2) Stimulate emission: $A_X^* + \gamma \rightarrow A_X^{*\prime} + 2\gamma$
3) Photo-absorption: $A_X^{*\prime} + \gamma \rightarrow A_X^*$

$$\Gamma^{\gamma}_{u,\mathrm{st}} = \sum_l B_{ul} B_{\nu_{ul}}(T) = \sum_l \frac{2Z_A^2\alpha}{m_A}\frac{g_l}{g_u} f_{lu} \frac{E_{ul}^2}{\exp(E_{ul}/T)-1}$$

$$\sim \frac{2N_l Z_A^2\alpha}{m_A} T\overline{E_{ul}}$$

$$= 9.21\times 10^{13}\ \mathrm{s}^{-1}\left(\frac{N_l}{226}\right)\left(\frac{\overline{E_{ul}}}{218\ \mathrm{eV}}\right)\left(\frac{T_9}{0.4}\right)\left(\frac{Z_A}{4}\right)^2\left(\frac{m_A}{6.53\ \mathrm{GeV}}\right)^{-1}$$

$$\overline{E_{ul}} \equiv \textstyle\sum_l[(g_l/g_u)f_{lu}E_{ul}]/N_l$$

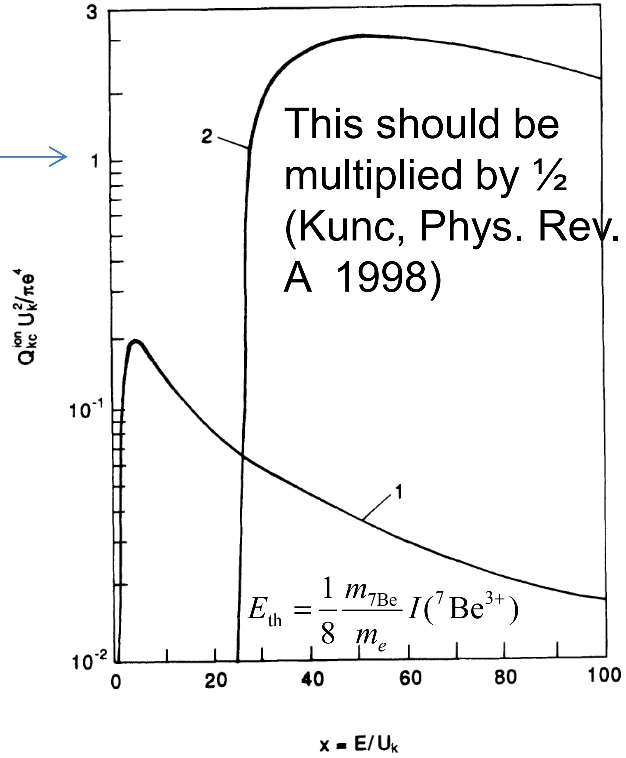
II) destruction

1) Collisional ionization: $A_X^* + e^{\pm} \rightarrow A + X^- + e^{\pm}$
2) Charge exchange: $A_X^* + e^- \rightarrow A^{(Z-1)+} + X^-$
3) Photo-ionization: $A_X^* + \gamma \rightarrow A + X^-$

4 × (Atomic radius)^2
$=\pi[2n^2/(Z_1Z_2\alpha\mu)]^2$

$$\Gamma_{\mathrm{Be}^*_X,\mathrm{de}} = (n_{e^-} + n_{e^+})\langle\sigma v\rangle$$
$$= 2.79\times 10^{10}\ \mathrm{s}^{-1}\left(\frac{T_9}{0.4}\right)^2 \frac{\mathrm{e}^{-m_e/T}}{2.97\times 10^{-7}}$$
$$\times \frac{(1+E_{\mathrm{th}}/T)}{11.1}\frac{\mathrm{e}^{-E_{\mathrm{th}}/T}}{4.13\times 10^{-5}}\left(\frac{n}{113}\right)^4, \qquad (29)$$

Kunc, Phys. Rev. A (1994)



This should be multiplied by ½ (Kunc, Phys. Rev. A 1998)

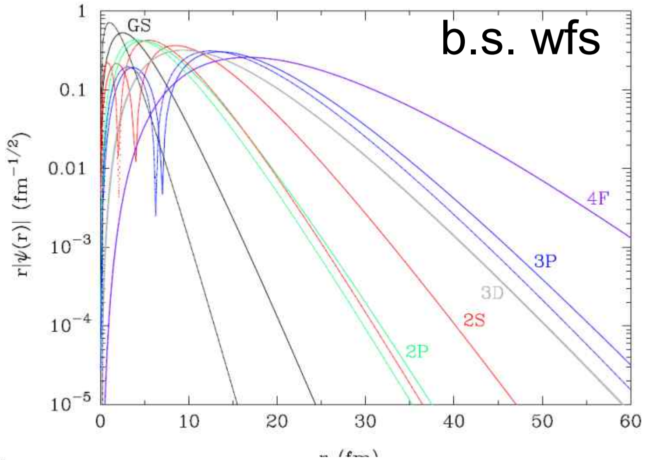
$$E_{\mathrm{th}} = \frac{1}{8}\frac{m_{7\mathrm{Be}}}{m_e} I(^7\mathrm{Be}^{3+})$$

GS $^7\mathrm{Be}^{3+}$ is only available path
←ESs $^7\mathrm{Be}^{3+*}$ are converted to highly ESs $^7\mathrm{Be}_X^*$ ($n \gtrsim 226$)
which are predominantly destroyed and
do not transit to $^7\mathrm{Be}_X$

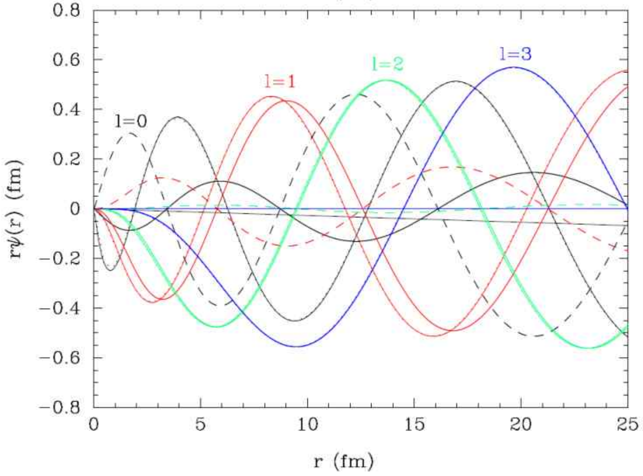
3. Recombination cross sections

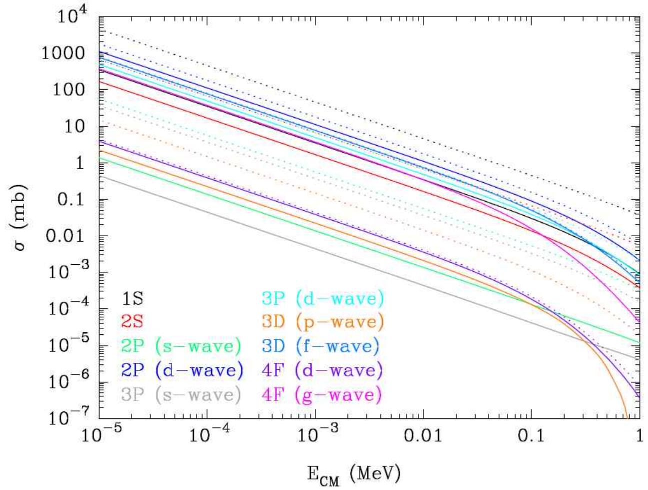
- ➢Resonant and nonresonant cross sections for (^{7}Be, ^{7}Li, ^{9}Be, ^{4}He)

ex)
^{7}Be$_X$ (m_X=1 TeV)



cont. wfs





Nonresonant cross section

- ✓ Finite size of nuclear charge
  →difference from point charge

- ➢cross sections for other nuclei are approximately given by those for the point-charge case into ground states.

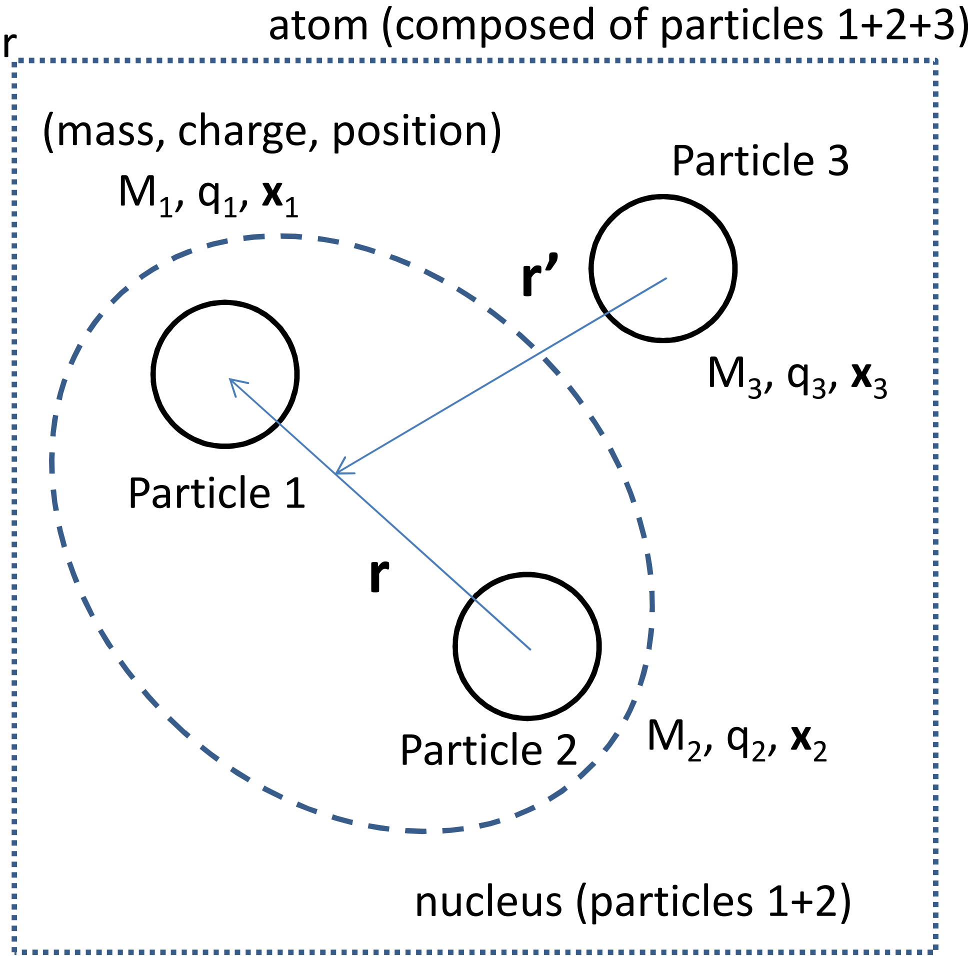
$$\sigma \approx \frac{2^9 \pi^2 e_1^2}{3\exp(4)} \frac{E_{\mathrm{bin}}}{\mu^3 v^2}$$

Transition from bound states of the first nuclear excited state of ^{7}Be* and heavy exotic particle X^{-} to those of the ground state ^{7}Be and X^{-}

→$^{7}\mathrm{Be}=\alpha+{}^{3}\mathrm{He}$: two-body bound state
α, ^{3}He, X^{-} are called 1, 2, 3, respectively.
Consider the situation show in the figure.

- Assume that the atom can be described as the sum of two wave functions for
  1) nucleus (1+2 two-body system), and
  2) atom ([1+2]+3 two-body system)
  …Eq. (A. 2)

- The electric dipole (E1) moment is written with Jacobi coordinates



$$\mathbf{d} = \sum_{i=1}^{3} q_i \mathbf{x}_i = \frac{(q_1+q_2)M_3 - q_3(M_1+M_2)}{M_1+M_2+M_3}\mathbf{r}' + \frac{M_2 q_1 - M_1 q_3}{M_1+M_2}\mathbf{r}$$

- Matrix element for the E1 transition is derived as integration over **r** and **r'** of E1 moment multiplied by wave functions of initial and final states. The E1 operator is the sum of a term of **r** and **r'**. By orthogonalities of wave functions, matrix element is zero for a transition to a final state whose nuclear and atomic states are different from those of the initial state

- ex) initial state $\boxed{\Psi^{\mathrm{n}m_1}_{j_1}(\boldsymbol{r})}\boxed{\Psi^{\mathrm{a}}{}_{n_\mathrm{i}l_\mathrm{i}m_\mathrm{i}}(\boldsymbol{r}')}$

  final state $\boxed{\Psi^{\mathrm{n}m_2}_{j_2}(\boldsymbol{r})}\boxed{\Psi^{\mathrm{a}}{}_{n_\mathrm{f}l_\mathrm{f}m_\mathrm{f}}(\boldsymbol{r}')}$

atomic w.f.

nuclear w.f.

Matrix element for the transition is

$$\langle \mathrm{f} | O(\mathrm{E1},\mu) | \mathrm{i} \rangle = \int d\boldsymbol{r} \int d\boldsymbol{r}'$$

$$\times \Psi^{\mathrm{n}m_2 *}_{j_2}(\boldsymbol{r})\Psi^{\mathrm{a}*}{}_{n_\mathrm{f}l_\mathrm{f}m_\mathrm{f}}(\boldsymbol{r}') \left[\underline{q_{r'} r' Y_{1\mu}(\hat{r'})} + \underline{q_r r Y_{1\mu}(\hat{r})}\right] \Psi^{\mathrm{n}m_1}_{j_1}(\boldsymbol{r})\Psi^{\mathrm{a}}{}_{n_\mathrm{i}l_\mathrm{i}m_\mathrm{i}}(\boldsymbol{r}')$$

r -independent→out of **r**-integration **r'**-independent→out of **r'**-integration

$$\langle \mathrm{f} | O(\mathrm{E1},\mu) | \mathrm{i} \rangle = \underline{\langle \Psi^{\mathrm{n}m_2}_{j_2} \mid \Psi^{\mathrm{n}m_1}_{j_1} \rangle} \int d\boldsymbol{r}' \Psi^{\mathrm{a}*}{}_{n_\mathrm{f}l_\mathrm{f}m_\mathrm{f}}(\boldsymbol{r}') \left[q_{r'} r' Y_{1\mu}(\hat{r'})\right] \Psi^{\mathrm{a}}{}_{n_\mathrm{i}l_\mathrm{i}m_\mathrm{i}}(\boldsymbol{r}')$$

Overlap of nuclear w.fs.= { 1 (if the initial and final nuclear states are identical)
0 (otherwise)

$$+ \underline{\langle \Psi^{\mathrm{a}}{}_{n_\mathrm{f}l_\mathrm{f}m_\mathrm{f}} \mid \Psi^{\mathrm{a}}{}_{n_\mathrm{i}l_\mathrm{i}m_\mathrm{i}} \rangle} \int d\boldsymbol{r} \Psi^{\mathrm{n}m_2 *}_{j_2}(\boldsymbol{r}) \left[q_r r Y_{1\mu}(\hat{r})\right] \Psi^{\mathrm{n}m_1}_{j_1}(\boldsymbol{r})$$

Overlap of atomic w.fs.= { 1 (if the initial and final atomic states are identical)
0 (otherwise)

- Then, E1 matrix element is zero for a transition to a final state whose atomic and nuclear states are different from those of the initial state.

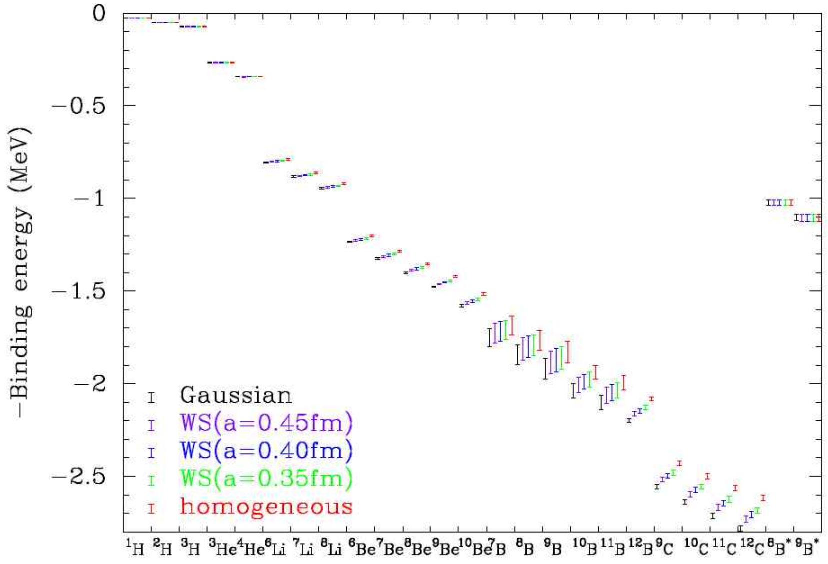
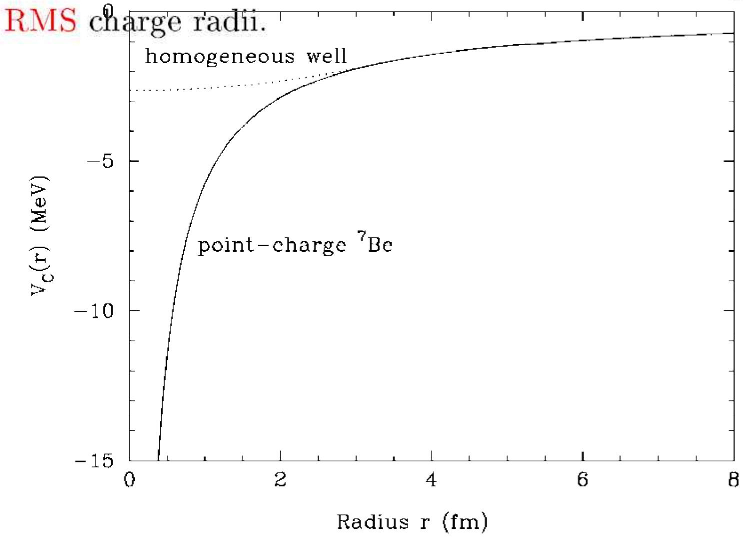


FIG. 1: Binding energies of nuclei and X^- particle with m_X = 100 TeV for different charge distributions of Gaussian (black lines), Woods-Saxon type with diffuseness parameters a = 0.45 fm (purple lines), 0.40 fm (blue lines), and 0.35 fm (green lines), and homogeneous well (red lines). Error bars indicate uncertainties determined from uncertainties in experimental RMS charge radii.



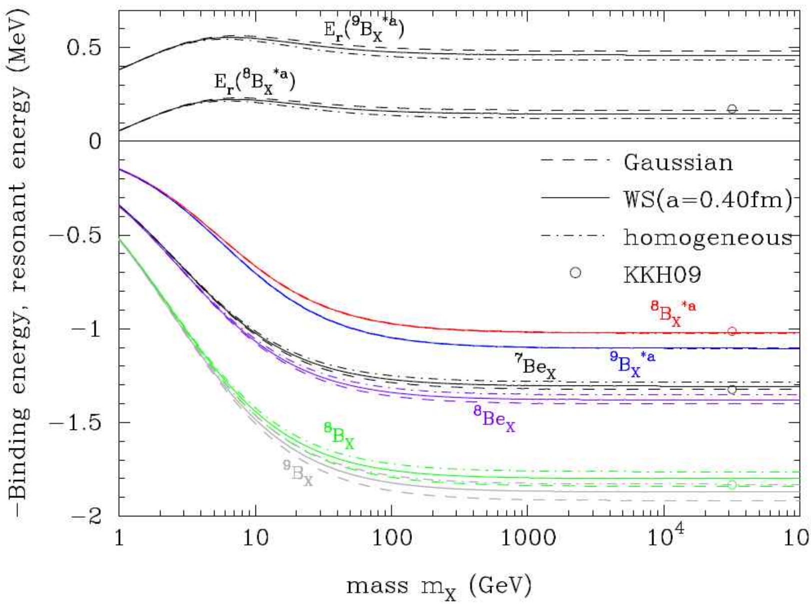
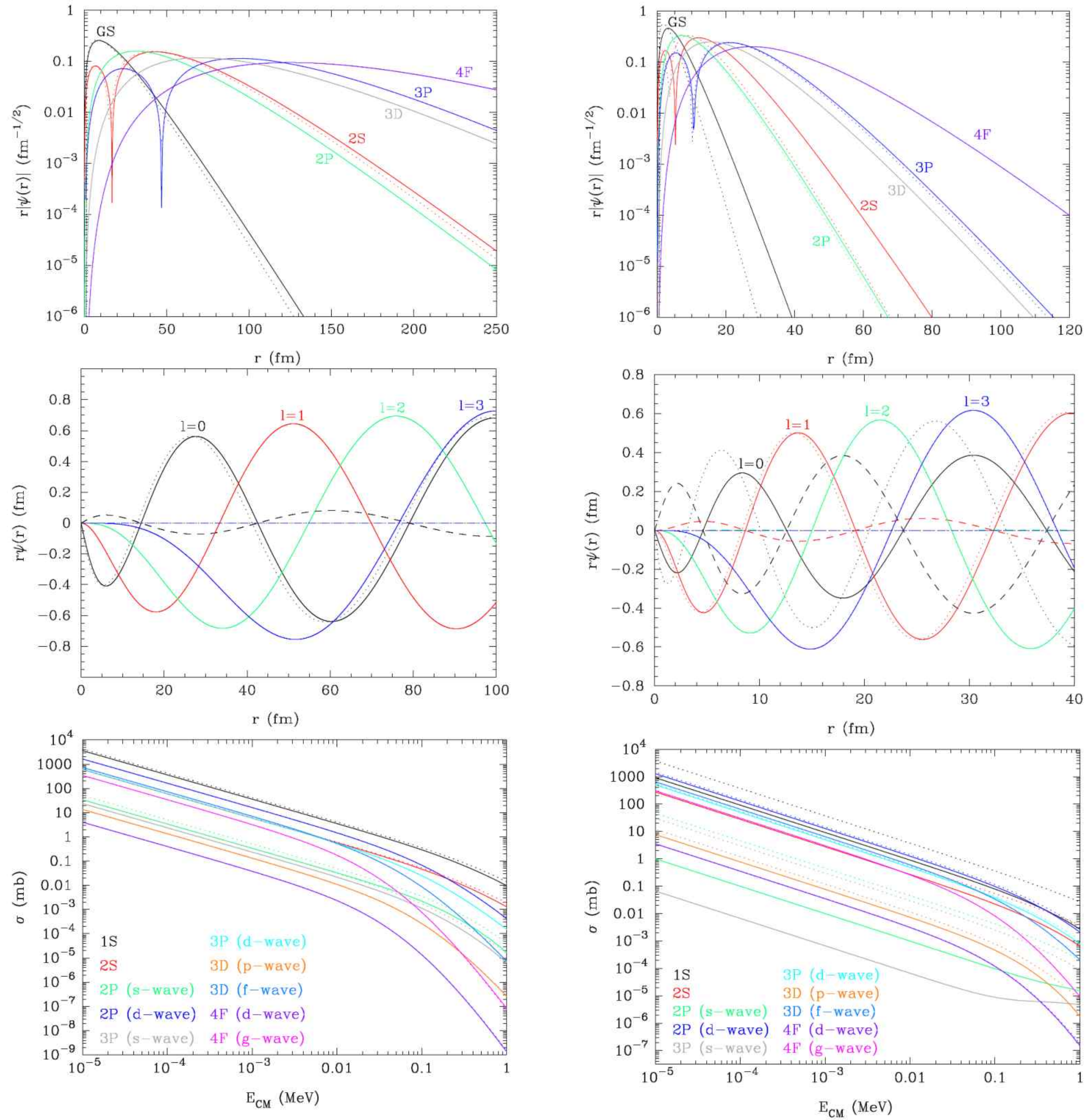


FIG. 2: Binding energies and resonance energies as a function of m_X. The upper black lines show resonance energies in the reactions $^7\mathrm{Be}_X(p, \gamma)^8\mathrm{B}_X$ and $^8\mathrm{Be}_X(p, \gamma)^9\mathrm{B}_X$. The lower lines show binding energies of $^7\mathrm{Be}_X$ (black lines), $^8\mathrm{Be}_X$ (purple lines), $^8\mathrm{B}_X$ (green lines), $^9\mathrm{B}_X$ (gray lines), and the first atomic excited states $^8\mathrm{B}_X^{*a}$ (red lines) and $^9\mathrm{B}_X^{*a}$ (blue lines). Results for different nuclear charge distributions of Gaussian (dashed lines), Woods-Saxon type with diffuseness parameter a = 0.40 fm (solid lines), and homogeneous well (dot-dashed lines) are drawn. Open circles show energy heights derived by the quantum many-body calculation for $m_X = \infty$ [39].

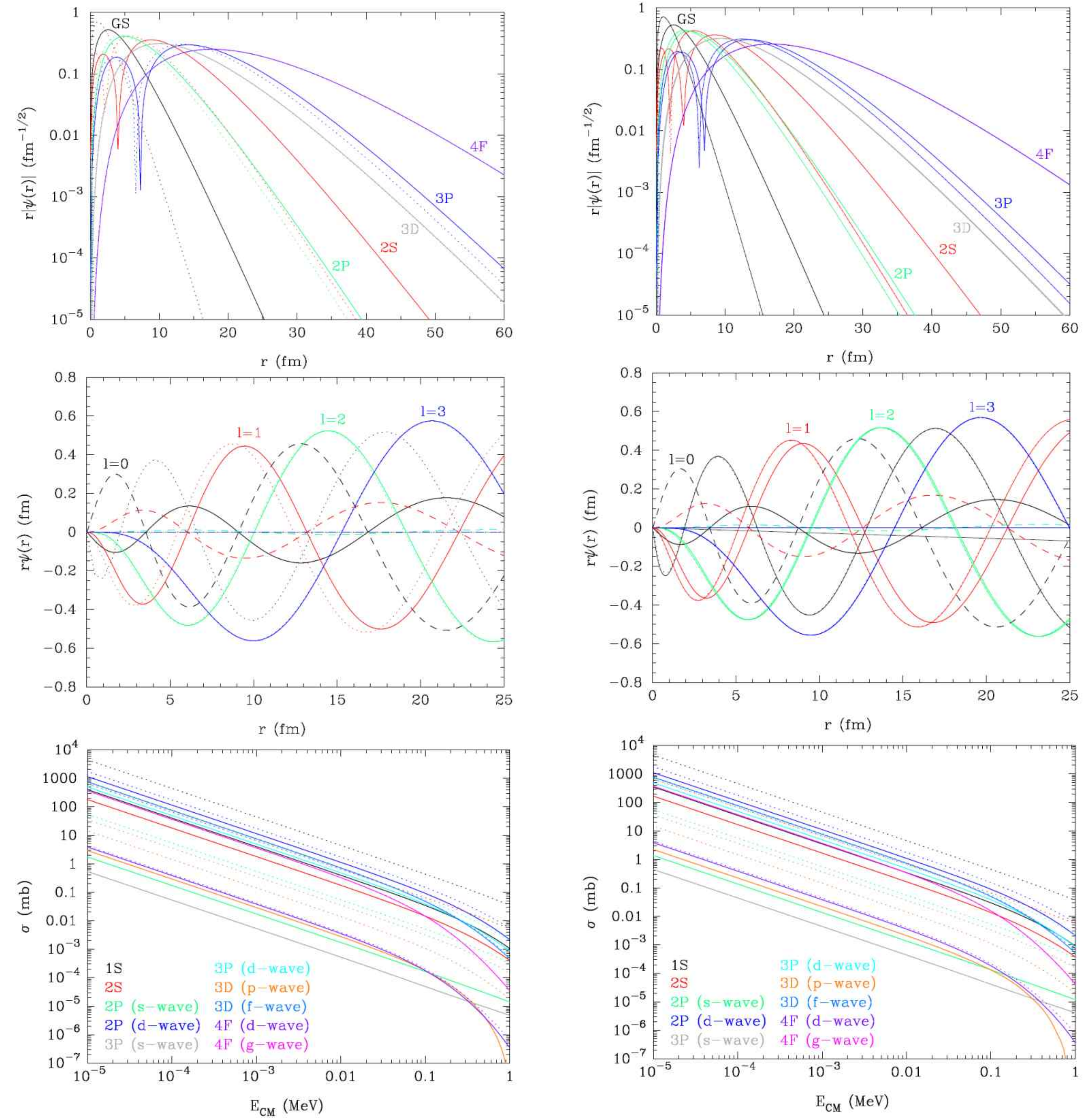
^7Be_X recombination

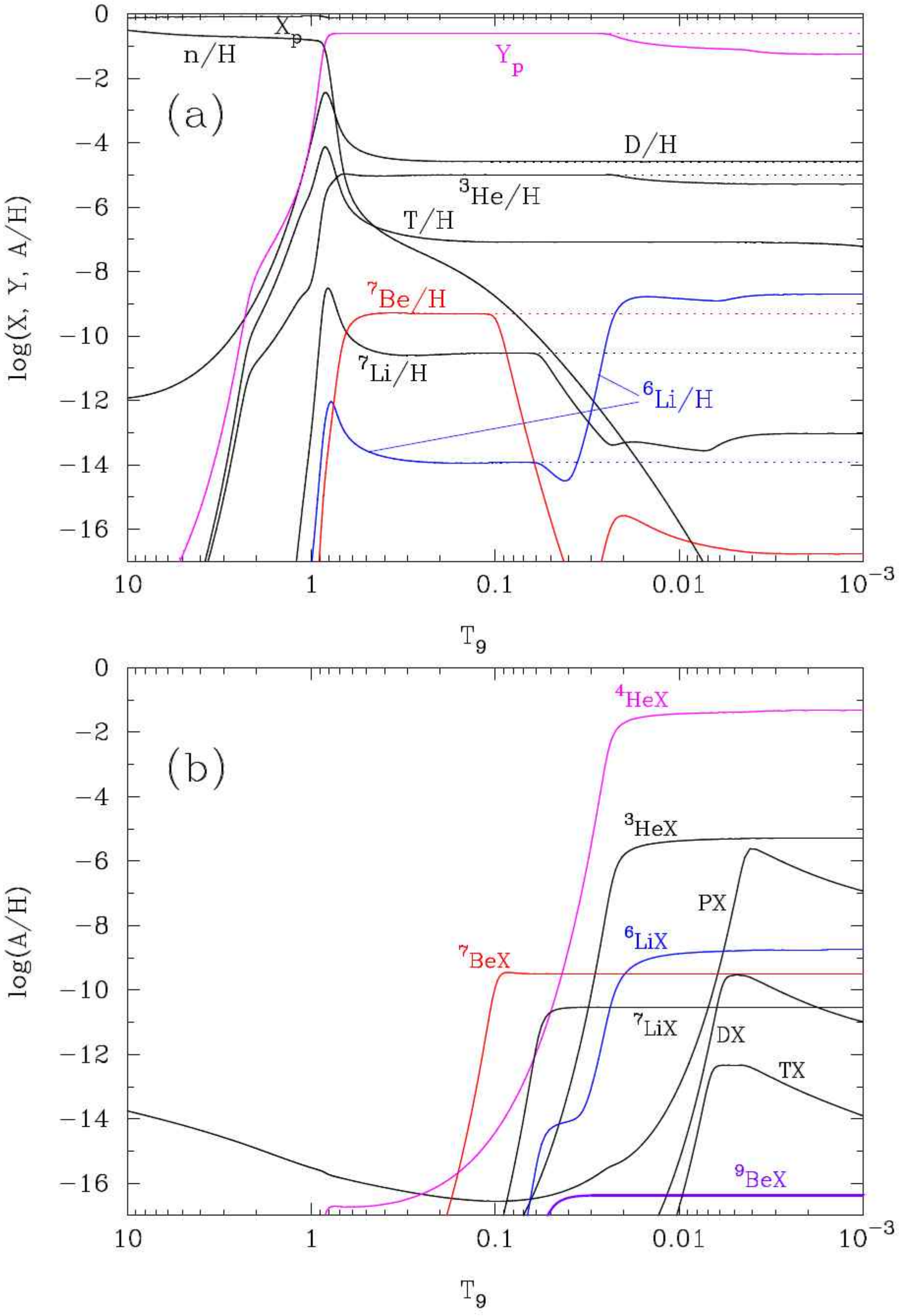
m_X=1 GeV,
10 GeV



^7Be_X recombination

m_X=100 GeV,
1000 GeV




(a)
X_p
n/H
Y_p
D/H
³He/H
T/H
⁷Be/H
⁷Li/H
⁶Li/H
log(X, Y, A/H)
0
−2
−4
−6
−8
−10
−12
−14
−16
10
1
0.1
0.01
10⁻³
T₉
(b)
⁴HeX
³HeX
PX
⁶LiX
⁷BeX
⁷LiX
DX
TX
⁹BeX
log(A/H)
0
−2
−4
−6
−8
−10
−12
−14
−16
10
1
0.1
0.01
10⁻³
T₉

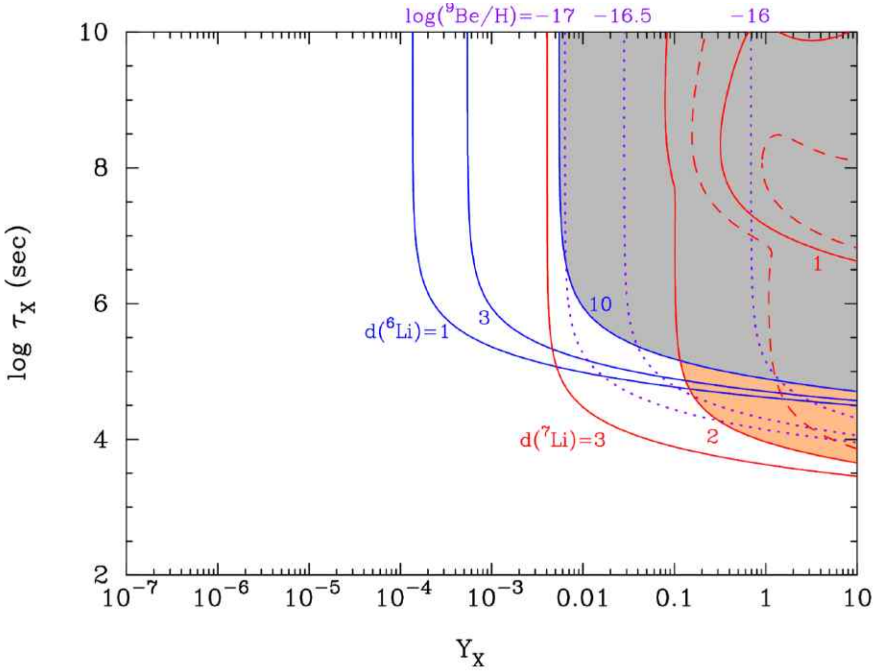


FIG. 23: Contours of constant lithium abundances relative to observed values in MPSs, i.e., $d(^6\mathrm{Li})={}^6\mathrm{Li}^{\mathrm{Cal}}/{}^6\mathrm{Li}^{\mathrm{Obs}}$ (blue lines) and $d(^7\mathrm{Li})={}^7\mathrm{Li}^{\mathrm{Cal}}/{}^7\mathrm{Li}^{\mathrm{Obs}}$ (red lines) for the case of $m_X = 1$ GeV. The adopted observational constraint on the ^{7}Li abundance is the center value of $\log(^7\mathrm{Li/H}) = -12 + (2.199 \pm 0.086)$ derived in a 3D NLTE model [9], while that of ^{6}Li is taken from the two sigma upper limit of G64-12 (NLTE model with 5 parameters), $^6\mathrm{Li/H}=(0.9 \pm 4.3) \times 10^{-12}$ [33]. Dashed lines around the line of $d(^7\mathrm{Li})=1$ correspond to the 2 sigma uncertainty in the observational constraint. The gray region which locates right from contours of $d(^6\mathrm{Li})=10$ or the 2 sigma lower limit, $d(^7\mathrm{Li})=0.67$, are excluded by overproduction of ^{6}Li and underproduction of ^{7}Li, respectively. The orange region is interesting parameter region in which a significant ^{7}Li reduction realizes without an overproduction of ^{6}Li. Purple lines are contours of the abundance ratio ^{9}Be/H predicted when the unknown rate of the reaction $^7\mathrm{Li}_X(d,\ X^-)^9\mathrm{Be}$ is assumed as described in text.

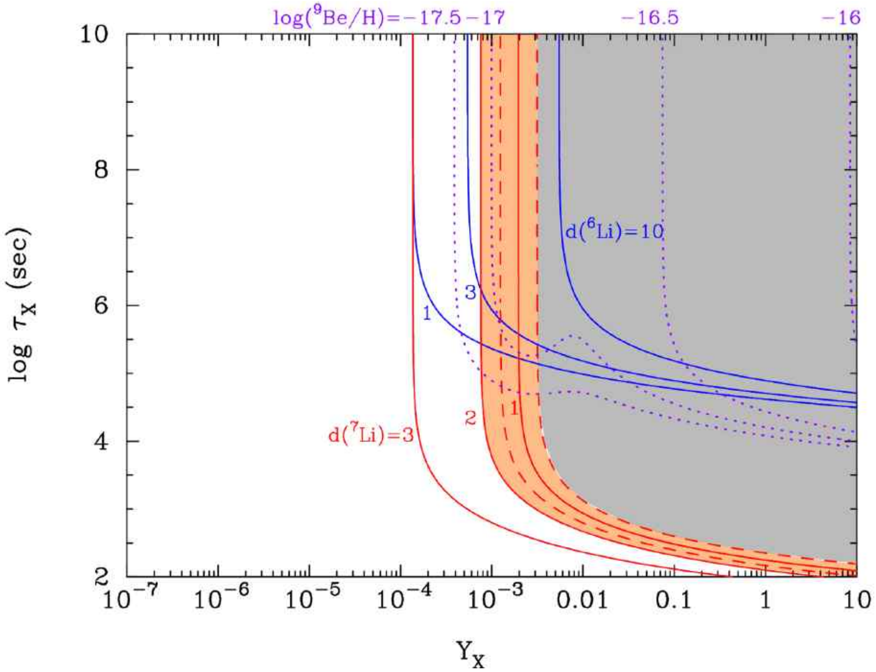
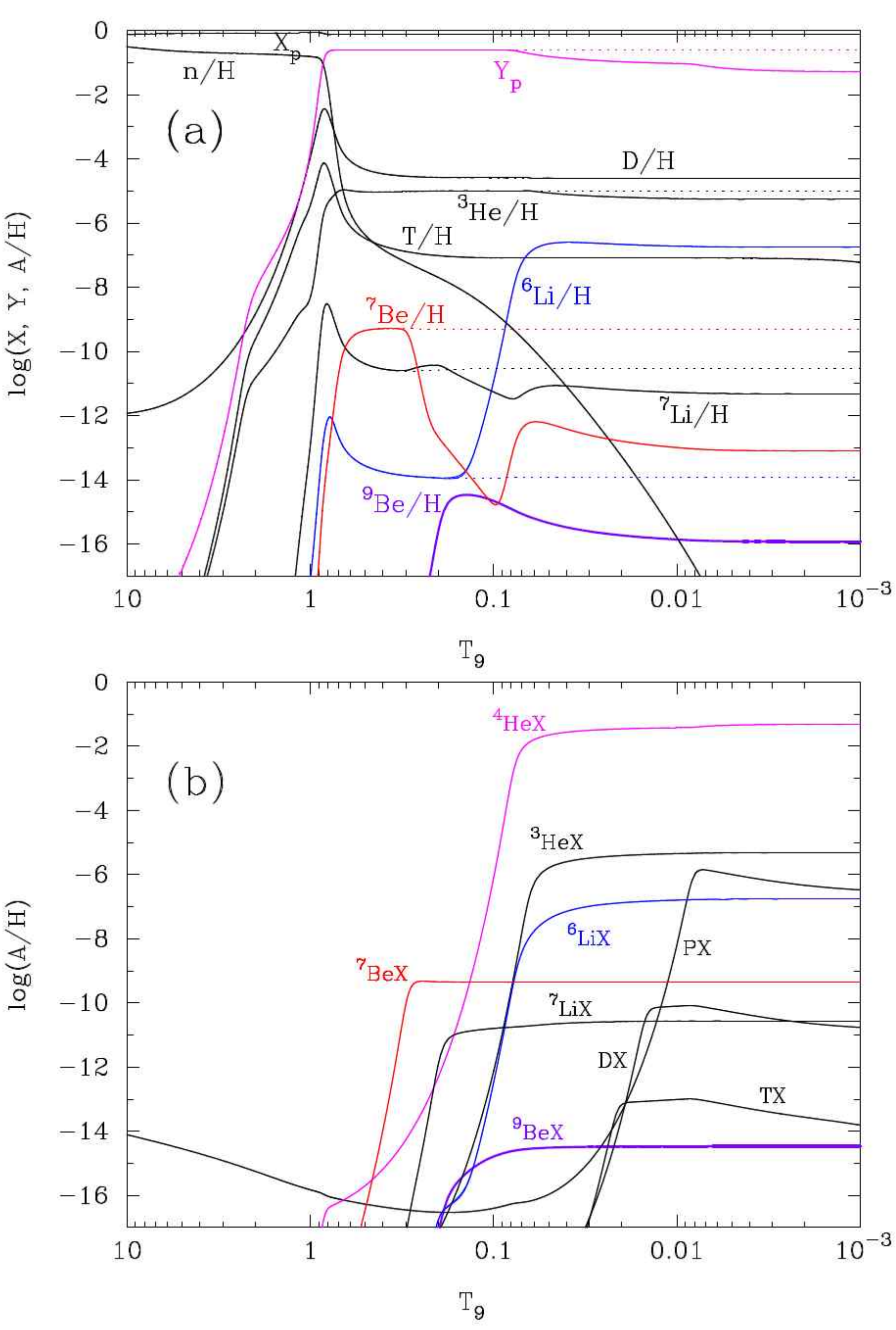
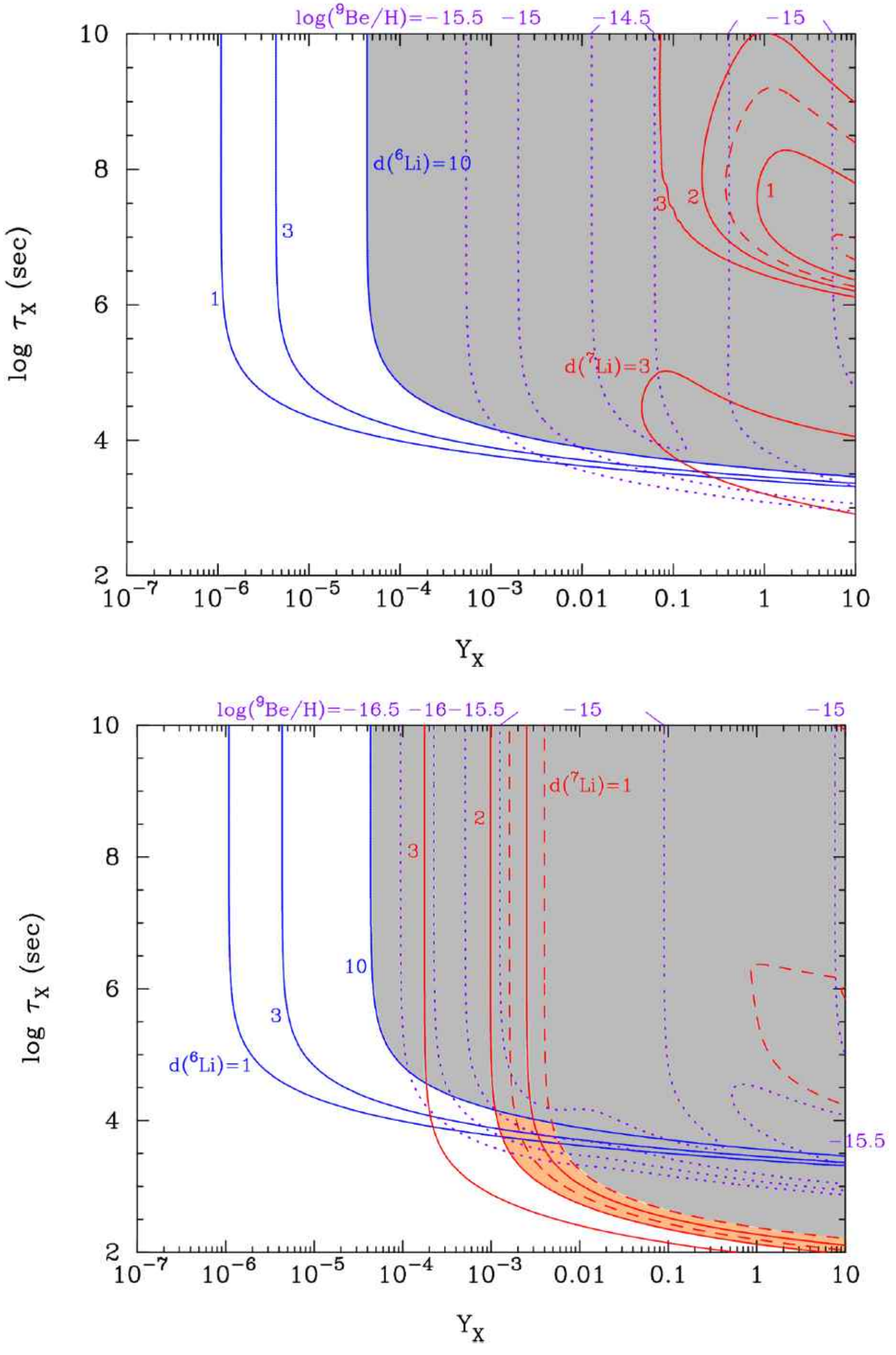
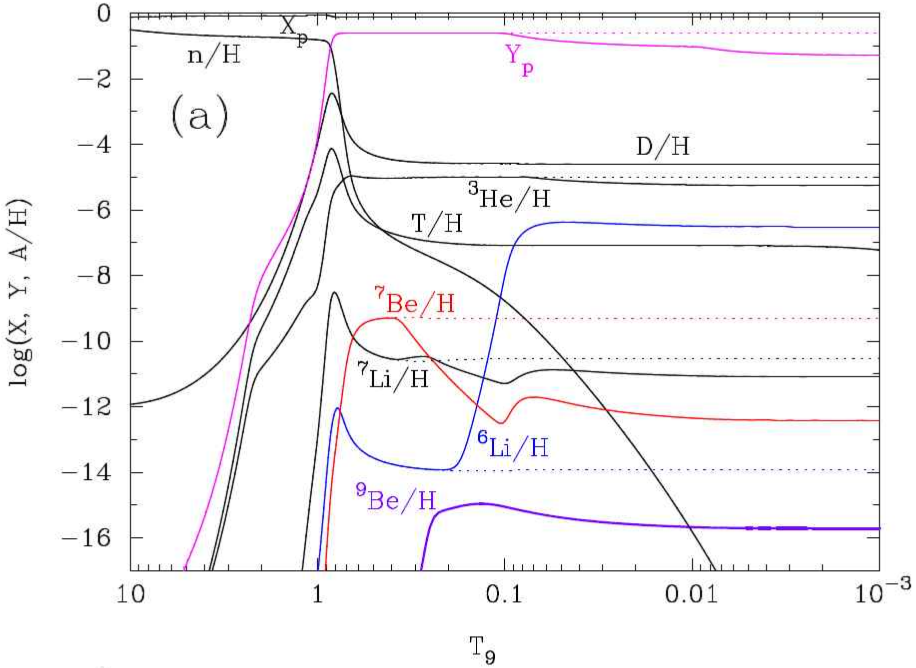
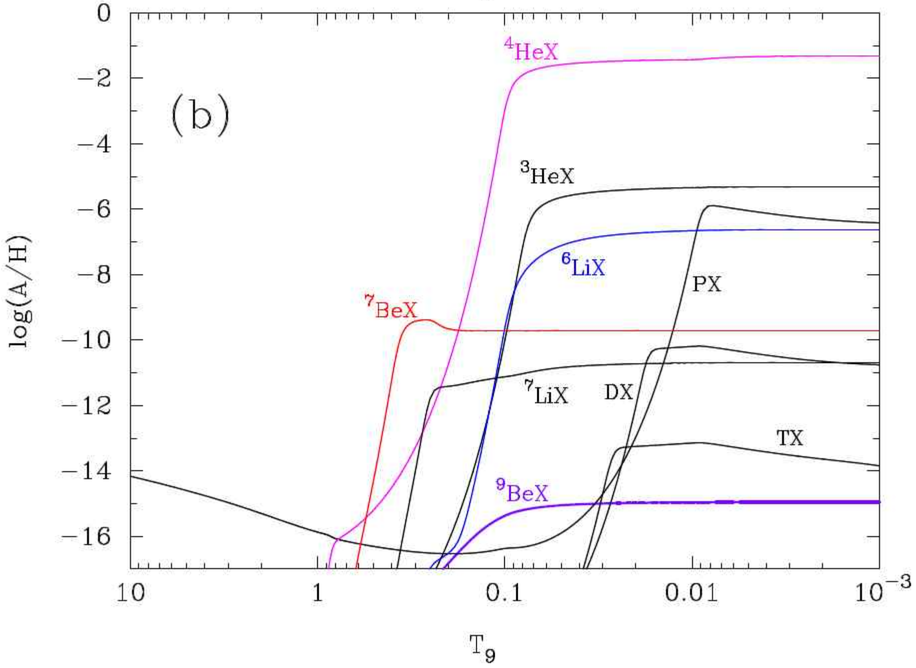


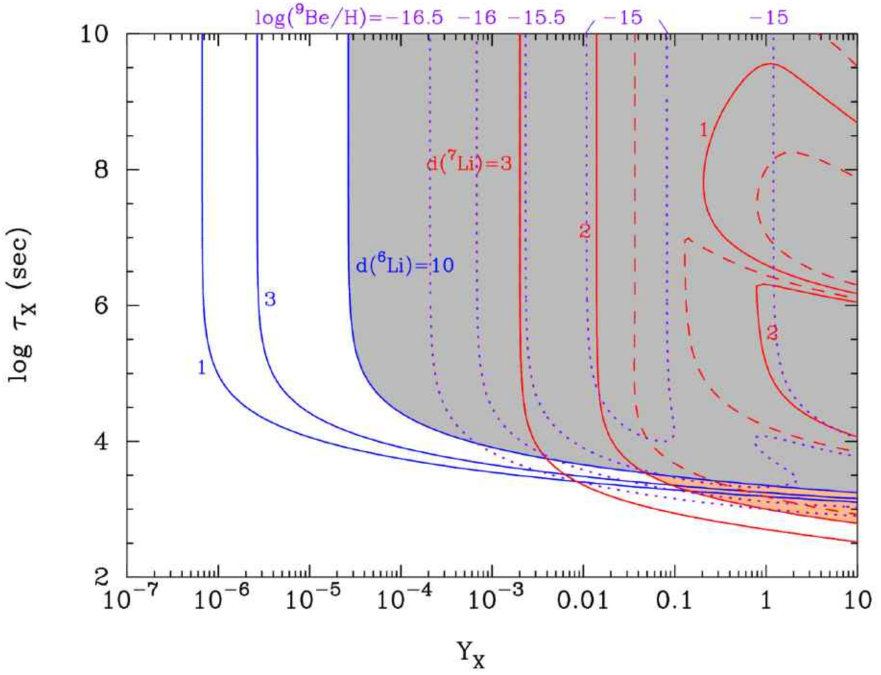
FIG. 24: Same as in Fig. 23 when the charged-current decay of $^7\mathrm{Be}_X \rightarrow {}^7\mathrm{Li}+X^0$ is included.

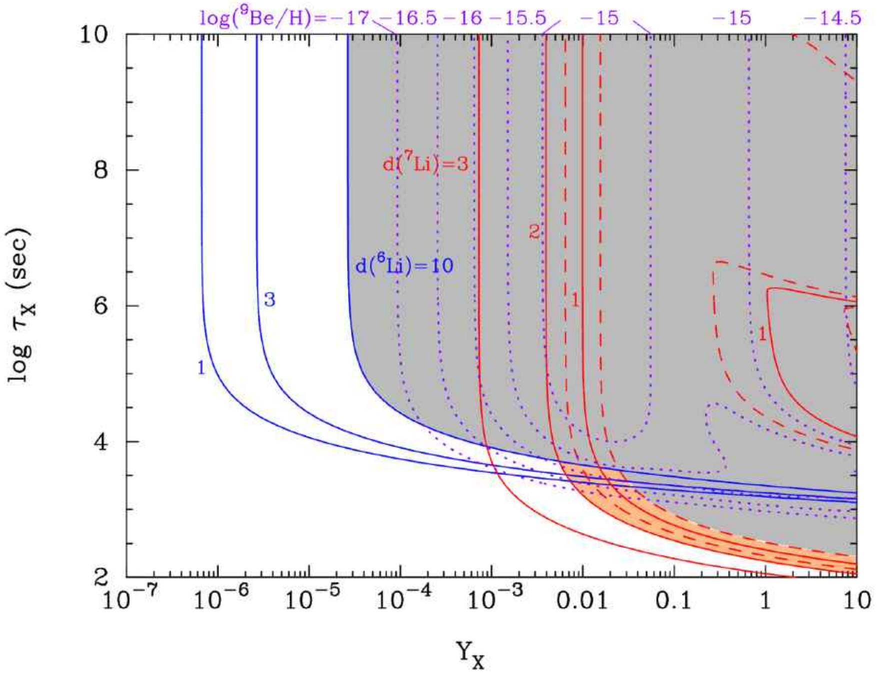

(a)
log(X, Y, A/H)
n/H
X_p
Y_p
D/H
^{3}He/H
T/H
^{6}Li/H
^{7}Be/H
^{7}Li/H
^{9}Be/H
T_9
(b)
log(A/H)
^{4}HeX
^{3}HeX
^{6}LiX
PX
^{7}BeX
^{7}LiX
DX
TX
^{9}BeX
T_9
0
−2
−4
−6
−8
−10
−12
−14
−16
10
1
0.1
0.01
10^{-3}

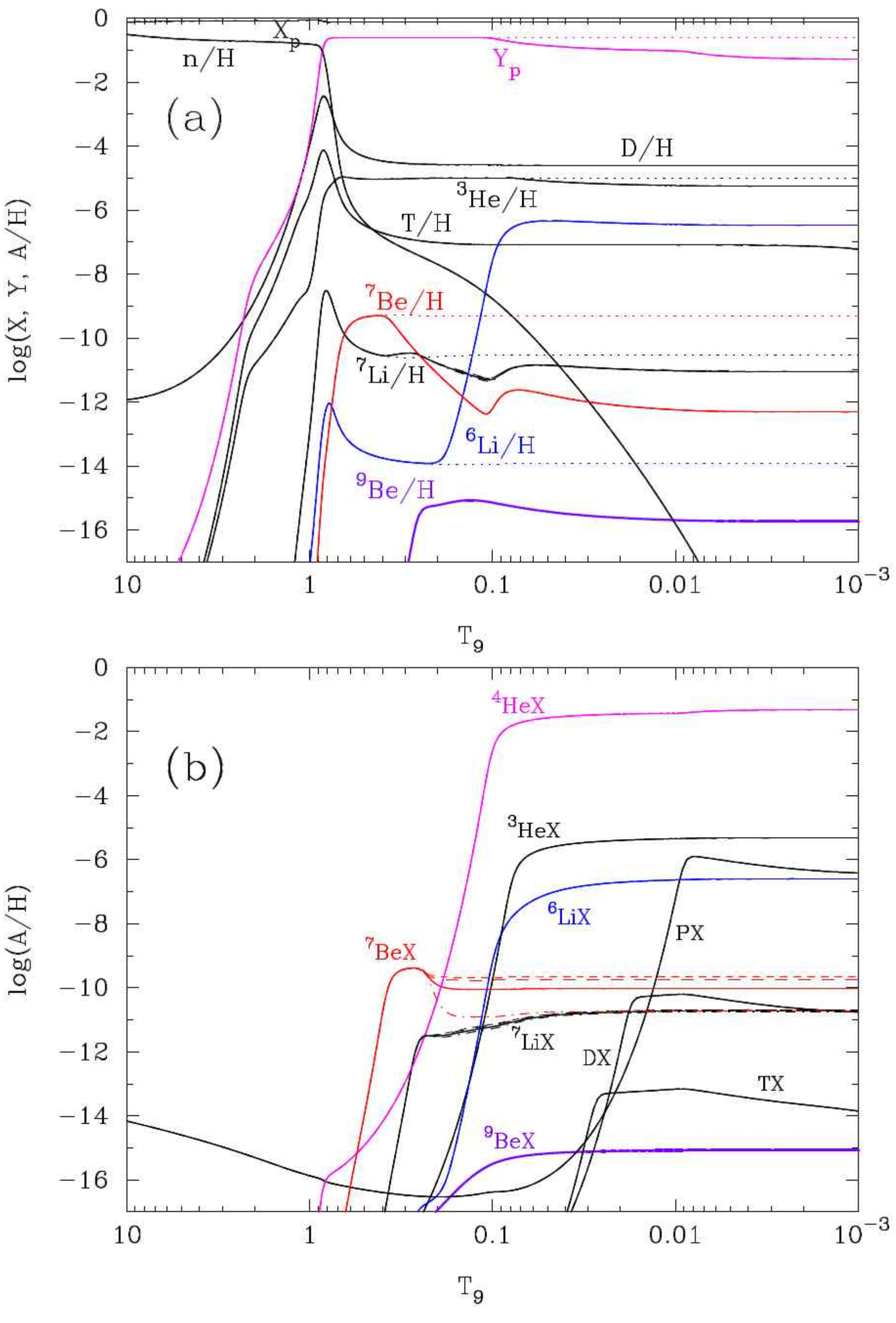

log(^{9}Be/H)=−15.5
−15
−14.5
−15
d(^{6}Li)=10
3
1
3
2
1
d(^{7}Li)=3
log τ_X (sec)
Y_X
log(^{9}Be/H)=−16.5
−16
−15.5
−15
−15
d(^{7}Li)=1
3
2
10
3
d(^{6}Li)=1
−15.5
2
4
6
8
10
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
0.01
0.1
1
10

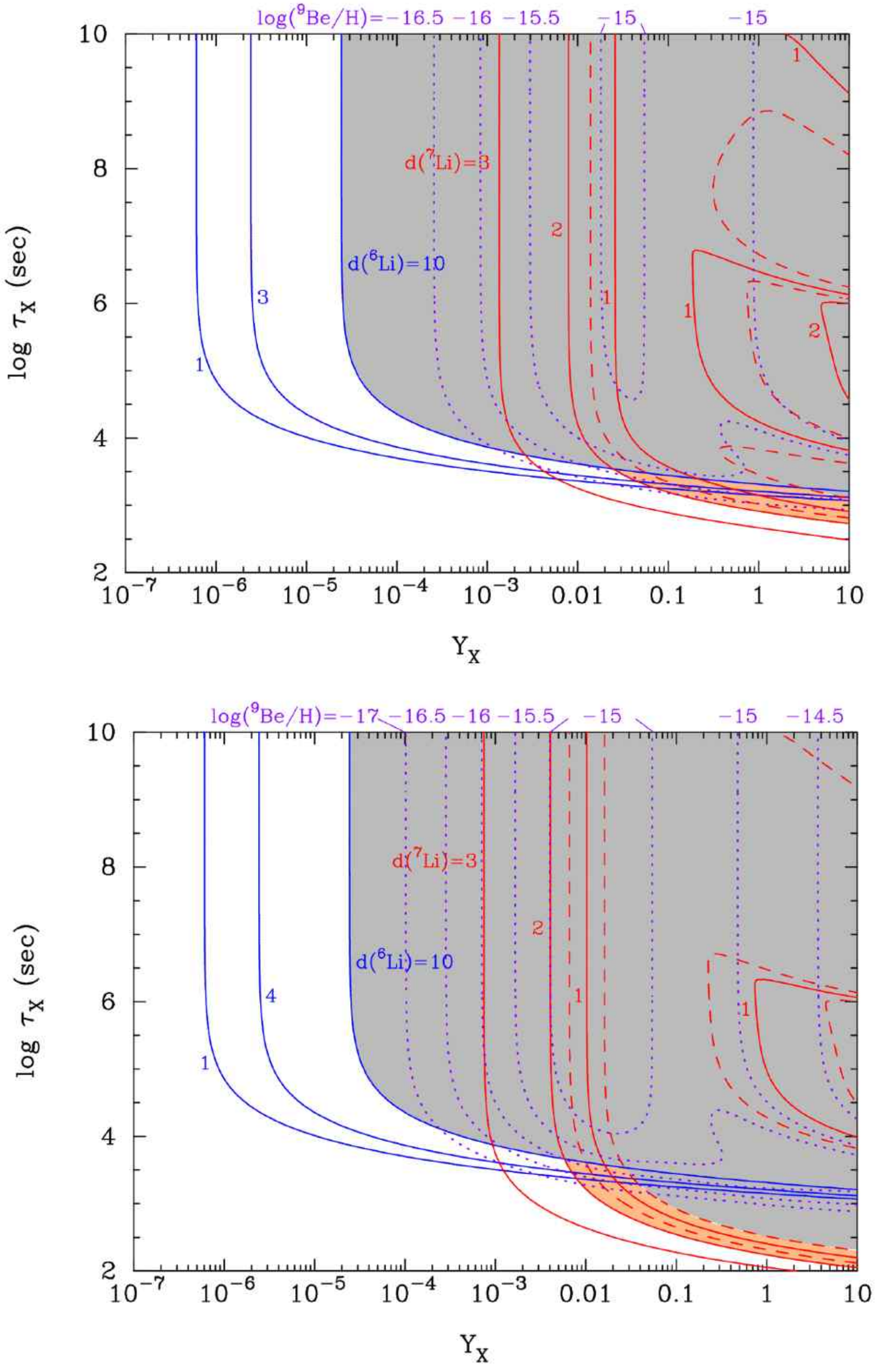

(a)
log(X, Y, A/H)
T$_9$
10
1
0.1
0.01
10^{-3}
0
−2
−4
−6
−8
−10
−12
−14
−16
X$_p$
n/H
Y$_p$
D/H
^{3}He/H
T/H
^{7}Be/H
^{7}Li/H
^{6}Li/H
^{9}Be/H


(b)
log(A/H)
T$_9$
10
1
0.1
0.01
10^{-3}
0
−2
−4
−6
−8
−10
−12
−14
−16
^{4}HeX
^{3}HeX
^{6}LiX
PX
^{7}BeX
^{7}LiX
DX
TX
^{9}BeX


log(^{9}Be/H)=−16.5 −16 −15.5 −15 −15
log τ$_X$ (sec)
Y$_X$
10
8
6
4
2
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
0.01
0.1
1
10
d(^{7}Li)=3
2
1
2
d(^{6}Li)=10
3
1


log(^{9}Be/H)=−17 −16.5 −16 −15.5 −15 −15 −14.5
log τ$_X$ (sec)
Y$_X$
10
8
6
4
2
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
0.01
0.1
1
10
d(^{7}Li)=3
2
1
1
d(^{6}Li)=10
3
1


(a)
log(X, Y, A/H)
T9
n/H
Xp
Yp
D/H
3He/H
T/H
7Be/H
7Li/H
6Li/H
9Be/H
(b)
log(A/H)
4HeX
3HeX
6LiX
7BeX
7LiX
PX
DX
TX
9BeX
10
1
0.1
0.01
10−3
0
−2
−4
−6
−8
−10
−12
−14
−16


log(9Be/H)=−16.5 −16 −15.5 −15 −15
d(7Li)=3
2
1
1
2
1
d(6Li)=10
3
1
log τX (sec)
YX
log(9Be/H)=−17 −16.5 −16 −15.5 −15 −15 −14.5
d(7Li)=3
2
1
1
1
d(6Li)=10
4
1
log τX (sec)
YX
10−7
10−6
10−5
10−4
10−3
0.01
0.1
1
10
2
4
6
8
10

Processes affecting elemental abundances

	Model	^{7}Li problem solved ?	Signatures on other nuclides ?
Existence of particle [z~10^{9}]	**sub-SIMP X^{0}**	**✓[1]**	**^{6}Li [2], ^{9}Be [1]**
	SIMP X^{0} [3]	no	**^{9}Be and/or ^{10}B**
	CHAMP X^{-} *	**✓[4,5,6]**	^{6}Li [7], **^{9}Be [6]**
Early stars [z~O(10)]	Early cosmic ray	no	^{6}Li [8], ^{9}Be & ^{10,11}B [9, 10]

[1] Kawasaki, MK (2011)
[2] MK, Kawasaki (2012)
[3] MK, Kajino, Yoshida, Mathews (2009)
[4] Bird, Koopmans, Pospelov (2008) [stronger]
[5] MK, Kajino, Boyd, Yoshida, Mathews (2007) [weaker]
[6] MK, Kim, Cheoun, Kajino, Mathews (in preparation)
[7] Pospelov (2007)
[8] Rollinde, Vangioni, Olive (2006)
[9] Rollinde, Maurin, Vangioni, Olive, Inoue (2008); MK (2008)
[10] MK, Kawasaki (2013)