

Searching for new physics with the electron, muon & tau g-2

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Preamble: today's values

$$a_e = 11596521807.3 \text{ (2.8)} \times 10^{-13}$$

0.24 parts per billion !! (Hanneke et al., PRL100 (2008) 120801)

$$a_\mu = 116592089 \text{ (63)} \times 10^{-11}$$

0.5 parts per million !! (E821 – Final Report: PRD73 (2006) 072003)

$$a_\tau = -0.018 \text{ (17)}$$

Well, not much yet.... (PDG 2013)

Outline

- ➊ 1. Lepton magnetic moments: the basics
- ➋ 2. μ : The muon g-2: a quick update
- ➌ 3. e : Testing new physics with the electron g-2
- ➍ 4. τ : The tau g-2: opportunities & challenges (fantasies?)

1. Lepton magnetic moments: the basics

- Uhlenbeck and Goudsmit in 1925 proposed:

$$\vec{\mu} = g \frac{e}{2mc} \vec{s}$$
$$g = \underline{2} \quad (\text{not } 1!)$$

- Dirac 1928:

$$(i\partial_\mu - eA_\mu) \gamma^\mu \psi = m\psi$$

- A Pauli term in Dirac's eq would give a deviation...

$$a \frac{e}{2m} \sigma_{\mu\nu} F^{\mu\nu} \psi \rightarrow g = 2(1 + a)$$

...but there was no need for it! g=2 stood for ~20 yrs.

Theory of the g-2: Quantum Field Theory

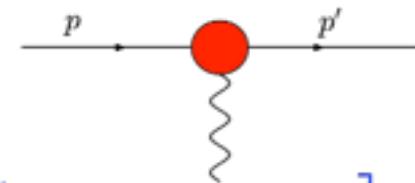
- Kusch and Foley 1948:

$$\mu_e^{\text{exp}} = \frac{e\hbar}{2mc} (1.00119 \pm 0.00005)$$

- Schwinger 1948 (triumph of QED!):

$$\mu_e^{\text{th}} = \frac{e\hbar}{2mc} \left(1 + \frac{\alpha}{2\pi}\right) = \frac{e\hbar}{2mc} \times 1.00116$$

- Keep studying the lepton- γ vertex:



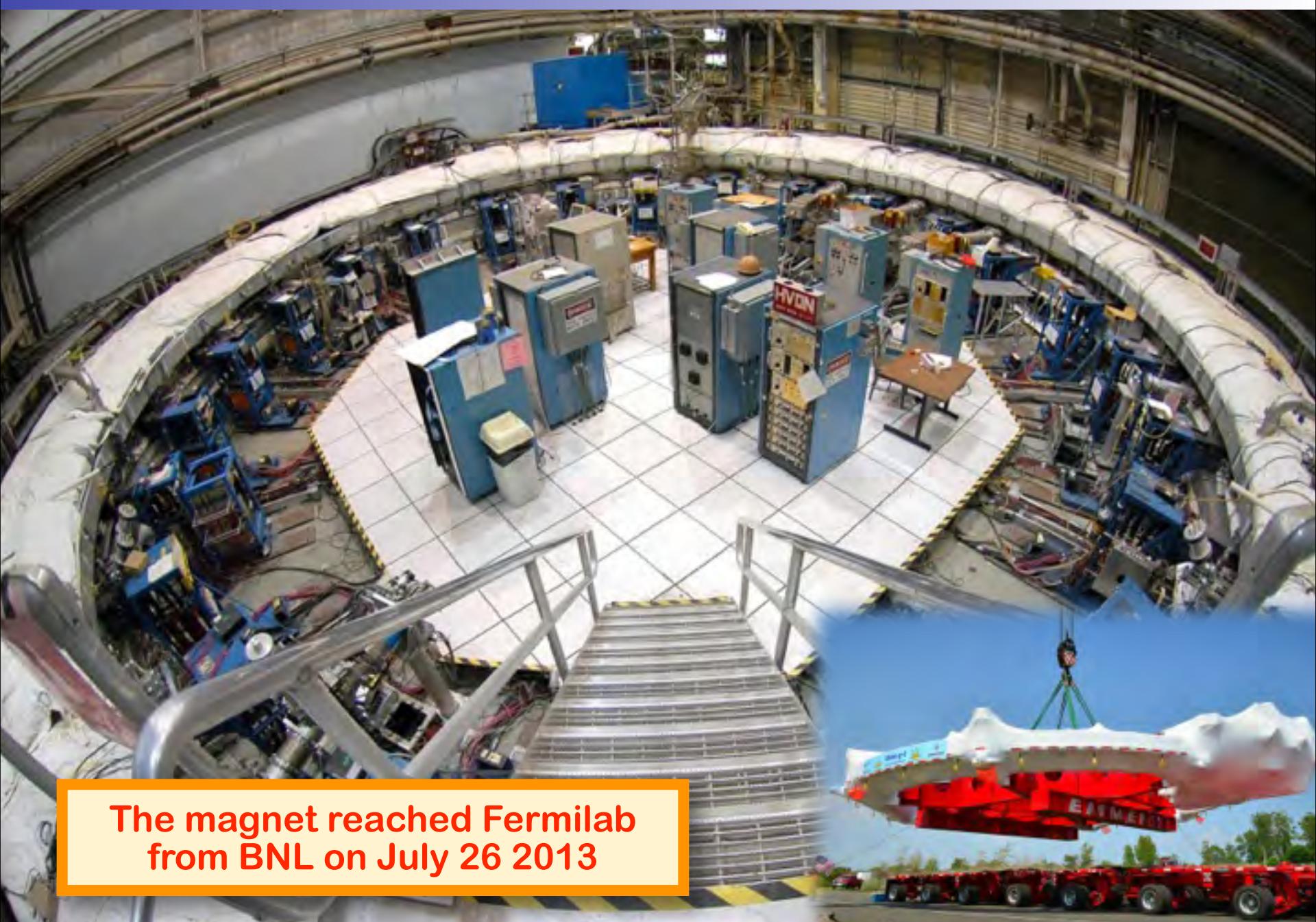
$$\bar{u}(p') \Gamma_\mu u(p) = \bar{u}(p') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m} F_2(q^2) + \dots \right] u(p)$$

$$F_1(0) = 1 \quad F_2(0) = a_l$$

A pure “quantum correction” effect!

2. The muon g-2: a quick update

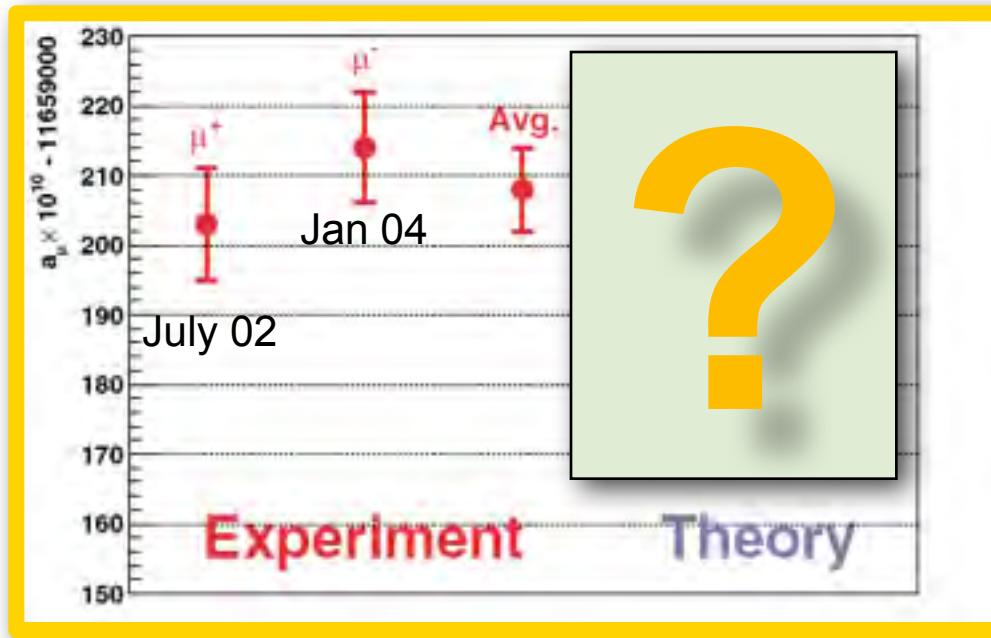
The old experiment E821



The magnet reached Fermilab
from BNL on July 26 2013



The muon g-2: the experimental result



- Today: $a_\mu^{\text{EXP}} = (116592089 \pm 54_{\text{stat}} \pm 33_{\text{sys}}) \times 10^{-11}$ [0.5 ppm].
- Future: new muon g-2 experiments proposed at:
 - Fermilab E989, aiming at $\pm 16 \times 10^{-11}$, ie 0.14 ppm
 - J-PARC aiming at 0.1 ppm
- Are theorists ready for this (amazing) precision? No(t yet)

Data in (late) 2016?

The muon g-2: the QED contribution

$$a_\mu^{\text{QED}} = (1/2)(\alpha/\pi) \quad \text{Schwinger 1948}$$

$$+ 0.765857426 (16) (\alpha/\pi)^2$$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

$$+ 24.05050988 (28) (\alpha/\pi)^3$$

Remiddi, Laporta, Barbieri ... ; Czarnecki, Skrzypek; MP '04;
Friot, Greynat & de Rafael '05, Mohr, Taylor & Newell 2012

$$+ 130.8796 (63) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012,
Steinhauser et al. 2013 (analytic, in progress).

$$+ 753.29 (1.04) (\alpha/\pi)^5 \text{ COMPLETED!}$$

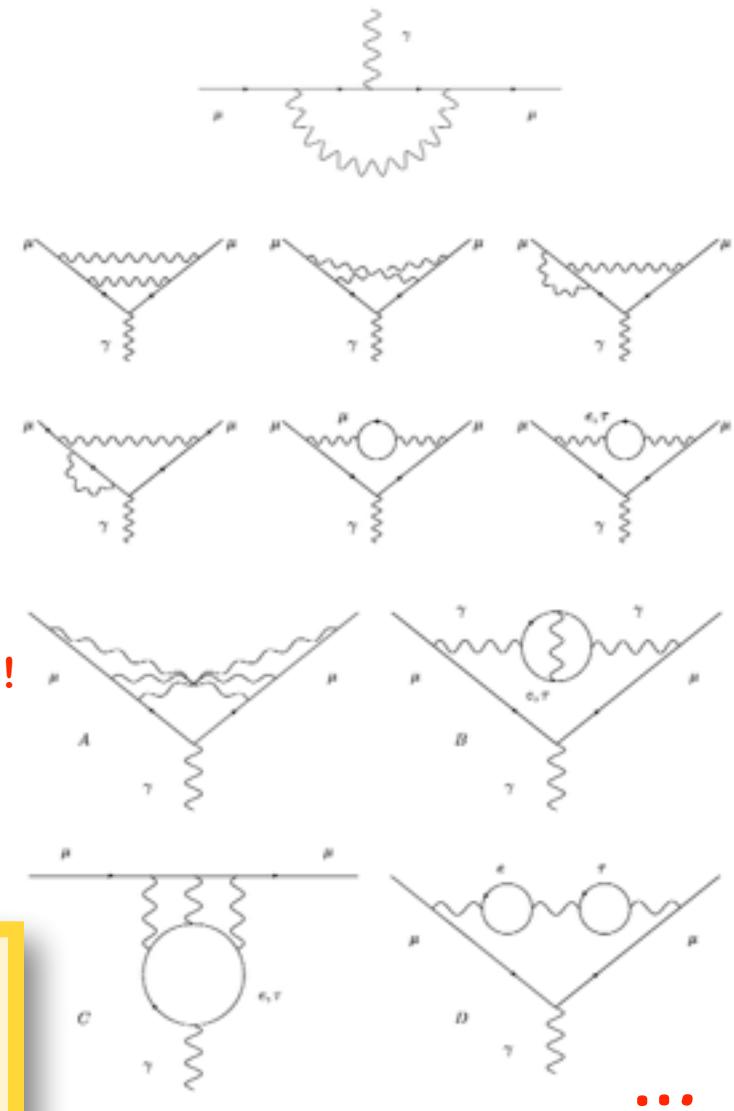
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,
Karshenboim,..., Kataev, Kinoshita & Nio '06, Kinoshita et al. 2012

Adding up, we get:

$$a_\mu^{\text{QED}} = 116584718.951 (22)(77) \times 10^{-11}$$

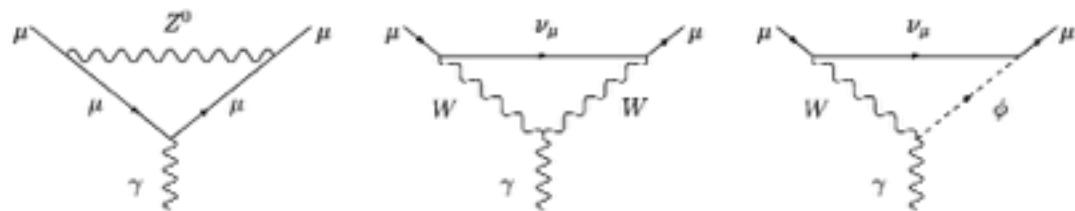
from coeffs, mainly from 4-loop unc from $\delta\alpha(\text{Rb})$

$$\text{with } \alpha = 1/137.035999049(90) [0.66 \text{ ppb}]$$



The muon g-2: the electroweak contribution

● One-loop term:



$$a_\mu^{\text{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2} \left[1 + \frac{1}{5} (1 - 4\sin^2\theta_W)^2 + O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right) \right] \approx 195 \times 10^{-11}$$

1972: Jackiw, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;
Studenikin et al. '80s

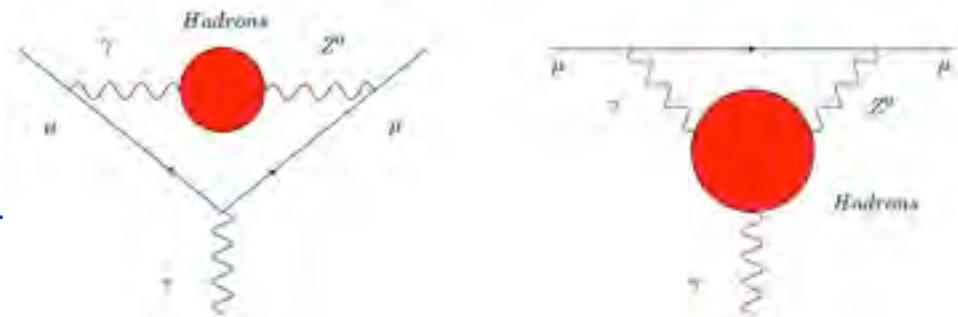
● One-loop plus higher-order terms:

$$a_\mu^{\text{EW}} = 153.6 (1) \times 10^{-11}$$

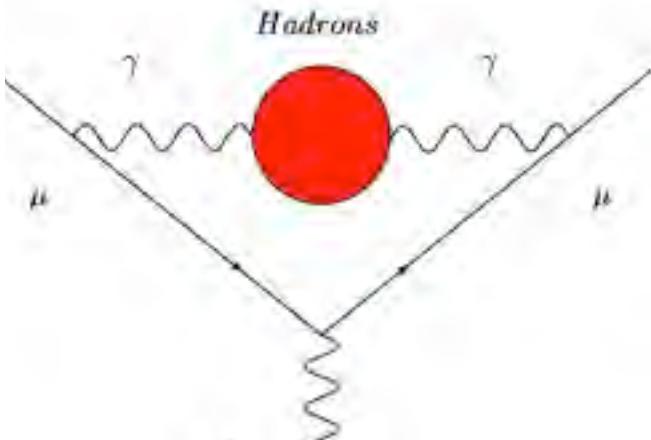
with $M_{\text{Higgs}} = 125.6 (1.5) \text{ GeV}$

Hadronic loop uncertainties
and 3-loop nonleading logs.

Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013



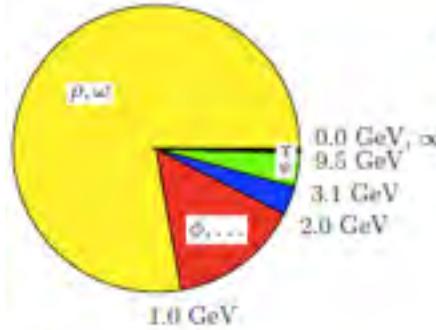
The muon g-2: the hadronic LO contribution (HLO)



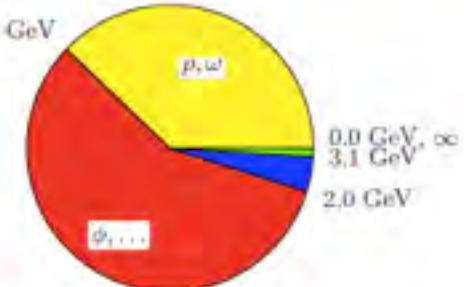
$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

$$a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^\infty ds K(s) \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^\infty \frac{ds}{s} K(s) R(s)$$

Central values



Errors²



F. Jegerlehner and A. Nyffeler, Phys. Rept. 477 (2009) 1

$$a_\mu^{\text{HLO}} = 6903 (53)_{\text{tot}} \times 10^{-11}$$

F. Jegerlehner, A. Nyffeler, Phys. Rept. 477 (2009) 1

$$= 6923 (42)_{\text{tot}} \times 10^{-11}$$

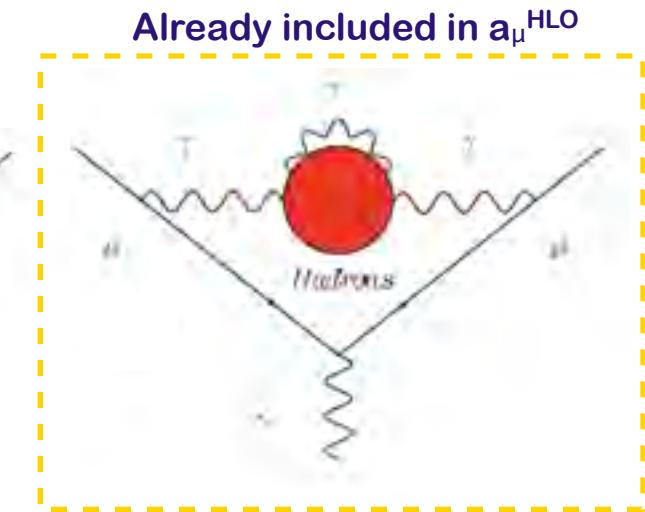
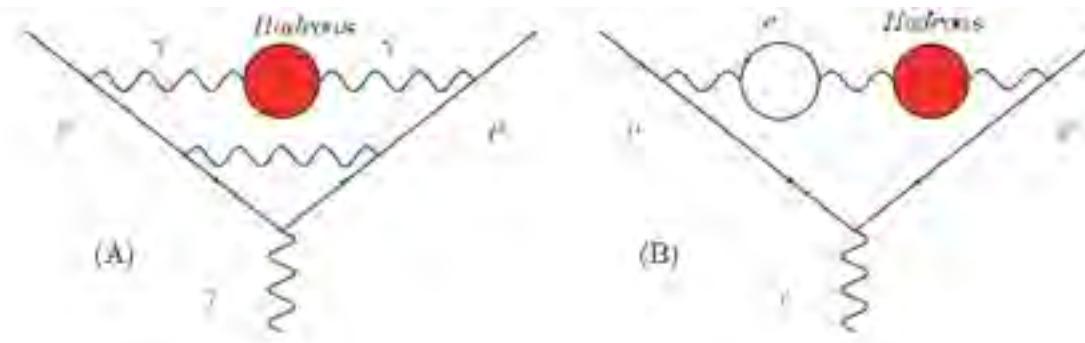
Davier et al, EPJ C71 (2011) 1515 (incl. BaBar & KLOE10 2π)

$$= 6949 (37)_{\text{exp}} (21)_{\text{rad}} \times 10^{-11}$$

Hagiwara et al, JPG 38 (2011) 085003



• HHO: Vacuum Polarization



$\mathcal{O}(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:

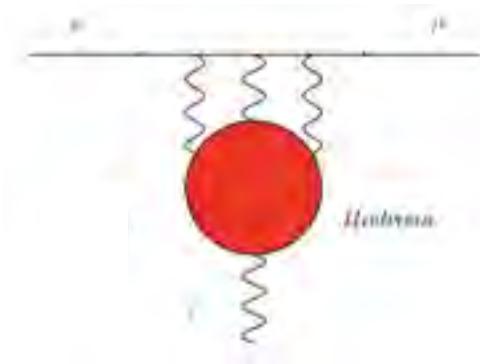
$$a_\mu^{\text{HHO(vp)}} = -98 (1) \times 10^{-11}$$

Krause '96, Alemany et al. '98, Hagiwara et al. 2011

- HHO: Light-by-light contribution**

📌 Unlike the HLO term, for the hadronic l-b-l term we must rely on theoretical approaches.

📌 This term had a troubled life! Latest values:



$$a_\mu^{\text{HHO}}(|l|) = +80(40) \times 10^{-11} \quad \text{Knecht \& Nyffeler '02}$$

$$a_\mu^{\text{HHO}}(|l|) = +136(25) \times 10^{-11} \quad \text{Melnikov \& Vainshtein '03}$$

$$a_\mu^{\text{HHO}}(|l|) = +105(26) \times 10^{-11} \quad \text{Prades, de Rafael, Vainshtein '09}$$

$$a_\mu^{\text{HHO}}(|l|) = +116(39) \times 10^{-11} \quad \text{Jegerlehner \& Nyffeler '09}$$

Results based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02

- 📌 “Bound” $a_\mu^{\text{HHO}}(|l|) < \sim 160 \times 10^{-11}$ Erler, Sanchez '06, Pivovarov '02; also Boughezal, Melnikov '11
- 📌 Lattice? Very hard... in progress. M. Golterman @ PhiPsi 2013; T. Blum @ Lattice 2012
- 📌 Pion exch. in holographic QCD agrees. D.K.Hong, D.Kim '09; Cappiello, Catà, D'Ambrosio '11
- 📌 Had lbl is likely to become the ultimate limitation of the SM prediction

The muon g-2: SM vs. Experiment

Adding up all contributions, we get the following SM predictions and comparisons with the measured value:

$$a_\mu^{\text{EXP}} = 116592089 (63) \times 10^{-11}$$

E821 – Final Report: PRD73
(2006) 072 with latest value
of $\lambda = \mu_\mu / \mu_p$ from CODATA'06

$a_\mu^{\text{SM}} \times 10^{11}$	$\Delta a_\mu = a_\mu^{\text{EXP}} - a_\mu^{\text{SM}}$	σ
116 591 793 (66)	$296 (91) \times 10^{-11}$	3.2 [1]
116 591 813 (57)	$276 (85) \times 10^{-11}$	3.2 [2]
116 591 839 (58)	$250 (86) \times 10^{-11}$	2.9 [3]

with the “conservative” $a_\mu^{\text{HHO}}(|\delta|) = 116 (39) \times 10^{-11}$ and the LO hadronic from:

[1] Jegerlehner & Nyffeler, Phys. Rept. 477 (2009) 1

[2] Davier et al, EPJ C71 (2011) 1515 (includes BaBar & KLOE10 2π)

[3] Hagiwara et al, JPG38 (2011) 085003 (includes BaBar & KLOE10 2π)

Note that the th. error is now about the same as the exp. one

The muon g-2: connection with the SM Higgs mass

- Δa_μ can be explained in many ways: errors in LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or, we hope, New Physics! (for the MSSM explanation, see Hyejung Kim's talk)
- Can Δa_μ be due to mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta \alpha_{\text{had}}^{(5)}(M_Z)$.
- Consider:

$$\begin{aligned} a_\mu^{\text{HLO}} &\rightarrow a = \int_{4m_\pi^2}^{s_u} ds f(s) \sigma(s), \quad f(s) = \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2, \\ \Delta \alpha_{\text{had}}^{(5)} &\rightarrow b = \int_{4m_\pi^2}^{s_u} ds g(s) \sigma(s), \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}$$

and the increase

$$\Delta \sigma(s) = \epsilon \sigma(s)$$

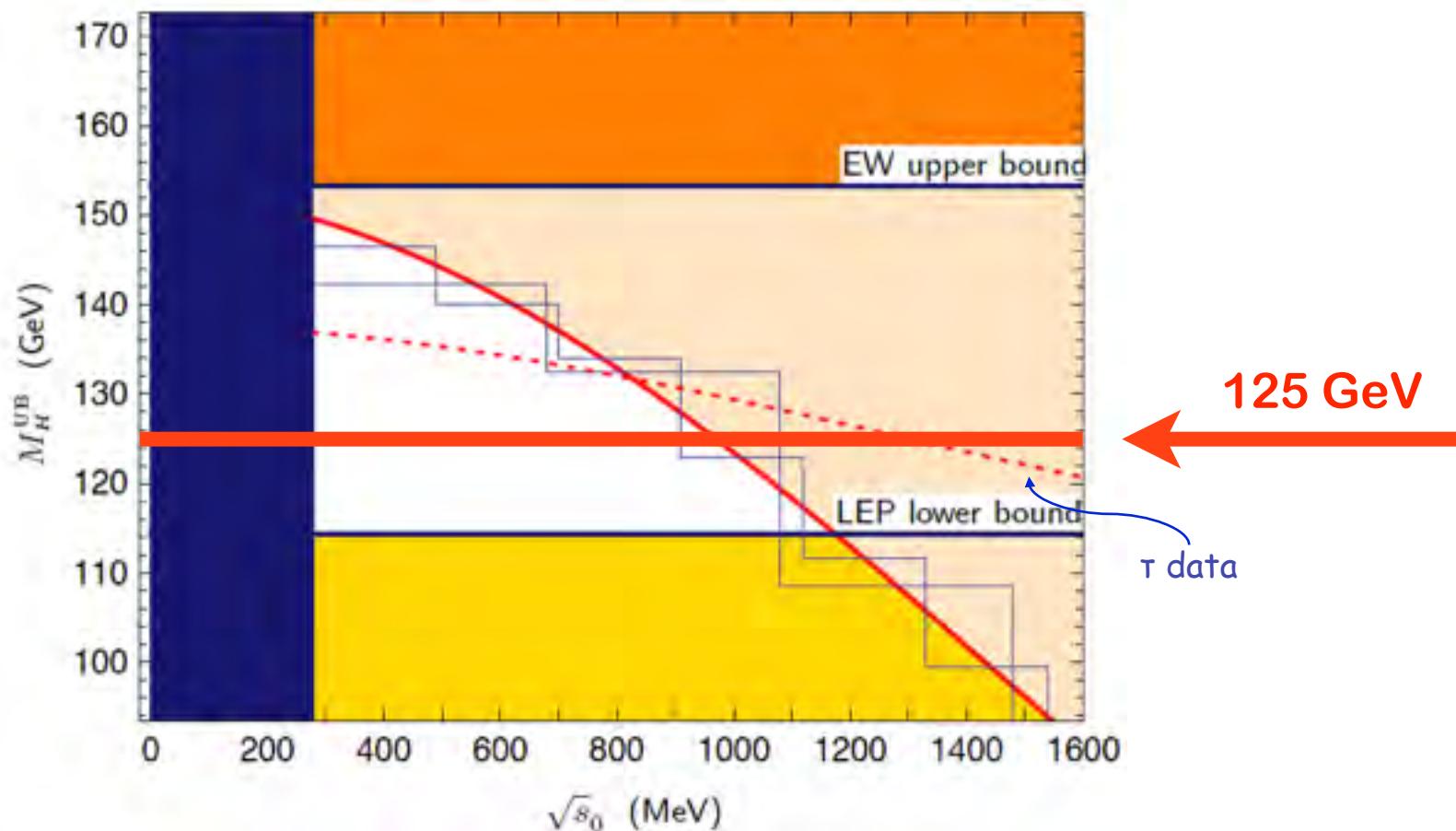
($\epsilon > 0$), in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$



The muon g-2: connection with the SM Higgs mass (2)

- How much does the M_H upper bound from the EW fit change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$] to accommodate Δa_μ ?



W.J. Marciano, A. Sirlin, MP, 2008 & 2010

The muon g-2: connection with the SM Higgs mass (3)

- Given the quoted exp. uncertainty of $\sigma(s)$, the possibility to explain the muon g-2 with these very large shifts $\Delta\sigma(s)$ appears to be very unlikely.
- Also, given a **125 GeV SM Higgs**, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energy (below ~ 1 GeV).
- Vice versa, assuming we now have a SM Higgs with $M_{Higgs} = 125$ GeV, if we bridge the M_{Higgs} discrepancy in the EW fit via changes in the low-energy hadronic cross section, **the muon g-2 discrepancy increases**.

W.J. Marciano, A. Sirlin, MP, 2008 & 2010 (and work in progress)

3. Testing new physics with the electron g-2

G.F. Giudice, P. Paradisi, MP

JHEP 1211 (2012) 113

The QED prediction of the electron g-2

$$a_e^{\text{QED}} = + (1/2)(\alpha/\pi) - 0.328\ 478\ 444\ 002\ 55(33) (\alpha/\pi)^2$$

Schwinger 1948 Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; CODATA Mar '12

$$A_1^{(4)} = -0.328\ 478\ 965\ 579\ 193\ 78\dots$$

$$A_2^{(4)} (m_e/m_\mu) = 5.197\ 386\ 68 (26) \times 10^{-7}$$

$$A_2^{(4)} (m_e/m_\tau) = 1.837\ 98 (33) \times 10^{-9}$$

$$+ 1.181\ 234\ 016\ 816 (11) (\alpha/\pi)^3$$

Kinoshita; Barbieri; Laporta, Remiddi; ... , Li, Samuel; MP '06; Giudice, Paradisi, MP 2012

$$A_1^{(6)} = 1.181\ 241\ 456\ 587\dots$$

$$A_2^{(6)} (m_e/m_\mu) = -7.373\ 941\ 62 (27) \times 10^{-6}$$

$$A_2^{(6)} (m_e/m_\tau) = -6.5830 (11) \times 10^{-8}$$

$$A_3^{(6)} (m_e/m_\mu, m_e/m_\tau) = 1.909\ 82 (34) \times 10^{-13}$$

$$- 1.9097 (20) (\alpha/\pi)^4$$

Kinoshita & Lindquist '81, ... , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio 2012

$$+ 9.16 (58) (\alpha/\pi)^5 \quad \text{COMPLETED! (12672 mass independent diagrams!)}$$

Aoyama, Hayakawa, Kinoshita, Nio, PRL 109 (2012) 111807.

The SM prediction of the electron g-2

The SM prediction is:

$$a_e^{\text{SM}}(\alpha) = a_e^{\text{QED}}(\alpha) + a_e^{\text{EW}} + a_e^{\text{HAD}}$$

The EW (1&2 loop) term is: Czarnecki, Krause, Marciano '96 [Codata 2012]

$$a_e^{\text{EW}} = 0.2973(52) \times 10^{-13}$$

The Hadronic contribution is: Nomura & Teubner '12, Jegerlehner & Nyffeler '09; Krause'97

$$a_e^{\text{HAD}} = 16.82(16) \times 10^{-13}$$

Which value of α should we use to compute a_e^{SM} and compare it with a_e^{EXP} ?? Not the PDG/Codata one (obtained equating $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$)! Use atomic-physics measurements of alpha.

The electron g-2 gives the best determination of alpha

- The 2008 measurement of the electron g-2 is:

$$a_e^{\text{EXP}} = 11596521807.3 \text{ (2.8)} \times 10^{-13} \quad \text{Hanneke et al, PRL100 (2008) 120801}$$

vs. old (factor of 15 improvement, 1.8σ difference):

$$a_e^{\text{EXP}} = 11596521883 \text{ (42)} \times 10^{-13} \quad \text{Van Dyck et al, PRL59 (1987) 26}$$

- Equate $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$ → best determination of alpha (2012):

$$\alpha^{-1} = 137.035\ 999\ 173 \text{ (34)} \quad [0.25 \text{ ppb}]$$

- Compare it with other determinations (independent of a_e):

$$\alpha^{-1} = 137.036\ 000\ 0 \text{ (11)} \quad [7.7 \text{ ppb}] \quad \text{PRA73 (2006) 032504 (Cs)}$$

$$\alpha^{-1} = 137.035\ 999\ 049 \text{ (90)} \quad [0.66 \text{ ppb}] \quad \text{PRL106 (2011) 080801 (Rb)}$$

Excellent agreement → beautiful test of QED at 4-loop level!

The electron g-2: SM vs. Experiment

- Using $\alpha = 1/137.035\ 999\ 049\ (90)$ [^{87}Rb , 2011], the SM prediction for the electron g-2 is

$$a_e^{\text{SM}} = 115\ 965\ 218\ 17.8\ (0.6)\ (0.4)\ (0.2)\ (7.6) \times 10^{-13}$$

δC_4^{qed} δC_5^{qed} δa_e^{had} from $\delta \alpha$

- The EXP-SM difference is:

$$\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -10.5\ (8.1) \times 10^{-13}$$

The SM is in very good agreement with experiment (1.3σ).

NB: The 4-loop contrib. to a_e^{QED} is $-5.56 \times 10^{-11} \sim 70 \Delta a_e$!
(the 5-loop one is 6.2×10^{-13})

The electron g-2 sensitivity and NP tests

- The present sensitivity is $\delta\Delta a_e = 8.1 \times 10^{-13}$, ie (10⁻¹³ units):

$$(0.6)_{\text{QED4}}, \quad (0.4)_{\text{QED5}}, \quad (0.2)_{\text{HAD}}, \quad (7.6)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\text{EXP}}}$$

$\underbrace{\qquad\qquad\qquad}_{(0.7)_{\text{TH}}} \leftarrow \text{may drop to 0.2 or 0.3}$

- The $(g-2)_e$ exp. error may soon drop below 10⁻¹³ and work is in progress for a significant reduction of that induced by $\delta\alpha$.
→ a sensitivity of 10⁻¹³ may be reached with ongoing exp work.
- In a broad class of BSM theories, contributions to a_1 scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}} \right)^2 \quad \text{This Naive Scaling leads to:}$$

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 0.8 \times 10^{-6}$$

The electron g-2 sensitivity and NP tests (2)

- The experimental sensitivity in Δa_e is not far from what is needed to test if the discrepancy in $(g-2)_\mu$ also manifests itself in $(g-2)_e$ under the naive-scaling hypothesis.
- BSM scenarios exist which violate Naive Scaling. They can lead to larger effects in Δa_e (& Δa_τ) and contributions to EDMs, LFV or observables violating lepton universality.
- Example: In the MSSM with non-degenerate but aligned sleptons (vanishing flavor mixing angles), Δa_e can reach $\sim 10^{-12}$ (at the limit of the present exp sensitivity). For these values one typically has violations of lepton universality at the few per mil level (within future exp reach).

4. The tau g-2: opportunities & challenges

Work in progress in collaboration with
S. Eidelman, D. Epifanov, M. Fael, L. Mercolli

arXiv:1301.5302
arXiv:1310.1081

The QED contribution to the tau g-2

$$a_{\tau}^{\text{QED}} = (1/2)(\alpha/\pi) + 2.057\,457\,(93) (\alpha/\pi)^2$$

Schwinger 1948

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66;
Li, Mendel & Samuel '93; Narison '01; MP '06

$$A_1^{(4)} = -0.328\,478\,965\,579\dots$$

$$A_2^{(4)}(m_\tau/m_e) = 2.024\,284\,(55)$$

$$A_2^{(4)}(m_\tau/m_\mu) = 0.361\,652\,(38)$$

$$+ 57.9315\,(27) (\alpha/\pi)^3$$

Kinoshita, Barbieri, Laporta, Remiddi, ... ; Samuel, Li & Mendel '91; Narison '01; MP '06

$$A_1^{(6)} = 1.181\,241\,456\,587\dots$$

$$A_2^{(6)}(m_\tau/m_e) = 46.3921\,(15)$$

$$A_2^{(6)}(m_\tau/m_\mu) = 7.01021\,(76)$$

MP 2006

$$A_3^{(6)}(m_\tau/m_e, m_\tau/m_\mu) = 3.347\,97\,(41)$$

$$+ ?(?) (\alpha/\pi)^4$$

Who? When?

Adding up:

$$a_{\tau}^{\text{QED}} = = 117324\,(2) \times 10^{-8}$$

2×10^{-8} is my estimate of the missing 4-loop

The EW and Hadronic contributions to the tau g-2

EW (1- and 2-loop) corrections

$$a_\tau^{\text{EW}}(\text{1 loop}) = \frac{5G_\mu m_\tau^2}{24\sqrt{2}\pi^2} \left[1 + \frac{1}{5} (1 - 4 \sin^2 \theta_W)^2 + O\left(\frac{m_\tau^2}{M_{Z,W,H}^2}\right) \right] = 55.1(1) \times 10^{-8}$$

Studenikin '90, included

Higgs mass (and τ mass error)

$$a_\tau^{\text{EW}} \text{ (1-loop)} = 55.09 \times 10^{-8}$$

$$a_\tau^{\text{EW}} \text{ (2-loop frm)} = -4.68 \times 10^{-8}$$

$$a_\tau^{\text{EW}} \text{ (2-loop bos)} = -3.06 \times 10^{-8}$$

$$a_\tau^{\text{EW}} = 47.4(5) \times 10^{-8}$$

Eidelman & MP
2007

Samuel, Li, Mendel '91; Czarnecki, Krause, Marciano '95;
Czarnecki, Krause '97; Narison '01

Higgs mass & Hadronic loop uncertainties
(plus M_{top} error & missing 3-loop terms)

Hadronic corrections (HLO & HHO):

- **HLO:** $a_\tau^{\text{HLO}} = 360(30)(10) \times 10^{-8}$ Samuel, Li & Mendel 1991

- $a_\tau^{\text{HLO}} = 343.3(9.1) \times 10^{-8}$ Eidelman & Jegerlehner '95

- $a_\tau^{\text{HLO}} = 353.6(4.0) \times 10^{-8}$ Narison 2001

- $a_\tau^{\text{HLO}} = 337.5(3.7) \times 10^{-8}$ Eidelman & MP 2007

- **HHO:** $a_\tau^{\text{HHO}} \text{ (vac)} = 7.6(2) \times 10^{-8}$ Krause 1996

- $a_\tau^{\text{HHO}} \text{ (lbl)} = 27.6(5.8) \times 10^{-8}$ Narison 2001

- $a_\tau^{\text{HHO}} \text{ (lbl)} = 5(3) \times 10^{-8}$ Eidelman & MP 2007

The SM prediction of the tau g-2

The Standard Model prediction of the tau g-2 is:

$$\begin{aligned} a_{\tau}^{\text{SM}} = & 117324 \quad (2) \quad \times 10^{-8} \quad \text{QED} \\ & + \quad 47.4 \quad (0.5) \quad \times 10^{-8} \quad \text{EW} \\ & + \quad 337.5 \quad (3.7) \quad \times 10^{-8} \quad \text{HO} \\ & + \quad 7.6 \quad (0.2) \quad \times 10^{-8} \quad \text{HHO (vac)} \\ & + \quad 5 \quad (3) \quad \times 10^{-8} \quad \text{HHO (lbl)} \end{aligned}$$

$$a_{\tau}^{\text{SM}} = 117721 \quad (5) \times 10^{-8}$$

Eidelman & MP
2007

$(m_{\tau}/m_{\mu})^2 \sim 280$: great opportunity to look for New Physics,
and a “clean” NP test too...

	Muon	Tau
$a^{\text{EW}}/a^{\text{HAD}}$	1/45	1/7
$a^{\text{EW}}/\delta a^{\text{HAD}}$	3	10

... if only we could measure it!!

The tau g-2: experimental bounds

- The very short lifetime of the tau makes it very difficult to determine a_τ measuring its spin precession in a magnetic field.
- DELPHI's result, from $e^+e^- \rightarrow e^+e^-\tau^+\tau^-$ total cross-section measurements at LEP 2 (the PDG value):

$$a_\tau = -0.018 (17)$$

PDG 2013

- With an effective Lagrangian approach, using data on tau lepton production at LEP1, SLC, and LEP2:

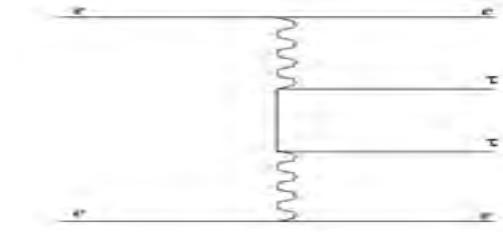
$$-0.004 < a_\tau^{\text{NP}} < 0.006 \quad (95\% \text{ CL})$$

Escribano & Massó 1997

$$-0.007 < a_\tau^{\text{NP}} < 0.005 \quad (95\% \text{ CL})$$

González-Sprinberg et al 2000

- Bernabéu et al, propose the measurement of $F_2(q^2=M_Y^2)$ from $e^+e^- \rightarrow \tau^+\tau^-$ production at B factories. NPB 790 (2008) 160



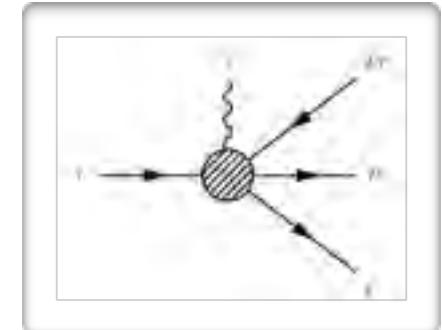
The tau g-2 via tau radiative leptonic decays: a proposal

- SM (unpolarized) tau radiative leptonic decays at LO:

$$\frac{d^3\Gamma}{dx dy d\cos\theta} = \frac{\alpha M_\tau^5 G_F^2 y \sqrt{x^2 - 4r^2}}{2\pi(4\pi)^6} G_0(x, y, \cos\theta, r) \quad x = \frac{2E_l}{M_\tau}, y = \frac{2E_\gamma}{M_\tau}, r = \frac{m_l}{M_\tau}$$

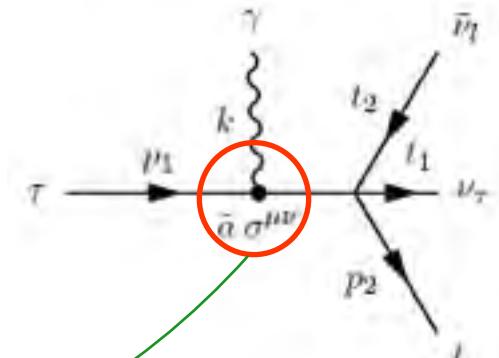
Kinoshita & Sirlin PRL2(1959)177; Kuno & Okada, RMP73(2001)151

$$\begin{aligned} \left. \frac{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10 \text{ MeV}} &= 1.836\% \quad \text{vs} \quad 1.75(18)\% \\ \left. \frac{\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau \gamma)}{\Gamma_{\text{total}}} \right|_{E_\gamma > 10 \text{ MeV}} &= 0.367\% \quad \text{vs} \quad 0.361(38)\% \end{aligned}$$



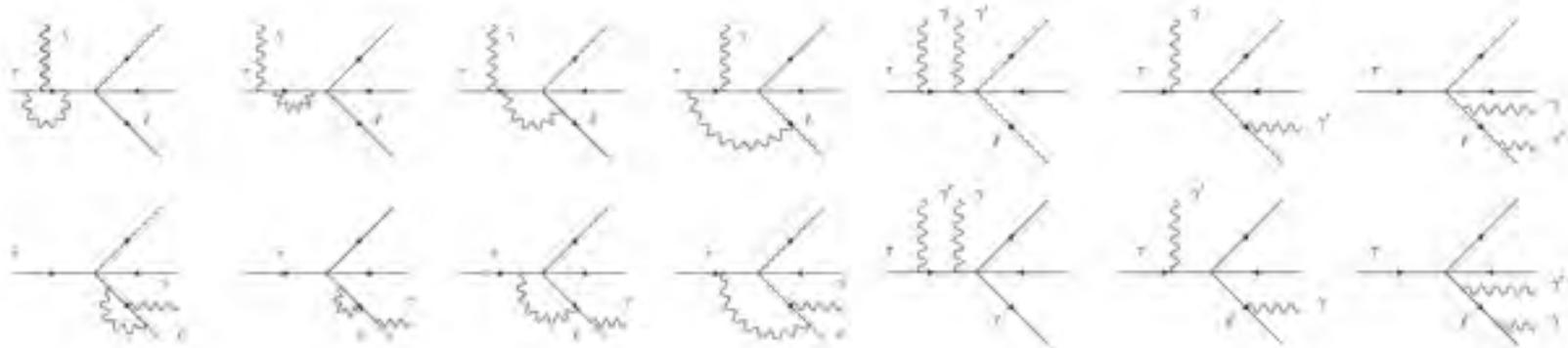
- Add the contributions of the effective g-2 operator and SM corrections:

$$G_0 \rightarrow G_0 + \tilde{a}_\tau G_a + \frac{\alpha}{\pi} G_{RC} + \frac{m_\tau^2}{M_W^2} G_W$$

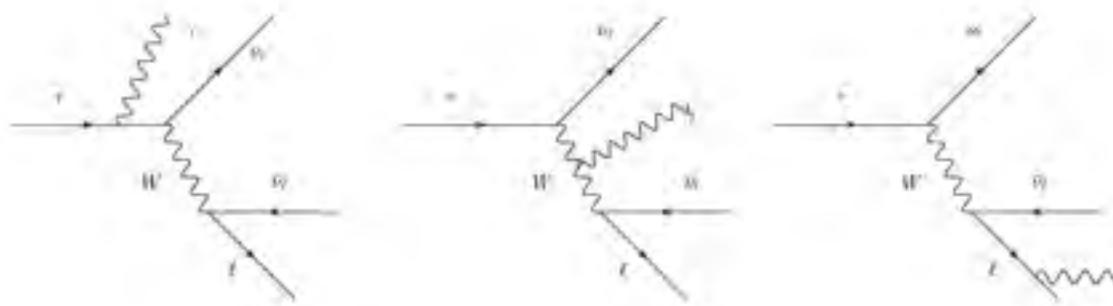


The tau g-2 via tau radiative leptonic decays: a proposal (2)

- Since we aim at determining \tilde{a}_τ with a precision of $O(10^{-3})$, must include QED radiative corrections at NLO Arbuzov & Scherbakova 2004



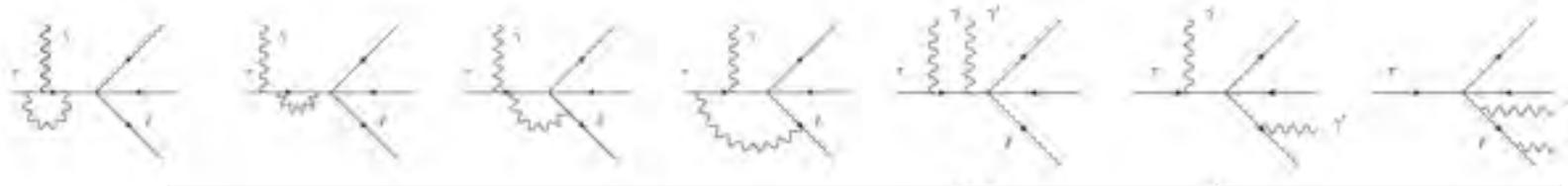
& W-propagator corrections ($m_\tau^2/M_W^2 \sim 0.5 \times 10^{-3}$) Fael, Mercolli, MP 2013



- Add τ polarization and measure $d^6\Gamma$ precisely at Belle II to get \tilde{a}_τ !

The tau g-2 via tau radiative leptonic decays: a proposal (2)

- Since we aim at determining \tilde{a}_τ with a precision of $O(10^{-3})$, must include QED radiative corrections at NLO Arbuzov & Scherbakova 2004



& W-p

WORK IN

PROGRESS

• 2013

- Add τ polarization and measure $d^6\Gamma$ precisely at Belle II to get \tilde{a}_τ !

Conclusions

- ➊ The lepton g-2 provide beautiful examples of interplay between theory and experiment.
- ➋ The discrepancy is $\Delta a_\mu \sim 3 \div 3.5 \sigma$. Is it NP? New g-2 experiment: ring now in Fermilab! QED & EW terms ready for the challenge; work needed for the LO hadronic one. Future of hadronic LBL??
- ➌ Could Δa_μ be due to mistakes in the hadronic $\sigma(s)$? Very unlikely. Also, given a 125 GeV SM Higgs, these hypothetical shifts $\Delta\sigma(s)$ could only occur at very low energies (below ~ 1 GeV).
- ➍ The sensitivity of the electron g-2 has improved. It may soon be possible to test if Δa_μ manifests itself also in the electron g-2! A robust and ambitious exp program is needed to improve α & a_e .
- ➎ The tau g-2 is essentially unknown: we propose to measure it at Belle II via its radiative leptonic decays.

The End