Q. This is a problem on the atomic periodic table.

1. Explain in what sense the basic patterns in the atomic periodic table, with magical periods of two, eight, eighteen, etc., emerge from the rotational symmetry (or the group O(3)) of the Coulomb-like potential and the Pauli exclusion principle.

2. Now imagine a world which has only two spatial directions (i.e., x and y). In such a world, the (attractive) Coulomb potential may be taken to have a logarithmic form, i.e.,

$$V(r) = (\text{const}) \ln \left(\frac{r}{r_0} \right),$$

when $r \equiv \sqrt{x^2 + y^2}$ and r_0 is a suitably introduced scale constant. Then, in this two-dimensional world, discuss how the patterns of the atomic periodic table will change.