Electromagnetic Radiation from an Accelerating Charged Particle

An accelerated, electrically charged, classical particle radiates electromagnetic waves. The energy that comes out with this radiation is known to be proportional, in the nonrelativistic limit, to the square of the acceleration. Naturally, one is lead to believe this lost energy must be accounted for by the kinetic energy loss of the particle itself, and this leads to a radiation damping force, known to be proportional to the time-derivative of the acceleration.

Review these two facts, which can be found in many graduate-level textbook on classical electrodynamics. Compute the radiation energy as a function of the acceleration and, from this, deduce the damping force.

The, consider the charge particle being accelerated uniformly, which means that the acceleration has no time-derivative. In this case, radiation energy remain constant and nontrivial, yet the damping force is zero. Try to explain how this can be consistent with energy conservation, or what kind of computation or reasoning would be needed to understand this strange-looking phenomenon.

The last puzzle is a famous problem that was resolved by many famous people independently in the 2nd half of the 20th century, although the problem itself was known in the early half. None of these resolutions are elementary, so please do not take this problem lightly as a project if you don't know where to start.