Constraints on the (Primordial) Non-Gaussianity Through Galaxy Clustering Topology

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Non-Gaussianity From Large Scale Structures

- LSS arose from primordial density fluctuations during inflation (Bardeen, Steinhardt & Turner 1983): the primordial field fluctuation is described statistically by a (*nearly*) *Gaussian* random field.
- Departure from the Gaussianity (Non-Gaussianity, NG) in the observed LSS:
    - NG in the initial density field: primordial NG
    - Non-linear gravitational evolution
    - Galaxy biasing
    - Shot noise, redshift-space distortion, survey mask, etc.

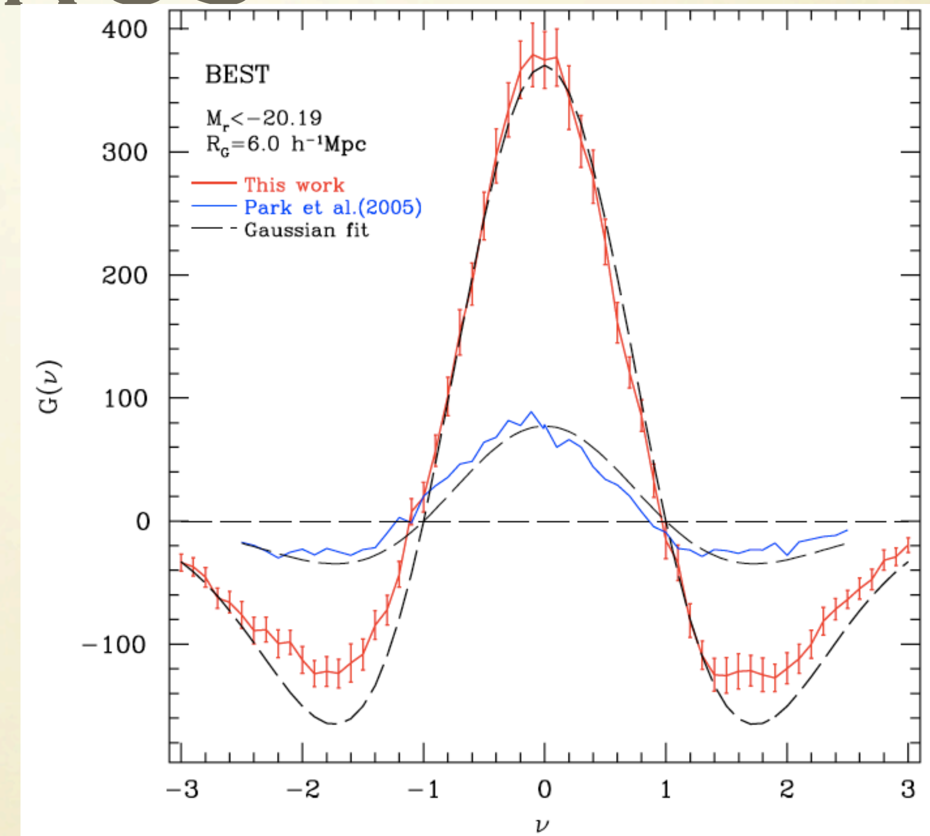
Why study Topology?

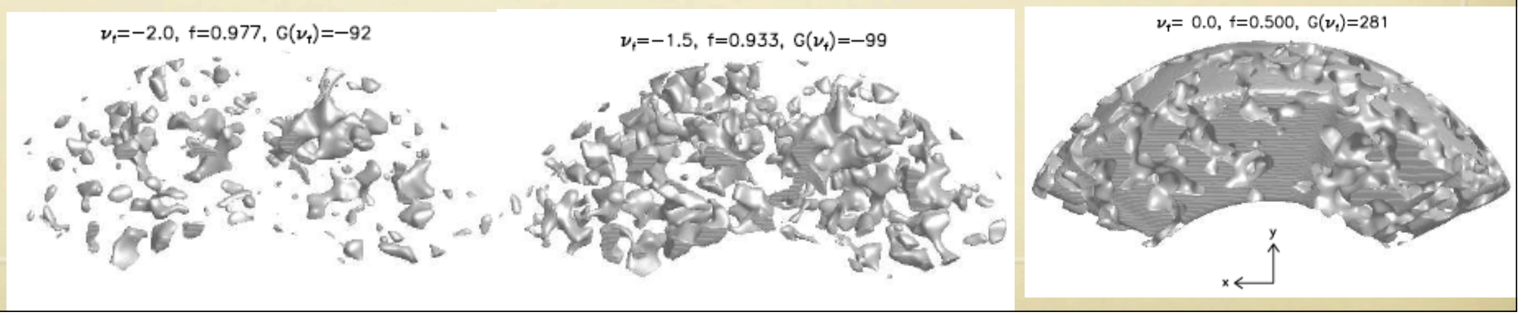
- **Intuitive measurement**: the degree of connectivity of the smoothed matter distribution in the Universe.
- **Easy to measure**: Integration of local curvature of a surface is related with its topological ***genus***.
- **Known topology** for the Gaussian fields: a good NG measure.
- **Relatively insensitive** to non-linear gravitational evolution, redshift distortion, and galaxy biasing: topology is independent of monotonic deformation of shape.

Topology measure: Genus

- A measure of the degree of connectivity of the smoothed galaxy density field.
- G = # of holes in iso-density contour surface - # of isolated regions (Gott et al. 1986, etc.)
- Gaussian field:

$$G(\nu) = \frac{1}{(2\pi)^2}\left(\frac{\sigma_1}{\sqrt{3}\,\sigma_0}\right)^3 e^{-\nu^2/2}(1-\nu^2)$$





Non-Gaussianity

Kim, Choi, et al. (2014) ApJS submitted

- Non-linear gravitational or local f_{NL}-type primordial non-Gaussianity (Matsubara 1993; Matsubara 2003; Hikage et al. 2006)

$$G(\nu)=\frac{1}{(2\pi)^2}\left(\frac{\sigma_1}{\sqrt{3}\,\sigma_0}\right)^3 e^{-\nu^2/2}\left(1-\nu^2+[(S^{(1)}-S^{(0)})(\nu^3-3\nu)+(S^{(2)}-S^{(0)})\nu]\sigma_0\right)$$

- Finite pixel size (Hamilton et al. 1986)

$$C=-\frac{1}{\pi}\left[\frac{-\xi''(0)}{\xi(0)}\right]^{3/2} e^{-\nu^2/2}\left[1-\nu^2-\frac{r_{12}^2\xi''(0)}{256\xi(0)}\left\{19(3-6\nu^2+\nu^4)-26\frac{\xi(0)\xi^{(4)}(0)}{\xi''(0)^2}(1-\nu^2)+\left[\frac{\xi(0)\xi^{(4)}(0)}{\xi''(0)^2}\right]^2\right\}\right]$$

- Shot noise

$$G=\frac{1}{4\pi}\left[\frac{<k^2>}{3}+\frac{\sigma_{\rm shot}^2}{\sigma_{\rm true}^2+\sigma_{\rm shot}^2}\left(\frac{1}{2R_G^2}-\frac{<k^2>}{3}\right)\right]^{3/2}$$

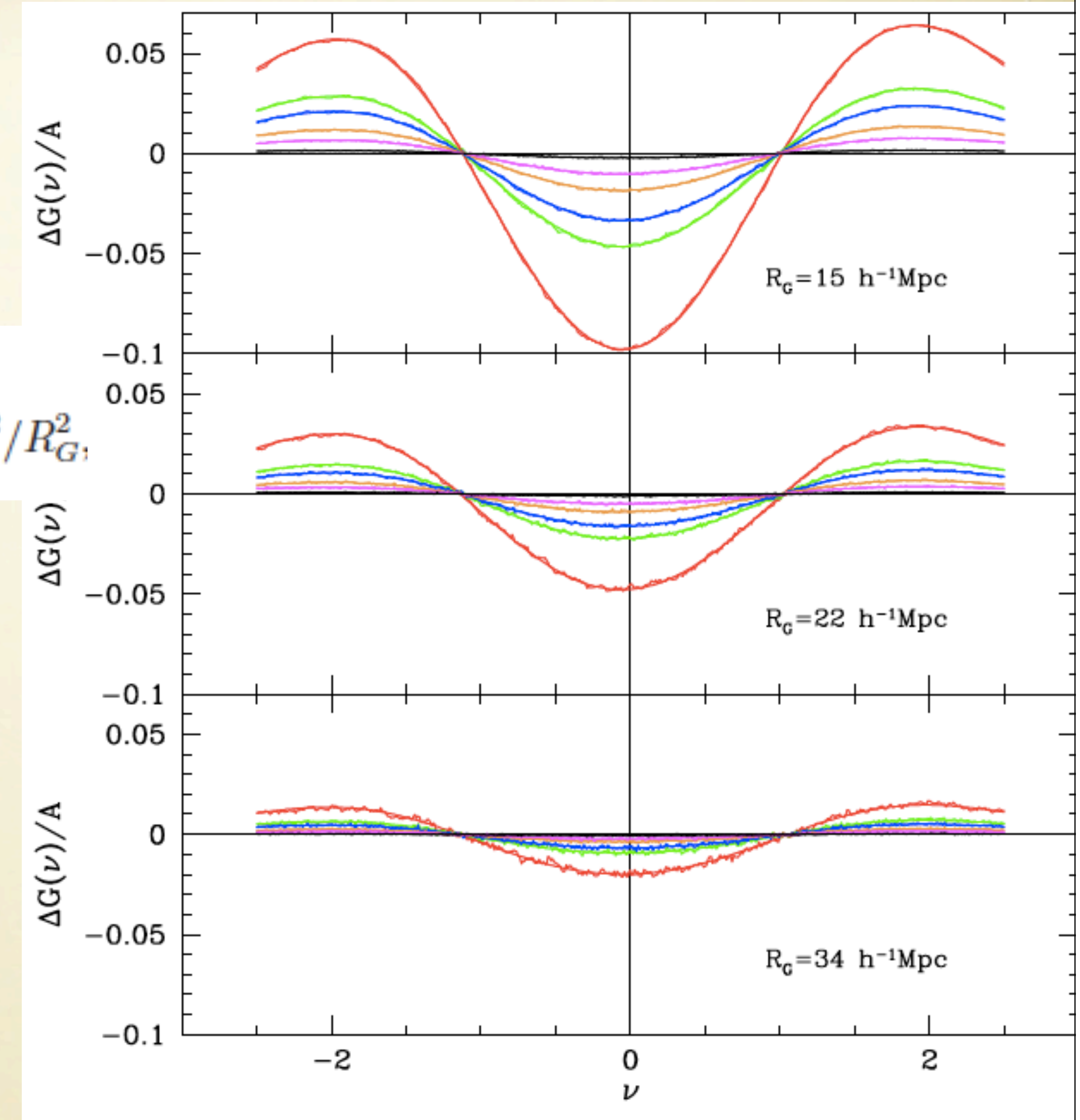
- Redshift-space distortion (Matsubara 1996)

$$G^{(s)}(\nu)=\frac{3\sqrt{3}}{2}\sqrt{C}(1-C)G^{(r)}(\nu)$$

- Galaxy/Halo bias effects (Park & Gott 1991; Park et al. 2005; Choi et al. 2010)

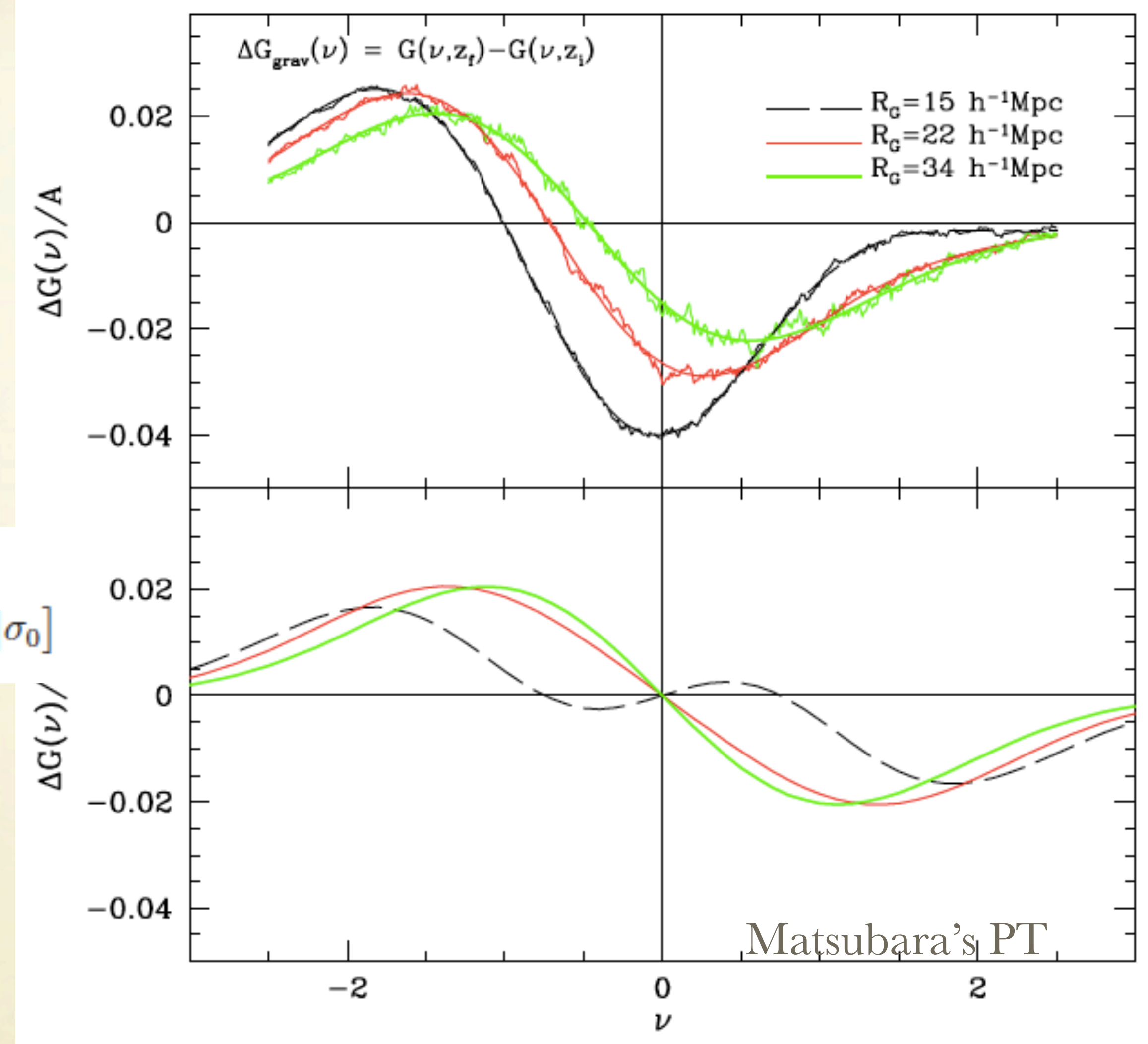
- Finite Pixel Size Effects
  - HR2 (WMAP5, 6000^3, $7200h^{-1}$Mpc) Matter density at z=0

$$\Delta G_{\rm pixel}(\nu) = Ae^{-\nu^2/2} \times [aH_0(\nu) + bH_1(\nu) + cH_2(\nu) + dH_4(\nu)]p^2/R_G^2$$



- Non-linear Gravitational Evolution
- HR2 (WMAP5, 6000^3, $7200h^{-1}$Mpc) Matter density at z=0 & ∞

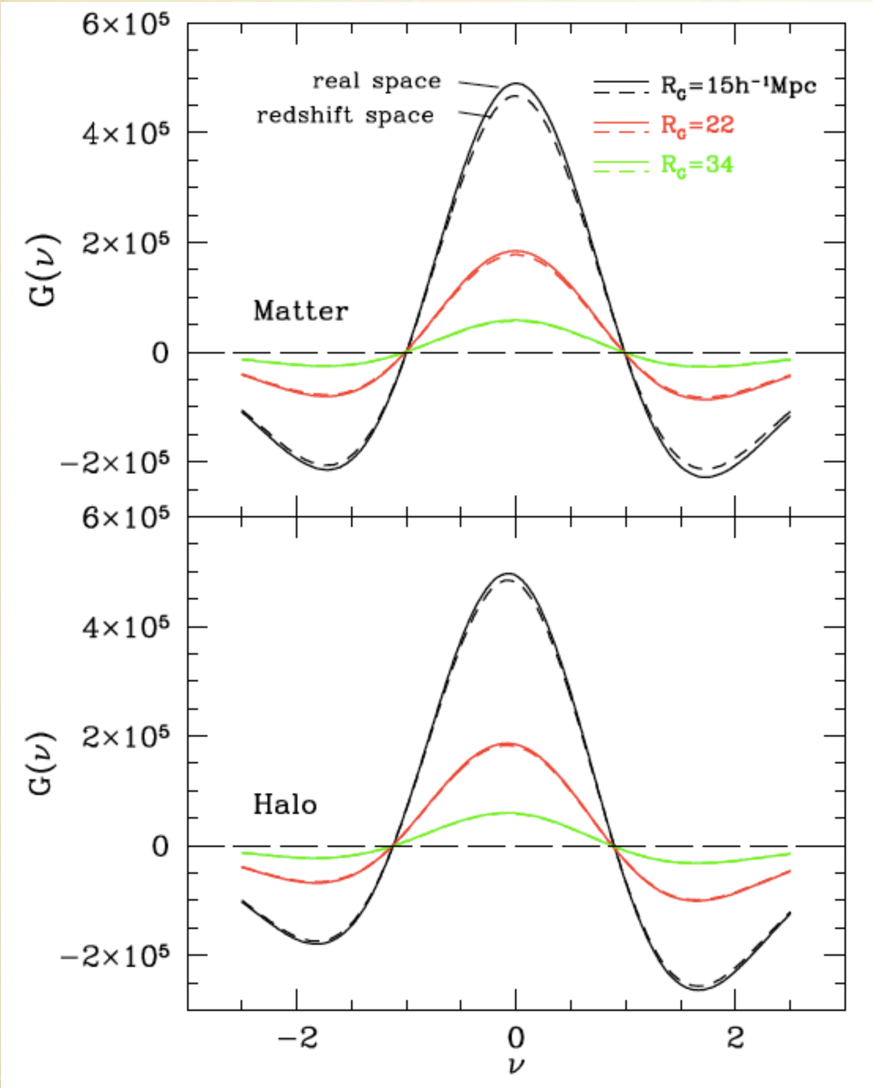
$$G(\nu) = -Ae^{-\nu^2/2} \times$$
$$[H_2(\nu) + [(S^{(1)} - S^{(0)})H_3(\nu) + (S^{(2)} - S^{(0)})H_1(\nu)]\sigma_0]$$

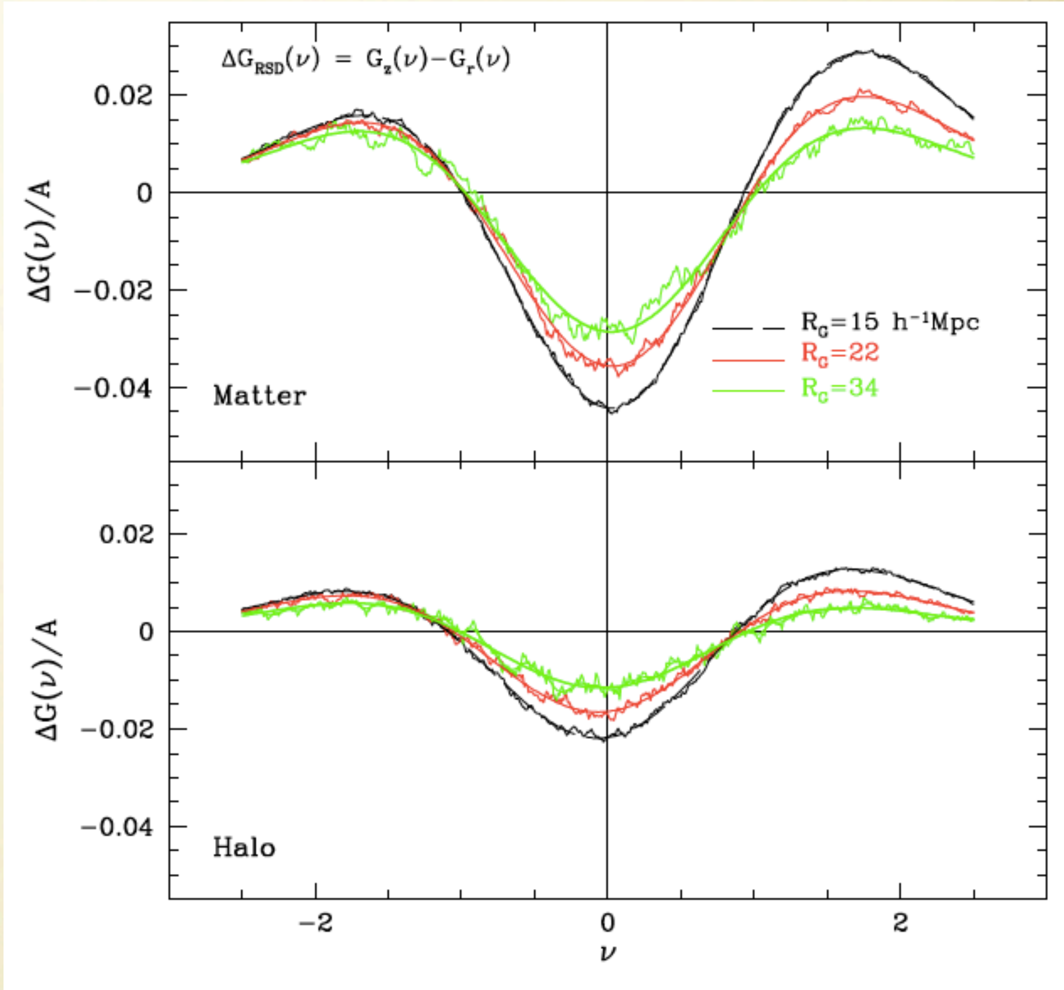


$$\Delta G_{grav}(\nu) = Ae^{-\nu^2/2} \times$$
$$[(bH_1(\nu) + dH_3(\nu))\sigma_0 + (aH_0(\nu) + cH_2(\nu))\sigma_0^2].$$

Redshift-space Distortion

- HR2 (WMAP5, 6000^3, $7200h^{-1}$Mpc) Matter density at z=0. cf. Halo density at z=0



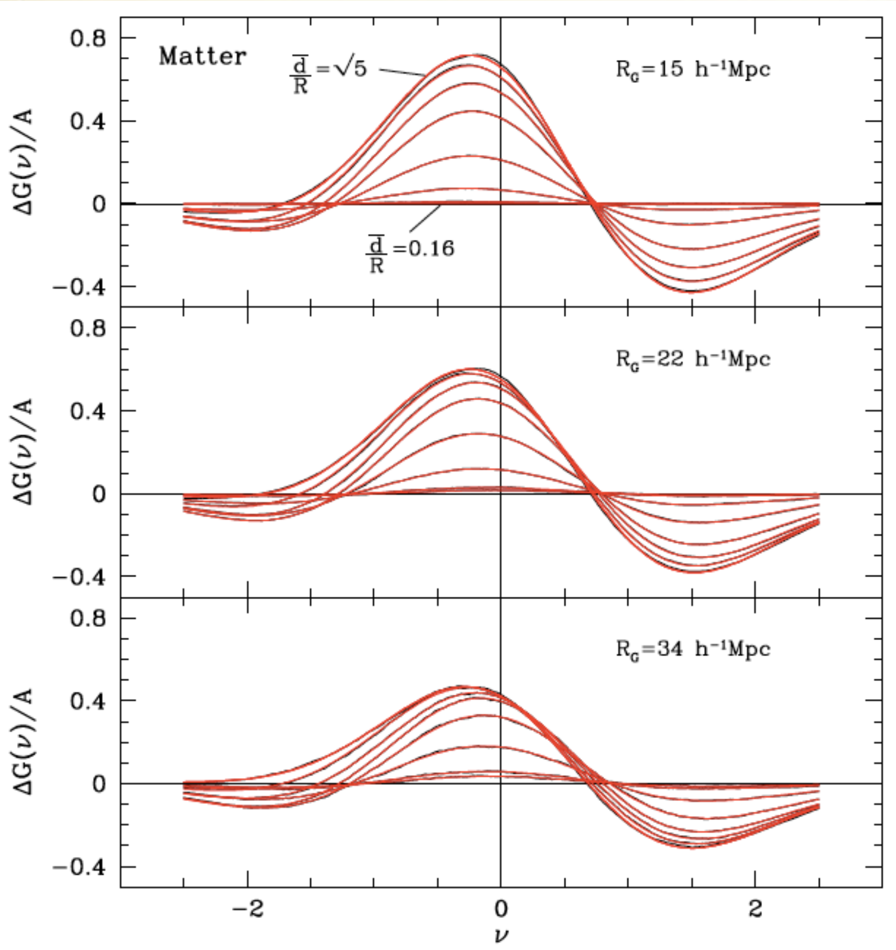


$$\Delta G_{\rm RSD}(\nu) = Ae^{-\nu^2/2} \times [aH_0(\nu) + bH_1(\nu) + cH_2(\nu) + dH_3(\nu)]$$

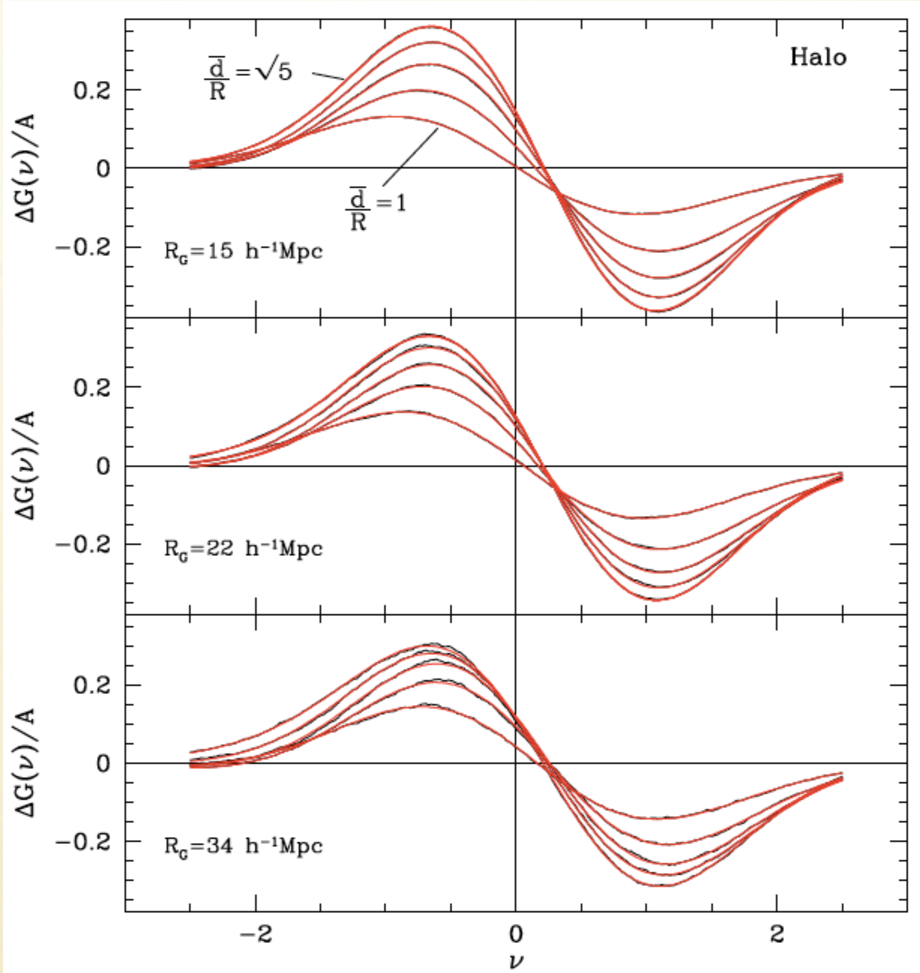
Shot Noise & Halo Biasing

- HR2 (WMAP5, 6000^3, $7200h^{-1}$Mpc) Matter density & Halo density at z=0

- Discrete sampling of underlying density at finite number of points



$$\Delta G_{\rm h,shot}(\nu, \bar{d}/R_G) = G_{\rm h}(\nu, \bar{d}/R_G) - G_{\rm m}(\nu, \bar{d}/R_G = 0).$$



- Most of the systematics comes from both the nonlinearity and stochasticity of halo biasing.

$$
\begin{aligned}
\Delta G_{\rm pixel}(\nu) &= Ae^{-\nu^2/2} \times \\
&\quad [aH_0(\nu) + bH_1(\nu) + cH_2(\nu) + dH_4(\nu)]p^2/R_G^2 \\
\Delta G_{\rm grav}(\nu) &= Ae^{-\nu^2/2} \times \\
&\quad [(bH_1(\nu) + dH_3(\nu))\sigma_0 + (aH_0(\nu) + cH_2(\nu))\sigma_0^2] \\
\Delta G_{\rm RSD}(\nu) &= Ae^{-\nu^2/2} \times \\
&\quad [aH_0(\nu) + bH_1(\nu) + cH_2(\nu) + dH_3(\nu)] \\
\Delta G_{\rm h,shot}(\nu) &= Ae^{-\nu^2/2} \times \\
&\quad [aH_0(\nu) + bH_1(\nu) + cH_2(\nu) + dH_3(\nu) + eH_4(\nu)]
\end{aligned}
$$

All systematic effects can be modeled by a sum of H_i up to i=4.

Mock Galaxy SAmple

- Very large volume simulation & Galaxy formation model
  - Horizon Run 3 (initially Gaussian ΛCDM model, WMAP 5, 7210^3 particles, $10.815h^{-1}$Gpc box): 27 all-sky surveys along the past light cone in redshift-space, having nearly non-overlapping survey volumes: *realistic* uncertainties due to cosmic variance.
  - Gravitationally bound subhalo finding (Kim & Park, 2006) and subhalo abundance matching (the most massive subhalos are identified as mock galaxies.)

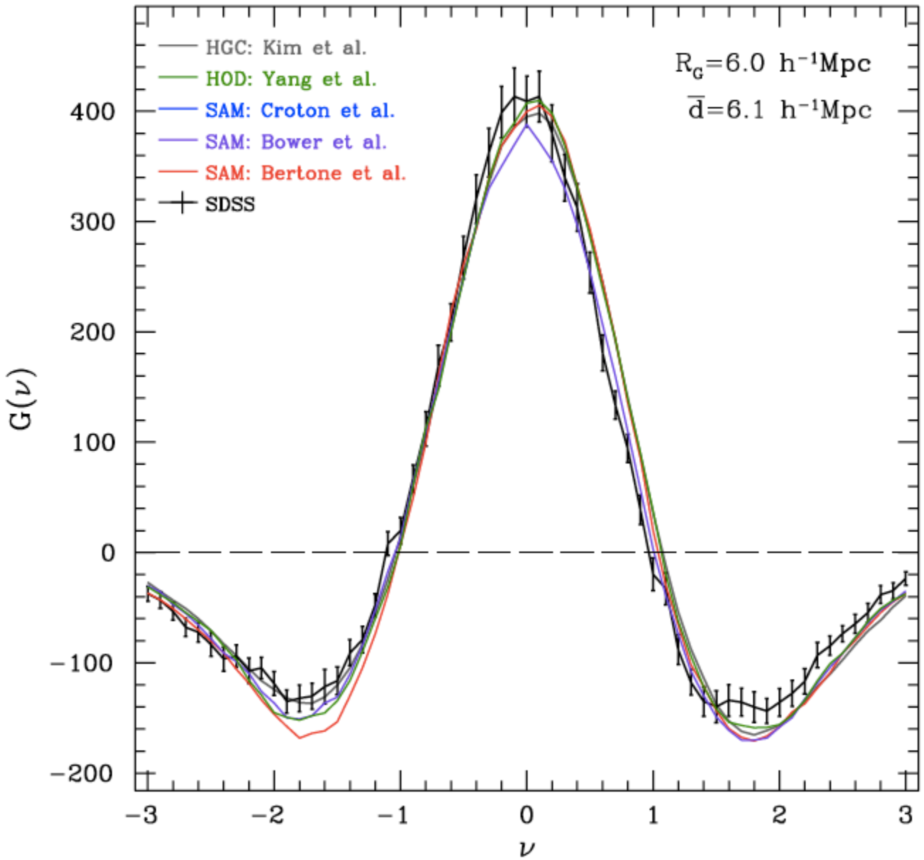
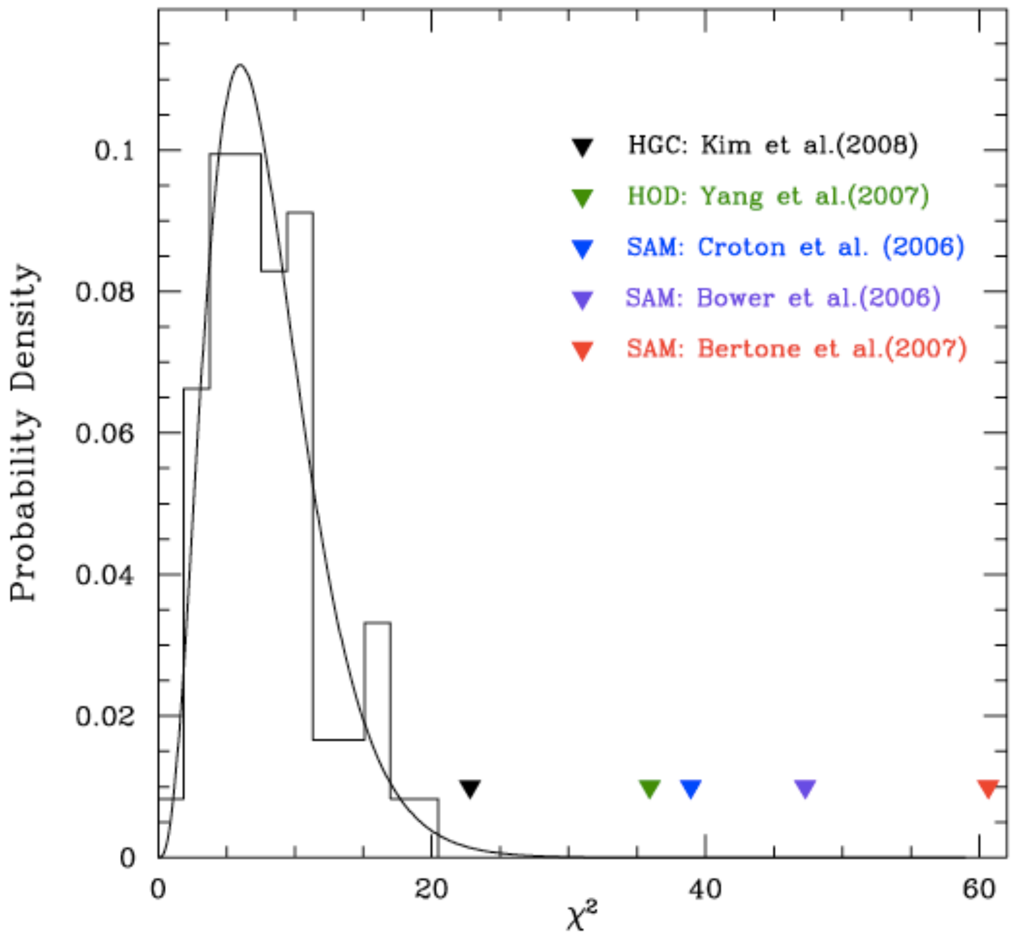
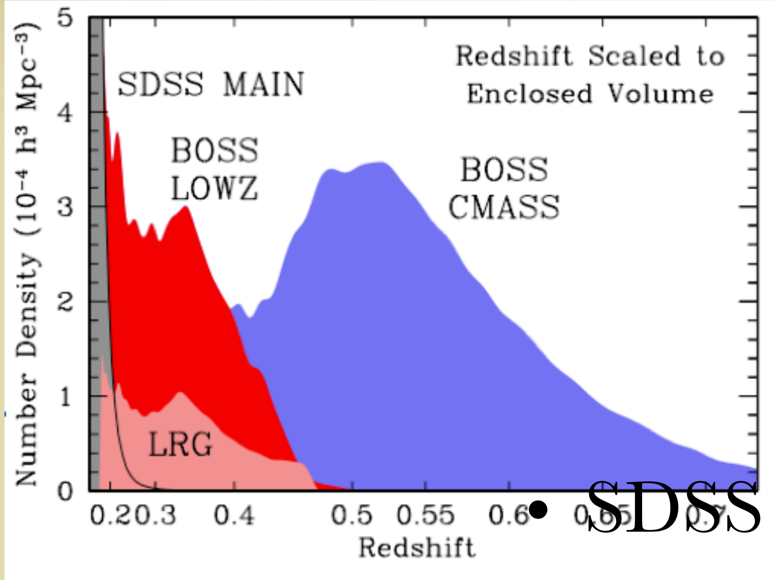


Figure 15. Genus curves for the SDSS galaxies (black solid line with error bars) in the BEST sample and five sets of mock galaxies. The curve for the observed sample is corrected for the systematic biases.

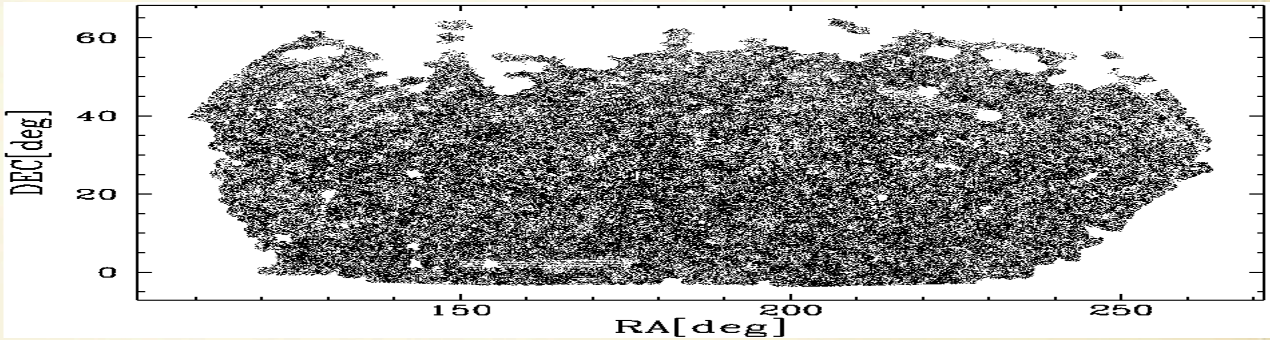




Data

- SDSS DR7 spectroscopic LRG sample
  - A volume-limited sample covering ~17% of sky : $0.16<z<0.36$, $-21.2<M_g<-23.2$, ~62k galaxies, mean separation of $22h^{-1}$Mpc
- The Baryon Oscillation Spectroscopic Survey **C**onstant**MASS** Sample
  - $0.43<z<0.7$, extending LRG CutII to more *fainter* and *bluer* galaxies. More complete sample at high stellar masses. A volume-limited Sample: 297,396 galaxies with $M_i<-21.4$ and $0.45<z<0.589$, galaxy mean separation: $16h^{-1}$Mpc

BOSS



- the Baryon Oscillation *Spectroscopic* Survey
  - SDSS-III (2008 July - 2014 July)
  - survey area- 10,000 deg^2
  - 1000 fibers per plate, $R=\lambda/\Delta\lambda$=1300-3000, 900s exp.
  - Targets: 1.5×10^6 massive galaxies, z<0.7, i<19.9

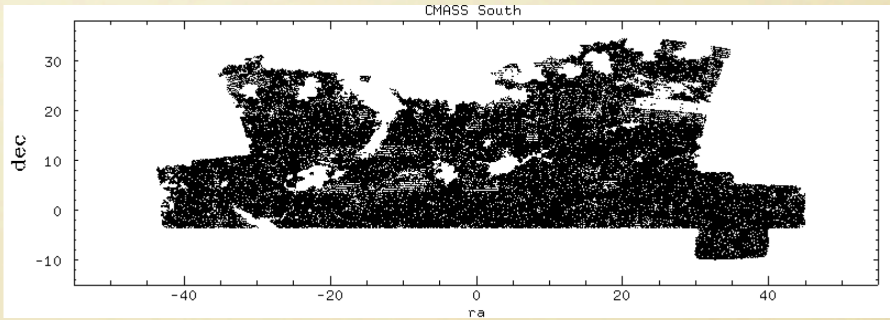
    1.5×10^5 quasars, z>2.2, g<22.0 (20% of fiber)

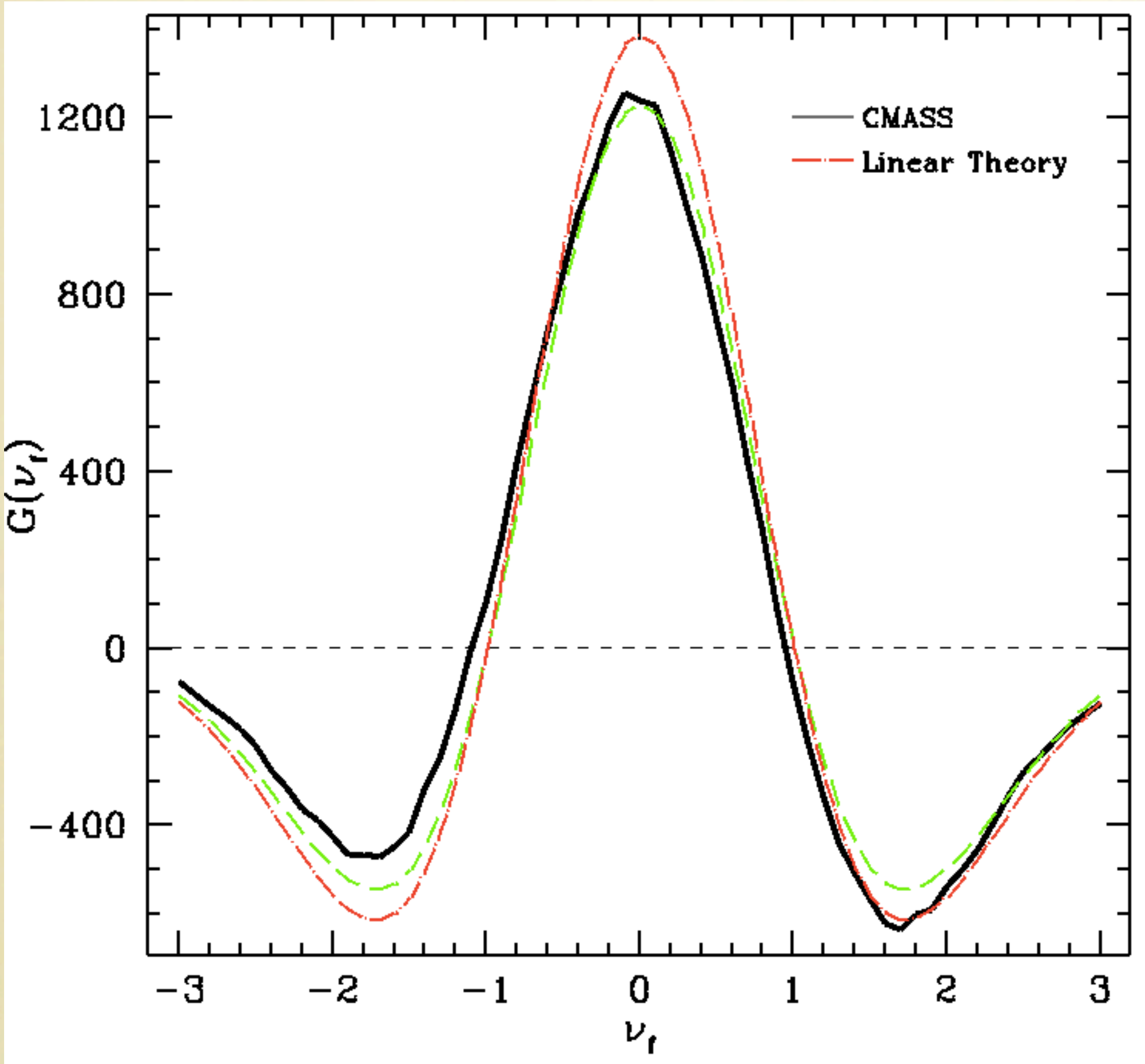
    100,000 ancillary targets

DR11 (2014 July) CMASS

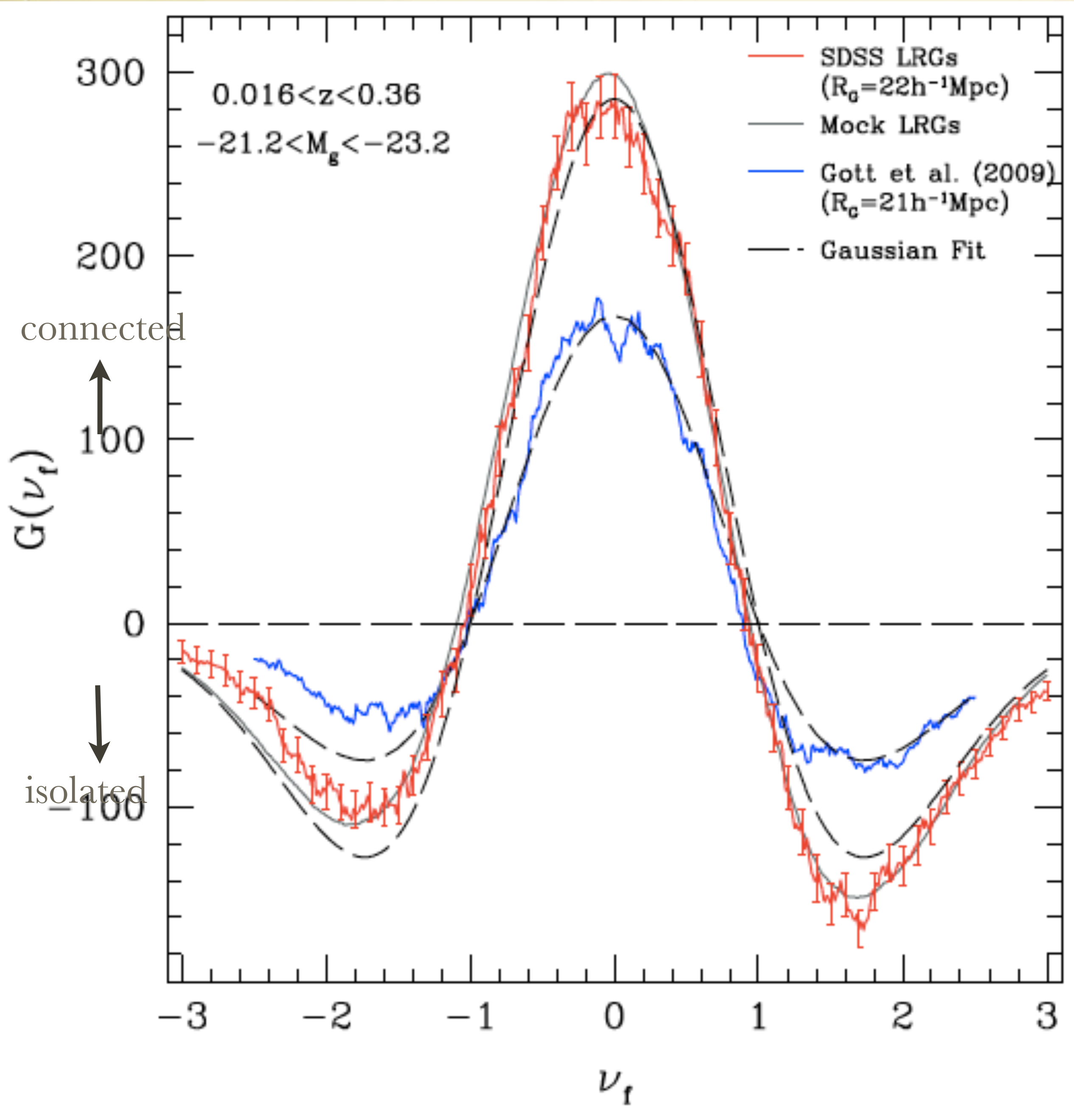
North Galactic Cap: 0.63million galaxies from 6846 deg^2 (DR12: 7600 deg^2)

South Galactic Cap: 0.19 million galaxies from 2020 deg^2 (DR12: 3100 deg^2)





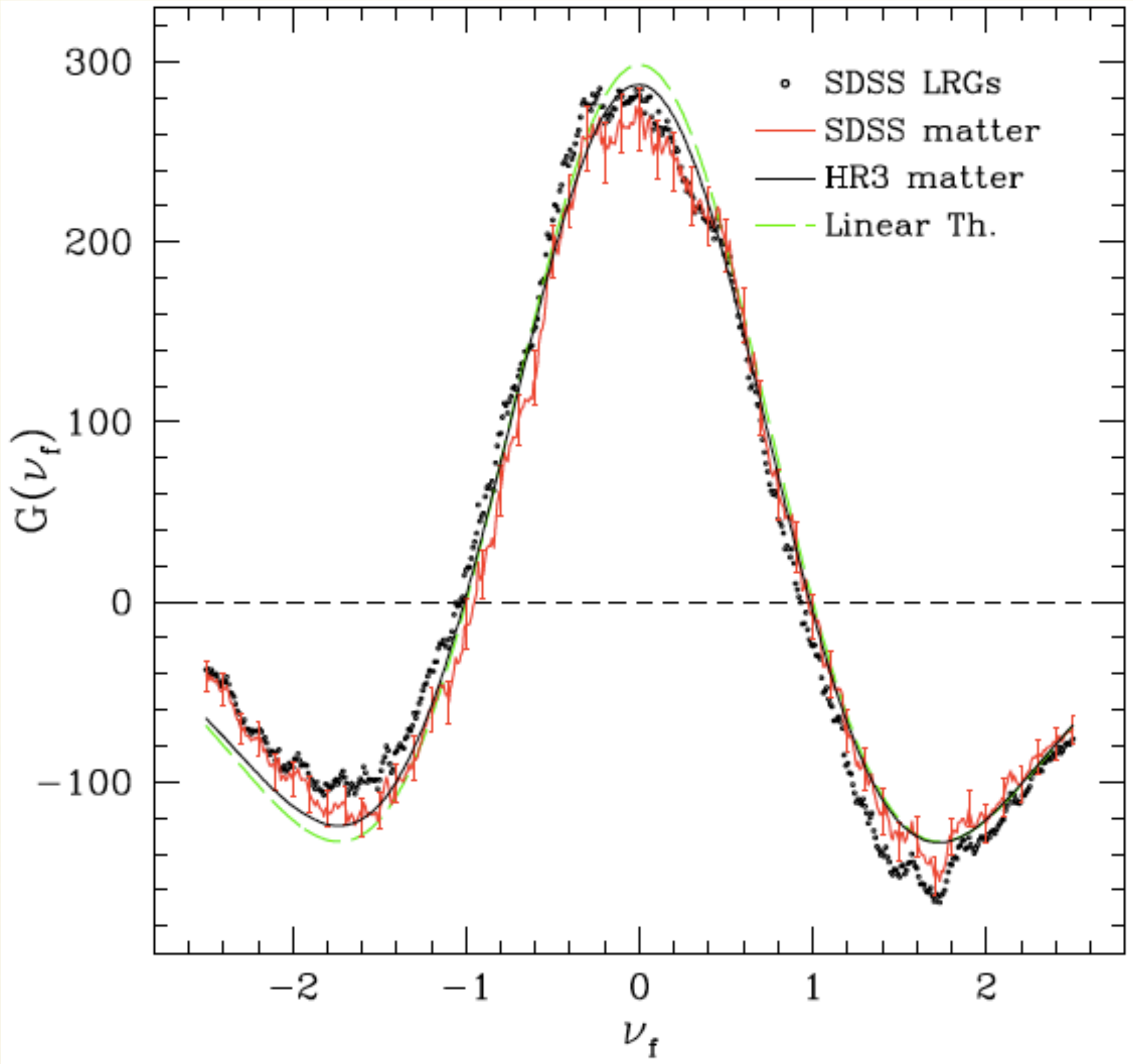
CMASS
Linear Theory
1200
800
400
0
−400
$G(\nu_f)$
−3
−2
−1
0
1
2
3
ν_f



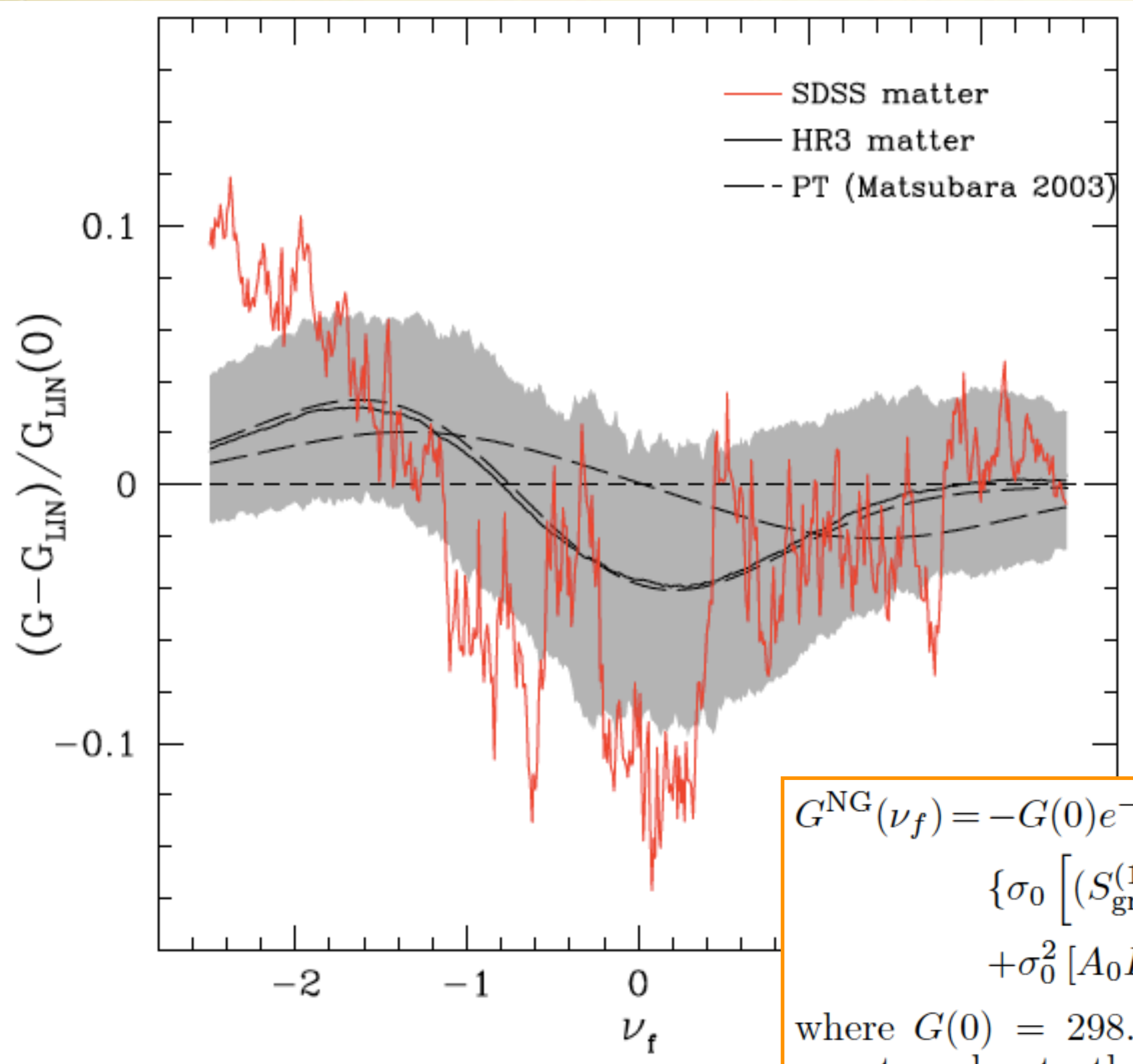
high density

Our initially Gaussian ΛCDM model + galaxy formation model successfully reproduce the observed topology of LRGs at 22h^{-1}Mpc scales except for the void abundance in very low density regions filling ~3.5% of survey volume.

Where does the non-Gaussianity come from?



SDSS LRGs
SDSS matter
HR3 matter
Linear Th.
300
200
100
0
−100
−2
−1
0
1
2
$G(\nu_f)$
ν_f



The discrepancy: the different biasing schemes of galaxy formation in between the simulation and the real universe, and **cosmic variance**.

$$G^{\rm NG}(\nu_f) = -G(0)e^{-\nu_f^2/2} \times \{\sigma_0 \left[(S^{(1)}_{\rm gr} - S^{(0)}_{\rm gr})H_3(\nu_f) + (S^{(2)}_{\rm gr} - S^{(0)}_{\rm gr})H_1(\nu_f)\right] + \sigma_0^2 \left[A_0 H_0(\nu_f) + A_2 H_2(\nu_f)\right]\}, \quad (27)$$

where $G(0) = 298.5$, $\sigma_0 = 0.00837$, the skewness parameters due to the non-linear gravitational evolution, $S^{(0)}_{\rm gr} = 3.422$, $S^{(1)}_{\rm gr} = 3.472$ and $S^{(2)}_{\rm gr} = 3.695$ which are derived from Equation (13)-(16). Coefficients A_0 and A_2

Measuring Primordial Non-Gaussianity

- f_{NL}: standard parameterization of the primordial non-Gaussianity when the local type non-Gaussianity is assumed - amplitude of a quadratic correction to the potential, ϕ,

$$\Phi = \phi + f_{NL}(\phi^2 - \langle \phi^2 \rangle)$$

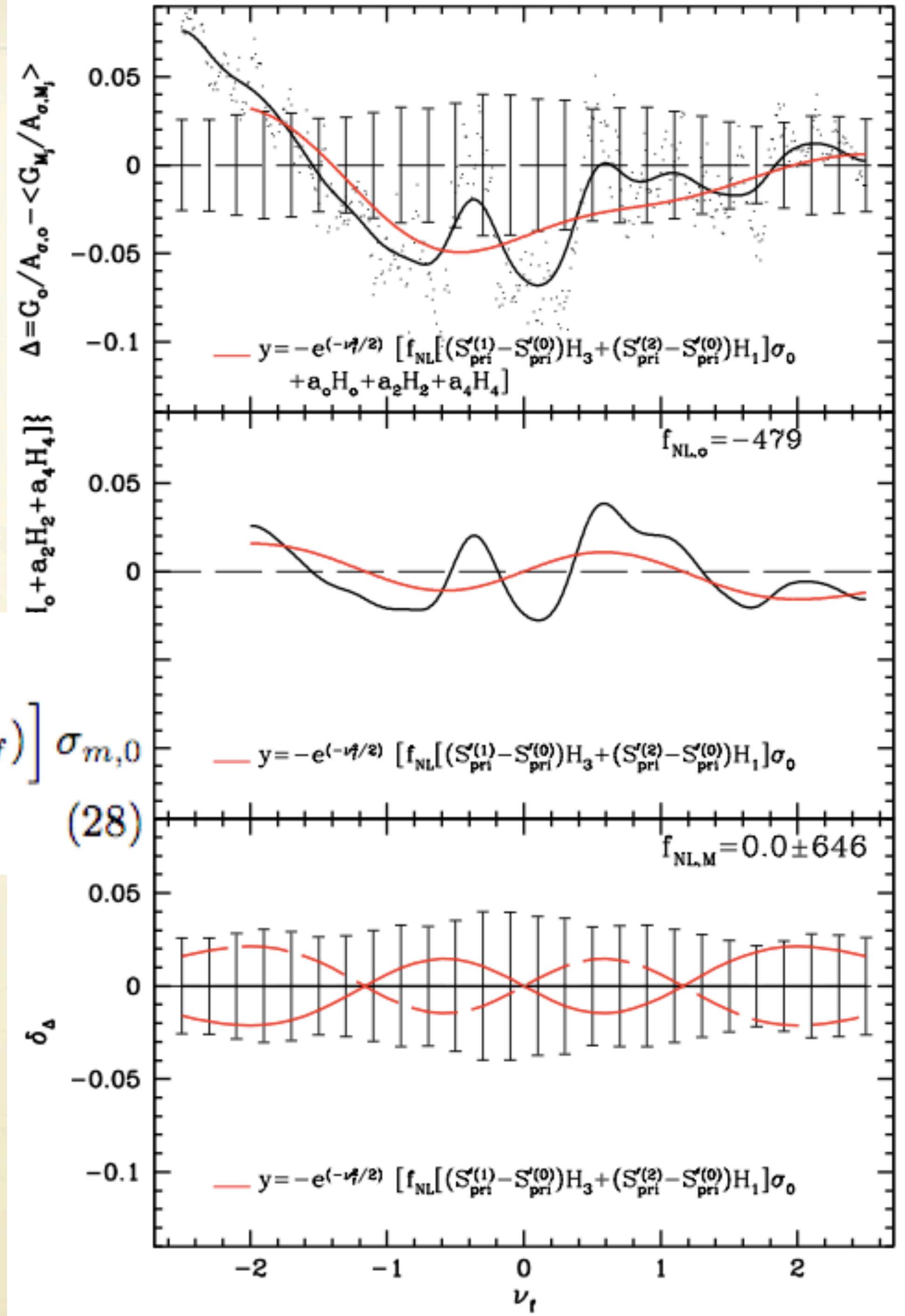
  - Typical value of f_{NL} for standard slow roll inflation is of order 10^{-2}

- For CMB, $-10 < f_{NL} < 74$ (WMAP 7, 95% confidence, Komatsu et al. 2011)
- For LSS, $-29 < f_{NL}$ f<70 (95% conf. combination of galaxy & quasar clustering measurements, Slosar et al. 2008)
- For LSS of SDSS photometric LRG, $-268 < f_{NL}$ f<164 (Slosar et al. 2008), $-81 < f_{NL} < 351$ (Xia et al. 2011), $-168 < f_{NL}$ f<364 (Ross et al. 2012; SDSS DR9 CMASS sample)

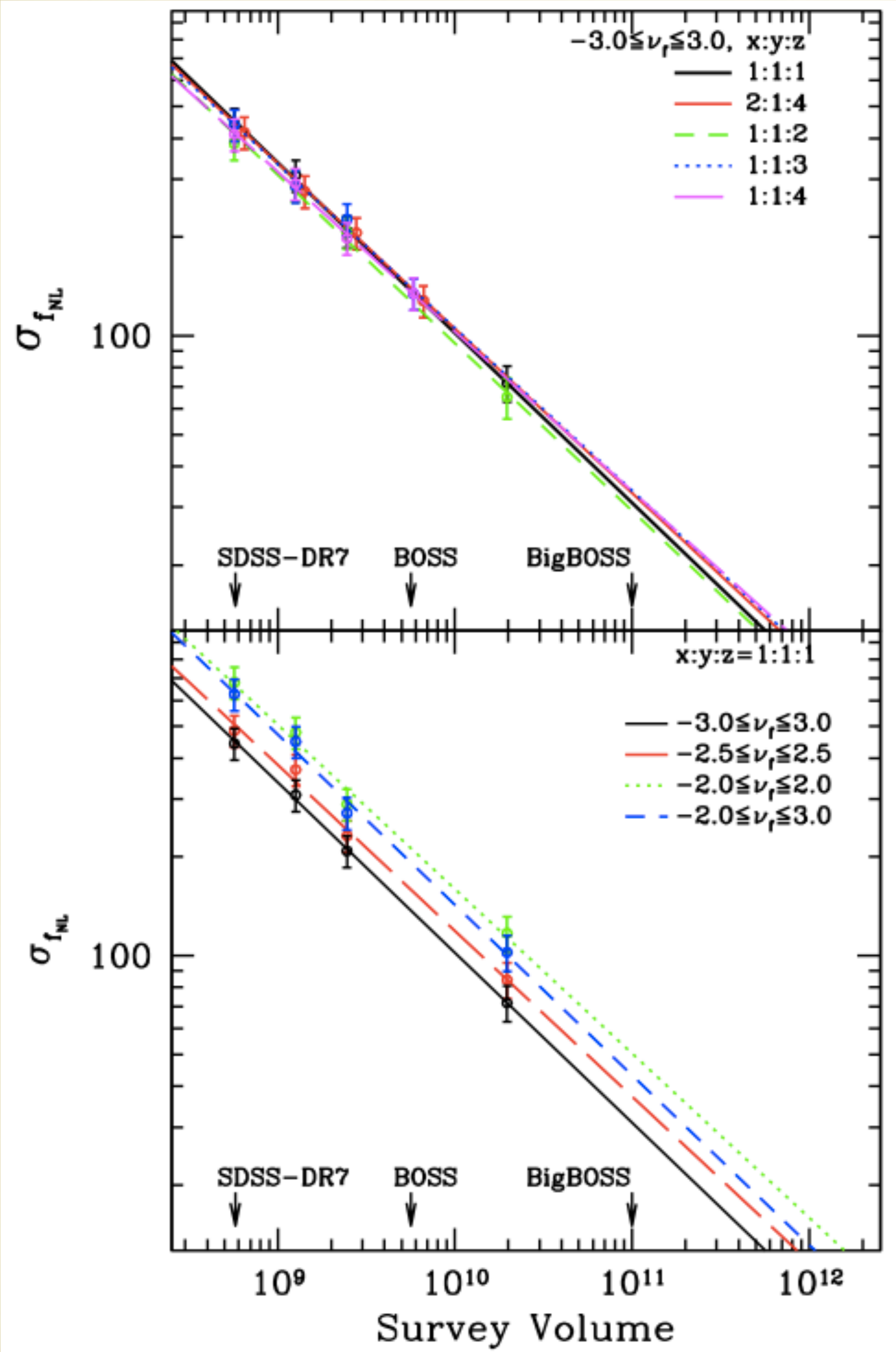
Constraints on the Primordial Non-Gaussianity via Topology-based Method

$$\Delta = G_o/A_{\sigma,o} - \frac{\sum_{i=1}^{N} G_{M_j}/A_{\sigma,M_j}}{N}$$

$$\Delta(\nu) = -e^{-\nu^2/2} \times$$
$$\{f_{NL}\left[(S'^{(1)}_{pri} - S'^{(0)}_{pri})H_3(\nu_f) + (S'^{(2)}_{pri} - S'^{(0)}_{pri})H_1(\nu_f)\right]\sigma_{m,0}$$
$$+a_0H_0(\nu) + a_2H_2(\nu) + a_4H_4(\nu)\}. \quad (28)$$



Δ: the perturbation predictions without any corrections of systematics (ref. Hikage et al. 2006, 2008)

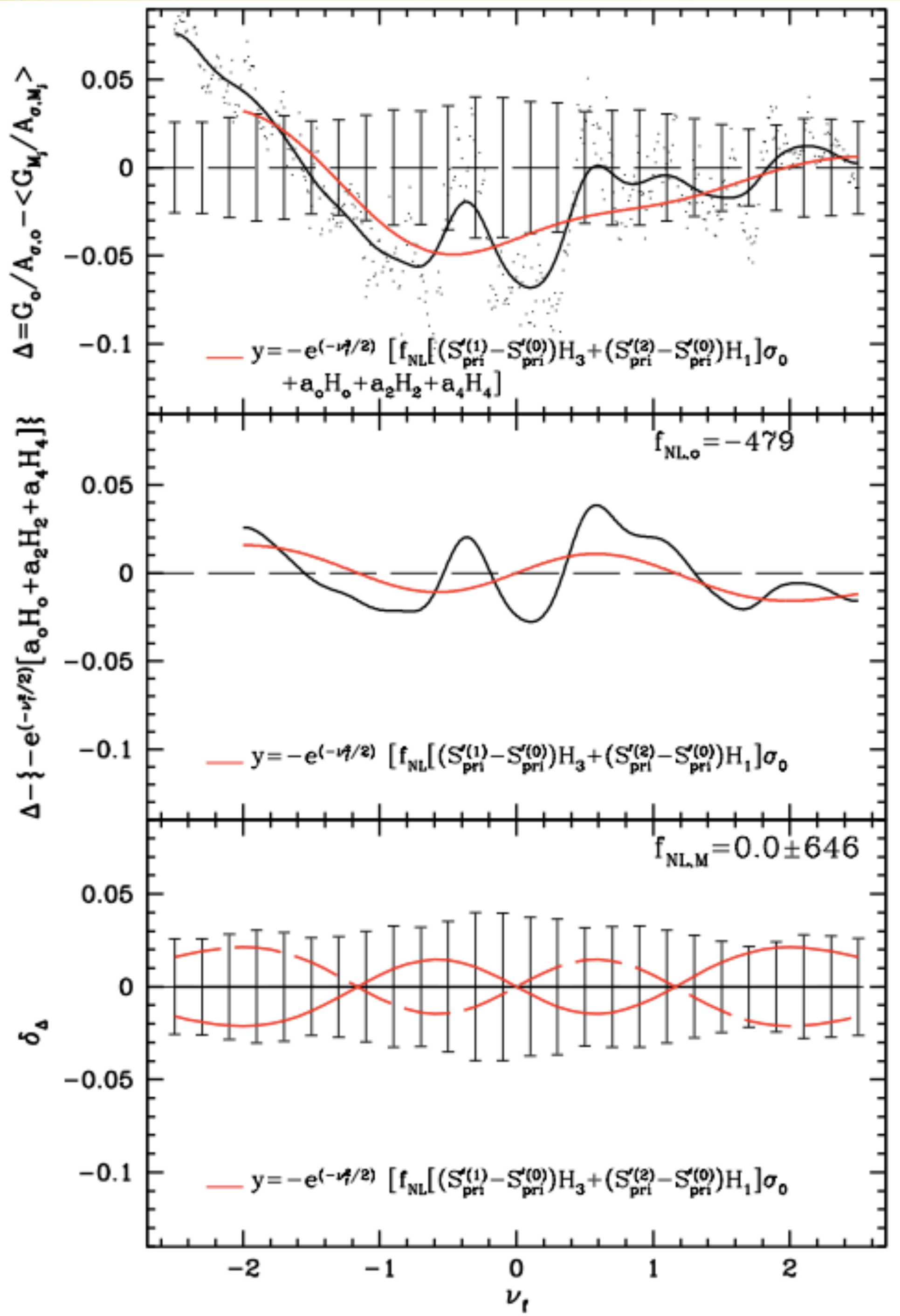


Measurement error of F_{NL} Due to the cosmic variance

- The volume of the SDSS DR9 sample (Ross et al. 2012) will approximately 3.2 times as large as that of the DR7 LRG sample and thus approximately halves the statistical uncertainty: from ~550 (when -3<ν<3) to *~270*: for the Ross et al. 2012, $-82 < f_{NL} < 245$ (68% conf) → isn't is too optimistic?

Summary

- Our initially Gaussian ΛCDM model + galaxy formation model successfully reproduce the observed topology of LRGs at $22h^{-1}$Mpc scales except for the void abundance in very low density regions filling 3.5% of the survey volume.
- Accurate estimation of systematic effects on the genus.
- Constraint on local-type f_{NL} from SDSS DR7 LRG with Δf_{NL} ~550 (68% conf.): the uncertainty limit will be Δf_{NL} ~130 for the final BOSS LRG sample.
- Cosmic variance is the crucial limitation in constraining primordial non-Gaussianity via topology-based methods.
- We obtain realistic uncertainties by using the largest simulation. Our topology-based results suggest that tighter constraints on non-Gaussianity from LSS previously quoted in the literature may be too optimistic and thus severely underestimating the contribution of cosmic variance is needed.



$\Delta = G_o/A_{\sigma,o} - \langle G_{M_i}/A_{\sigma,M_i} \rangle$
$y = -e^{(-\nu_f^2/2)}\,[f_{NL}[(S'^{(1)}_{pri} - S'^{(0)}_{pri})H_3 + (S'^{(2)}_{pri} - S'^{(0)}_{pri})H_1]\sigma_0 + a_0H_0 + a_2H_2 + a_4H_4]$
$\Delta - \{-e^{(-\nu_f^2/2)}[a_0H_0 + a_2H_2 + a_4H_4]\}$
$f_{NL,o} = -479$
$y = -e^{(-\nu_f^2/2)}\,[f_{NL}[(S'^{(1)}_{pri} - S'^{(0)}_{pri})H_3 + (S'^{(2)}_{pri} - S'^{(0)}_{pri})H_1]\sigma_0$
δ_Δ
$f_{NL,M} = 0.0 \pm 646$
$y = -e^{(-\nu_f^2/2)}\,[f_{NL}[(S'^{(1)}_{pri} - S'^{(0)}_{pri})H_3 + (S'^{(2)}_{pri} - S'^{(0)}_{pri})H_1]\sigma_0$
ν_f