ER=EPR,
change of EE & S-matrix

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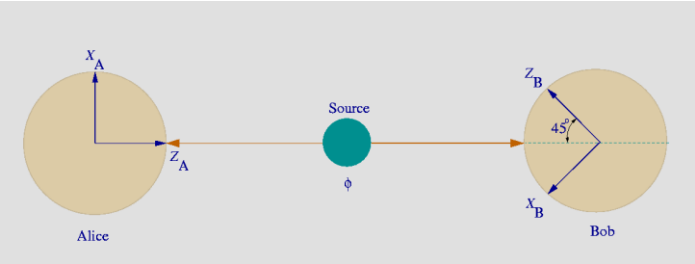
PLAN

- Motivation: Entanglement is least understood quantum phenomena.
- EPR/ ER/ AMPS
- Maldacena–Susskind
- Karch-Jesen / Sonner
- Seki-Sin

Einstein-Podolski-Rosen

- Measurement of A → state collapses → determine other side. Determine the M of B.



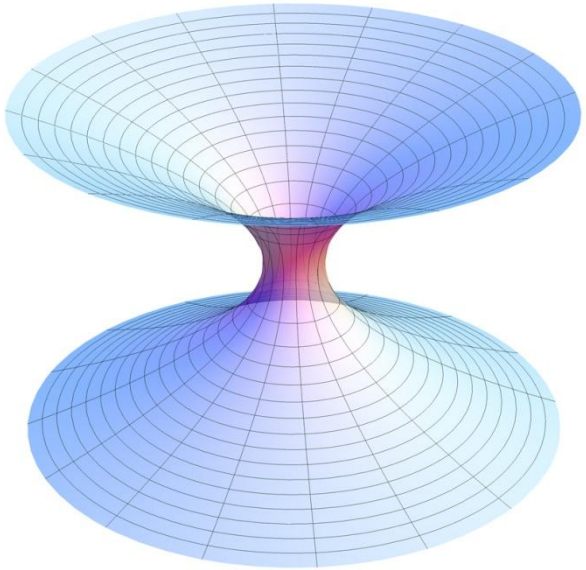
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle \otimes |-z\rangle - |-z\rangle \otimes |+z\rangle \right)$$

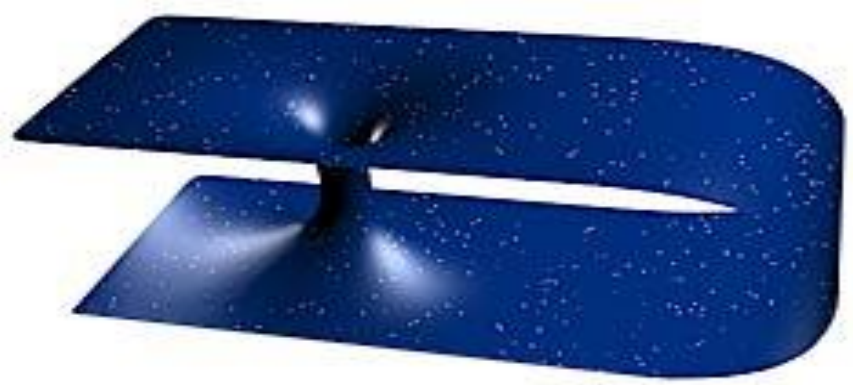
- Non-locality in QM or
  Hidden variable (Info there from the beginning) → Bell's inequality → experiment disproved.

- No information transfer.
  Causality is OK although Locality is lost.

Einstein-Rosen Bridge (wormhole)

- ER= (BH+BH) : space-time short cut



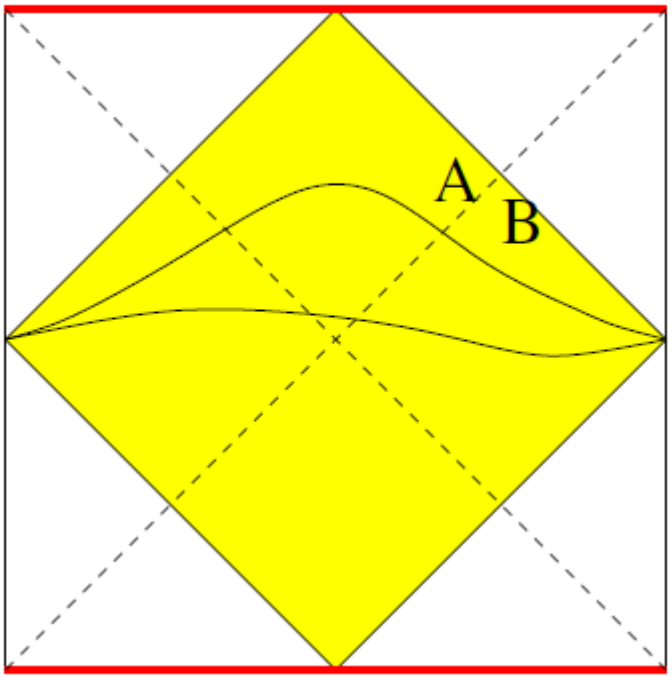


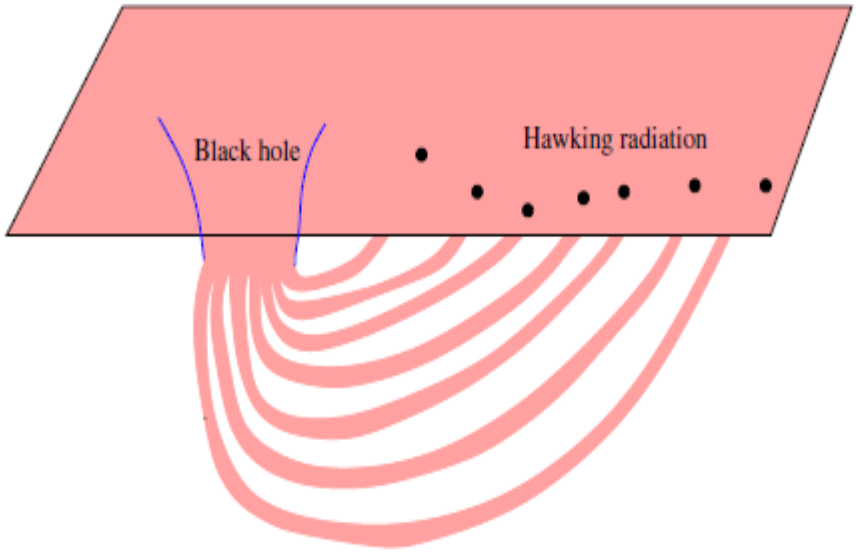
- Again, non-locality!
- Again, Nothing can go through. (NCV)
- However QM might utilize its presence.

EPR = ER
(Entanglement = wormhole)

- Wheeler: e+ e- =wormhole.
- Ramsdonk : If any two quantum subsystems are entangled→ they are connected by a wormhole.
- Maldacena and Susskind (MS) used this to resolve AMPS puzzle:
- Protect quantum monogamy→ BH-photon E must be broken →Freefall obs burn outside the BH.
- MS→ the entanglement needn't be broken in the presence of BH-photon wormhole.

Wormhole





New type of WH that resolves AMPS puzzle

$$|\Psi_t\rangle \sim \sum_n e^{-\beta E_n/2} e^{-2iE_n t} |n, n\rangle$$

World sheet version (Karch-Jesen)

- Entangled quark-anti quark

Xiao
0804.1343

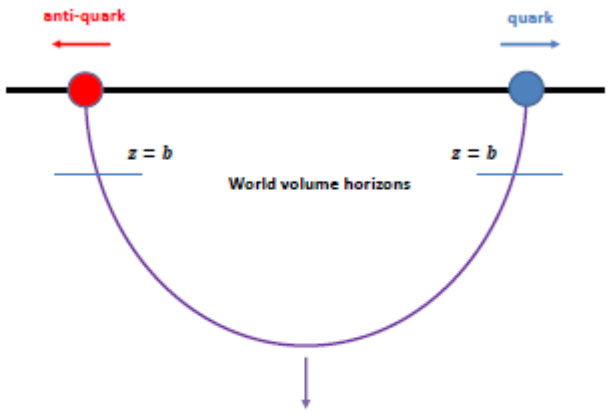


FIG. 2. The holographic q-$\bar{q}$ system entangled into a color-neutral EPR pair.

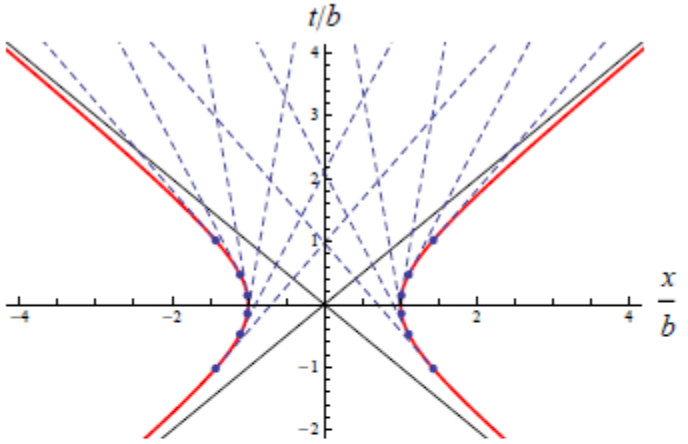


FIG. 3. The casual structure on the string worldsheet. The thick red lines indicate the worldlines of the quark and anti-quark, and the string worldsheet fills the universe in between. The solid lines indicate the worldsheet horizons, which happens to be the location of the Rindler horizons for each of the quarks. The solid dots denote events where light rays are emitted from the quark and anti-quark into the dual worldsheet, and the dotted lines indicate the resulting lightlike trajectories. The string worldsheet clearly has the same causal structure of an eternal AdS black hole as in Figure 1.

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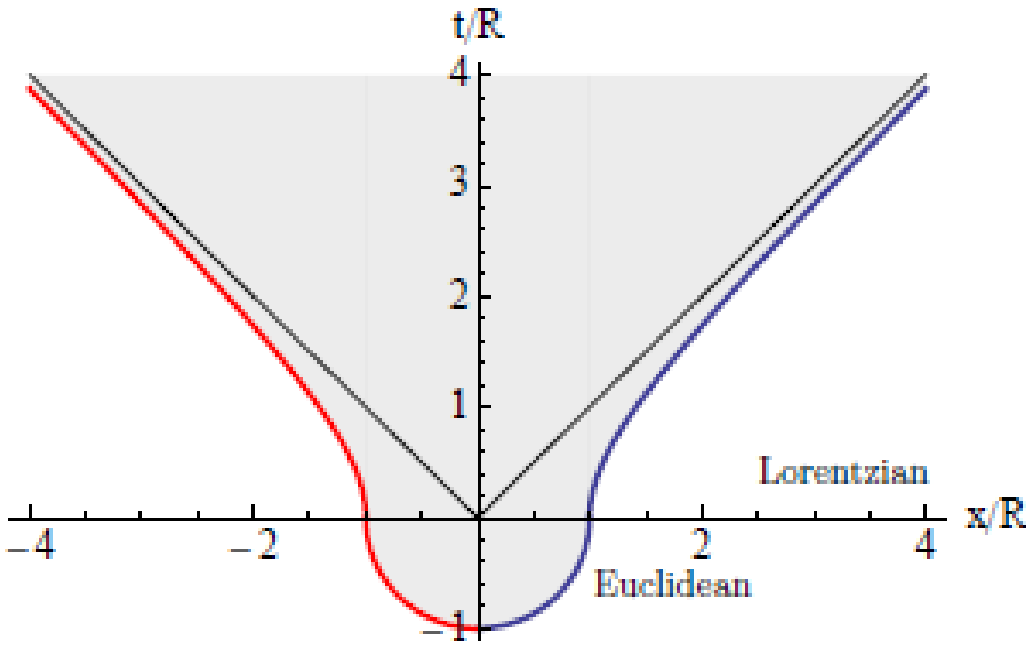
Sonner



Holographic Schwinger Effect and the Geometry of Entanglement
Julian Sonner Phys.Rev.Lett. 111 (2013) 211603

Seki+sjs

- More examples of ER=EPR wormhole/entanglement
- S-matrix v.s Entaglement.

1. Some other example

- 4-Gluon scattering amplitude
- Minimal surface by Alday-Maldacena
- In momentum space,

$$ds^2 = \frac{R^2}{r^2}\left(\eta_{\mu\nu}dy^\mu dy^\nu + dr^2\right), \quad \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1), \tag{1}$$

and have found the minimal surface solution corresponding to the gluon scattering [12],

$$\begin{aligned} r &= \frac{\alpha}{\mathrm{ch}\,u_1\,\mathrm{ch}\,u_2 + \beta\,\mathrm{sh}\,u_1\,\mathrm{sh}\,u_2}, \\ y_0 &= r\sqrt{1+\beta^2}\,\mathrm{sh}\,u_1\,\mathrm{sh}\,u_2, \quad y_3 = 0, \\ y_1 &= r\,\mathrm{sh}\,u_1\,\mathrm{ch}\,u_2, \quad y_2 = r\,\mathrm{ch}\,u_1\,\mathrm{sh}\,u_2, \end{aligned} \tag{2}$$

T-dual (x-space)

$$\partial_m y^\mu = \frac{R^2}{z^2}\epsilon_{mn}\partial_n x^\mu\,, \quad z = \frac{R^2}{r}\,,$$

$$ds^2 = (R^2/z^2)(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2).$$

$$\begin{aligned}
z &= \frac{R^2}{2\alpha}\left[(1+\beta)\,\mathrm{ch}\,u_+ + (1-\beta)\,\mathrm{ch}\,u_-\right]\,,\\
x_0 &= -\frac{R^2}{2\alpha}\sqrt{1+\beta^2}\,\mathrm{sh}\,u_+\,\mathrm{sh}\,u_-\,, \quad x_3 = 0\,,\\
x_+ &= -\frac{R^2}{2\sqrt{2}\alpha}\left[(1+\beta)u_- + (1-\beta)\,\mathrm{ch}\,u_+\,\mathrm{sh}\,u_-\right]\,,\\
x_- &= \frac{R^2}{2\sqrt{2}\alpha}\left[(1-\beta)u_+ + (1+\beta)\,\mathrm{sh}\,u_+\,\mathrm{ch}\,u_-\right]\,,
\end{aligned}$$

T-dual

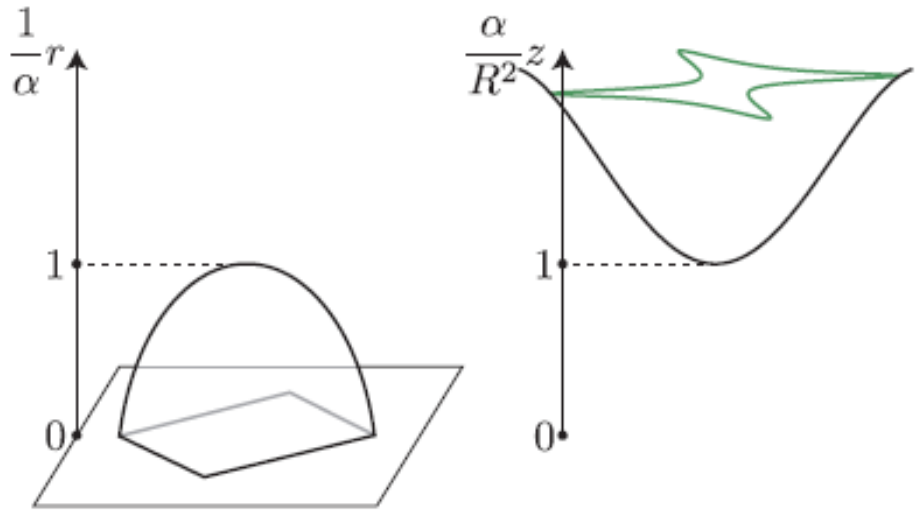


FIG. 1. The minimal surfaces in momentum space (left) and in position space (right).

Causal structure on ws

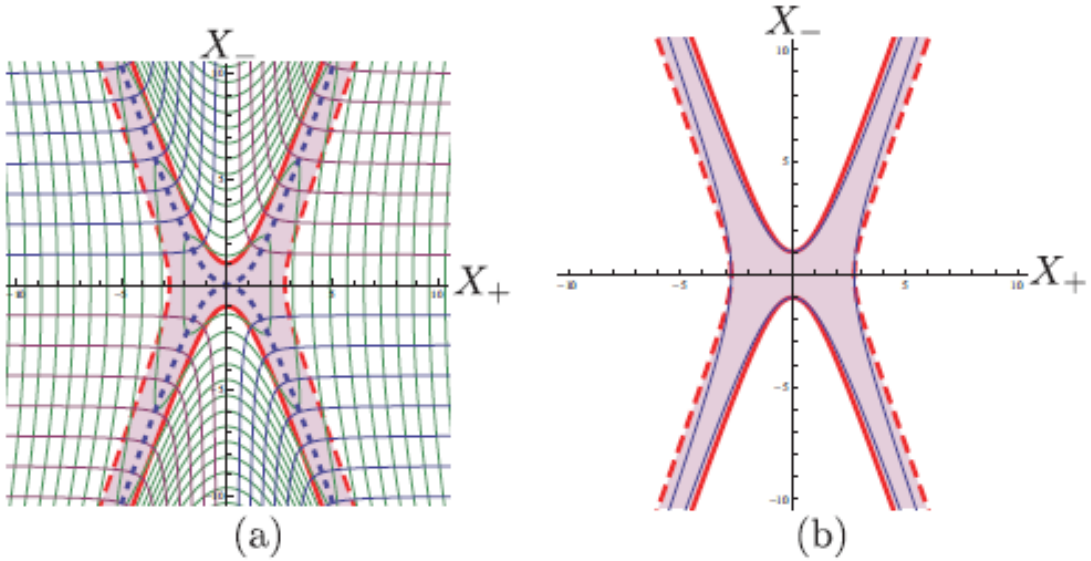


FIG. 2. (a) The causal structure on the minimal surface in position space ($\beta = 1/2$). (b) The blue lines are the singularity.

regions. Therefore these white regions are Lorentzian, and are separated by the (red) Euclidean region, that is, a wormhole.

Induced metric on ws

$$ds^2_{\rm ws} = R^2 \left(g_{++} du_+^2 + 2 g_{+-} du_+ du_- + g_{--} du_-^2 \right) ,$$

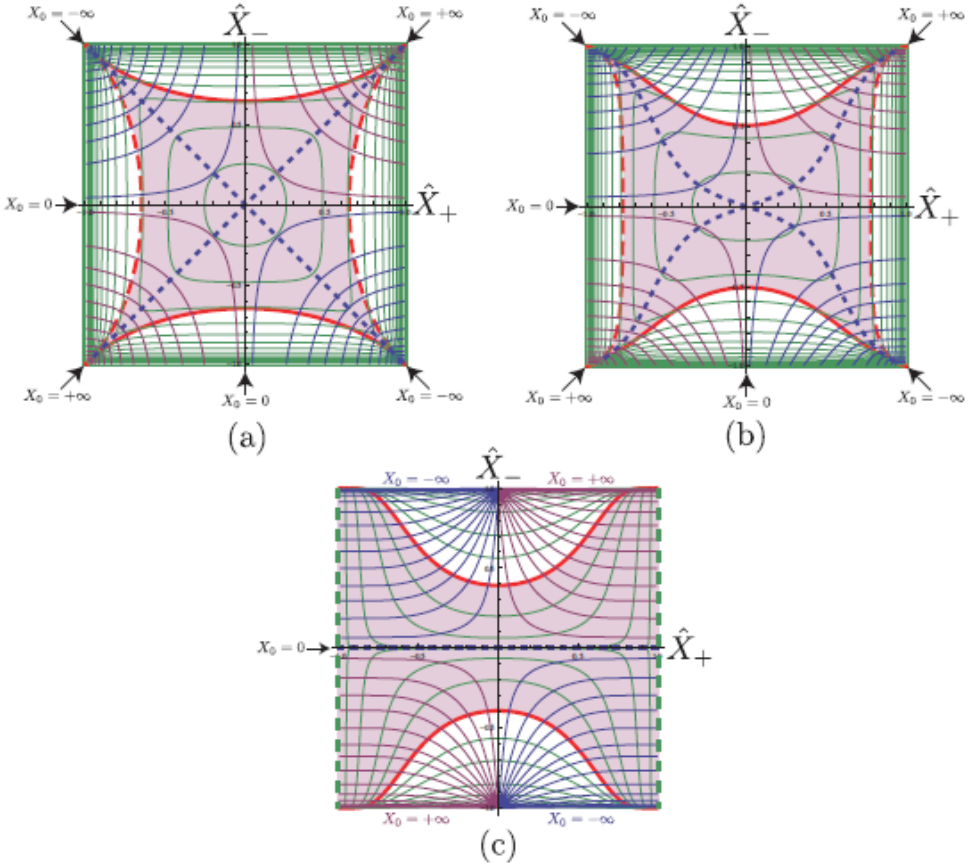
with

$$\begin{aligned} g_{\pm\pm} &= \frac{2}{\left[(1+\beta) \operatorname{ch} u_+ + (1-\beta) \operatorname{ch} u_- \right]^2} \left[(1 \pm \beta)^2 \operatorname{sh}^2 u_\pm \right. \\ &\quad \left. + (1+\beta^2) - 4^{-1} \left((1 \pm \beta) \operatorname{ch} u_\pm - (1 \mp \beta) \operatorname{ch} u_\mp \right)^2 \right] , \\ g_{+-} &= \frac{2(1-\beta^2) \operatorname{sh} u_+ \operatorname{sh} u_-}{\left[(1+\beta) \operatorname{ch} u_+ + (1-\beta) \operatorname{ch} u_- \right]^2} . \end{aligned} \tag{8}$$

two kinds of horizons:

$$g_{--} = 0, \qquad g_{++} = 0,$$

FIG. 3. The causal structure on world-sheet. (a) $\beta = 0$, (b) $\beta = 1/2$, (c) $\beta = 1$.



$$-s\,(2\pi)^2 = \frac{8\alpha^2}{(1-\beta)^2}\,, \quad -t\,(2\pi)^2 = \frac{8\alpha^2}{(1+\beta)^2}\,,$$

4-gluon ws wormhole

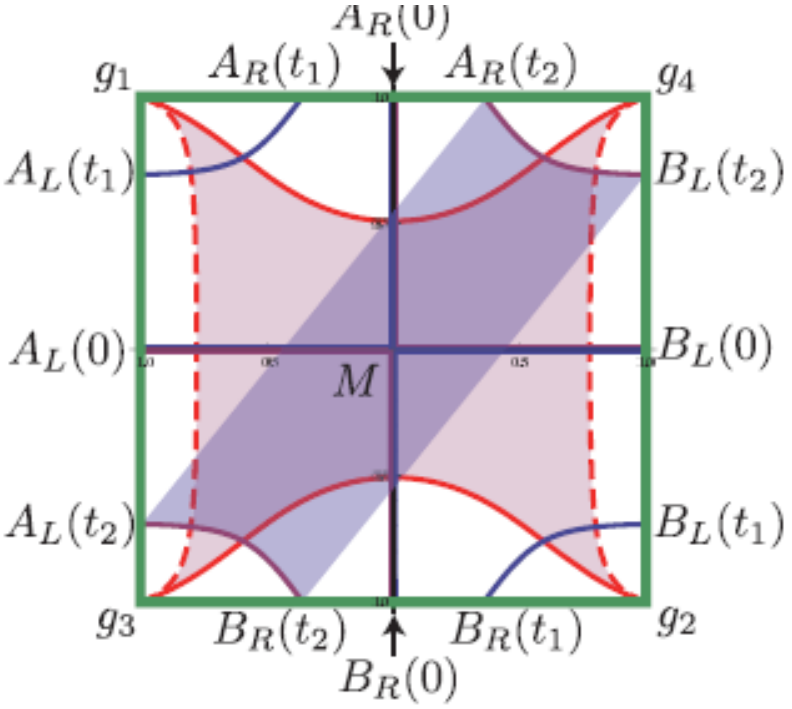


FIG. 4. The gluon scattering world-sheet projected onto $(\hat{X}_+, \hat{X}_-)$. The boundary is denoted by the green box. The red region is a wormhole.

S-matrix and EE

- Consider scattering process with unentangled initial state $|i\rangle = |i_1\rangle \otimes |i_2\rangle$

time evolution is given by the evolution operator $U = \exp[-iT(H_1 + H_2 + H_{\rm int})]$ or S-matrix:

$$|\psi\rangle = \lim_{T\to\infty} U|i\rangle = \sum_f |f\rangle\langle f|S|i\rangle = \sum_f |f\rangle S_{fi} \qquad (16)$$

EE of Final state = change of EE over the whole process.

Why Only one worldsheet?: Semi-classical nature due to large N.

Change of EE and S-matrix

$$S_E = (1 - c\lambda\partial_\lambda)\log\langle W\rangle$$

A. Lewkowycz and J. Maldacena, arXiv:1312.5682 [hep-th].

the EE itself is associated with a quantum state at a time while the Wilson loop $\langle W\rangle$ depends on the entire time dependent process. Therefore we should consider the left hand side of above mentioned equation as the *change* of the EE, ΔS_E. So the gluon scattering amplitude [12] is related to the change of the EE in leading order of large λ by

$$\begin{aligned}\Delta S_E &\sim \frac{(1-\frac{1}{2}c)\sqrt{\lambda}}{8\pi}\left(\log\frac{s}{t}\right)^2 \\ &= \frac{(1-\frac{1}{2}c)\sqrt{\lambda}}{2\pi}\left(\log\frac{1+\beta}{1-\beta}\right)^2 , \qquad (13)\end{aligned}$$

Conclusion

- World sheet wormhole in semiclassical (dominant world sheet).
- Change of Entanglement entropy is related to the scattering Amplitude.
- Minimal area prescription ~ EE
- Minimal world sheet area ~ change of EE

Thank you for your attention.